An Alternative Ratio-Cum-Product Estimator of Finite Population Mean Using Coefficient of Kurtosis of Two Auxiliary Variates in Two-Phase Sampling

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Abstract

This paper deals with the problem of estimation of population mean in two-phase sampling. A ratio-product estimator of population mean using known coefficient of kurtosis of two auxiliary variates has been proposed. In fact, it is a two-phase sampling version of Tailor et al. (2010) estimator and its properties are studied. Proposed estimator has been compared with usual unbiased estimator, classical ratio and product estimator in two-phase sampling, and two-phase sampling versions of Singh (1967) and Singh et al. (2004) estimators respectively. To judge the merits of the proposed estimator over other estimators an empirical study is also carried out.

Keywords: Population mean, Coefficient of kurtosis, Two-phase sampling, Bias and Mean squared error.

1. Introduction

Auxiliary information plays a very important role in improving the efficiencies of estimator(s) of population parameter(s). Ratio, product and regression methods are good examples in this context. These methods require knowledge of population mean of auxiliary. Use of coefficient of kurtosis of auxiliary variate has also been in practice for improving the efficiency of the estimators of finite population mean. In some practical situations population coefficient of variation and coefficient of kurtosis of auxiliary variate x are known. [1]Ajagaonkar (1975) and [12]Sisodia and Dwivedi (1982) discussed double sampling procedure using single auxiliary variate whereas [3]Khan and Tripathi (1967), [6]Rao (1975) and [7]Singh and Namjoshi (1988) considered the use of multiauxiliary variates in double sampling.

[10]Singh (1967) used information on two auxiliary variates and defined a ratio-product estimator assuming that population mean of auxiliary variates are known. In the line of [11]Sisodia and Dwivedi (1981) [9]Singh et al. (2004) proposed a ratio type estimator using coefficient of kurtosis. [18]Tailor et al. (2010) suggested a ratio-cum-product estimator using coefficient of kurtosis of two auxiliary variates in simple random sampling while [16]Tailor and Sharma (2013) suggested a ratio-cum-product estimator using double sampling. In this research paper authors have suggested [18]Tailor et al. (2010) ratio-product estimator in two phase sampling.

Let us consider a finite population $U = \{U_1, U_2, ..., U_N\}$ of size N. Let y, x_1 and x_2 be the study variate and auxiliary variates taking values y_i x_{1i} and x_{2i} respectively on $U_i\,(i = 1,2,...,N)$. Let the auxiliary variables x_1 and x_2 be positively and negatively correlated with the study variate y respectively.

Let us suppose that $\bar{Y} = \sum_{i=1}^{N} y_i / N$, $\bar{X}_1 = \sum_{i=1}^{N} x_{1i} / N$ and $\bar{X}_2 = \sum_{i=1}^{N} x_{2i} / N$ be the population mean of the study variate y and auxiliary variates (x_1, x_2) respectively.

Let $\bar{y} = \sum_{i=1}^{n} y_i / n$, $\bar{x}_1 = \sum_{i=1}^{n} x_{1i} / n$ and $\bar{x}_2 = \sum_{i=1}^{n} x_{2i} / n$ be the unbiased estimators of population mean $\bar{Y}$, $\bar{X}_1$ and $\bar{X}_2$ respectively.

The classical ratio and product estimators for estimating population mean $\bar{Y}$ are respectively defined by

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right) \tag{1.1}$$

and

$$\hat{\bar{Y}}_P = \bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right). \tag{1.2}$$

Assuming that population means $\bar{X}_1$ and $\bar{X}_2$ of the auxiliary variables x_1 and x_2 are known, [9]Singh et al. (2004) defined a ratio and product type estimators using coefficient of kurtosis $\beta_2(x_1)$ and $\beta_2(x_2)$ respectively as

$$\hat{\bar{Y}}_{SR} = \bar{y}\left(\frac{\bar{X}_1 + \beta_2(x_1)}{\bar{x}_1 + \beta_2(x_1)}\right), \tag{1.3}$$

$$\hat{\bar{Y}}_{SP} = \bar{y}\left(\frac{\bar{x}_2 + \beta_2(x_2)}{\bar{X}_2 + \beta_2(x_2)}\right), \tag{1.4}$$

