Availability, MTTF and Cost analysis of complex system under Preemptive resume repair policy using copula distribution

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Abstract

In the present paper authors have focused on the study of complex system consisting two subsystems, in series configuration and handling by a human operator. The subsystem (1) has three units at super priority, priority and ordinary & the subsystem (2) has one unit in series configuration with the subsystem-1. The whole system is operated by a human operator and human failure can also appear at different state where system is in operational mode. Initially super priority unit starts function and failure during of super priority unit the priority unit start functioning and super priority unit goes under repair. The primitive resume repair policy is employed for repair of subsystem-1. The all failure rates are assumed to constants and follow exponential distribution but repair follow general and Gumbel-Hougaard family copula distribution. The system is studied by supplementary variable technique and Laplace transform. Various measure of reliability such as availability, state transition probabilities, mean time to system failure(M.T.T.F) and profit function has been discussed for available maintenance cost for all time and profit incurred by unit time for given interval. Some particular cases have been discussed for different values of different rates.

Keywords: Reliability, Availability. Human failure, Preemptive resume repair policy, super priority, M.T.T.F. and profit function.

I. Introduction

Many researcher whose works referred as Alistair (2007), Cox (1995), Dilip (2014) and Oliverira et al. (2005) studied repairable complex system and proclaimed their validation in the field of reliability by taking different failure rates and common assumptions that one type of (general) repair is possible between two transition state. Thus whenever the system fails, one type of repair is employed to repair the system which takes more times for repair of failed unit, resulting the industry suffers with a great loss. Usually the researchers consider that only one repair that is possible between two adjacent states i.e. failed state and operational state. Many authors including Govil, A. (1974), Ram. M & Singh (2008 &2010), and Cai. X et al. (2005) studied the complex systems under different type of failure and preemptive resume repair policy. The researchers Dhillon et al. (1992 & 1993), Vanderperre. E. J. (1990) & Ram. M and.et al. (2013) studied reliability chactristic for a complex system with comman cause failure and reliability of duplex standby system by suplimentary variable technique (Cox, 1995) and Laplace transform. But there are many situations in real life where more than one repair is require between two transition states for quick maintenance of failed unit. When such type of situation arises, the system is studies by using Copula (Gennheimer. H, 2002 and Nelson, 2006). The authors (Singh et al. 2010) studied a complex system which have three units superpriorty, priorty and ordinary under preemptive resume repair policy using different types of failure and repair by using Gumbel-Hougaard family Copula distribution. If the system is running under minor partial failure and the system is in operation mode general repair can be employed but whenever the system is in complete failure mode the system is to be repaired using Copula [Gumbel-Hougaard family copula] distribution. Refered to this stratregy Ghasemi. A et al. (2010) and Ram et al (2013) studies the complex system having two subsystem with controller and standby complex system with waiting repair policy using Gumbel-Hougaard family copula distribution. The prime aim in any industry or organization is to gain more profit with a least expenditure. There for the minor partially failed state when a system is in working with degraded mode, the general repair is enough but if system is in completely failure mode the copula distribution should be employed for quick maintenance of the system. Here in the system the authors have considered a complex system which consists of two subsystem, subsystem-1 & subsystem-2. The subsystem-1 has three units super priority, priority and ordinary units & is working under preemitive resume repair policy & the subsystem-2 has one unit attached with subsystem-1 in series configuration. The system is operated by a human operator. Initially the super priority unit of subsystem-1 starts working and other units-priority and ordinary unit remains in warm standby mode. When super priority unit fails the priority unit starts of working and the failed unit gets for repair. If super priority unit repair before failure of priority unit then super priority unit starts functioning and the priority unit will go for standby mode. In case priority unit fail before repair of super priority unit then ordinary unit start functioning and priority unit will have to wait for repair. As soon as super priority unit is repaired, it start function and priority unit will go for repair and after get repair it goes for standby mode. Thus super priority unit will never be in standby mode. The system is operated by human operator; the human error can arise at any stage when the system is in operational mode. The system will completely fail in following situation: 1) ordinary unit of subsystem-1 fails before the repair of the super priority unit. 2) subsystem-2 fails whenever subsystem-1 is in operational mode. 3) Human failure can appear at any stage when subsystem-1 is in operational mode.

