Ridge Regression: A tool to forecast wheat area and production

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Abstract

This research study is designed to develop forecasting models for acreage and production of wheat crop for Chakwal district of Rawalpindi region keeping in view the assumptions of OLS estimation. The forecasting models are developed on the basis of 15 years data from 1984-85 to 1998-99 then wheat area and production for next five years from 1999-2000 to 2003-04 is forecasted through the models and compared with the actual figures. After evaluating the accuracy of the models, final models are developed on the basis of 20 years data for the period 1984-85 to 2003-04. These linear models can be used to forecast wheat area and production of next five years. The Urea fertilizer, DAP fertilizer and manures plays a significant role to enhance the production of wheat crop. Number of ploughs in the wheat fields is significant factor to increase the production of wheat crop. Good rains in the month of October and November significantly contributes to increase the production of wheat crop and mean maximum temperature in the month of March is a significant factor to reduce the production of wheat crop.

1. Introduction

Agriculture sector is the backbone of our economy and the success of economic polices and plans in this sector depend on the timely and reliable estimates of crops. In Punjab, the Crop Reporting Service (CRS), Agriculture Department, is playing a vital role in producing such estimates. These estimates are based on visual judgment, Growers Opinion Survey, Crop Acreage Survey and Crop Cutting Experiments.

Wheat is one of the major crops whose production influences the economy of our country. Accurate and advance estimates of wheat area and production are essential for food security.

No doubt, various studies concerning forecasting models do exist in the available literature but they are limited in scope to some extent, neglecting the assumptions of OLS estimation. This research study is designed to develop forecasting models for acreage and production of wheat crop for Chakwal district of Rawalpindi region keeping in view the assumptions of OLS estimation. This study will certainly play a key role in the development of wheat area and production forecasting models for other districts of the region.

2. Methodology

Different parts of the Punjab province have different levels of rainfall during various periods. Temperature also varies from area to area. Soil texture and cropping patterns are also different in different parts of the province. Thus impact of these factors on wheat crop differs from place to place. To have homogeneous area, with respect to impact of these factors, rain fed area of Chakwal district of Rawalpindi region, Punjab has been selected for this study.

All-important factors such as rainfall, temperature, chemical fertilizes, manure and cultural practices affecting the area and production of wheat crop are considered for the development of the forecasting model. Filled in Wheat Yield Estimation Survey Forms for last twenty years from1984-85 to 2003-04 were collected in respect of 43 sample villages of district: Chakwal from CRS.

The detail of variables used for wheat area forecasting models is given below:

Y=	Current year wheat area (in '000' acres)	X_5=	Last year rapeseed and mustard area (in '000' acres)
X_1=	Last year wheat area (in '000' acres)	X_6=	Last year rapeseed and mustard production (in '000' maunds).
X_2=	Last year wheat production (in '000' maunds)	X_7=	Total rainfall for the month of August (in mm)
X_3=	Last year lentil area (in '000' acres)	X_8=	Total rainfall for the months of September and October (in mm)
X_4=	Last year lentil production (in '000' maunds)	X_9=	Total rainfall for the month of November and December (in mm)

The detail of variables used for wheat production forecasting models is given below:

Y=	Per plot average yield in Kg.	X_6=	Total rainfall in the months of October amd November (in mm)
X_1=	Average Urea fertilizer used in Kg/acre	X_7=	Total rainfall in the months of December and January (in mm)
X_2=	Average DAP fertilizer used in Kg/acre	X_8=	Total rainfall in the months of February and March (in mm)
X_3=	Manures: 1 for used and 0 for not used	X_9=	Total rainfall in the month of April (in mm)
X_4=	Number of ploughs in the selected wheat field	X_{10}=	Mean maximum temperature in the month of March (in centigrade)
X_5=	0 for fellow and 1 for cultivated in last kharif season	X_{11}=	Mean maximum temperature in the month of April (in centigrade)

Violation of basic assumptions of OLS estimation in the data is examined. Problem of multicollinearity is found in the data for wheat area model. Ridge regression is used to correct multicollinearity problem in this data. The forecasting models are developed on the basis of 15 years data from 1984-85 to

1998-99 then wheat area and production for next five years from 1999-2000 to 2003-04 is forecasted through the models and compared with the actual figures. After evaluating the accuracy of the models, final models are developed on the basis of 20 years data for the period 1984-85 to 2003-04. These linear models can be used to forecast wheat area and production of next five years.

