Fuzzy Goal Programming Approach in Selective Maintenance Reliability Model

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Abstract

In the present paper, we have considered the allocation problem of repairable components for a parallel-series system as a multi-objective optimization problem and have discussed two different models. In first model the reliability of subsystems are considered as different objectives. In second model the cost and time spent on repairing the components are considered as two different objectives. These two models is formulated as multi-objective Nonlinear Programming Problem (MONLPP) and a Fuzzy goal programming method is used to work out the compromise allocation in multi-objective selective maintenance reliability model in which we define the membership functions of each objective function and then transform membership functions into equivalent linear membership functions by first order Taylor series and finally by forming a fuzzy goal programming model obtain a desired compromise allocation of maintenance components. A numerical example is also worked out to illustrate the computational details of the method.

Keywords: Reliability, Fuzzy Goal Programming, Compromise allocation, Selective Maintenance, Multi-objective programming.

1. Introduction

All the Industries depend on the reliable performance of repairable systems for the successful completion of missions. In every industry, systems are used in the production of goods. If such systems deteriorate or fail, the effect can be wide spread. Indeed, system deterioration is often reflected in higher production cost, time, lower product quality and also quantity. The system maintenance decision is taken on the basis of the state condition of the system (*i.e.* whether the system is good or bad). Due to the limitations in maintenance resources, a maintenance decision maker must decide how to allocate available resources. This allocation falls within the domain of selective maintenance. Selective maintenance is defined as the process of identifying the subset of maintenance activities to perform from a set of desired maintenance actions.

The selective maintenance decisions have been used for the following scenario – A system has just completed a mission and will begin its next mission soon. Maintenance cannot be performed during missions; therefore, the decision-maker must decide which components to maintain prior to the next mission. The selective maintenance models considered to date treat decision-making relative to a single, future mission. If a system is required to perform a sequence of missions, then the selective maintenance decisions directly affect system reliability for the next mission and indirectly affect the system reliability for later missions.

The selective maintenance operation is an optimal decision-making activity for system consisting of several components under limited maintenance resources. The main objective of the selective maintenance operation is to select the most important component within subsystems. Rice et al. (1998) were the first to deal with the selective maintenance problem. They define a system that must complete a series of missions where maintenance is performed only during finite breaks between missions. Due to the limited maintenance time, it may not be possible to repair all failed components before the next mission. A nonlinear, discrete selective maintenance optimization model is developed which is designed to maximize system reliability for the next mission. The numbers of components to repair are the decision variables, and the limitation on maintenance time serves as the primary functional constraint.

Cassady *et al.* (2001a, 2001b) extend the work of Rice *et al.* (1998) in several ways. First, more complex systems are analyzed. Specifically, systems are comprised of independent subsystems connected in series with the individual components in each subsystem connected in any fashion. Next, the selective maintenance model is extended to consider the case where both time and cost are constrained. This leads to the development of three different selective maintenance models. These models include maximizing system reliability subject to both time and cost constraints; minimizing system repair costs subject to a time constraint and a minimum required reliability level; and minimizing total repair time subject to both cost and reliability constraints.

Cassady *et al.* (2001c) extend the work of Rice *et al*. (1998) in two other ways. First, system components are assumed to have Weibull life distributions. This assumption permits systems to experience an increasing failure rate (IFR) and requires monitoring of the age of components. Second, the selective maintenance model is formulated to include three maintenance actions: minimal repair of failed components, replacement of failed components, and preventive maintenance.

Chen *et al.* (1999) extend the work of Rice *et al*. (1998) and Cassady *et al.* (2001a) by considering systems in which each component and the system may be in $K + 1$ possible states, 0, 1, … , K. They use an optimization model to minimize the total cost of maintenance activities subject to minimum required system reliability.