All failure rates are constants and follows exponential distribution, however repair follows two types of distribution namely: General and Gumbel-Hougaard family copula distribution. Whenever the system is in partially failed state [S_0, S_1, S_2, S_3, S_5, S_6, S_7] the general repair is employed but whenever the system is in completely failed state [S_4, S_8, S_9], it is repaired by Gumbel-Hougaard family copula distribution. The system is studied by supplementary variable technique & Laplace Transform. Various measures of reliability such as availability, state transition probability, mean time to system failure MTSF profit function have been discussed. Some particular cases also have been highlighted for different value of failure rates. The results are demonstrated by graphs and conclusion have been drawn.

II. State description

State	State Description
S_1	The state indicates that the super priorty unit has been failed, priorty unit start functioning and superpriorty unit is under repair.
S_2	The priorty unit has also been failed and supper priorty unit is running under repair, system is in operational mode.
S_3	The super priorty unit has been repaired and is in operational mode, priorty unit is running under repair.
S_4	The system has completely failed due to failure of subsystem-1.
S_5	In this state the super priorty unit has start functioning after repaired and priorty unit is running repair.
S_6	The super priorty unit is in operational mode, priorty unit is in standby mode and ordinary unit in running under repair.
S_7	In state S_7 the superpriorty unit of subsystem-1 has fail and priorty unit is in operational mode, the system is in operational mode.
S_8	In this state the system is in a completely failed due to failure of subsystem-2.
S_9	The system has completely failed due to human failure of operator.

The state description highlight that S_0 is a state where the system is in perfect state where both subsystems are in good working condition. S_1, S_2, S_3, S_5, S_6, S_7, are the states where the system is in degraded mode and the repair is being employed, states S_4, S_8, and S_9 are the states where the system is in completely failure mode.

III. Assumption

The following assumptions are taken throughout the discussion of the model:

(i) Initially the system is in S_0 state and all units of subsystem-1 and subsystem-2 are in good working condition.

(ii) The subsystem-1 works successfully till ordinary unit is in good working condition.

(iii) Subsystem-1 fails if ordinary unit fail before repair of superpriorty unit.

(iv) The units of subsystem-1 are in warm stated by mode and ready to start within a negligible period of time after failure of any unit of subsystem-1.

(v) The system can be repaired when it is in degraded state or completely failed state.

(vi) All failure rates are constant and they follow an exponential distribution.

(vii) Human failure /complete failure system needs immediate repairing (by Gumbel-Hougaard family copula).

(viii) A repaired system works like a new system and there will be no damage done due to repair.

(ix) As soon as the failed unit gets repair it ready to perform the task.

IV. Notations

T	Time variable on time scale.
S	Laplace transform variable.
λ_h / λ_B	Failure rates of due to human failure/ failure rate of subsystem-2.
$\lambda_s / \lambda_p / \lambda_o$	Failure rates of superpriorty/ priorty/ and ordinary units of subsystem-1.
$\eta(x)$/ $\phi(y)$/ $\Psi(x)$	Repair rate for superpriorty/ priorty/ and ordinary unit.
$P_i(t)$	The probability that the system is in S_i state at instant 't' for i=0 to 9.
$\overline{P}(s)$	Laplace transformation of P (t).
P_j (x, t)	The probability that a system is in state S_o for j=1 to 9; the system is running under repair and elapsed repair time is x, t.
$E_p(t)$	Expected profit during the interval [0, t).
K_1, K_2	Revenue and service cost per unit time respectively.
$\mu_0(x)$= $C_\theta(u_1(x),u_2(x))$	The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula is given as $C_\theta(u_1(x),u_2(x)) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where θ is a parameter.