When the method of least squares is applied to the data with multicollinearity problem, the variance of least squares estimates of the regression coefficients may be considerably inflated, and the length of least squares parameter estimates is too long on the average. This implies that the absolute value of the least squares estimates are too large and that they are very unstable, that is, their magnitudes and signs may change considerably given a different sample.

The problem with the method of least squares is the requirement that $\hat{\beta}$ be an unbiased estimator of β. The Gauss-Markov property assures us that the least squares have minimum variance in the class of unbiased linear estimators, but there is no guarantee that this variance will be small. The variance of $\hat{\beta}$ is large implying that the confidence intervals on β would be wide and the point estimate $\hat{\beta}$ is very unstable. One way to alleviate this problem is to drop the requirement that the estimator of β be unbiased. If we can find a biased estimator of β, say $\hat{\beta}^*$, that has a smaller variance than the unbiased estimator $\hat{\beta}$. By allowing a small amount of bias in $\hat{\beta}^*$, the variance of $\hat{\beta}^*$ can be made small such that the MSE of $\hat{\beta}^*$ is less than the variance of the unbiased estimator $\hat{\beta}$. Consequently, confidence intervals on β would be much narrow using the biased estimator. The small variance for the biased estimator also implies that $\hat{\beta}^*$ is more stable estimator of β than the unbiased estimator $\hat{\beta}$.

A number of procedures have been developed for obtaining biased estimators of regression coefficients. One of these procedures is ridge trace, first proposed by Hoerl (1962) and discussed at length by Hoerl and Kennard (1970a). They also discussed with examples (1970b). In its simplest form the procedure is as follows:

Correlation transformation is used for controlling the round off error and for expressing the regression coefficients in the same units. The ridge standardized regression estimators are obtained by introducing into the least squares normal equation a biasing constant c, ranging from 0 to 1, in the following form of eq. (2.1)

$$b^R = (r_{xx} + cI)^{-1} r_{xy} \qquad (2.1)$$

Where b^R is the vector of standardized ridge regression coefficients b_k^R and I is the (p-1) x (p-1) identity matrix. The constant c reflects the amount of bias in the estimators. When c = 0 reduces to the ordinary least squares regression coefficients in the standardized form and when c > 0, the ridge regression coefficients are biased but tend to more stable than ordinary least squares estimators. A commonly used method for determining the biasing constant c explained by Kutner et *al.* (2005) is based on ridge trace and the variance inflation factor $(VIF)_k$. The ridge trace is simultaneous plot of the values of the p-1 estimated ridge standardized regression coefficients for the different values of c, usually between 0 an 1. Extensive experience has indicated that the estimated regression coefficients b_k^R may fluctuate widely as c is changed slightly from 0, and some may change signs. Gradually, however, these wide fluctuations cease and the magnitudes of the regression coefficients tend to move slowly towards zero as c is increased further. At the same time, the values of $(VIF)_k$ for k regression coefficients on different biasing constants c tends to fall rapidly as c is changed from zero, and gradually the $(VIF)_k$ values also tend to change only moderately as c increased further. Whereas $(VIF)_k$ is defined by Gruber (1998) given in eq. (2.2) after correlation transformation

$$(VIF)_k = (r_{xx} + cI)^{-1} r_{xx} (r_{xx} + cI)^{-1} \qquad (2.2)$$

Usually the plot of the estimated ridge standardized regression coefficients becomes stabilize and the $(VIF)_k$ value for each X variable becomes approximately equal to one for the same value of biasing constant c. Hoeral et *al.* (1975) suggested another possible automatic way of choosing c. They argued that a reasonable choice might be given in eq. (2.3)