[10]Singh (1967) suggested a ratio-product estimator using information on two auxiliary variates x_1 and x_2 to estimate population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{SRP} = \bar{y}\left(\frac{\bar{X}_1}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{X}_2}\right), \tag{1.5}$$

The problem of estimating population mean $\bar{Y}$ of y when the population means $\bar{X}_1$ and $\bar{X}_2$ of x_1 and x_2 are known, has been dealt at a great length in the literature see [8]Singh and Tailor (2005), [17]Tailor and Tailor (2008) and many others. However, in many practical situations when no information is available on the population means $\bar{X}_1$ and $\bar{X}_2$ of x_1 and x_2 in advance before starting the survey, we estimate $\bar{Y}$ from a sample obtained through a two phase selection. Adopting simple random sampling without replacement (SRSWOR) scheme at each phase, the two-phases (or double) sampling scheme is as follows:

i. A first phase sample S_1 of fixed size n' is drawn form U to observe only x_1 and x_2 to estimate $\overline{X}_1$ and $\overline{X}_2$ respectively.

ii. A second phase sample S_2 of fixed size n is drawn from S_1 to observe y only or second phase sample may be drawn independently to the first sample i.e. two cases, designated as-

Case I : As a sub sample from the first phase sample,

Case II : Draw independently to the first phase sample.

In two-phase or double sampling, the usual ratio and product estimators of population mean $\bar{Y}$ are respectively defined as

$$\hat{\bar{Y}}_R^{(d)} = \bar{y}\left(\frac{\bar{x}_1'}{\bar{x}_1}\right), \tag{1.6}$$

$$\hat{\bar{Y}}_P^{(d)} = \bar{y}\left(\frac{\bar{x}_2}{\bar{x}_2'}\right), \tag{1.7}$$

where $\bar{y}$, $\bar{x}_1$ and $\bar{x}_2$ are sample means based on second phase sample of size n whereas $\bar{x}_1' = \frac{1}{n_1'}\sum_{i=1}^{n_1'} x_{1i}$ and $\bar{x}_2' = \frac{1}{n_2'}\sum_{i=1}^{n_2'} x_{2i}$ are the first phase sample means of x_1 and x_2, which are unbiased estimates of population means $\overline{X}_1$ and $\overline{X}_2$ respectively of auxiliary variate x.

Two-phase sampling versions of [9]Singh et al. (2004) ratio and product type estimators of population mean $\overline{Y}$ are defined by

$$\hat{\bar{Y}}_{SR}^{(d)} = \bar{y}\left(\frac{\bar{x}_1' + \beta_2(x_1)}{\bar{x}_1 + \beta_2(x_1)}\right), \tag{1.8}$$

$$\hat{\bar{Y}}_{SP}^{(d)} = \bar{y}\left(\frac{\bar{x}_2 + \beta_2(x_2)}{\bar{x}_2' + \beta_2(x_2)}\right), \tag{1.9}$$

Two-phase sampling version of [10]Singh (1967) ratio-product estimator of population mean $\overline{Y}$ is defined by

$$\hat{\bar{Y}}_{RP}^{(d)} = \bar{y}\left(\frac{\bar{x}_1'}{\bar{x}_1}\right)\left(\frac{\bar{x}_2}{\bar{x}_2'}\right). \tag{1.10}$$

We obtain the bias and mean squared error of two-phase sampling versions of estimators considered in this section to the first degree of approximation.

