On Balanced Repeated Measurements Designs

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Abstract

Repeated Measurements designs are concerned with scientific experiments in which each experimental unit is assigned more than once to a treatment either different or identical. This class of designs has the property that the unbiased estimators for elementary contrasts among direct and residual effects are obtainable. Afsarinejad (1983) provided a method of constructing balanced Minimal Repeated Measurements designs $p < t$, when t is an odd or prime power, one or more than one treatment may occur more than once in some sequences and designs so constructed no longer remain uniform in periods. In this paper an attempt has been made to provide a new method to overcome this drawback. Specifically, two cases have been considered RM[t, t(t-t)/(p-1), p], $\lambda_2=1$ for balanced minimal repeated measurements designs and RM[t, 2t(t-t)/(p-1), p], $\lambda_2=2$ for balanced repeated measurements designs. In addition, a method has been provided for constructing extra-balanced minimal designs for special case RM [t, $t^2/(p-1)$, p], $\lambda_2=1$.

Keywords and Phrases: Balanced Minimal Repeated Measurements design; Extra-balanced Repeated Measurements design; Uniform Repeated Measurements design; Cross-over designs; direct effects; residual effects.

1. Introduction

An experimental design in which experimental units are used repeatedly by exposing them to a sequence of different or identical treatments is called Repeated Measurements design (RM designs). In Repeated Measurements designs we administer t treatments during p periods to n experimental units and these designs are usually denoted as RM (t, n, p). Repeated Measurements designs find application in many branches of scientific inquiry, such as agriculture, animal husbandry, food science, biology, market research, medicine, pharmacology, social engineering, etc.

In some experiments, the nature of treatments or the experimental units may generate responses, which are free from residual effects. Some times the experimenter can introduce a rest period and allow the experimental units to even up. As many researchers such as Cochran and Cox (1986), Sheeh and Bross (1961) Westlake (1974) Afsarinejad (1989) and Afsarinejad (1990) have pointed out that, it is not always possible or desirable to use such a design. If residual effects exist, then the methods applicable to conventional designs are not valid. The experimenter has to design the experiment such that unbiased estimator for elementary contrasts among direct and residual effects can be obtained.

2. Notations, Definitions and Terminology

The following notations and definitions due to Hedayat and Afsarinejad (1975) and (1978) provide a useful language in which to express the results of this paper.

In the rest of this paper by an RM (t, n, p) design, we mean a Repeated Measurements design based on t treatments, n experimental units each being used for t distinct treatments in p periods. Let the rows correspond to periods and columns to experimental units. The total number of observations in this setup are "pn". For analysis of these pn observations, we assume the fixed effects model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \tau_{d(i,j)\tau} + \rho_{d(i-1,j)} + \varepsilon_{ij}$$

i =1, 2, 3, ………, p and j =1, 2, 3, ………, n

Where

y_{ij} is the response of the *j*th experimental unit in the ith period

α_i is the ith period effect

β_j is the *j*th experimental unit effect

$\tau_{d(i,j)}$ is direct effect of d(i, j) treatment

$\rho_{d(i-1,j)}$ is the residual effect of d(i, j) treatment and

ε_{ij} is the random error normally distributed with mean zero and variance σ^2

Definition 2.1. A design d in RM (t, n, p) is said to be uniform on the experimental units if d (i, j) ≠ d (i, j), i ≠ i´ for all j.

Definition 2.2. A design d in RM (t, n, p) is said to be uniform on the periods if, in each period, d assigns the same number of experimental units to each treatment.

Definition 2.3. A design d in RM (t, n, p) is said to be uniform if it is uniform on both the experimental units and periods.

Definition 2.4. A RM (t, n, p) design is said to be balanced with respect to sets of direct and residual effects if (i) each treatment is tested equally frequently λ_1 times in each period, (ii) in the order of application each treatment is preceded by each other treatment equally frequently λ_2 times.