$$C = \frac{(p-1)S^2}{\{b^*(0)\}'\{b^*(0)\}} \qquad (2.3)$$

Where (p-1) is the number of parameter in the model excluding intercept, S^2 is the residual mean squares in the ANOVA table obtained from standard least square fit and $\{b^*(0)\}'$ is given in eq. (2.4)

$$\{b^*(0)\}' = \{b_1^*(0), b_2^*(0), ..., b_{(p-1)}^*\} = \{\sqrt{S_{11}}\, b_1(0), \sqrt{S_{22}}\, b_2(0), ..., \sqrt{S_{(p-1)(p-1)}}\, b_{(p-1)}(0) \qquad (2.4)$$

After finalizing a forecasting model for wheat area significance of the explanatory factors is checked through modified t test explained by Gruber (1998) and given in eq. (2.5)

$$t = \frac{\hat{b}_i^R}{S_{ii}} \qquad (2.5)$$

Sii is the i'th diagonal element of the dispersion matrix of the ridge estimator given in eq. (2.6)

$$D = \sigma^2 (r_{xx} + cI)^{-1} r_{xx} (r_{xx} + cI)^{-1} \qquad (2.6)$$

Where σ^2 is given in eq. (2.7)

$$\sigma^2 = \frac{\hat{e}'\hat{e}}{n-(p-1)} \qquad (2.7)$$

Mallows C_p Statistic

C. Mallows proposed a statistic, *Cp*, for variable selection in multiple regression. Gorman and Toman (1966) published the statistic, its derivation, and several examples of its use. Kennard (1971) showed that there is one-to-one correspondence between C*p* and the adjusted R^2.

The statistic is given by the formula:

$$Cp = (SSEp / MSEm) - (n-2p) \quad (2.8)$$

Where SSEp is SSE for the best model with p parameters (including the intercept, if it is in the equation), and MSEm is the mean square error for the model with all m predictors. In general, we look for models where Cp is small and is also close to p. This is used in selection of best subset of predictors for forecasting model of Wheat production.

3. Results and Discussion

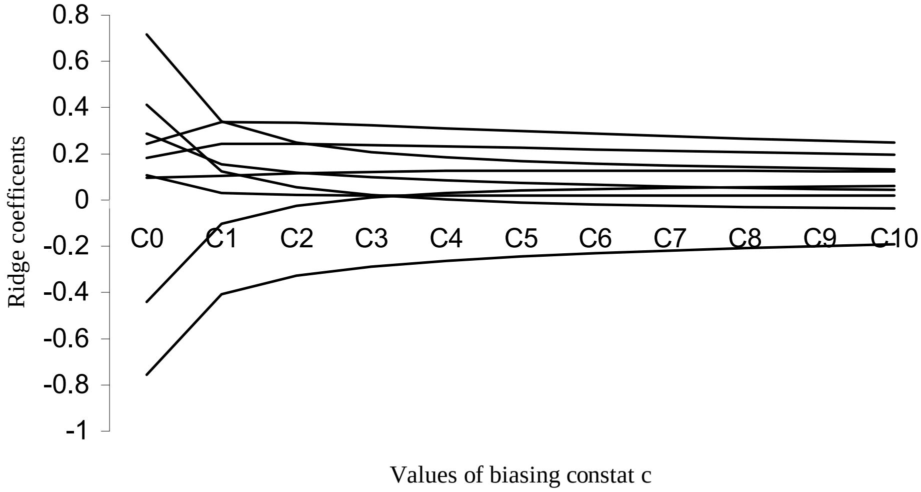
Assumptions of OLS estimation are checked. Ridge regression is used to correct multicollinearity problem in the data for wheat area models. The forecasting models are developed on the basis of 15 years data from 1984-85 to 1998-99 then wheat area and production for next five years from 1999-2000 to 2003-04 is forecasted through these models and compared with the actual figures. After evaluating the accuracy of models, final models are developed on the basis of 20 years for the period 1984-85 to 2003-04. These forecasting models can be used to forecast wheat area and production of next five years.

