An Improved Estimator of Finite Population Mean Using Auxiliary Attribute(s) in Stratified Random Sampling Under Non-Response

Rajesh Singh
Department of Statistics
BHU, Varanasi (U.P.), India
rsinghstat@yahoo.com

Sachin Malik
Department of Statistics
BHU, Varanasi (U.P.), India
sachinkurava999@gmail.com

Abstract

In the present study, we propose a new estimator for population mean using Singh et al. (2007) and Malik and Singh (2013) estimators in the case of stratified random sampling when the information is available in form of attributes under non-response. Expressions for the mean squared error (MSE) of the proposed estimators are derived up to the first degree of approximation. The theoretical conditions have also been verified by a numerical example. It has been shown that the proposed estimators are more efficient than the traditional estimators.

Keywords: Stratified random sampling, Combined exponential ratio type estimator, bias, Mean Square error, Auxiliary attribute, Efficiency.

1. Introduction

The problem of estimating the population mean in the presence of an auxiliary variable has been widely discussed in finite population sampling literature. Diana (1993) suggested a class of estimators of the population mean using one auxiliary variable in the stratified random sampling and examined the MSE of the estimators up to the kth order of approximation. Kadilar and Cingi (2003), Singh and Vishwakarma (2005), Singh et al. (2009), Koyuncu and Kadilar (2008, 2009) proposed estimators in stratified random sampling. Singh (1965) and Perri (2007) suggested some ratio cum product estimators in simple random sampling. Bahl and Tuteja (1991) and Singh et al. (2007) suggested some exponential ratio type estimators. There are some situations when in place of one auxiliary attribute, we have information on two qualitative variables. For illustration, to estimate the hourly wages we can use the information on marital status and region of residence (see Gujrati and Sangeeta (2007). Here we assume that both auxiliary attributes have significant point bi-serial correlation with the study variable and there is significant phi-correlation (see Yule (1912)) between the two auxiliary attributes.

In surveys covering human populations, information in most cases is not obtained from all the units in the survey even after some call-backs. An estimate obtained from such incomplete data may be misleading especially when the respondents differ from the non-respondents because the estimate can be biased. Hansen and Hurwitz (1946) envisaged a simple technique of sub-sampling the non-respondents in order to adjust for the non-response in a mail survey. In estimating population parameters like the mean, total or ratio, sample survey experts sometimes use auxiliary information to improve precision of the estimates.

Consider a finite population of size N and is divided into L strata such that $\sum_{h=1}^{L} N_h = N$ where N_h is the size of h^{th} stratum (h=1,2,...,L). We select a sample of size n_h from each stratum by simple random sample without replacement sampling such that $\sum_{h=1}^{L} n_h = n$, where n_h is the stratum sample size. Let (y_{hi}, ψ_{jhi}) (j=1,2) denote the values of the study variable (*y*) and the auxiliary attributes ψ_j, (j=1,2) respectively in the h^{th} stratum. Suppose that n_{h1} units will respond and n_{h2} will not respond such that $n_{h1} + n_{h2} = n_h$. We select a sub-sample of size $r_h = \frac{n_{h2}}{k_h}(k_h \geq 1)$ from n_{h2} units and it is assumed that all the selected units will respond.

Suppose

$$\psi_{jhi} = \begin{cases} 1, \text{ if } i^{th} \text{ unit in the stratum h possesses the attribute } \psi_j \\ 0, \text{ otherwise} \end{cases}$$

Let $P_j = \sum_{h=1}^{L} W_h P_{jh}$ and $p_{jst} = \sum_{h=1}^{L} W_h p_{jh}$ (j=1, 2) are the population and sample proportions of units of the auxiliary attributes where $P_{jh} = \frac{A_{jh}}{N_h}$, $p_{jh} = \frac{a_{jh}}{n_h}$ and $A_{jh} = \sum_{i=1}^{N_h} \psi_{jhi}$ and $a_{jh} = \sum_{i=1}^{n_h} \psi_{jhi}$.