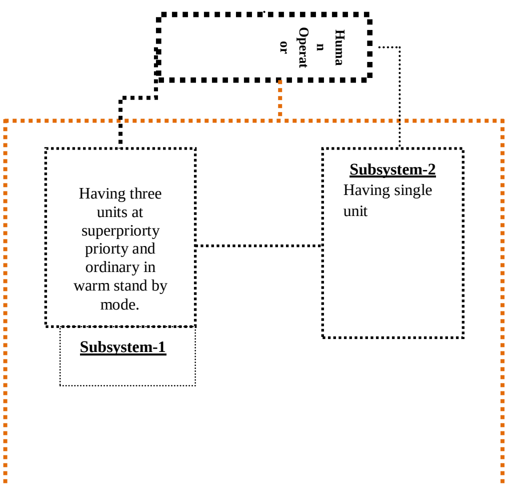

Fig.1 System Configuration

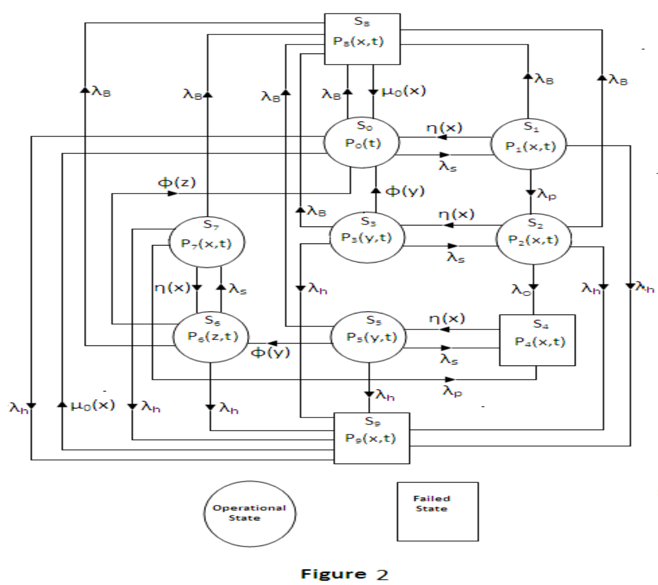

Fig.2 State Transition Diagram

V. Formulation and solution of Mathematical Model

By the probability of considerations and continuity arguments, the following set of difference differential equations are associated by the present mathematical model.

$$\left[\frac{\partial}{\partial t}+\lambda_s+\lambda_B+\lambda_h\right]P_0(t)=\int_0^\infty \eta(x)P_1(x,t)dx+\int_0^\infty \mu_0(x)P_8(x,t)dx+\int_0^\infty \phi(z)P_6(z,t)dz$$
$$+\int_0^\infty \phi(y)P_3(y,t)dy+\int_0^\infty \mu_0(x)P_9(x,t)dx \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_p+\lambda_B+\lambda_h+\eta(x)\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_o+\lambda_B+\lambda_h+\eta(x)\right]P_2(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_s+\lambda_B+\lambda_h+\phi(y)\right]P_3(y,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta(x)\right]P_4(x,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_s+\lambda_B+\lambda_h+\phi(y)\right]P_5(y,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_s+\lambda_B+\lambda_h+\phi(z)\right]P_6(z,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_p+\lambda_B+\lambda_h+\eta(x)\right]P_7(x,t)=0 \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_8(x,t)=0 \quad (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_9(x,t)=0 \quad (10)$$

Boundary conditions

$$P_1(0,t)=\lambda_s P_0(t) \quad (11)$$

$$P_2(0,t)=\lambda_p P_1(0,t)+\lambda_s P_3(0,t) \quad (12)$$

$$P_3(0,t)=\int_0^\infty \eta(x)P_2(x,t)dx \quad (13)$$

$$P_4(0,t)=\lambda_0 P_2(0,t)+\lambda_s P_5(0,t)+\lambda_p P_7(0,t) \quad (14)$$

$$P_5(0,t) = \int_0^\infty \eta(x) P_4(x,t)dx \tag{15}$$

$$P_6(0,t) = \int_0^\infty \eta(x) P_7(x,t)dx + \int_0^\infty \phi(y) P_5(y,t)dy \tag{16}$$

$$P_7(0,t) = \lambda_s P_6(0,t) \tag{17}$$

$$P_8(0,t) = \lambda_B (P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) + P_6(0,t) + P_7(0,t)) \tag{18}$$

$$P_9(0,t) = \lambda_h (P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) + P_6(0,t) + P_7(0,t)) \tag{19}$$