Tests for the assumptions of OLS estimation are performed and only the problem multicollinearity is found.

The ridge regression coefficients for biasing factor at $0 \le C \le 1$ are given in Table No.1.

Table 1: Ridge Regression Coefficients

C	b_1^R	b_2^R	b_3^R	b_4^R	b_5^R	b_6^R	b_7^R	b_8^R	b_9^R
0.00	0.716	-0.441	-0.757	0.287	0.413	0.108	0.242	0.183	0.096
0.10	0.340	-0.104	-0.407	0.156	0.125	0.030	0.338	0.243	0.105
0.20	0.208	0.010	-0.287	0.099	0.021	0.020	0.324	0.238	0.123
0.30	0.185	0.029	-0.262	0.084	0.002	0.020	0.311	0.232	0.126
0.40	0.169	0.041	-0.244	0.074	-0.011	0.020	0.298	0.226	0.127
0.50	0.158	0.048	-0.230	0.065	-0.019	0.020	0.286	0.219	0.127
0.60	0.150	0.053	-0.218	0.058	-0.025	0.020	0.276	0.213	0.127
0.70	0.143	0.056	-0.208	0.053	-0.030	0.020	0.265	0.208	0.126
0.80	0.137	0.058	-0.199	0.048	-0.033	0.020	0.256	0.202	0.125
0.90	0.132	0.059	-0.191	0.044	-0.036	0.020	0.248	0.197	0.124
1.00	0.716	-0.441	-0.757	0.287	0.413	0.108	0.242	0.183	0.096



Ridge trace shows that the ridge standardized regression coefficients are stabilized between C_1=0.10 and C_2=0.20.

VIF for all ridge standardized regression coefficients at different values of C_i between 0.10 and 0.20 are given in table No. 2.

Table 2: VIF For C=0.10 to C=0.19

C	VIF_1	VIF_2	VIF_3	VIF_4	VIF_5	VIF_6	VIF_7	VIF_8	VIF_9	$\sum VIF_i$
0.11	1.422	1.498	1.352	1.374	1.603	1.335	1.244	1.346	1.507	12.681
0.12	1.297	1.391	1.251	1.301	1.489	1.265	1.179	1.284	1.379	11.836
0.13	1.190	1.297	1.163	1.235	1.389	1.202	1.120	1.227	1.268	11.091
0.14	1.098	1.214	1.086	1.175	1.299	1.145	1.067	1.175	1.172	10.429
0.15	1.017	1.140	1.018	1.120	1.218	1.092	1.019	1.126	1.088	9.837
0.16	0.946	1.073	0.958	1.069	1.145	1.044	0.975	1.080	1.013	9.304
0.17	0.884	1.013	0.904	1.023	1.080	0.999	0.935	1.038	0.947	8.823
0.18	0.828	0.958	0.856	0.980	1.020	0.958	0.898	0.998	0.889	8.385
0.19	0.779	0.909	0.813	0.940	0.966	0.920	0.863	0.961	0.836	7.986

Above table shows that better value of C lies between 0.16 and 0.17.
Therefore again VIF_i at different values of C_i between 0.16 and 0.17 are calculated and achieved an improved value of C = 0166.

Other way to choose C given at equation No. 3 is use where $\{b^*(0)\}^/$ = {28.39493, -17.47910, -29.99067, 11.37353, 16.35693, 4.26636, 9.60239, 7.24485, 3.79672}, S^2 = 51.8976 and p-1 = 9, therefore C= 0.181.

The standardized ridge regression coefficients at C = 0.166 are {0.27136, -0.04356, -0.34607, 0.12900, 0.07175, 0.02269, 0.33840, 0.24465, 0.11375} with R^2=94.22% and adjusted R^2 = 86.51%, whereas the standardized ridge regression coefficients at C = 0.181 are {0.26089, -0.03449, -0.33666, 0.12448, 0.06344, 0.021190, 0.33734, 0.24433, 0.11524} with R^2 = 93.95% and adjusted R^2 = 85.88%. The values R^2 and adjusted R^2 shows that biasing factor C=0.166 is better than C=0.181.