Let $S_{yh}^2 = \frac{\sum_{i=1}^{N_h}(y_{hi} - \overline{Y}_h)^2}{N_h - 1}$ and $S_{\psi jh}^2 = \frac{\sum_{i=1}^{N_h}(\psi_{jhi} - P_{jh})^2}{N_h - 1}$ be the population variances of the study variable and the auxiliary attributes in the h^{th} stratum. Where,

$$\overline{Y}_h = \sum_{i=1}^{N_h} \frac{y_{hi}}{N_h},\ S_{y\psi_{jh}} = \sum \frac{(y_{hi} - \overline{Y}_h)(\psi_{jhi} - P_{jh})}{N_h - 1} \quad \text{and } \rho_{y\psi_{jh}} = \frac{S_{y\psi_{jh}}}{S_{\psi_{jh}} S_{yh}}$$

be the population bi-covariance and point bi-serial correlation between the study variable and the auxiliary attributes respectively in the h^{th} stratum. Also $S_{\psi_1\psi_2 h} = \sum_{i=1}^{N_h} \frac{(\psi_{1hi} - P_{1h})(\psi_{2hi} - P_{2h})}{N_h - 1}$ and $\rho_{\psi_1\psi_2 h} = \frac{S_{\psi_1\psi_2 h}}{S_{\psi_{1h}} S_{\psi_{2h}}}$ be the population bi-covariance and phi-correlation coefficient between the auxiliary attributes in the h^{th} stratum respectively.

To estimate $\overline{Y} = \sum_{h=1}^{L} W_h \overline{Y}_h$, we assume that P_{jh} (j=1, 2) are known.

Now adapting the Hansen-Hurwitz (1946) methodology, an unbiased estimator of population mean $\overline{Y}$ is given by $\overline{y}_{st}^{*} = \sum_{h=1}^{L} W_h \overline{y}_h^{*}$, where $W_h = \frac{N_h}{N}$ is known as stratum weight and $\overline{y}_h^{*} = \frac{n_{h1}\overline{y}_{1h} + n_{h2}\overline{y}_{2h}}{n_h}$. Here, $\overline{y}_{1h}$ and $\overline{y}_{2h}$ are the sample means of n_{h1} responding units n_{h2} sub-sampled units respectively.

The variance of the estimator $\overline{y}_{st}^{*}$ is given as

$$var\left(\overline{y}_{st}^{*}\right) = \sum_{h=1}^{L} W_h^2 f_h S_{yh}^2 + \sum_{h=1}^{L} W_h^2 \frac{(k_h - 1)}{n_h} W_{h2} S_{yh2}^2$$

Where, $W_{h2} = \frac{N_{h2}}{N_h}$ is the known non-response rate in the h^{th} stratum and $f_h = \frac{1}{n_h} - \frac{1}{N_h}$.

2. Estimators in literature

In order to have an estimate of the study variable y, assuming the knowledge of the population proportion P, Naik and Gupta (1996) and Singh et al. (2007) respectively proposed following estimators

$$t_1 = \overline{y}_{st}^{*}\left(\frac{P_1}{p_{1st}}\right) \tag{2.1}$$

$$t_2 = \overline{y}_{st}^{*} \exp\left(\frac{P_1 - p_{1st}}{P_1 + p_{1st}}\right) \tag{2.2}$$

where,

$$P_1 = \sum_{h=1}^{L} W_h P_{1h} \text{ and } p_{1st} = \sum_{h=1}^{L} W_h p_{1h}.$$

The Bias and MSE expressions of the estimator's t_i (i=1, 2) up to the first order of approximation are, respectively, given by

$$B(t_1) = \frac{1}{P_1}\sum_{h=1}^{L} W_h^2 f_h \left(R_1 S_{\psi_{1h}}^2 - \rho_{y\psi_{1h}} S_{yh} S_{\psi_{1h}}\right) \tag{2.3}$$

$$B(t_2)=\frac{1}{2P_1}\sum_{h=1}^{L}W_h^2 f_h\left(\frac{3}{4}S_{\psi_{1h}}^2-\rho_{y\psi_{1h}}S_{yh}S_{\psi_{1h}}\right) \tag{2.4}$$

$$MSE(t_1)=\sum_{h=1}^{L}W_h^2 f_h\left[S_{yh}^2+R_1^2S_{\psi_{1h}}^2-2R_1\rho_{y\psi_{1h}}S_{yh}S_{\psi_{1h}}\right]+\sum_{h=1}^{L}W_h^2\frac{(k_h-1)}{n_h}W_{h2}S_{yh2}^2 \tag{2.5}$$