Solution of the Model

Taking Laplace transformation of equations (1)-(19) and using equation with help of initial condition, $P_0(0)=1$, one can obtain.

$$\begin{aligned}[s + \lambda_s + \lambda_B + \lambda_h]\overline{P}_0(s) = 1 + \int_0^\infty \eta(x)\overline{P}_1(x,s)dx + \int_0^\infty \mu_0(x)\overline{P}_8(x,s)dx + \int_0^\infty \mu_0(x)\overline{P}_8(x,s)dx + \\ \int_0^\infty \mu_0(x)\overline{P}_9(x,s)dx + \int_0^\infty \phi(y)\overline{P}_3(y,s)dy + \int_0^\infty \phi(z)\overline{P}_6(z,s)dx\end{aligned} \tag{20}$$

$$\left[s + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_h + \eta(x)\right]\overline{P}_1(x,s) = 0 \tag{21}$$

$$\left[s + \frac{\partial}{\partial x} + \lambda_0 + \lambda_B + \lambda_h + \eta(x)\right]\overline{P}_2(x,s) = 0 \tag{22}$$

$$\left[s + \frac{\partial}{\partial y} + \lambda_s + \lambda_B + \lambda_h + \phi(y)\right]\overline{P}_3(y,s) = 0 \tag{23}$$

$$\left[s + \frac{\partial}{\partial x} + \eta(x)\right]\overline{P}_4(x,s) = 0 \tag{24}$$

$$\left[s + \frac{\partial}{\partial y} + \lambda_s + \lambda_B + \lambda_h + \phi(y)\right]\overline{P}_5(y,s) = 0 \tag{25}$$

$$\left[s + \frac{\partial}{\partial z} + \lambda_s + \lambda_B + \lambda_h + \phi(z)\right]\overline{P}_6(z,s) = 0 \tag{26}$$

$$\left[s + \frac{\partial}{\partial x} + \lambda_p + \lambda_B + \lambda_h + \eta(x)\right]\overline{P}_7(x,s) = 0 \tag{27}$$

$$\left[s + \frac{\partial}{\partial x} + \mu_0(x)\right]\overline{P}_8(x,s) = 0 \tag{28}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_9(x,s)=0 \tag{29}$$

Laplace transform of boundary conditions

$$\overline{P}_1(0,s)=\lambda_s\overline{P}_0(s) \tag{30}$$

$$\overline{P}_2(0,s)=\lambda_p\overline{P}_1(0,s)+\lambda_s\overline{P}_3(0,s) \tag{31}$$

$$\overline{P}_3(0,s)=\int_0^\infty \eta(x)\overline{P}_2(x,s)dx \tag{32}$$

$$\overline{P}_4(0,s)=\lambda_p\overline{P}_7(0,s)+\lambda_s\overline{P}_5(0,s)+\lambda_0\overline{P}_2(0,s) \tag{33}$$

$$\overline{P}_5(0,s)=\int_0^\infty \eta(x)\overline{P_4}(x,s)dx \tag{34}$$

$$\overline{P}_6(0,s)=\int_0^\infty \phi(y)\overline{P}_5(y,s)dy+\int_0^\infty \eta(x)\overline{P}_7(y,s)dx \tag{35}$$

$$\overline{P}_7(0,s)=\lambda_s\left[\int_0^\infty \phi(y)\overline{P}_5(y,s)dy+\int_0^\infty \eta(x)\overline{P}_7(y,s)\,dx\right] \tag{36}$$

$$\overline{P}_8(0,s)=\lambda_B(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)+\overline{P}_3(0,s)+\overline{P}_5(0,s)+\overline{P}_7(0,s)+\overline{P}_6(0,s)) \tag{37}$$

$$\overline{P}_9(0,s)=\lambda_h(\overline{P}_0(s)+\overline{P}_1(0,s)+\overline{P}_2(0,s)+\overline{P}_3(0,s)+\overline{P}_5(0,s)+\overline{P}_7(0,s)+\overline{P}_6(0,s)) \tag{38}$$