Schneider and Cassady (2004) formulate an optimization model to extend the work of Rice *et al.* (1998) by defining a selective maintenance model for a set of systems that must perform a set of missions with system maintenance performed only between sets of missions. Three models are formulated. The first model maximizes the probability that all systems within the set successfully complete the next mission, where as the second model minimizes the variable cost associated with maintenance. A special case of the second model allows the user to maximize the expected value of the number of successful missions in the next set. The third model permits cancellation of a mission based on costs associated with the risk of failure. Recently Ali *et al.* (2011a, 2011b, 2011c, 2012a, 2012b, 2013), Faisal and Ali (2012) have been studied the problem of selective maintenance and solved it by various single and multi-objective optimization techniques. Fuzzy programming offers a powerful means of handling optimization problems with fuzzy parameters. Fuzzy programming has been used in different ways and in different fields in the past. In reliability Park (1987), Mahapatra and Roy (2006), Huang (1997), Dhingra (1992), Rao and Dhingra (1992) Ravi et al. (2000) and Ali and Hasan (2012c) have used fuzzy multi-objective optimization method to solve reliability optimization problem having several conflicting objectives and many others.

In the present paper the problem of finding the optimum compromise allocation of maintenance components is formulated as multi-objective Nonlinear Programming Problem (MONLPP) and a Fuzzy goal programming method is used to work out the compromise allocation in multi-objective selective maintenance reliability model in which we define the membership functions of each objective function and then transform them into equivalent linear membership functions by first order Taylor series and finally by forming a fuzzy goal programming model obtain a desired compromise allocation of maintenance components. A numerical example is also worked out to illustrate the computational details of the method. The numerical example is solved by "Fuzzy Goal programming algorithm" using software package LINGO. LINGO is a user's friendly package for constrained optimization developed by LINDO Systems Inc. A user's guide- LINGO User's Guide (2001) is also available. For more information one can visit the site http://www.lindo.com.

2. Hypothetical Series-Parallel System Problem

We consider a Hypothetical system which is a series arrangement of *m* subsystems and performing a sequence of identical production runs after every given (fixed) period.

Suppose that after completion of particular production runs, each component in the system is either functioning or failed. Ideally all the failed components in the subsystems are repaired and then replaced back prior to the beginning of the next production runs. However, due to the constraints on the time and cost, it may not be possible to repair all the failed components in the system. In such situation, a method is needed to decide which failed components should be repaired and replaced back prior to the next production run and the rest be left in a failed condition. This process is referred as selective maintenance (See Rice et al. 1998). In the selective maintenance the number of components available for the next production run in the i^{th} subsystem will be

$$(n_i - a_i) + d_i\ ,\ i = 1, 2, \ldots, m \tag{1}$$

The reliability of the given system is defined as

$$R = \prod_{i=1}^{m} \left\{ 1 - (1 - r_i)^{n_i - a_i + d_i} \right\} \tag{2}$$

The repair time constraint for the system is given as

$$\sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] \le T_0 \qquad (3)$$

where t_i is the time required to repair a component in $i-th$ subsystem and $\exp(\theta_i\, a_i)$ is the additional time spent due to the interconnection between parallel components (Wang *et al.* (2009)).

The repair cost constraint for the system is defined as

$$\sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i d_i)\right] \le C_0 \qquad (4)$$

where $\exp(\beta_i\, a_i)$ is the additional cost spent due to the interconnection between parallel components (Wang *et al.* (2009)).

<u>Model 1</u>: However, in the event when the reliability of each subsystems are of equally serious concern. Let us consider, for instance, the following multi-objective problem (see Ali et al. (2011c)):

$$\left.\begin{array}{lll} & Maximize\left(R_1,\, R_2,\, \ldots,\, R_m\right) & (i) \\ & \text{subject to} & \\ & \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i)\right] \le T_0 & (ii) \\ & \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i\, d_i)\right] \le C_0 & (iii) \\ \text{and} & 0 \le d_i \le a_i\, , i = 1,2,\ldots, m \text{ and integers} & (iv) \end{array}\right\} \qquad (5)$$