In balanced RM (t, n, p) design the following relations hold for design parameters t, n, p, λ_1 and λ_2

$$n = \lambda_1 t, \qquad n(p-1) = \lambda_2 t(t-1) \qquad 2.1$$

For given t and p the above conditions lead to the following definition (Patterson, 1952)

Definition 2.5. For the given t and p, a balanced RM (t, n, p) design is said to be minimal if

$$n = t(t-1)/(p-1) \text{ and } (t-1)/(p-1) = \lambda_1 \text{ is an integer}$$

The following theorem which was first proved by Houston (1966)

Theorem 2.1. *A balanced minimal repeated measurements design satisfying the equations (2.1) and $p < t$ exists if $\lambda_1 p \times t$ tables can be constructed cyclically from $\lambda_1 p \times 1$ columns with the property that the differences between pairs adjacent elements of their first column are distinct and nonzero (mod t).*

Definition 2.6. Afsarinejad (1983) defined Extra–balanced Repeated Measurement designs as the extension of balanced RM design in which each treatment is allowed to be immediately preceded by itself as well. So for the Extra-balanced Repeated Measurements design amongst t, n, p, λ_1, λ_2 the following relations hold:

$$n = \lambda_1 t, \qquad n(p-1) = \lambda_2 t^2 \qquad 2.2$$

Definition 2.7. For every t and p an Extra-balanced Repeated Measurements design is minimal if $n = t^2 / (p-1)$ and $t / (p-1) = \lambda_1$ is an integer

***Theorem** 2.2. An Extra-balanced Minimal Repeated Measurements design satisfying the equations (2.2) and $p < t$ exists if λ_1 $p \times t$ tables can be constructed cyclically from λ_1 $p \times 1$ columns with the property that the differences between pairs adjacent elements of their first column are distinct (mod t).*

The class of balanced minimal RM (t, n, p) design can be divided into two families. *Family* (I), where t = p and *Family* (II), where p < t.
Family (I) p = t:

For this family λ_1 achieves its minimum value viz., 1. Note that for this family n = t and each of its number can be represented by an RM (t, t, t) design.

Williams (1949) introduced and constructed minimal RM designs for those experimental situations in which the number of periods is equal to the number of treatments whenever the number of treatments is even. Houston (1966) showed that it is impossible to construct a balanced minimal RM design based on a cyclic group when the number of treatments is odd. Mendelsohn (1968) constructed a balanced minimal RM design for 21 treatments based on non-cyclic group. Bradley (1953), Gordon (1961), Sheehe and Brass (1961) have all considered and constructed the balanced minimal RM (t, t, t) for all even t's. Mir *et al.,* (2005) introduced the balanced minimal Repeated Measurements designs uniform on periods, when the number of periods is equal to the number of treatments whenever the number of treatments is odd. Mir *et al.,* (2008) constructed a series of balanced minimal Repeated Measurements designs when t is an odd prime or prime power.

Our main concern is to study *Family* (II), where p < t: Patterson (1952) considered and constructed balance minimal Repeated Measurements designs whenever p< t. Patterson & Lucas (1962) gave a catalogue of Repeated Measurements designs in which the number of periods is equal to or less than the number of treatments. Atkinson (1961), Davis & Hall (1969), Hedayat & Afsarinejad (1975), Constantine & Hedayat (1982), Afsarinejad (1983), Gharde (2007), Dey *et al.,* (1995) and Varghese & Sharma (2000) studied the cross-over designs in the presence of first order residual effects. Stufken (1991) constructed some families of optimal and efficient repeated measurements designs. Hedayat and Young (2004) studied universal optimality for selected crossover designs. Sharma *et al.,* (2003) have constructed some minimal balanced Repeated Measurements designs for the situations whenever p < t. Iqbal *et al.,* (2009) constructed Circular strongly balanced repeated measurements designs and again Iqbal *et al.,* (2010) studied Circular first- and second-order strongly balanced repeated measurements designs. Mir *et al.,* (2009) introduced and constructed a new class of balanced Repeated Measurements designs for two-crop system involving two different sets of treatments.

Afsarinejad (1983) have given a construction method for balanced minimal Repeated Measurements design when t is an odd number. But this method has a drawback that when the number of treatments t is an odd prime or prime power, one or more than one treatment occurs more than once in any sequence and thereby design no longer remain uniform. In this paper an attempt has been made to provide a new method to overcome this drawback. Specifically, two cases have been considered

RM [t, t (t-t) / (p-1), p], λ_2=1 for balanced minimal repeated measurements designs and RM [t, 2t (t-t)/(p-1), p], λ_2=2 for balanced repeated measurements designs. In addition, an approach has been provided for constructing extra-balanced minimal designs for special case RM [t, t^2 / (p-1), p], λ_2=1.

3. Construction of Balanced minimal Repeated Measurements designs

3.1: RM [t, t(t-1)/(p-1), p], λ_2 = 1

For every t that is prime or prime power the design RM [t, t (t-1) / (p-1), p], λ_2 =1 can always be constructed where (t-1) / (p-1) is an integer, wherein no treatment occur more than once in any sequence.