Solving (21)-(29), with help of equations (30) to (38) one may get,

$$\overline{P}_0(s)=\frac{1}{D(s)} \tag{39}$$

$$\overline{P}_1(s)=\frac{\lambda_s}{D(s)}\frac{(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)} \tag{40}$$

$$\overline{P}_2(s)=\frac{\lambda_s}{D(s)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)} \tag{41}$$

$$\overline{P}_3(s)=\frac{\lambda_s\lambda_p S_\eta(s+\lambda_B+\lambda_o+\lambda_h)}{D(s)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)} \tag{42}$$

$$\overline{P}_4(s)=\frac{\lambda_o\lambda_p\lambda_s H}{D(s)(F.H-C)A}\frac{(1-S_\phi(s))}{(s)} \tag{43}$$

$$\overline{P}_5(s)=\frac{\lambda_o\lambda_p\lambda_s H}{D(s)(F.H-C)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_s+\lambda_h))}{(s+\lambda_B+\lambda_s+\lambda_h)} \tag{44}$$

$$\overline{P}_6(s)=\frac{\lambda_o C}{D(s)(F.H-C)A}\frac{(1-S_\phi(s+\lambda_B+\lambda_s+\lambda_h))}{(s+\lambda_B+\lambda_s+\lambda_h)} \tag{45}$$

$$\overline{P}_7(s)=\frac{\lambda_o\lambda_s C}{D(s)(F.H-C)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_p+\lambda_h))}{(s+\lambda_B+\lambda_p+\lambda_h)} \tag{46}$$

$$\overline{P}_8(s)=\overline{P_8}(0,s)\frac{(1-S_{\mu_0}(s))}{(s)} \tag{47}$$

$$\overline{P}_9(s)=\overline{P_9}(0,s)\frac{(1-S_{\mu_0}(s))}{(s)} \tag{48}$$

$$\begin{aligned}D(s)=s+\lambda_s+\lambda_B+\lambda_h-(1-H)-(1+\lambda_s)(\lambda_B+\lambda_h)S\mu_0(s)\\
\begin{pmatrix}\lambda^2{}_B\lambda_0\lambda_s S_\eta(s)+\lambda_B\lambda_0 C+\lambda_B\lambda_0\lambda_s C+\\ \lambda_h\lambda_0\lambda_p HS_\eta(s)+\lambda_0\lambda_h C+\lambda_h\lambda_0\lambda_s C\end{pmatrix}\frac{S\mu_0(s)}{(FH-C)A}-\\
(\lambda_B\lambda_P\lambda_s+\lambda_B\lambda_h\lambda_p)(1+S_\eta(s+\lambda_B+\lambda_0+\lambda_h))S\mu_0(s)-\\
\left\{\frac{\lambda_0 C}{(FH-C)}+\lambda_P\lambda_s S_\eta(s+\lambda_B+\lambda_0+\lambda_h)\right\}\frac{S_\phi(s+\lambda_B+\lambda_S+\lambda_h)}{A}\end{aligned}$$

Where, $A=(1-\lambda_s S_\eta(s+\lambda_B+\lambda_0+\lambda_h))$, F= $(1-\lambda_s S_\eta(s))$,
C= $\lambda_p\lambda_s S_\eta(s)S_\phi(s+\lambda_B+\lambda_s+\lambda_h)$,

