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Enhancing Food Security Analysis in South Sulawesi Using Robust Mixed Geographically and Temporally Weighted Regression with M-Estimator

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Abstract

MGTWR (Mixed Geographically and Temporally Weighted Regression) combines a global linear regression model with GTWR by incorporating spatial and temporal dimensions. However, it remains sensitive to outliers, which can reduce accuracy. To address this limitation, a robust regression approach with the M-Estimator was applied to model the food security index in South Sulawesi Province from 2018 to 2022. The resulting Robust MGTWR (RMGTWR) model demonstrated improved performance, with a lower AIC (414.9719) and a high explanatory power ($R^2 = 99.4815\%$). Key factors influencing food security include the ratio of normative consumption per capita to net production, the percentage of households with a proportion of expenditure on food more significant than 65% of total spending, the percentage of households without access to electricity, the percentage of households without access to clean water, and the percentage of stunting toddlers. These findings highlight the effectiveness of RMGTWR with M-Estimator in addressing data irregularities and provide valuable insights for policymakers in designing targeted strategies to strengthen food security in South Sulawesi Province.

Key Words: RMGTWR, M-Estimator, Outliers, Robust Regression, Food Security Index

1. Introduction

Spatial analysis is a statistical method for analyzing data that is based on geographic structure and utilizes a coordinate system in the form of latitude and longitude (Paramita et al., 2021). Analysis of spatial data requires special attention because the conditions of one observation location will be different from other observation locations so spatial heterogeneity will arise (Debataraja et al., 2021). To address this, Geographically Weighted Regression (GWR) was introduced as a method that accommodates spatial heterogeneity by producing location-specific parameters (Fotheringham et al., 2002). The GWR model is a regression analysis that considers spatial elements so that its parameters only apply to certain locations and differ from others (Lee et al., 2024). However, GWR accounts only for spatial variation and neglects the temporal dimension, which is equally important in understanding social and environmental processes (Ma et al., 2018; Fotheringham et al. 2015). As a result, the GWR model was expanded into the Geographically and Temporally Weighted Regression (GTWR) model. Including the time component in GWR will result in a more representative model for each location and time (Huang et al., 2010).

The GTWR model generates parameters specific to each location, but not every variable in the model affects the response spatially. Some variables exert a global influence, while others retain their spatial impact. As a result, the GTWR model was further advanced into the Mixed Geographically and Temporally Weighted Regression (MGTWR) model, which combines a global linear regression model with the GTWR model (Djuraidah et al., 2021). However, according to Asianingrum et al. (2020), the MGTWR model is susceptible to outliers, resulting in bias and inaccuracy in the regression relationship. This research gap highlights the need for a more robust approach.

To overcome the occurrence of outliers in the spatial regression model, a robust regression method was developed to produce model estimates resistant to outliers (Rahman and Widodo, 2018). One estimation method in robust regression is the M-Estimator, which is applied to minimize the objective function of the residual (Begashaw and Yohannes, 2020). This estimation method is highly efficient in handling outliers and is the most straightforward approach in terms of computation and theory (Huber and Ronchetti, 1981). Integrating this approach into MGTWR yields the Robust Mixed Geographically and Temporally Weighted Regression (RMGTWR) model, which combines spatial–temporal flexibility with robustness against outliers. The estimation of the RMGTWR model is performed through an iterative procedure, initialized with the MGTWR parameter estimates, and iterated until convergence is reached using the M-Estimator.

The RMGTWR method is used and applied to the food security index data in South Sulawesi Province in 2018–2022. The food security index is a measurement tool used to evaluate a region's food security level. The value of the food security index in each district or city in South Sulawesi Province varies, and the factors that influence the food security index also vary depending on the characteristics of each region. However, some factors may have a global influence on all observation locations. Hence, with the integration of robust estimation, RMGTWR is expected to yield more accurate and reliable insights into the determinants of food security across spatial and temporal dimensions in South Sulawesi Province for the period 2018–2022.

2. Materials and Methods

2.1 Data

This study utilizes secondary data acquired from the National Food Agency publication. The observation unit used is the district or city level in South Sulawesi Province, which consists of 24 districts or cities in the 2018–2022 period. The variables employed are detailed in Table 1.

Table 1: Data Identification.