$B(.)_{I}$, $B(.)_{II}$, $MSE(.)_{I}$ and $MSE(.)_{II}$ denote the bias and the mean squared error under case I and II respectively which are given as

$$B(\hat{\bar{Y}}_{R}^{(d)})_{I} = \bar{Y} f_3 C_{x_1}^2 (1 - K_{01}), \tag{1.11}$$

$$B(\hat{\bar{Y}}_{P}^{(d)})_{I} = \bar{Y} f_3 C_{x_2}^2 (1 + K_{02}), \tag{1.12}$$

$$B(\hat{\bar{Y}}_{SR}^{(d)})_{I} = \bar{Y} f_3 t_1 C_{x_1}^2 (t_1 - K_{01}), \tag{1.13}$$

$$B(\hat{\bar{Y}}_{SP}^{(d)})_{I} = \bar{Y} f_3 t_2 C_{x_1}^2 (t_2 + K_{02}), \tag{1.14}$$

$$B(\hat{\bar{Y}}_{SRP}^{(d)})_{I} = \bar{Y} f_3 \left[C_{x_1}^2 (1 - K_{01}) - C_{x_2}^2 (K_{02} + K_{01}) \right], \tag{1.15}$$

$$B(\hat{\bar{Y}}_{R}^{(d)})_{II} = \bar{Y} f_1 C_{x_1}^2 (1 - K_{01}), \tag{1.16}$$

$$B(\hat{\bar{Y}}_{P}^{(d)})_{II} = \bar{Y} C_{x_2}^2 (f_2 + f_1 K_{02}), \tag{1.17}$$

$$B(\hat{\bar{Y}}_{SR}^{(d)})_{II} = \bar{Y} f_1 t_1 C_{x_1}^2 (t_1 - K_{01}), \tag{1.18}$$

$$B(\hat{\bar{Y}}_{SP}^{(d)})_{II} = \bar{Y} t_2 C_{x_2}^2 (f_2 t_2 + f_1 K_{02}), \tag{1.19}$$

$$B(\hat{\bar{Y}}_{SRP}^{(d)})_{I} = \bar{Y} \left[f_1 C_{x_1}^2 (1 - K_{01}) + C_{x_2}^2 (f_2 - K_{12} + f_1 K_{02}) \right], \tag{1.20}$$

$$MSE(\hat{\bar{Y}}_{R}^{(d)})_{I} = \bar{Y}^2 \left[f_1 C_y^2 + f_3 C_{x_1}^2 (1 - 2K_{01}) \right], \tag{1.21}$$

$$MSE(\hat{\bar{Y}}_{P}^{(d)})_{I} = \bar{Y}^2 \left[f_1 C_y^2 + f_3 C_{x_2}^2 (1 + 2K_{02}) \right], \tag{1.22}$$

$$MSE(\hat{\bar{Y}}_{SR}^{(d)})_{I} = \bar{Y}^2 \left[f_1 C_y^2 + f_3 t_1 C_{x_1}^2 (t_1 - 2K_{01}) \right], \tag{1.23}$$

$$MSE(\hat{\bar{Y}}_{SP}^{(d)})_{I} = \bar{Y}^2 \left[f_1 C_y^2 + f_3 t_2 C_{x_2}^2 (t_2 + 2K_{02}) \right], \tag{1.24}$$

$$MSE(\hat{\bar{Y}}_{SRP}^{(d)})_{I} = \bar{Y}^2 \left[f_1 C_y^2 + f_3 C_{x_1}^2 (1 - 2K_{01}) + f_3 C_{x_2}^2 (1 + 2K_{02} - 2K_{12}) \right], \tag{1.25}$$

$$MSE(\hat{\bar{Y}}_{R}^{(d)})_{II} = \bar{Y}^2 \left[f_1 C_y^2 + C_{x_1}^2 \left\{ (f_1 + f_2) - 2 f_1 K_{01} \right\} \right], \tag{1.26}$$

$$MSE(\hat{\bar{Y}}_{P}^{(d)})_{II} = \bar{Y}^2 \left[f_1 C_y^2 + C_{x_2}^2 \left\{ (f_1 + f_2) + 2 f_1 K_{02} \right\} \right], \tag{1.27}$$