where $R_i = \left\{1-(1-r_i)^{n_i - a_i + d_i}\right\},\ i = 1, 2, \ldots, m.$

<u>Model 2</u>: Ali et al. (2011c) also discussed the situation in which time taken and the cost spent on system maintenance are minimized simultaneously for the required reliability R^* (say). The mathematical model of the problem is defined as:

$$\left.\begin{array}{ll} & MinC = \sum_{i=1}^{m} c_i \left[d_i + \exp\left(\beta_i d_i\right)\right] \\ & \textit{and} \\ & MinT = \sum_{i=1}^{m} t_i \left[d_i + \exp\left(\theta_i d_i\right)\right] \\ & \text{subject to} \\ & \prod_{i=1}^{m} 1-\left(1-r_i\right)^{n_i - a_i + d_i} \ge R^* \\ \text{and} & 0 \le d_i \le a_i\, , i = 1,2,\ldots, m \text{ and integers} \end{array}\right\} \qquad (6)$$

2. The Fuzzy Goal Programming Approach

To solve multi-objective allocation problem of repairable components define in equation (5), we apply the fuzzy goal programming approach consideration of multi-objective vector maximum problem.

Algorithm:

The Fuzzy Goal Programming algorithm for solving MONLPP can be outlined as given below:

Step 1: Find the ideal solution of objective functions by optimizing each objective subject to the system constraints.

Step 2: Formulate the payoff matrix using the ideal solutions. Then define upper and lower tolerance limits of each objective function as

$$L_i = \min_i R_i(d_i^*) \quad and \quad U_i = \max_i R_i(d_i^*)$$

Step 3: Construct non-linear membership function

$$\mu_i(d_i) = \begin{cases} 0 & R_i < L_i \\ \dfrac{R_i - L_i}{U_i - L_i} & L_i < R_i < U_i \\ 1 & R_i \geq U_i \end{cases}$$

If $\mu_i(d_i) = 1$; then R_i is perfectly achieved,

= 0; R_i is nothing achieved,

$0 \leq \mu_i(d_i) \leq 1$; then R_i is partially achieved.

Step 4: Find the individual best solution of the non-linear membership functions $\mu_i(d_i)$ subject to the system constraints.

Step 5: Transform the non-linear membership functions $\mu_i(d_i)$ into equivalent linear membership functions ξ_i respectively at the individual best solution point by first order Taylor series as

$$\mu_i \cong \mu_i(d_i^*) + (d_1 - d_1^*)\frac{\partial}{\partial d_1}\mu_i(d_i^*) + \cdots + (d_L - d_L^*)\frac{\partial}{\partial d_L}\mu_i(d_i^*) = \xi_i$$

Step 6: The maximum value of a membership function is unity (one), so for the defined membership functions in step (3), the flexible membership goals having the aspiration level unity can be presented as:

$$\xi_i + \delta_i = 1, \; i = 1,2,\ldots,m.$$

Here $\delta_i \geq 0$ are the deviational variables.

Now the FGP model can be formulated as follows:

$$\left.\begin{array}{ll} Minimize\ \sum_{i=1}^{m}\delta_i & (i) \\ \quad \text{subject to} & \\ \qquad \xi_i+\delta_i=1 & (ii) \\ \qquad \sum_{i=1}^{m} t_i\left[d_i+\exp(\theta_i d_i)\right]\le T_0 & (iii) \\ \qquad \sum_{i=1}^{m} c_i\left[d_i+\exp(\beta_i\, d_i)\right]\le C_0 & (iv) \\ \text{and}\quad 0\le d_i\le a_i\,, i=1,2,\ldots,m \text{ and integers} & (v) \end{array}\right\} \qquad (7)$$

where $R_i=\left\{1-(1-r_i)^{n_i-a_i+d_i}\right\},\ i=1,2,\ldots,m.$

<u>Step 7</u>: Solve the FGP model using LINGO software.