Variables	Descriptions	Units
Y	Food security index	Index
X_1	The ratio of normative consumption per capita to net production	Ratio
X_2	The percentage of the population living below the poverty line	%
X_3	The percentage of households with a proportion of food expenditure of more than 65% of total expenditure	%
X_4	The percentage of households without access to electricity	%
X_5	The percentage of households without access to clean water	%
X_6	Life expectancy at birth	Year
X_7	The percentage of stunting toddlers	%

2.2 Heterogeneity Test

Spatial heterogeneity is a condition that occurs when the same predictor variable produces varying responses at different locations within a single research area (Caraka and Yasin, 2017). Thus, spatial heterogeneity can produce different parameters in each observation location. According to Anselin (2013), testing for spatial heterogeneity is conducted using the Breusch-Pagan (BP) test with the following hypotheses:
$H_0 : \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_n^2 = \sigma^2$ (there is no spatial heterogeneity present)
H_1 : There was at least one $\sigma_i^2 \neq \sigma_j^2$ for $i \neq j$ (there is spatial heterogeneity)
The value of the BP test is calculated using Equation (1).

$$BP = \frac{1}{2}\mathbf{f}^{\mathbf{T}}\mathbf{Z}(\mathbf{Z}^{\mathbf{T}}\mathbf{Z})^{-1}\mathbf{Z}^{\mathbf{T}}\mathbf{f} \tag{1}$$

When vector f, the element, is:

$$f_i = \left(\frac{e_i^2}{\sigma^2} - 1\right)$$

e_i^2 is the residual for the i^{th} observation from the regression estimation using OLS, and Z is a matrix of $n \times (p+1)$ predictor variables. Decision-making in the BP test is if the BP value $> \chi^2_{(\alpha,p)}$ or $p - value < \alpha$ then H_0 is rejected, which means spatial heterogeneity occurs.

2.3 Mixed Geographically and Temporally Weighted Regression

The MGTWR model combines the global linear regression model with the GTWR model. The MGTWR model is composed of p predictor variables with global effects and q predictor variables with local effects, assuming the model's intercept is local. Liu et al. (2017) mathematically expressed the MGTWR model as Equation (2).

$$y_i = \beta_0(u_i, v_i, t_i) + \sum_{k=1}^{q} \beta_l(u_i, v_i, t_i)\, x_{ik} + \sum_{k=q+1}^{p} \beta_g\, x_{ik} + \varepsilon_i \tag{2}$$

with $\beta_0(u_i, v_i, t_i)$ represents the intercept at the observation location (u_i, v_i) and time t_i, β_g denotes the global regression coefficient, $\beta_l(u_i, v_i, t_i)$ represents the local regression coefficient for the k^{th} predictor variable at the i^{th} observation location and time t_i, ε_i is the residual observation at the i^{th} location. Fotheringham et al. (2002) state that there are two estimation processes involved in estimating the MGTWR model parameters: $\boldsymbol{\beta}_g$ dan $\boldsymbol{\beta}_l(u_i, v_i, t_i)$ estimation, which is represented in Equations (3) and (4).

$$\widehat{\boldsymbol{\beta}}_g = \left[\boldsymbol{X}_g^T(\boldsymbol{I} - \boldsymbol{S}_l)^T(\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{X}_g\right]^{-1}\boldsymbol{X}_g^T(\boldsymbol{I} - \boldsymbol{S}_l)^T(\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{Y} \tag{3}$$

$$\widehat{\boldsymbol{\beta}}_l(u_i, v_i, t_i) = [\boldsymbol{X}_l^T\boldsymbol{W}(u_i, v_i, t_i)\boldsymbol{X}_l]^{-1}\boldsymbol{X}_l^T\boldsymbol{W}(u_i, v_i, t_i)\left(\boldsymbol{Y} - \boldsymbol{X}_g\widehat{\boldsymbol{\beta}}_g\right) \tag{4}$$

where $\boldsymbol{W}(u_i, v_i, t_i) = diag(w_{i1}, w_{i2}, \dots, w_{in})$ and $W(u_i, v_i, t_i)$ is the weight matrix at observation (u_i, v_i) and time t_i, while n represents the number of observations. Huang et al. (2010) defined the spatial-temporal distance function for the MGTWR model in Equation (5).