$$MSE(t_2)=\sum_{h=1}^{L}W_h^2 f_h\left[S_{yh}^2+\frac{R_1^2}{4}S_{\psi_{1h}}^2-R_1\rho_{y\psi_{1h}}S_{yh}S_{\psi_{1h}}\right]+\sum_{h=1}^{L}W_h^2\frac{(k_h-1)}{n_h}W_{h2}S_{yh2}^2 \tag{2.6}$$

where, $R_1=\frac{\bar{Y}}{P_1}$

Following Bahl and Tuteja (1991), Shagir and Shabbir (2012) proposed a ratio type exponential estimator as

$$t_3=\bar{y}_{st}^{*}\exp\left(\frac{P_1-p_{1st}}{P_1+(a-1)p_{1st}}\right)\exp\left(\frac{P_2-p_{2st}}{P_2+(b-1)p_{2st}}\right) \tag{2.7}$$

The Bias and MSE expressions of the estimators t_3 up to the first order of approximation is

$$B(t_3)=\sum_{h=1}^{L}W_h^2 f_h\left(\frac{(2a-1)}{2a^2}\frac{S_{\psi_{1h}}^2}{P_1^2}+\frac{(2b-1)}{2b^2}\frac{S_{\psi_{2h}}^2}{P_2^2}-\frac{\rho_{y\psi_{1h}}S_{yh}S_{\psi_{1h}}}{a\bar{Y}P_1}-\frac{\rho_{y\psi_{2h}}S_{yh}S_{\psi_{2h}}}{b\bar{Y}P_2}+\frac{\rho_{\psi_1\psi_2h}S_{\psi_{1h}}S_{\psi_{2h}}}{abP_1P_2}\right) \tag{2.8}$$

and

$$MSE(t_3)=\sum_{h=1}^{L}W_h^2 f_h\left[S_{yh}^2+\frac{R_1^2}{a^2}S_{\psi_{1h}}^2+\frac{R_2^2}{b^2}S_{\psi_{21h}}^2-2\frac{R_1}{a}\rho_{y\psi_{1h}}S_{yh}S_{\psi_{1h}}-2\frac{R_1}{b}\rho_{y\psi_{2h}}S_{yh}S_{\psi_{2h}}\right.$$
$$\left.+2\frac{R_1R_2}{ab}\rho_{\psi_1\psi_2h}S_{\psi_{1h}}S_{\psi_{2h}}\right]+\sum_{h=1}^{L}W_h^2\frac{(k_h-1)}{n_h}W_{h2}S_{yh2}^2 \tag{2.9}$$

where, $R_2=\frac{\bar{Y}}{P_2}$

3. The proposed estimator

Following Malik and Singh (2013), we propose an estimator as

$$t_4=\bar{y}_{st}^{*}\exp\left(\frac{P_1-p_{1st}}{P_1+p_{1st}}\right)^{\alpha_1}\exp\left(\frac{P_2-p_{2st}}{P_2+p_{2st}}\right)^{\alpha_2}+b_1(P_1-p_{1st})+b_2(P_2-p_{2st}) \tag{3.1}$$

where, α_1 and α_2 are real constants.

To obtain the bias and MSE of t_4 to the first degree of approximation, we define

$$e_0=\frac{\bar{y}_{st}^{*}-\bar{Y}}{\bar{Y}},\quad e_1=\frac{p_{1st}-P_1}{P_1},\quad e_2=\frac{p_{2st}-P_2}{P_2}$$

Such that, $E(e_i)=0;\quad i=0,1,2.$

And $E\left(e_0^2\right)=\dfrac{\sum_{h=1}^{L}W_h^2 f_h S_{yh}^2}{\bar{Y}^2},\quad E\left(e_1^2\right)=\dfrac{\sum_{h=1}^{L}W_h^2 f_h S_{\psi_1 h}^2}{P_1^2},\ E\left(e_2^2\right)=\dfrac{\sum_{h=1}^{L}W_h^2 f_h S_{\psi_2 h}^2}{P_2^2},$

$$E\left(e_0 e_1\right)=\frac{\sum_{h=1}^{L}W_h^2 f_h S_{y\psi_1 h}^2}{\bar{Y}P_1},\qquad E\left(e_0 e_2\right)=\frac{\sum_{h=1}^{L}W_h^2 f_h S_{y\psi_2 h}^2}{\bar{Y}P_2},\ \textit{and}\ \ E\left(e_1 e_2\right)=\frac{\sum_{h=1}^{L}W_h^2 f_h S_{\psi_1\psi_2 h}^2}{P_1 P_2}$$