H= $(1-\lambda_s S_\eta(s+\lambda_B+\lambda_p+\lambda_h))$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_5(s)+\overline{P}_6(s)+\overline{P}_7(s)$$

$$=\frac{1}{D(s)}\left[\begin{array}{l}1+\frac{\lambda_s(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)}+\frac{\lambda_s}{A}\frac{(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)}+\\ \frac{\lambda_s\lambda_p S_\eta(s+\lambda_B+\lambda_o+\lambda_h)}{A}\frac{(1-S_\eta(s+\lambda_B+\lambda_o+\lambda_h))}{(s+\lambda_B+\lambda_o+\lambda_h)}+\\ \frac{\lambda_o\lambda_p\lambda_s H}{(F.H-C)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_s+\lambda_h))}{(s+\lambda_B+\lambda_s+\lambda_h)}+\frac{\lambda_o C}{(F.H-C)A}\\ \frac{(1-S_\phi(s+\lambda_B+\lambda_s+\lambda_h))}{(s+\lambda_B+\lambda_s+\lambda_h)}+\frac{\lambda_o\lambda_s C}{(F.H-C)A}\frac{(1-S_\eta(s+\lambda_B+\lambda_p+\lambda_h))}{(s+\lambda_B+\lambda_p+\lambda_h)}\end{array}\right] \tag{49}$$

$$\overline{P}_{down}(s)=1-\overline{P}_{up}(s) \tag{50}$$

Particular Cases

A. Availability

When repair follows exponential distribution, setting $S_{\mu_0}(s) = \bar{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \dfrac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}$,

$\bar{S}_\phi(s) = \dfrac{\phi}{s+\phi}$, $\bar{S}_\eta(s) = \dfrac{\eta}{s+\eta}$, and taking the values of different parameters as λ_s=0.01, λ_B=0.02, λ_h = 0.03, λ_p = 0.015, λ_o = 0.04, $\phi = 1$, θ = 1, x = 1, in (49), then taking the inverse Laplace transform, one can obtain,

$$\begin{aligned} P_{up}(t) = {} & 0.016357e^{(-2.768488t)} - 0.001031e^{(-2.718000t)} - 0.000253e^{(-2.217303t)} - \\ & (0.000571\cos(0.010681t) + 0.002655\sin(0.010681t))e^{(-1.077369t)} \\ & + 0.982709e^{(-0.000565t)} + (0.003458\cos(0.001951t) - 0.001781 \\ & \sin(0.001951t))e^{(-1.062871t)} - 0.005004e^{(-1.049877t)} + 0.001296e^{(-0.988884t)} \end{aligned} \tag{51}$$

For, *t*= 0, 2, 4, 6, 8, 10, 12, 14, 16, 18; units of time, one may get different values of P_{up}(t) with the help of (51) as shown in Fig.3 calculated).

Time(t)	Availability
0	1.0000
2	0.9816
4	0.9801
6	0.9794
8	0.9782
10	0.9771
12	0.9761
14	0.9750
16	0.9739
18	0.9728

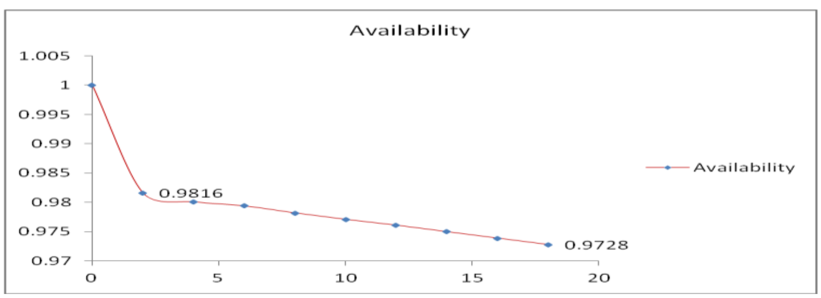

Fig. 3: Availability as function of time

B. Mean Time to Failure (M.T.T.F.)

Taking all repairs zero and the limit as *s* tends to zero in (49) for the exponential distribution, one can obtain the expression for M.T.T.F. as:

$$M.T.T.F. = \lim_{s\to 0} \bar{P}_{up}(s) \; \frac{1}{(\lambda_s + \lambda_B + \lambda_h)}\left(1 + \frac{\lambda_s(1+\lambda_p)}{\lambda_p + \lambda_B + \lambda_h}\right) \tag{52}$$

Setting λ_B=0.02, λ_h=0.03, λ_p=0.015 and varying λ_s, λ_p, λ_h, λ_B one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10 in (52), one may obtain the variation of M.T.T.F. with respect to failure rates as shown in Fig.4.