$$MSE(\hat{\bar{Y}}_{SR}^{(d)})_{II} = \bar{Y}^2 \left[f_1 C_y^2 + t_1 C_{x_1}^2 \left\{ t_1 (f_1 + f_2) - 2 f_1 K_{01} \right\} \right], \tag{1.28}$$

$$MSE(\hat{\bar{Y}}_{SP}^{(d)})_{II} = \bar{Y}^2 \left[f_1 C_y^2 + t_2 C_{x_2}^2 \left\{ t_2 (f_1 + f_2) + 2 f_1 K_{02} \right\} \right], \tag{1.29}$$

$$MSE(\hat{\bar{Y}}_{SRP}^{(d)})_{II} = \bar{Y}^2 \left[f_1 C_y^2 + C_{x_1}^2 \left\{ f_1 + f_2 - 2K_{01}) \right\} + C_{x_2}^2 \left\{ (f_1 + f_2 + 2K_{02} - 2K_{12}) \right\} \right], \tag{1.30}$$

where $t_1 = \dfrac{\bar{X}_1}{(\bar{X}_1 + \beta_2(x_1))}$, $t_2 = \dfrac{\bar{X}_2}{(\bar{X}_2 + \beta_2(x_2))}$,

$f_1 = \left(\dfrac{1}{n} - \dfrac{1}{N} \right)$, $f_2 = \left(\dfrac{1}{n'} - \dfrac{1}{N} \right)$ and $f_3 = f_1 - f_2$.

It is well known under Simple random sampling without replacing (SRSWOR) variance of unbiased estimator is defined as

$$V(\bar{y}) = \bar{Y}^2 f_1 C_y^2 \tag{1.31}$$

2. Proposed Ratio-Product Estimator

[18]Tailor et al. (2010) proposed ratio-product estimator of population mean $\bar{Y}$ using information on coefficient of kurtosis $\beta_2(x_1)$ & $\beta_2(x_2)$ of auxiliary variates x_1 and x_2 as

$$\hat{\bar{Y}}_T = \bar{y}\left(\frac{\bar{X}_1+\beta_2(x_1)}{\bar{x}_1+\beta_2(x_1)}\right)\left(\frac{\bar{x}_2+\beta_2(x_2)}{\bar{X}_2+\beta_2(x_2)}\right) \tag{2.1}$$

The estimator $\hat{\bar{Y}}_T$ requires the knowledge of $\bar{X}_1$ and $\bar{X}_2$. When information is not available, we define $\hat{\bar{Y}}_T$ in two-phase sampling as

$$\hat{\bar{Y}}_T^{(d)} = \bar{y}\left(\frac{\bar{x}_1'+\beta_2(x_1)}{\bar{x}_1+\beta_2(x_1)}\right)\left(\frac{\bar{x}_2+\beta_2(x_2)}{\bar{x}_2'+\beta_2(x_2)}\right) \tag{2.2}$$

To obtain the bias and mean squared error of $\hat{\bar{Y}}_T^{(d)}$ we write

$\bar{y} = \bar{Y}(1+e_0)$, $\bar{x} = \bar{X}(1+e_1)$, $\bar{x}_1' = \bar{X}_1(1+e_1')$ and $\bar{x}_2 = \bar{X}_2'(1+e_2')$ such that

$E(e_0)=E(e_1)=E(e_2)=E(e_3)=E(e_4)=0$, $E(e_0^2)=f_1 C_y^2$,

$E(e_1^2)=f_1 C_{x_1}^2$, $E(e_2^2)=f_1 C_{x_2}^2$, $E(e_1'^2)=f_2 C_{x_1}^2$, $E(e_2'^2)=f_2 C_{x_2}^2$,