Expressing equation (3.1) in terms of e's, we have

$$t_4=\bar{Y}\left(1+e_0\right)\left(\exp\left(\frac{-e_1}{2+e_1}\right)^{\alpha_1}\exp\left(\frac{-e_2}{2+e_2}\right)^{\alpha_2}\right)-b_1 e_1 P_1-b_2 e_2 P_2$$

$$=\bar{Y}\left[1+e_0-\frac{\alpha_1 e_1}{2}+\frac{\alpha_1^2 e_1^2}{4}-\frac{\alpha_2 e_2}{2}-\frac{\alpha_1\alpha_2 e_1 e_2}{4}+\frac{\alpha_2^2 e_2^2}{4}-\frac{\alpha_2 e_0 e_2}{2}-\frac{\alpha_1 e_0 e_1}{2}\right]-b_1 e_1 P_1-b_2 e_2 P_2 \tag{3.2}$$

Retaining the term's up to single power of e's in (3.2) we have

$$t_4-\bar{Y}=\left\{\bar{Y}\left[e_0-\frac{\alpha_1 e_1}{2}-\frac{\alpha_2 e_2}{2}\right]-b_1 e_1 P_1-b_2 e_2 P_2\right\} \tag{3.3}$$

Squaring both sides of (3.3) and then taking expectations, we get the MSE of the estimator t_4 up to the first order of approximation, as

$$\begin{aligned}MSE(t_4)=\sum_{h=1}^{L}W_h^2 f_h\Bigg\{\Bigg[&S_{yh}^2+\frac{\alpha_1^2 R_1^2 S_{\psi_1 h}^2}{4}+\frac{\alpha_2^2 R_2^2 S_{\psi_2 h}^2}{4}+\frac{\alpha_1\alpha_2 R_1 R_2 \rho_{\psi_1\psi_2 h} S_{\psi_{1h}} S_{\psi_{2h}}}{2}-\alpha_1 R_1 \rho_{y\psi_{1h}} S_{yh} S_{\psi_{1h}}\\ &-\alpha_2 R_2 \rho_{y\psi_{2h}} S_{yh} S_{\psi_{2h}}\Bigg]+B_1^2 S_{\psi_{1h}}^2+B_2^2 S_{\psi_{2h}}^2+2B_1 B_2 \rho_{\psi_1\psi_2 h} S_{\psi_{1h}} S_{\psi_{2h}}\\ &-2\Bigg[B_1 R_1 S_{\psi_{1h}}^2+B_2 R_2 S_{\psi_{2h}}^2\\ &-\frac{\alpha_1 B_1 R_1 S_{\psi_{1h}}^2}{2}-\frac{\alpha_2 B_2 R_2 S_{\psi_{2h}}^2}{2}-\frac{\alpha_1 B_2 R_2 S_{\psi_{2h}}^2}{2}-\frac{\alpha_2 B_1 R_1 S_{\psi_{2h}}^2}{2}\Bigg]\Bigg\}\\ &+\sum_{h=1}^{L}W_h^2\frac{\left(k_h-1\right)}{n_h}W_{h2}S_{yh2}^2\end{aligned} \tag{3.4}$$

Where, $B_1=\dfrac{\sum_{h=1}^{L}W_h^2 f_h \rho_{y\psi 1h} S_{yh} S_{\psi 1h}}{\sum_{h=1}^{L}W_h^2 f_h S_{\psi_1}^2}$ and $B_2=\dfrac{\sum_{h=1}^{L}W_h^2 f_h \rho_{y\psi 2h} S_{yh} S_{\psi 2h}}{\sum_{h=1}^{L}W_h^2 f_h S_{\psi_2}^2}$