Failure Rate	MTTF λ_s	MTTF λ_p	MTTF λ_h	MTTF λ_B
.01	19.2692	19.4722	30.6389	23.6909
.02	18.7472	19.0953	23.6909	19.2692
.03	18.3557	18.8125	19.2692	16.2190
.04	18.0512	18.5926	16.2190	13.9926
.05	17.8076	18.4167	13.9926	12.2982
.06	17.6083	18.2728	12.2982	10.9667
.07	17.4423	18.1528	10.9667	9.8933
.08	17.3017	18.0513	9.8933	9.0100
.09	17.1813	17.9643	9.0100	8.2707
.10	17.0767	17.8889	8.2707	7.6429

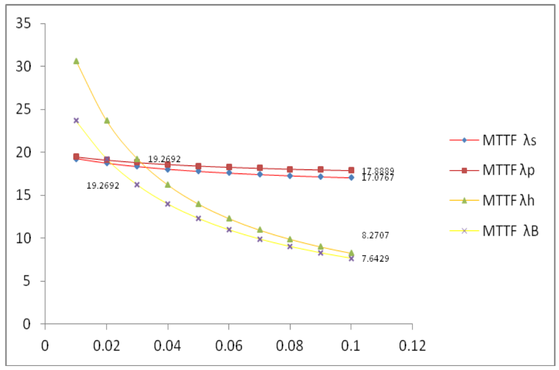

Fig. 4: M.T.T.F. as function of Failure rate

C. Cost Analysis

If the service facility be always available, then expected profit during the interval [0, t) is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{53}$$

For the same set of parameter of (49), one can obtain (53).

Therefore

$$\begin{aligned} E_p(t) = K_1(&-0.006023e^{(-2.768488t)} + 0.000233e^{(-2.718000t)} - 0.000325e^{(-2.217303t)} + \\ &(0.000554\cos(0.010681t) + 0.002459\sin(0.010681t))e^{(-1.077369t)} \\ &-1737.876752e^{(-0.000565t)} + (0.003251\cos(0.001951t) - 0.001682 \\ &\sin(0.001951t))e^{(-1.062871t)} + 0.004767e^{(-1.049877t)} - 0.001310e^{(-0.988884t)} \\ &+1737.881835) - K_2\, t \end{aligned} \tag{54}$$

Setting K_1= 1and K_2= 0.5, 0.25, 0.15, 0.10 and 0.05 respectively and varying t =0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 units of time, the results for expected profit can be obtain as shown in Fig.5.

Time(t)	$E_p(t)$; $K_2=0.5$	$E_p(t)$; $K_2=0.25$	$E_p(t)$; $K_2=0.15$	$E_p(t)$; $K_2=0.10$	$E_p(t)$; $K_2=0.05$
0	0	0	0	0	0
2	0.4874	0.7374	0.8374	0.8874	0.9374
4	0.9695	1.4695	1.6695	1.7695	1.8895
6	1.4507	2.2007	2.5007	2.6507	2.8007
8	1.9315	2.9315	3.3315	3.5315	3.7315
10	2.4117	3.6617	4.1617	4.4117	4.6617
12	2.8916	4.57	4.9914	5.2914	5.5914
14	3.3705	5.1205	5.833	6.1705	6.5205
16	3.849	5.849	6.800	7.049	7.449
18	4.327	6.577	7.4717	7.9217	8.3717
20	4.8085	7.3045	8.3045	8.8045	9.3045