In similar way, we solve multi-objective allocation problem of repairable components define in equation (6); we apply the fuzzy goal programming approach consideration of multi-objective vector minimum problem. At first, we find the lower bound L_r (best) and upper bound U_r (worst) for corresponding objective function Z_r where $r=1,2,\ldots,k$.

Let L_r = aspiration level of achievement for objective r,

U_r = highest acceptable level of achievement for objective r,

$\delta_r = U_r - L_r$ = the degradation allowance for objective r,

when the aspiration level and degradation allowance for each objective are specified.

Now construct membership function for model (2) define in equation (6) as,

$$\Omega_{Z_r}(d_{ij})=\begin{cases}1 & Z_r<L_r\\ \dfrac{U_r-Z_r}{U_r-L_r} & L_r<Z_r<U_r\\ 0 & Z_r\ge U_r\end{cases}$$

If $\Omega_{Z_r}(d_{ij})=1$; then Z_r is perfectly achieved,

$=0$; Z_r is nothing achieved,

$0\le\Omega_{Zr}(d_{ij})\le 1$; then Z_r is partially achieved.

Now transform the non-linear membership function as discussed in above algorithm and finally we get the desired FGP model for equation (6) as follows:

$$\left.\begin{aligned}
&Minimize\ \sum_{i=1}^{m}\delta_i && (i)\\
&\quad\text{subject to}\\
&\qquad \xi_i+\delta_i=1 && (ii)\\
&\qquad \prod_{i=1}^{m}1-(1-r_i)^{n_i-a_i+d_i}\geq R^* && (iii)\\
&\text{and}\quad 0\leq d_i\leq a_i\ , i=1,2,\dots,m \text{ and integers}
\end{aligned}\right\} \qquad (8)$$

3. Numerical Illustration

Consider a system consisting of 3 subsystems. The available time between two production runs for repairing and replacing back the components is 60 time units. Let the given maintenance cost of the system be 90 units. The other parameters for the various subsystems are given in table 1.

Table 1: The parameters for the numerical example

Subsystem	**1**	**2**	**3**
n_i	10	8	12
r_i	0.55	0.45	0.50
a_i	7	5	8
c_i	8	7	8
t_i	3	4	3
θ_i	0.25	0.25	0.25
β_i	0.25	0.25	0.25

Solution by Using Fuzzy Goal Programming Approach

Model 1: Using the values given in Table 1 the NLPP (5) and their optimal solutions $\underline{d}^{(i)}$; $i=1,\ 2\ and\ 3$ with the corresponding values of R_i^* are listed below. These values are obtained by software LINGO.

$$\left.\begin{aligned}
&Maximize\ R_i=\left\{1-(1-r_i)^{n_i-a_i+d_i}\right\}\\
&\quad\text{subject to}\\
&\qquad \sum_{i=1}^{5}t_i\left[d_i+\exp(\theta_i d_i)\right]\leq 60\\
&\qquad \sum_{i=1}^{5}c_i\left[d_i+\exp(\beta_i\, d_i)\right]\leq 90\\
&\text{and}\quad 1\leq d_i\leq a_i\ , i=1,2,\dots,5 \text{ and integers}
\end{aligned}\right\} i=1,2,\dots,m \qquad (9)$$

Using the equation (9) construct a pay-off matrix, according to every objective with respect to each solution the pay-off matrix in the main program gives the set of non dominated solution which shown in the following table

$$\begin{bmatrix} & Z_1 & Z_2 & Z_3 \\ d^{(1)} & 0.9996595 & 0.9084938 & 0.9589938 \\ d^{(2)} & 0.9589938 & 0.9916266 & 0.9589938 \\ d^{(3)} & 0.9589938 & 0.9084938 & 0.9998468 \end{bmatrix}$$

The upper bounds for the given model 1 are

$U_1 = 0.9996595, U_2 = 0.9916266, U_3 = 0.9998486$ and lower bound
$L_1 = 0.9589938,\ L_2 = 0.9084938, L_3 = 0.9589938$.