Afsarinejad (1983) have given a construction method for balanced minimal Repeated Measurements design when t is an odd number. But this method has a drawback that when the number of treatments t is an odd prime or prime power, one or more than one treatment occurs more than once in some sequences and thereby designs no longer remain uniform over periods. For example RM (7, 21, 3), the initial sequences obtained for this design by the method given by Afsarinejad (1983) is reproduced below in Table No.3.1

Table No. 3.1: λ1= 3 Initial Sequences

	Sequences no.		
	I	II	III
Elements	1	**3**	3
	7	5	7
	3	**3**	1

It is evident that on developing above initial sequences treatments will be repeated more than once. So new method of construction is introduced which does not suffer from this drawback.

Theorem 3.1: *Balanced minimal RM [t, t (t-1) / (p-1), p], λ_2 =1 where t is a prime or prime power and (t-1)/(p-1) is an integer can always be constructed.*

Construction Method 3.1: Let t be a prime or prime power, say t = S^h, where S is a prime number and h is an integer. Let (t -1) ≡ 0 mod (p – 1) i.e. (t-1) / (p-1) = λ_1 is an integer. Let x is the primitive root of Gloise Field GF (t) and we denote the elements of GF (t) as $\alpha_0 = 0$, $\alpha_1 = x^0$, $\alpha_2 = x^1$,…, $\alpha_l = x^{l-1}$,……, $\alpha_{t-1} = x^{t-2}$

Let RM [t, t (t-1) / (p-1), p] consisting of n = t (t-1) / (p-1) sequences each of p elements (treatments) be represented as p x n array say A of p rows n columns. Let the p rows be numbered as 1, 2, 3,……., pth row and n = λ_1 t column be numbered as 0, 1, 2, 3, ..., t-1, t-0, t+1, t+2, ……, t+(t-1),…., (k-1)t+0, (k-1)t+1,…, (k-1)t+(t-1),…, (λ_1-1)t+0, (λ_1-1)t+1,….., (λ_1-1)t+(t-1).

The entries of the array A will be the treatments of the experiment. The element in the ith row and mth column, will be an element of GF (t) equal to

$$[\alpha_i]\, [\alpha_{(k-1)(p-1)+1}]+ \alpha_j = [x^{i-1}][x^{(k-1)(p-1)}] + \alpha_j$$

where m = (k-1)t + j, i = 1, 2, 3,….., p ; j = 0, 1, 2, 3,……, t-1 ; and k = 1, 2, 3,…, $\lambda_{1.}$

The array A is a balanced minimal RM [t, t (t-1) / (p-1), p], where

λ_1 = (t-1) / (p-1) and λ_2 = 1 according to the definition of Hedayat and Afsarinejad (1975).

The construction method 3.1 is illustrated by the following example 3.1

Example 3.1. Let t = 7 (prime number) and p = 3. Then the above procedure produces balanced minimal RM (7, 21, 3), λ_2=1 as shown in Table 3.2 and the design so obtained is reproduced in the Table 3.2

Table No. 3.2: Balanced minimal RM (7, 21, 3), λ_1= 3, λ_2=1

Periods	Experimental Units																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	4	3	0	5	6	2	3	5	4	1	6	0	4	5	0	6	3	1	2
2	3	4	6	5	2	0	1	6	0	2	1	5	3	4	5	6	1	0	4	2	3
3	2	3	5	4	1	6	0	4	5	0	6	3	1	2	1	2	4	3	0	5	6

Example3.2. Let t = 9 (prime power) and p =5. Then the above procedure produces balanced minimal RM (9, 18, 5), λ_2 =1 and the design so obtained is reproduced in the Table 3.3

Table No. 3.3: Balanced minimal RM (9, 18, 5), λ_1= 2, λ_2=1

Periods	Experimental Units																	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1	5	3	8	7	0	4	6	2	5	0	8	2	6	1	7	4	3
2	2	3	6	4	1	8	0	5	7	6	4	0	1	3	7	2	8	5
3	3	8	4	7	5	2	1	0	6	7	6	5	0	2	4	8	3	1
4	4	7	1	5	8	6	3	2	0	8	2	7	6	0	3	5	1	4
5	5	0	8	2	6	1	7	4	3	1	5	3	8	7	0	4	6	2

3.2: RM [t, 2t (t-1) / (p-1), p], $\lambda_2 = 2$

Theorem 3.2: Balanced RM [t, 2t(t-1)/(p-1), p] can always be constructed when t is a prime or prime power and $2(t-1)/(p-1) = \lambda_1$ is an integer with $\lambda_2 = 2$.