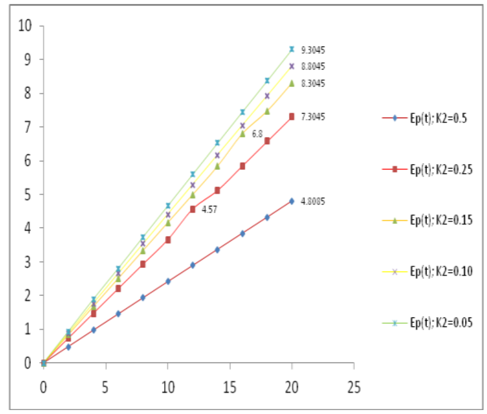

Fig. 5: Expected profit as function of time

VI. Conclusions

Fig. 3 provides information how the availability of the complex repairable system changes with respect to the time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_s = 0.01$, $\lambda_P = 0.015$, $\lambda_o = 0.04$, $\lambda_B = 0.02$, $\lambda_h = 0.03$, availability of the system decreases with at very high up to time t=2 but after ward the variation becomes slowly and probability of failure increases, with the passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Fig. 4, yields the mean-time-to-failure (M.T.T.F.) of the system with respect to variation in λ_s, λ_p, λ_h, and λ_B respectively when the other parameters have been taken as constant. The variation in MTTF corresponding to failure rates λ_s, λ_p are almost is very closure but corresponding to λ_B, λ_h the variation is very high which indicates that these both are more responsible to proper operation of the system.

When revenue cost per unit time K_1 is fixed at 1, service costs K_2 = 0.5, 0.25, 0.15, 0.10, 0.05, profit has been calculated and results are demonstrated by graphs in Fig.5. A critical examination from Fig.5 reveals that expected profit increases with respect to the time when the service cost K_2 fixed at minimum value 0.05. Finally, one can observe that as service cost increase, profit decrease. In general for low service cost, the expected profit is high in comparison to high service cost.

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Appendix

Appendix1: *For study the particular cases in solution part of paper the authors have use the maple 7 software for computation purpose such as taking inverse Laplace transform and computing the availability and MTTF and profit from the expression in equation 51, 52 & 54.*

Appendix 2: *The system to be in state S_i during time t and t+Δt is that the system should not move to any other state i.e the probability that the system will be in state i=0 (for illustration) say S_0 is;*

$$P_0(t+\Delta t) = (1-\lambda_s \Delta t)(1-\lambda_B \Delta t)(1-\lambda_h \Delta t)P_0(t) + \int_0^\infty \eta(x)P_1(x,t)\Delta t dx + \int_0^\infty \mu_0(x)P_8(x,t)\Delta t dx + \int_0^\infty \phi(z)P_6(z,t)\Delta t dz$$
$$+ \int_0^\infty \phi(y)P_3(y,t)\Delta t dy + \int_0^\infty \mu_0(x)P_9(x,t)\Delta t dx$$

$$Lt\ t \to 0 \frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} + (\lambda_s + \lambda_s + \lambda_s)P_0(t) + (\Delta t + (\Delta t)^2 + \ldots.)P_0(t) = \int_0^\infty \eta(x)P_1(x,t)dx + \int_0^\infty \mu_0(x)P_8(x,t)dx +$$
$$\int_0^\infty \phi(z)P_6(z,t)dz + \int_0^\infty \phi(y)P_3(y,t)dy + \int_0^\infty \mu_0(x)P_9(x,t)dx$$

OR Equation 1;

$$\left[\frac{\partial}{\partial t} + \lambda_s + \lambda_B + \lambda_h\right]P_0(t) = \int_0^\infty \eta(x)P_1(x,t)dx + \int_0^\infty \mu_0(x)P_8(x,t)dx + \int_0^\infty \phi(z)P_6(z,t)dz + \int_0^\infty \phi(y)P_3(y,t)dy + \int_0^\infty \mu_0(x)P_9(x,t)dx \qquad (1)$$