Now the linearized membership functions are:

$$\mu_1(7,1,1) = 1 + (d_1 - 7) \times 0.00012 = \xi_1$$
$$\mu_2(1,5,1) = 1 + (d_2 - 5) \times 0.00227 = \xi_2$$
$$\mu_3(1,1,8) = 1 + (d_3 - 8) \times 0.00005 = \xi_3$$

Using the equation (7), we can formulated the model 1 as

$$\left.\begin{array}{l} Minimize \ \sum_{i=1}^{3} \delta_i \\ \quad \text{subject to} \\ \quad\quad 1 + (d_1 - 7) \times 0.00012 + \delta_1 = 1 \\ \quad\quad 1 + (d_2 - 5) \times 0.00227 + \delta_2 = 1 \\ \quad\quad 1 + (d_3 - 8) \times 0.00005 + \delta_3 = 1 \\ \quad\quad \sum_{i=1}^{m} t_i \left[d_i + \exp(\theta_i d_i) \right] \le T_0 \\ \quad\quad \sum_{i=1}^{m} c_i \left[d_i + \exp(\beta_i\, d_i) \right] \le C_0 \\ \text{and} \quad 0 \le d_i \le a_i\ , i = 1,2,\ldots, m \text{ and integers} \end{array}\right\} \tag{10}$$

The above problem (10) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get $\delta_1 = 0.00036, \delta_2 = 0, \delta_3 = 0.00009$ and the compromise solution as $d_1^* = 4,\ d_2^* = 5,\ d_3^* = 6$. The optimal reliabilities of each subsystem are $R_1^* = 0.9962633,\ R_2^* = 0.9916266,\ R_3^* = 0.9992433$.

Model 2: Using the values given in Table 1 the NLPP (6) for the desired reliability requirement $R^* \ge 0.97$ has been solved and construct a pay-off matrix, according to every objective with respect to each solution the pay-off matrix in the main program gives the set of non dominated solution which shown in the following table for

$$\begin{bmatrix} & Z_1 & Z_2 \\ d^{(1)} & 81 & 43.06106 \\ d^{(2)} & 95.82010 & 37 \end{bmatrix}$$

The upper bound and lower bond for the model 2 are

$U_1 = 95.82010,\ U_2 = 43.06106$ and $L_1 = 81,\ L_2 = 37$.

Now the linearized membership functions are:

$$\mu_1(3,5,3) = 1-(d_1-3)\times 0.6978649-(d_2-5)\times 0.6259985-(d_3-3)\times 0.6978649 = \xi_1$$
$$\mu_2(4,4,3) = 1-(d_1-4)\times 0.6477293-(d_2-4)\times 0.8636390-(d_3-3)\times 0.6398898 = \xi_2$$

$$\left.\begin{aligned} & Minimize\ \sum_{i=1}^{2}\delta_i \\ & \quad \text{subject to} \\ & 1-(d_1-3)\times 0.6978649-(d_2-5)\times 0.6259985-(d_3-3)\times 0.6978649+\delta_1 = 1 \\ & 1-(d_1-4)\times 0.6477293-(d_2-4)\times 0.8636390-(d_3-3)\times 0.6398898+\delta_2 = 1 \\ & \prod_{i=1}^{m} 1-(1-r_i)^{n_i-a_i+d_i} \geq R^* \\ & \text{and} \quad 0 \leq d_i \leq a_i,\ i=1,2,\ldots,m \text{ and integers} \end{aligned}\right\} \qquad (11)$$

The above problem (11) is solved by the LINGO Software for obtaining the optimal solution of the problem. We get the compromise solution as $d_1^* = 4,\ d_2^* = 4,\ d_3^* = 3$.

4. Conclusion

The main purpose of this manuscript is to demonstrate the practical utility of the Fuzzy Goal programming approach in multi-objective selective maintenance reliability model.

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