Following is the wheat area model based on agricultural and meteorological data for the period 1984-85 to 1998-99 after transforming back to the original variables at biasing factor C = 0.166.

$$Y = 199.4309 + 0.23803X_1 - 0.00185X_2 - 7.41126X_3 - 0.71843X_4 + 0.64493X_5 + 0.03488X_6 + 0.22574X_7 + 0.11487X_8 + 0.08677X_9$$

The table No. 3 shows the forecasting ability of above wheat area model.

Table 3: Forecasting Wheat area

S. No.	Year	Area		Percentage
		Forecasted	Actual	Difference
1	1999-2000	319.000	320.099	0.34
2	2000-01	306.000	309.688	1.21
3	2001-02	267.000	277.547	3.95
4	2002-03	273.000	274.648	0.60
5	2003-04	300.000	295.281	-1.57

In the light of above research study, final ridge regression model based on agricultural and meteorological data for the period 1984-85 to 2003-04 after transforming back to the original variables at biasing factor C = 0.166 is given below.

$$Y = 206.230 + 0.215X_1 - 0.001X_2 - 6.626X_3 + 0.410X_4 + 0.6475X_5 + 0.054X_6 + 0.210X_7 + 0.129X_8 + 0.091X_9$$

The values of R^2 and adjusted R^2 for above model are 92.83% and 87.62% respectively. Therefore above model can be used successively for the estimation of wheat area of district Chakwal for next five years with greater accuracy, that is, for the period 2004-05 to 2008-9.

Modified t test is performed and the calculated values of t-statistic for standardized ridge regression coefficients at C = 0.166 are {40.9144, -5.1946, -61.1618, 21.9034, 11.2136, 11.9742, 61.8093, 37.4008, 18.5363}.
All explanatory variables playing significant role to explain area under wheat crop for Chakwal district.

Good rains in the months of August, September, October, November and December cause a significant increase in area of wheat crop. Last year wheat production and lentil area plays a significant role to reduce the area under wheat. Last year wheat area, last year lentil production and last year rapeseed and mustard area and production plays significant factors to increase the area under wheat crop. All explanatory variables involved in the model are playing their role to forecast the wheat area according to the prior expectations of agriculturists.

Wheat production model

Tests for assumptions of OLS estimation are conducted and violation of assumption under consideration is not found in the data.

Best subsets regression

R^2, Adj-R^2, C-p and S are given table No. 4.

Table 4: Best Subsets Regression-Attock Production

Vars	R^2	R^2- adj	C-p	S	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
2	64.2	61.5	95.6	0.44027										X	
2	53.6	50.0	127.3	0.50144								X			
3	76.1	72.1	62.3	0.37483				X						X	
3	75.6	71.5	63.8	0.37865	X									X	
4	87.5	84.1	30.3	0.28306	X					X				X	
4	85.0	80.9	37.7	0.30990	X					X					X
5	93.7	91.2	13.8	0.21070	X					X				X	X
5	93.1	90.4	15.5	0.21994	X	X				X				X	
6	95.3	92.7	11.0	0.19191	X	X	X			X				X	
6	95.0	92.2	12.0	0.19842	X	X				X				X	X
7	96.9	94.6	8.2	0.16522	X	X	X	X		X				X	
7	96.4	93.6	9.8	0.17879	X	X	X		X	X				X	
8	97.9	95.8	7.2	0.14466	X	X	X	X	X	X				X	
8	97.7	95.5	7.7	0.15067	X	X	X	X		X			X	X	
9	98.7	97.0	6.9	0.12326	X	X	X	X		X			X	X	X
9	98.4	96.3	7.8	0.13695	X	X	X	X	X	X			X	X	
10	98.8	96.6	8.6	0.13020	X	X	X	X	X	X			X	X	X
10	98.8	96.6	8.6	0.13045	X	X	X	X		X		X	X	X	X
11	99.0	96.4	10.1	0.13508	X	X	X	X		X	X	X	X	X	X
11	98.8	96.0	10.4	0.14239	X	X	X	X	X	X		X	X	X	X
12	99.0	95.3	12.0	0.15374	X	X	X	X	X	X	X	X	X	X	X