$$\left(d_{ij}^{ST}\right)^2 = \lambda\left\{\left(u_i - u_j\right)^2 + \left(v_i - v_j\right)^2\right\} + \mu\left(t_i - t_j\right)^2 \tag{5}$$

where λ and μ serve as balancing parameters for the unit differences between location and time in spatial-temporal distance measurements. According to Liu et al. (2017), if τ represents the ratio of these parameters, defined as $\tau = \mu / \lambda$ with $\lambda \neq 0$, Equation (6) will be obtained.

$$\frac{\left(d_{ij}^{ST}\right)^2}{\lambda} = \left(u_i - u_j\right)^2 + \left(v_i - v_j\right)^2 + \tau\left(t_i - t_j\right)^2 \tag{6}$$

The parameters λ, μ, and τ are determined by optimizing the coefficient of determination using a Cross-Validation (CV) procedure based on the formula in Equation (7).

$$CV = \sum_{i=1}^{n}(y_{=i} - \hat{y}_{\neq i})^2 \tag{7}$$

2.4 Robust Mixed Geographically and Temporally Weighted Regression

The RMGTWR model is obtained by multiplying the robust weights with the spatial-temporal weights obtained in MGTWR modeling. The RMGTWR model also consists of global parameters and local parameters. The global parameters of the RMGTWR model containing outliers are shown in Equation (8).

$$\sum_{i=1}^{n}\rho(\boldsymbol{\varepsilon}) = \sum_{i=1}^{n}\rho\left((\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{Y} - (\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{X}_g\boldsymbol{\beta}_g\right) \tag{8}$$

Meanwhile, the local parameters of the RMGTWR model for the i^{th} data containing outliers are shown in Equation (9).

$$\textstyle\sum_{i=1}^{n}\rho(\boldsymbol{\varepsilon}) = \sum_{i=1}^{n}\rho\left(\widetilde{\boldsymbol{Y}} - \boldsymbol{X}_l\boldsymbol{\beta}_l(u_i, v_i, t_i)\right) \tag{9}$$

The robust regression process with the M-Estimator is carried out by minimizing the objective function of the residuals so that the estimation of the RMGTWR model parameters for global parameters is shown in Equation (10) and the local parameter estimator in Equation (11).

$$\widehat{\boldsymbol{\beta}}_g = \left[\boldsymbol{X}_g^T(\boldsymbol{I} - \boldsymbol{S}_l)^T\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{X}_g\right]^{-1}\boldsymbol{X}_g^T(\boldsymbol{I} - \boldsymbol{S}_l)^T\boldsymbol{B}(\boldsymbol{I} - \boldsymbol{S}_l)\boldsymbol{Y} \tag{10}$$

$$\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i) = [\boldsymbol{X}_l^T\boldsymbol{B}\boldsymbol{X}_l]^{-1}\boldsymbol{X}_l^T\boldsymbol{B}\widetilde{\boldsymbol{Y}} \tag{11}$$

where $\boldsymbol{B}$ is the initial weight matrix of size $n \times n$, with diagonal elements holding the weights $b_{i1}, b_{i2}, \dots, b_{in}$ in using the Tukey weight function. In addition, the estimation of model parameters is completed using Iteratively Reweighted Least Squares (IRLS). During this iteration, the value of b_i will change with each step, resulting in the calculation $\widehat{\boldsymbol{\beta}}_g^1, \widehat{\boldsymbol{\beta}}_g^2, \dots, \widehat{\boldsymbol{\beta}}_g^m$ and $\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^1, \widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^2, \dots, \widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^m$, where m is the number of iterations. Therefore, Equations (12) and (13) estimate the global and local parameters for m iterations.

$$\widehat{\boldsymbol{\beta}}_g^m = \left[\boldsymbol{X}_g^T(\boldsymbol{I}-\boldsymbol{S}_l^{m-1})^T\boldsymbol{B}^{m-1}(\boldsymbol{I}-\boldsymbol{S}_l^{m-1})\boldsymbol{X}_g\right]^{-1}\boldsymbol{X}_g^T(\boldsymbol{I}-\boldsymbol{S}_l^{m-1})^T\boldsymbol{B}^1(\boldsymbol{I}-\boldsymbol{S}_l^{m-1})\boldsymbol{Y} \tag{12}$$

$$\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^m = [\boldsymbol{X}_l^T\boldsymbol{B}^{m-1}\boldsymbol{X}_l]^{-1}\boldsymbol{X}_l^T\boldsymbol{B}^{m-1}\left(\boldsymbol{Y}-\boldsymbol{X}_g\widehat{\boldsymbol{\beta}}_g^{m-1}\right) \tag{13}$$