$E(e_0e_1)=f_1\rho_{yx_1}C_yC_{x_1}$, $E(e_0e_2)=f_1\rho_{yx_2}C_yC_{x_2}$, $E(e_0e_1')=f_2\rho_{yx_1}C_yC_{x_1}$,

$E(e_0e_2')=f_2\rho_{yx_2}C_yC_{x_2}$, $E(e_1e_1')=f_2 C_{x_1}^2$, $E(e_2e_2')=f_2 C_{x_2}^2$,

$E(e_1e_2)=f_1\rho_{x_1x_2}C_{x_1}C_{x_2}$ and $E(e_1'e_2)=E(e_1'e_2')=E(e_1e_2')=f_2\rho_{x_1x_2}C_{x_1}C_{x_2}$.

Expressing (2.2) in terms of $e_i's$ we have

$$\hat{\bar{Y}}_T^{(d)} = \bar{Y}(1+e_0)\left[(1+t_1e_1')(1+t_1e_1)^{-1}(1+t_2e_2)(1+t_2e_2')^{-1}\right], \tag{2.3}$$

Now using the standard technique, we get bias and mean squared error of suggested estimator $\hat{\bar{Y}}_T^{(d)}$ up to the first degree of approximation under case I and II are respectively obtained as

$$B(\hat{\bar{Y}}_T^{(d)})_I = \bar{Y}\Big[t_1^2\left(f_1C_{x_1}^2 + f_2C_{x_2}^2\right) - t_1f_3\rho_{yx_1}C_yC_{x_1} + t_2f_3\rho_{yx_2}C_yC_{x_2} - t_1t_2\left(f_2\rho_{x_1x_2}C_{x_1}C_{x_2} - f_2C_{x_2}^2\right) + t_1^2f_2\rho_{yx_1}C_yC_{x_1}\Big],$$

$$B(\hat{\bar{Y}}_T^{(d)})_I = \bar{Y}\left[t_1C_{x_1}^2\left(t_1f_1 - f_3K_{01} + f_2t_1K_{01}\right) + C_{x_2}^2\left\{\left(f_2t_1(t_1 - t_2K_{12}) + t_2(f_2t_1 + f_3K_{02})\right)\right\}\right] \tag{2.4}$$

$$B(\hat{\bar{Y}}_T^{(d)})_{II} = \bar{Y}\Big[t_1^2\left(f_1C_{x_1}^2 + f_2C_{x_2}^2\right) - t_1f_1\rho_{yx_1}C_yC_{x_1} + t_2f_2\rho_{yx_2}C_yC_{x_2} - t_1t_2f_1\rho_{x_1x_2}C_{x_1}C_{x_2} + t_1^2f_2C_{x_2}^2\Big],$$

$$B(\hat{\bar{Y}}_T^{(d)})_{II} = \bar{Y}\left[t_1C_{x_1}^2\left(t_1f_1 - t_1f_1K_{01}\right) + C_{x_2}^2\left(2t_1^2f_2 + t_2f_2K_{02} - t_2t_1f_1K_{12}\right)\right], \tag{2.5}$$

$$MSE(\hat{\bar{Y}}_T^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 t_1^2 C_{x_1}^2 - 2t_1 f_3 \rho_{yx_1} C_y C_{x_1} + t_2^2 f_3 C_{x_2}^2 + 2t_2 f_3 \left(\rho_{yx_2} C_y C_{x_2} - t_1 \rho_{x_1x_2} C_{x_1} C_{x_2}\right)\Big],$$

$$MSE(\hat{\bar{Y}}_T^{(d)})_I = \bar{Y}^2\Big[f_1 C_y^2 + f_3 C_{x_1}^2 t_1(t_1 - 2K_{01}) + f_3 C_{x_2}^2 t_2(t_2 + 2K_{02} - 2t_1 K_{12})\Big], \quad (2.6)$$