Keeping in view these results, the explanatory variables X_1, X_2, X_3, X_4, X_6, X_9 and X_{10} are recommended to forecasting of wheat production. Analysis of regression coefficients is given in table No. 5.

Table 5: Regression Analysis

Predictor	Coef	StDev	T	P
Constant	6.28700	1.51300	4.16	0.004
X_1	0.09555	0.02898	3.30	0.013
X_2	0.09038	0.02104	4.30	0.004
X_3	2.80000	0.98410	2.85	0.025
X_4	0.53360	0.19370	2.75	0.028
X_6	0.01430	0.00256	5.58	0.000
X_9	0.00895	0.00553	1.62	0.150
X_{10}	-0.30702	0.04742	-6.47	0.000

S = 0.1590 R-Sq = 97.7% R-Sq(adj) = 95.5%

Forecasting of wheat production in thousand maunds for the period 1998-99 to 2002-2003 and percentage increase/decrease of forecasted production over actual production is calculated and given in table No. 6.

Table 6: Forecasting of Wheat Production

	Area in '000' acres		*Average Yield in Mds./Acre		Production in '000' maunds		%
Year	Actual	Estimated	Actual	Estimated	Actual	Estimated	Difference
1999-00	319.000	320.180	11.870	12.096	3786.513	3872.905	2.28
2000-01	306.000	306.986	5.310	5.396	1624.935	1656.493	1.94
2001-02	267.000	270.358	6.280	6.401	1676.643	1730.530	3.21
2002-03	273.000	268.819	11.670	11.573	3185.837	3111.084	-2.35
2003-04	300.000	297.344	11.890	12.202	3567.087	3628.156	1.71

* Average yield in Mds./Acre = 3.89024 x Average yield per plot

Again the regression coefficients are calculated on the basis of agricultural and meteorological data for the period 1984-85 to 2003-04. The forecasting model with these regression coefficients is given below:

$$Y = 6.30000 + 0.09315X_1 + 0.09201X_2 + 2.88820X_3 + 0.54664X_4 + 0.01440X_6 + 0.00943X_9 - 0.31118X_{10}$$

This can be used successfully to forecast wheat production for the period 2004-05 to 2008-09.

3. Conclusion

The Urea fertilizer, DAP fertilizer and manures plays a significant role to enhance the production of wheat crop. Number of ploughs in the wheat fields is significant factor to increase the production of wheat crop. Good rains in the month of October and November significantly contributes to increase the production of wheat crop and mean maximum temperature in the month of March is a significant factor to reduce the production of wheat crop. All explanatory variables involved in the model playing their role to forecast the wheat production according to the prior expectations of agriculturists.

Recommendations

Latest, reliable and timely forecast of wheat area and production would help the policy makers for improving the accuracy and precision of planning regarding import, export, economic infrastructure including stability of prices. To achieve this goal it is recommended in national interest that a Research and Evaluation Section should be established in Crop Reporting Service, Agriculture Department on permanent basis. The function of this section should be:

- Collection, tabulation and computerization of agricultural and agro-meteorological statistics. The Agriculture Statistics includes results of acreage, yield estimation surveys, support prices, harvest prices and market prices of agricultural commodities and off-take of fertilizers whereas Agro-metrological Statistics include temperature, humidity, rainfall, air pressure, winds and storms.
- Level of impact of these factors on different stages of crop development should be quantified. On the basis of these factors area and production forecasting models should be developed for all major as well as minor crops.
- Validity of these forecasting models should be evaluated on the basis of the results of objective surveys. Day by day improvement of these models should be a regular job of this section.

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