Minimising equation (3.4) with respect to α_1 and α_2 we get the optimum values as

$$\alpha_1 = \frac{2R_2A_3A_4 - 4R_1A_2A_5}{R_1^2R_2\left[A_1A_4 - 4A_2^2\right]} \text{ and } \alpha_2 = \frac{2R_1A_1A_5 - 4R_2A_2A_3}{R_1R_2^2\left[A_1A_4 - 4A_2^2\right]}$$

where,

$$\left.\begin{aligned}
A_1 &= \sum_{h=1}^{L} W_h^2 f_h S_{yh}^2 \\
A_2 &= \sum_{h=1}^{L} W_h^2 f_h \rho_{\psi_1\psi_2 h} S_{\psi_{1h}} S_{\psi_{2h}} \\
A_3 &= \sum_{h=1}^{L} W_h^2 f_h \left[R_1 \rho_{y\psi_{1h}} S_{yh} S\psi_{1h} + B_1 R_1 S_{\psi 1h}^2 + B_2 R_2 S_{\psi 2h}^2\right] \\
A_4 &= \sum_{h=1}^{L} W_h^2 f_h S_{\psi 2h}^2 \text{and} \\
A_5 &= \sum_{h=1}^{L} W_h^2 f_h \left[R_2 \rho_{\psi_1\psi_2 h} S_{\psi_{1h}} S_{\psi_{2h}} + B_1 R_1 S_{\psi 2h}^2 + B_2 R_2 S_{\psi 2h}^2\right]
\end{aligned}\right\}$$

4. Empirical study

Source: [Murthy (1967)]

We randomly select a sample of size n_h from each stratum by using the Neyman allocation and consider the first 10%, 20% and 30% values in each stratum as non-response for W_{h2} =0.1, W_{h2} =0.2 and W_{h2} =0.3 respectively.

The population consists of village wise complete enumeration and data are obtained in 1951 and 1961 censuses for a Tehsil. The area of village is used to stratify the population into three strata.

Let *y* be the cultivated area in the village in hectares in 1951 and ψ_{jhi} (j=1,2) are given below

Let

$$\psi_{11i} = \begin{cases} 1, \text{if } i^{th} \text{village in the stratum 1 has area greater than 550 hectares} \\ 0, \text{otherwise} \end{cases}$$

$$\psi_{12i} = \begin{cases} 1, \text{if } i^{th} \text{ village in the stratum 2 has area greater than 1300 hectares} \\ 0, \text{otherwise} \end{cases}$$

$$\psi_{13i} = \begin{cases} 1, \text{if } i^{th} \text{village in the stratum 3 has area greater than 2500 hectares} \\ 0, \text{otherwise} \end{cases}$$

Let

$$\psi_{21i} = \begin{cases} 1, \text{if } i^{th} \text{ village in the stratum1has number of cultivators greater than 550} \\ 0, \text{otherwise} \end{cases}$$

$$\psi_{22i} = \begin{cases} 1, \text{if } i^{th} \text{village in the stratum2has number of cultivators greater than 700} \\ 0, \text{otherwise} \end{cases}$$

and

$$\psi_{22i} = \begin{cases} 1, \text{if } i^{th} \text{ village in the stratum3has number of cultivators greater than 1500} \\ 0, \text{otherwise} \end{cases}$$

Data are presented in Table 4.1 and results are given in Table 4.2.

Table 4.1: Data Statistics

Stratum no.	1	2	3
N_h	43	45	40
n_h	10	12	18
$\overline{Y}_h$	397.1425	760.1795	1234.6180
P_{1h}	0.5814	0.4444	0.4000
P_{2h}	0.39535	0.5333	0.4000
S^2_{yh}	39975.0569	61455.990	172425.900
$S^2_{\psi_{1h}}$	0.2492	0.2525	0.2462
$S^2_{\psi_{2h}}$	0.2448	0.2545	0.2462
$\rho_{y\psi_{1h}}$	0.6922	0.3750	0.5057
$\rho_{y\psi_{2h}}$	0.3956	0.2847	0.3261
$\rho_{\psi_1\psi_2 h}$	0.2040	0.3884	0.5833
S^2_{yh2} , For W_{h2}=0.1	16871.6298	5534.3610	174694.200
For W_{h2}=0.2	10951.1712	43003.2000	123887.767
For W_{h2}=0.3	15878.3587	49346.8000	152450.527

We compute the percent relative efficiency (PRE) of different estimators for different values of W_{h2} and k_h and use the given expression

$$PRE = \frac{var\left(\overline{y}^*_{st}\right)}{MSE\left(t_i\right)} x100 \text{ , (i=1, 2, 3, 4)}$$

Table 4.2: Percentage relative efficiency of different estimators with respect to $\bar{y}_{st}^{*}$