$$MSE(\hat{\bar{Y}}_T^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + (f_1 + f_2)(t_1^2 C_{x_1}^2 + t_2^2 C_{x_2}^2) - 2t_1 f_1 C_{x_1}^2 (K_{01} + t_2 K_{12}) + 2t_2 f_1 \rho_{yx_2} C_y C_{x_2}\Big],$$

$$MSE(\hat{\bar{Y}}_T^{(d)})_{II} = \bar{Y}^2\Big[f_1 C_y^2 + C_{x_1}^2 t_1\{t_1(f_1 + f_2\} - 2f_1 K_{01}) + C_{x_2}^2 t_2\{t_2(f_1 + f_2\} + 2f_1 K_{02} - 2t_1 f_1 K_{12})\Big] \quad (2.7)$$

3. Efficiency Comparisons of $(\hat{\bar{Y}}_T^{(d)})_I$

It is observed from (1.31) and (2.6) that the suggested estimator $\hat{\bar{Y}}_T^{(d)}$ under case I ($(\hat{\bar{Y}}_T^{(d)})_I$**)** is more efficient than the usual unbiased estimator $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_1(2K_{01} - t_1)}{t_2(t_2 + 2K_{02} - t_1 K_{12})} \quad (3.1)$$

It is noted from (1.21) and (2.6) that the suggested estimator $(\hat{\bar{Y}}_T^{(d)})_I$ is more efficient than usual two-phase sampling ratio estimator $\hat{\bar{Y}}_R^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1 - t_1)(1 + t_1 - 2K_{01})}{t_2(t_2 + 2K_{02} - t_1 K_{12})} \quad (3.2)$$

Comparison of (1.22) and (2.6) shows that the suggested estimator $(\hat{\bar{Y}}_T^{(d)})_I$ would be more efficient than usual two-phase sampling product estimator $\hat{\bar{Y}}_P^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_1(2K_{01} - t_1)}{(t_2 - 1)\{(1 + t_2 + 2K_{02}) - 2t_1 t_2 K_{12}\}} \quad (3.3)$$

Comparing (1.23) and (2.6) reveals that the suggested estimator $(\hat{\bar{Y}}_T^{(d)})_I$ is more efficient than the two-phase sampling version of Singh et al. (2004) ratio type estimator $\hat{\bar{Y}}_{SR}^{(d)}$ if

$$C_{x_2}^2 < 0 \quad \text{or} \quad K_{02} > \frac{2t_1 K_{12} - t_2}{2} \quad (3.4)$$

From (1.24) and (2.6) it is observed that the suggested estimator $(\hat{\bar{Y}}_T^{(d)})_I$ is more efficient than the two-phase sampling version of Singh et al. (2004) product type estimator $\hat{\bar{Y}}_{SP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{2K_{01} - t_1}{2t_2 K_{12}} \quad (3.5)$$

Comparison of (1.25) and (2.6) that the suggested estimator $(\hat{\bar{Y}}_T^{(d)})_I$ is more efficient than the two-phase sampling version of Singh (1967) ratio-cum-product type estimator $\hat{\bar{Y}}_{SRP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_1)(1+t_1-2K_{01})}{t_2(t_2+2K_{02}-t_1K_{12})-(1+2K_{02}-2K_{12})} \tag{3.6}$$

Expressions (3.1) to (3.6) are conditions for case I under which suggested estimator has less mean squared error than usual unbiased estimator $\bar{y}$, usual two-phase sampling ratio estimator $\bar{y}_R^{(d)}$, two-phase sampling product estimators $\bar{y}_P^{(d)}$, two-phase sampling versions of [9]Singh et al. (2004) estimators $\hat{\bar{Y}}_{SR}^{(d)}$ and $\hat{\bar{Y}}_{SP}^{(d)}$ respectively and the two-phase sampling version of [10]Singh (1967) estimator $\hat{\bar{Y}}_{SRP}^{(d)}$.