Construction Method 3.2: Let t be a prime or prime power of the form $t = S^h$, where S is a prime number and h is an integer. Let the number of rows (periods) p be such that 2(t-1) / (p-1) is an integer. Then an array A of p rows and n = 2t (t-1) / (p-1) columns will form a balanced RM [t, 2t (t-1) / (p-1), p] when the entries of the array are filled in the following manner.

Let the elements of GF (t) be obtained as $\alpha_0 = 0$, $\alpha_1 = x^0$, $\alpha_2 = x^1$, ……, $\alpha_{t-1} = x^{t-2}$, where x is the primitive root of GF(t) and hence $x^{t-1} = 1$. Let the p rows of the array A be numbered as 1, 2, 3,......., p and the n = 2t(t-1)/(p-1) columns be numbered as 0, 1, 2, ……., (n-1).

Then the element of array A in ith row (i= 1, 2, 3, ……, p) and lth column (l = 0, 1, 2, ……., n-1) will be

$$[\alpha_i] [\alpha_{(k-1)(p-1)+1}] + \alpha_j = [x^{l-1}][x^{(k-1)(p-1)}] + \alpha_j$$

where l = (k-1)t + j ; j = 0,1,2,3,……, t-1 ; and k = 1,2,3,…, λ_1.

The array A is a balanced RM [t, 2t (t-1) / (p-1), p], where $\lambda_1 = 2(t-1)/(p-1)$ is an integer and $\lambda_2 = 2$ according to the definition of Hedayat and Afsarinejad (1975).

The construction method 3.2 is illustrated by following example 3.3

Example3.3. Let t = 13 and p = 9, $\lambda_1 = 3$ and $\lambda_2 = 2$. Then the above procedure produces balanced RM (13, 39, 9), $\lambda_2 = 2$ and the design so obtained is reproduced in the Table 3.4

Table No. 3.4.: Balanced RM (13, 39, 9), $\lambda_1 = 3$, $\lambda_2 = 2$

Periods	Experimental Units																		
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1	2	3	5	9	4	7	0	12	10	6	11	8	9	10	11	0	4	12
2	2	3	4	6	10	5	8	1	0	11	7	12	9	5	6	7	9	0	8
3	4	5	6	8	12	7	10	3	2	0	9	1	11	10	11	12	1	5	0
4	8	9	10	12	3	11	1	7	6	4	0	5	2	7	8	9	11	2	10
5	3	4	5	7	11	6	9	2	1	12	8	0	10	1	2	3	5	9	4
6	6	7	8	10	1	9	12	5	4	2	11	3	0	2	3	4	6	10	5
7	12	0	1	3	7	2	5	11	10	8	4	9	6	4	5	6	8	12	7
8	11	12	0	2	6	1	4	10	9	7	3	8	5	8	9	10	12	3	11
9	9	10	11	0	4	12	2	8	7	5	1	6	3	3	4	5	7	11	6

Experimental Units																			
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
2	8	7	5	1	6	3	3	4	5	7	11	6	9	2	1	12	8	0	10
11	4	3	1	10	2	12	6	7	8	10	1	9	12	5	4	2	11	3	0
3	9	8	6	2	7	4	12	0	1	3	7	2	5	11	10	8	4	9	6
0	6	5	3	12	4	1	11	12	0	2	6	1	4	10	9	7	3	8	5
7	0	12	10	6	11	8	9	10	11	0	4	12	2	8	7	5	1	6	3
8	1	0	11	7	12	9	5	6	7	9	0	8	11	4	3	1	10	2	12
10	3	2	0	9	1	11	10	11	12	1	5	0	3	9	8	6	2	7	4
1	7	6	4	0	5	2	7	8	9	11	2	10	0	6	5	3	12	4	1
9	2	1	12	8	0	10	1	2	3	5	9	4	7	0	12	10	6	11	8

4. Construction of Extra-Balanced Minimal Repeated Measurements Designs

4.1: RM [t, t^2 / (p-1), p], $\lambda_2 = 1$

For every t which is prime power, the design RM [t, t^2 / (p-1), p] , $\lambda_2 = 1$ and $p < t$ can always be constructed where only one treatment in a sequence is occurring more than once.