W_{h2}	k_h	$PRE(t_1)$	$PRE(t_2)$	$PRE(t_3)$	$PRE(t_4)$
0.1	2.0	13.95	54.93	129.83	449.35
	2.5	14.45	55.93	128.31	435.30
	3.0	14.94	56.89	126.94	422.10
	3.5	15.42	57.80	125.69	409.00
0.2	2.0	15.01	57.02	126.75	422.10
	2.5	15.10	58.87	124.32	397.95
	3.0	16.97	60.57	122.30	376.43
	3.5	17.92	62.13	120.58	357.10
0.3	2.0	16.65	60.02	122.93	397.95
	2.5	18.39	62.86	119.82	366.51
	3.0	20.05	65.33	117.45	339.68
	3.5	21.65	67.49	115.59	316.50

From Table 4.2, we observe that the proposed estimator t_4 is more efficient as compared to the usual estimator $\bar{y}_{st}^{*}$, Shagir and Shabbir (2012) and the estimators t_1 and t_2. The efficiency of the estimators decreases with increase in the level of non-response in each stratum and k_h.

5. Conclusion

In this paper, we have proposed a combined exponential ratio type estimator t_4 using information on the auxiliary attribute(s) under non-response. Expressions for bias and MSE's of the proposed estimators are derived up to first degree of approximation. The proposed estimator is compared with usual mean estimator, Shagir and Shabbir (2012) estimator and estimators t_1 and t_2. A numerical study is carried out to support the theoretical results for different values of non-responses. The proposed estimator t_4 performed better than the usual sample mean estimator and estimators t_1, t_2 and t_3. The efficiency of the estimators decreases with increase rate of non-response W_{h2} and k_h.

References

1. Bahl, S. and Tuteja, R.K. (1991). Ratio and product type exponential estimator. *Journal of information and optimization sciences*, 12(1), 159-163.

2. Diana, G. (1993). A class of estimators of the population mean in stratified random sampling. *Statistica*, 53(1), 59–66.

3. Gujarati, D. N. and Sangeetha (2007). *Basic Econometrics*. Tata McGraw – Hill.

4. Hansen, M.H. and Hurwitz, W.N. (1946). The problem of non-response in sample surveys. *J. Am. Stat. Assoc.*, 41,517–529.

5. Kadilar, C. and Cingi, H. (2003). Ratio Estimators in Stratified Random Sampling. *Biometrical Journal*, 45 (2), 218-225.

6. Kadilar, C. and Cingi, H. (2005). A new estimator using two auxiliary variables. *Applied math and computation*, 162, 901–908.

7. Koyuncu, N. and Kadilar, C. (2008). Ratio and product estimators in stratified random sampling. *J. Statist. Plann. Inference*, 139(8), 2552-2558.

8. Koyuncu, N. and Kadilar, C. (2009). Family of estimators of population mean using two auxiliary variables in stratified random sampling. *Comm. In Stat. – Theory and Meth.*, 38(14), 2398-2417.

9. Malik S. and Singh R. (2013). An improved estimator using two auxiliary attributes. *Applied mathematics and computation.* 219, 10983-10986.

10. Murthy, M.N. (1967). *Sampling Theory and Methods.* Statistical Publishing Society, Calcutta.

11. Naik, V.D., Gupta, P.C. (1996). A note on estimation of mean with known population proportion of an auxiliary character. *Journal of the Indian Society of Agricultural Statistics*, 48(2), 151-158.

12. Perri, P. F. (2007). Improved ratio-cum-product type estimators. *Statist. Trans.* 8:51–69.

13. Shagir A. and Shabbir J. (2012). Estimation of Finite Population Mean in Stratified Random Sampling Using Auxiliary Attribute(s) under Non-Response. *Pak. j. of stat. oper. Res*., 8(1), 73-82.

14. Singh, H. P.; Vishwakarma, G. K. (2005). Combined Ratio-Product Estimator of Finite Population Mean in Stratified Sampling. *Metodologia de Encuestas*, 8, 35-44.

15. Singh, M.P. (1965). On the estimation of ratio and product of the population parameters. *Sankhya, B*, 27, 321-328.

16. Singh, R., Kumar, M., Chaudhary, M. K. and Kadilar, C. (2009). Improved Exponential Estimator in Stratified Random Sampling. *Pakistan Journal of Statistics and Operation Research*, 5(2), 67-82.

17. Yule, G. U. (1912). On the methods of measuring association between two attributes. *Jour. of the Royal Soc*., 75, 579-642.