4. Efficiency Comparisons of $(\hat{\bar{Y}}_T^{(d)})_{II}$

From (1.26), (1.27), (1.28), (1.29), (1.30), (1.31) and (2.7) it is observed that the suggested estimator $\hat{\bar{Y}}_T^{(d)}$ under case II ($(\hat{\bar{Y}}_T^{(d)})_{II}$ **)**, would be more efficient than

(i) usual unbiased estimator $\bar{y}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_1\{2f_1K_{01}-t_1(f_1+f_2)\}}{t_2\{t_2(f_1+f_2)+2f_1K_{02}-2f_1t_1K_{12}\}} \tag{4.1}$$

(ii) two-phase sampling ratio estimator $\hat{\bar{Y}}_R^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_1)\{(1+t_1)(f_1+f_2)-2f_1K_{01}\}}{t_2\{t_2(f_1+f_2)+2f_1K_{02}-2f_1t_1K_{12}\}} \tag{4.2}$$

(iii) two-phase sampling product estimator $\hat{\bar{Y}}_P^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{t_1\{2f_1K_{01}-t_1(f_1+f_2)\}}{[(t_2-1)\{(f_1+f_2)(t_2+1)+2f_1K_{02}\}]-2f_1t_1t_2K_{12}} \tag{4.3}$$

(iv) Singh et al's (2004) two-phase sampling ratio type estimator $\hat{\bar{Y}}_{SR}^{(d)}$ if

$$C_{x_2}^2 < 0 \quad \text{or} \quad K_{02} > \frac{2t_1K_{12}-(f_1+f_2)t_2}{2} \tag{4.4}$$

(v) Singh et al's (2004) two-phase sampling product type estimator $\hat{\bar{Y}}_{SP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{2f_1K_{01}-t_1(f_1+f_2)}{2f_1t_2K_{12}} \tag{4.5}$$

(vi) Singh (1967) two-phase sampling ratio-cum-product type estimator $\hat{\bar{Y}}_{SRP}^{(d)}$ if

$$\frac{C_{x_2}^2}{C_{x_1}^2} < \frac{(1-t_1)\{(1+t_1)(f_1+f_2)-2f_1K_{01})\}}{t_2\{t_2(f_1+f_2)+2f_1K_{02}-2f_1t_1K_{12}\}\{(f_1+f_2)-2(K_{02}-K_{12}\}} \tag{4.6}$$

Expressions (4.1) to (4.6) are the conditions in which proposed estimator $\hat{\hat{Y}}_T^{(d)}$ in case II would be more efficient than simple mean estimator $\bar{y}$, usual two-phase sampling ratio and product estimators $\bar{y}_R^{(d)}$ and $\bar{y}_P^{(d)}$, two-phase sampling versions of estimators suggested by [9]Singh et al. (2004) ($\hat{\hat{Y}}_{SR}^{(d)}$ and $\hat{\hat{Y}}_{SP}^{(d)}$) and [10]Singh (1967) two-phase sampling ratio-product type estimator $\hat{\hat{Y}}_{SRP}^{(d)}$ respectively.

5. Empirical Study

To analyze the performance of the proposed estimator of population mean $\bar{Y}$ in two-phase sampling in comparison to other estimators, one natural population data set is being considered. We have computed Percent relative efficiencies (PREs) of $\hat{\hat{Y}}_R^{(d)}$, $\hat{\hat{Y}}_P^{(d)}$, $\hat{\hat{Y}}_{SR}^{(d)}$, $\hat{\hat{Y}}_{SP}^{(d)}$ and $\hat{\hat{Y}}_{SRP}^{(d)}$, $(\hat{\hat{Y}}_T^{(d)})_I$ and $(\hat{\hat{Y}}_T^{(d)})_{II}$ with respect to $\bar{y}$. The description of the population is given below.