Construction Method 4.1: First construct the balanced RM (t, t, t), the jth sequence of which is given by $C_0 + \alpha_j\ E_{t,1}\ \ \forall\ \ j = 0, 1, 2, 3, \ldots, (t - 1)$ Where C_0 is an initial column vector of t element as $C_0 = [1= x^0, x^1, x^2, \ldots, x^{t-1} = 1]'$, x is primitive root of GF (t), α_j jth element of GF (t) and $E_{t,1}$ column vector of ones.

Let the number of periods p of the required design be such that t = 0 mod (p – 1) . Now construct the extra-balanced RM (t , t, t +1) by introducing one more period in balanced RM (t, t, t) at [(t+3)/2]th period in which same treatments applied as applied in the [(t+1)/2]th period and period number of all other succeeding periods of RM(t, t, t) is increased by integer 1 while keeping their entries same as those of the corresponding balanced RM (t, t, t) and denote it as [t x (t+1)] array, say A. Now form the arrays R_1, R_2, R_3,…….., $R_{\lambda 1}$, each of order (p x t) from the array A in the following manner: R_1 is array consisting of 1st to pth row of A; R_2 is an array consisting of pth row to (2p-1)th row of A; R_3 is an array consisting of (2p-1)th row to (3p – 2)th of A and in general R_i is an array consisting of [(i –1)(p -1) +1]th row for every i = 1, 2, 3, ……., λ_1. Now the juxtaposition of arrays R_1, R_2, R_3,…….., $R_{\lambda 1}$ will be an extra-balanced minimal repeated measurement design i.e. RM [t, $n= t^2/(p-1)$, p], $\lambda_2 = 1$.

The construction method 4.1 is illustrates by the following example 4.1

Example 4.1: **Let t = 9, p= 4 , λ_1= 3 and λ_2 = 1**

Balanced RM (9, 9, 9)

periods	Experimental Units								
	1	2	3	4	5	6	7	8	9
1	1	2	0	4	5	3	7	8	6
2	3	4	5	6	7	8	0	1	2
3	7	8	6	1	2	0	4	5	3
4	8	6	7	2	0	1	5	3	4
5	2	0	1	5	3	4	8	6	7
6	6	7	8	0	1	2	3	4	5
7	5	3	4	8	6	7	2	0	1
8	4	5	3	7	8	6	1	2	0
9	1	2	0	4	5	3	7	8	6

Extra-Balanced RM (9, 9, 10)

periods	Experimental Units								
	1	2	3	4	5	6	7	8	9
1	1	2	0	4	5	3	7	8	6
2	3	4	5	6	7	8	0	1	2
3	7	8	6	1	2	0	4	5	3
4	8	6	7	2	0	1	5	3	4
5	2	0	1	5	3	4	8	6	7
6	2	0	1	5	3	4	8	6	7
7	6	7	8	0	1	2	3	4	5
8	5	3	4	8	6	7	2	0	1
9	4	5	3	7	8	6	1	2	0
10	1	2	0	4	5	3	7	8	6

The array R_1, R_2 and R_3 each of order (4 x 9) are obtained from the Extra-balanced Repeated Measurements design RM (9, 9, 10), are as follows:

R_1 array;

1	2	0	4	5	3	7	8	6
3	4	5	6	7	8	0	1	2
7	8	6	1	2	0	4	5	3
8	6	7	2	0	1	5	3	4

R_2 array;

8	6	7	2	0	1	5	3	4
2	0	1	5	3	4	8	6	7
2	0	1	5	3	4	8	6	7
6	7	8	0	1	2	3	4	5

R_3 array;

6	7	8	0	1	2	3	4	5
5	3	4	8	6	7	2	0	1
4	5	3	7	8	6	1	2	0
1	2	0	4	5	3	7	8	6

Now the juxtaposition of arrays R_1, R_2 and R_3 forms an extra-balanced minimal Repeated Measurements design as given in table 4.1.

Table 4.1: Extra-balanced minimal RM (9, 27, 4), $\lambda_2 = 1$

periods	Experimental units																										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
1	1	2	0	4	5	3	7	8	6	8	6	7	2	0	1	5	3	4	6	7	8	0	1	2	3	4	5
2	3	4	5	6	7	8	0	1	2	2	0	1	5	3	4	8	6	7	5	3	4	8	6	7	2	0	1
3	7	8	6	1	2	0	4	5	3	2	0	1	5	3	4	8	6	7	4	5	3	7	8	6	1	2	0
4	8	6	7	2	0	1	5	3	4	6	7	8	0	1	2	3	4	5	1	2	0	4	5	3	7	8	6

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