Population: [Source: Steel and Torrie (1960, p.282)]

y = Log of leaf burn in seconds,

x_1 = Potassium percentage,

x_2 = Chlorine percentage,

N=30,	n=6,	n'=14,	
$\bar{Y}$ = 0.6860,	C_y = 0.4803,	$\beta_2(x_1)$ =1.56,	ρ_{01} = 0.1794,
$\bar{X}_1$ = 4.6537,	C_{x_1} = 0.2295,	$\beta_2(x_2)$ =1.40,	ρ_{02} = -0.4996,
$\bar{X}_2$ = 0.8077,	C_{x_2} = 0.7493,	and	ρ_{12} = 0.4074.

Table - 5.1: Percent relative efficiencies of $\hat{\hat{Y}}_R^{(d)}$, $\hat{\hat{Y}}_P^{(d)}$, $\hat{\hat{Y}}_{SR}^{(d)}$, $\hat{\hat{Y}}_{SP}^{(d)}$, $\hat{\hat{Y}}_{SRP}^{(d)}$ and $\hat{\hat{Y}}_T^{(d)}$ or $(\hat{\hat{Y}}_T^{(d)})_I$ (Under case-I) with respect to $\bar{y}$

	Estimator						
	$\bar{y}$	$\hat{\hat{Y}}_R^{(d)}$	$\hat{\hat{Y}}_P^{(d)}$	$\hat{\hat{Y}}_{SR}^{(d)}$	$\hat{\hat{Y}}_{SP}^{(d)}$	$\hat{\hat{Y}}_{SRP}^{(d)}$	$\hat{\hat{Y}}_T^{(d)}$ or $(\hat{\hat{Y}}_T^{(d)})_I$
PRE	100.00	96.01	61.54	100.02	121.16	81.18	141.60

Table - 5.2: Percent relative efficiencies of $\hat{\bar{Y}}_R^{(d)}$, $\hat{\bar{Y}}_P^{(d)}$, $\hat{\bar{Y}}_{SR}^{(d)}$, $\hat{\bar{Y}}_{SP}^{(d)}$, $\hat{\bar{Y}}_{SRP}^{(d)}$ and $\hat{\bar{Y}}_T^{(d)}$ or $(\hat{\bar{Y}}_T^{(d)})_{II}$ (Under case-II) with respect to $\bar{y}$

	Estimator						
	$\bar{y}$	$\hat{\bar{Y}}_R^{(d)}$	$\hat{\bar{Y}}_P^{(d)}$	$\hat{\bar{Y}}_{SR}^{(d)}$	$\hat{\bar{Y}}_{SP}^{(d)}$	$\hat{\bar{Y}}_{SRP}^{(d)}$	$\hat{\bar{Y}}_T^{(d)}$ or $(\hat{\bar{Y}}_T^{(d)})_{II}$
PRE	100.00	89.12	73.92	96.50	117.85	85.26	139.2

Conclusion

Table 5.1 and 5.2 exhibits that the suggested estimators $(\hat{\bar{Y}}_T^{(d)})_I$ and $(\hat{\bar{Y}}_T^{(d)})_{II}$ [Under case I and II] are more efficient than the usual unbiased estimator $\bar{y}$, ratio estimator in two-phase sampling $\bar{y}_R^{(d)}$, two-phase sampling product estimator $\bar{y}_P^{(d)}$, two-phase sampling versions of Singh et al. (2004) ratio and product type estimators $\hat{\bar{Y}}_{SR}^{(d)}$ and $\hat{\bar{Y}}_{SP}^{(d)}$ respectively, and two-phase sampling version of Singh (1967) estimator.

Larger gain in efficiency is observed by using proposed estimators over other estimators. It is also observed that $(\hat{\bar{Y}}_T^{(d)})_I$ **(**When samples taken as a sub sample from the first phase sample) is giving better result as compare to $(\hat{\bar{Y}}_T^{(d)})_{II}$ (When samples drawn independently to the first phase sample). Therefore suggested estimators $(\hat{\bar{Y}}_T^{(d)})_I$ and $(\hat{\bar{Y}}_T^{(d)})_{II}$ may be recommended for use in practice.

Conflict of Interest: None.

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