Then the weight $\boldsymbol{B}^m$ is recalculated, and the global and local parameter estimates for the next iteration are in Equations (14) and (15).

$$\widehat{\boldsymbol{\beta}}_g^{m+1} = \left[\boldsymbol{X}_g^T(\boldsymbol{I}-\boldsymbol{S}_l^{m})^T\boldsymbol{B}^{m}(\boldsymbol{I}-\boldsymbol{S}_l^{m})\boldsymbol{X}_g\right]^{-1}\boldsymbol{X}_g^T(\boldsymbol{I}-\boldsymbol{S}_l^{m})^T\boldsymbol{B}^m(\boldsymbol{I}-\boldsymbol{S}_l^{m})\boldsymbol{Y} \tag{14}$$

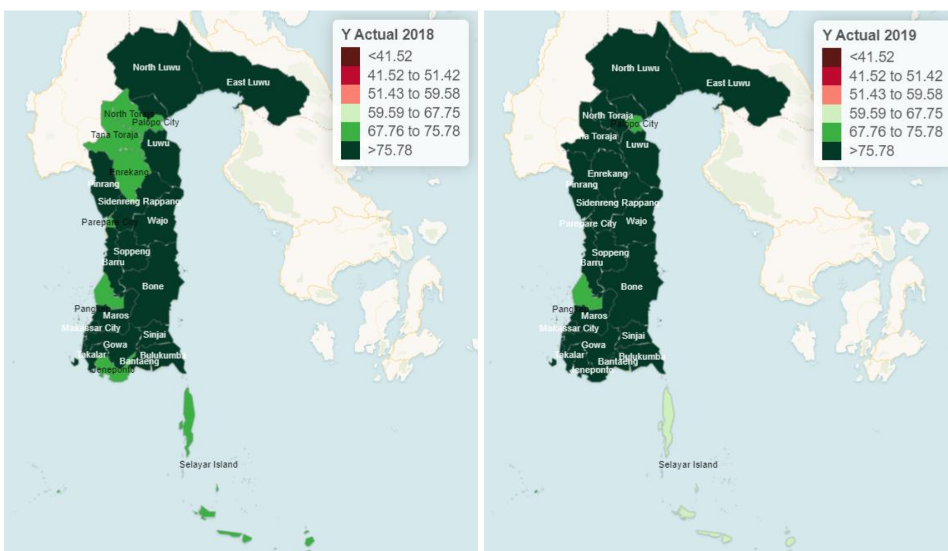
$$\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^{m+1} = [\boldsymbol{X}_l^T\boldsymbol{B}^{m}\boldsymbol{X}_l]^{-1}\boldsymbol{X}_l^T\boldsymbol{B}^{m}\left(\boldsymbol{Y}-\boldsymbol{X}_g\widehat{\boldsymbol{\beta}}_g^{m}\right) \tag{15}$$

The above calculation will continue to repeat until a convergent estimator is obtained, namely when the difference in the value of $\widehat{\boldsymbol{\beta}}_g^{m+1}$ with $\widehat{\boldsymbol{\beta}}_g^m$ approaches 0 and the difference in the value of $\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^{m+1}$ with $\widehat{\boldsymbol{\beta}}_l(u_i,v_i,t_i)^m$ approaches 0 (Asianingrum et al., 2020).

3. Main Results

3.1 Exploration of Food Security Index Data in South Sulawesi Province

The food security index of South Sulawesi Province has increased from year to year. From 2018 to 2022, South Sulawesi Province succeeded in achieving the title as the province with the best level of food security. In 2022, South Sulawesi Province had a food security level of 81.38 and succeeded in occupying the third position compared to other provinces in Indonesia. An overview of the level of food security of South Sulawesi Province from 2018-2022 is presented through a thematic map in Figure 1.



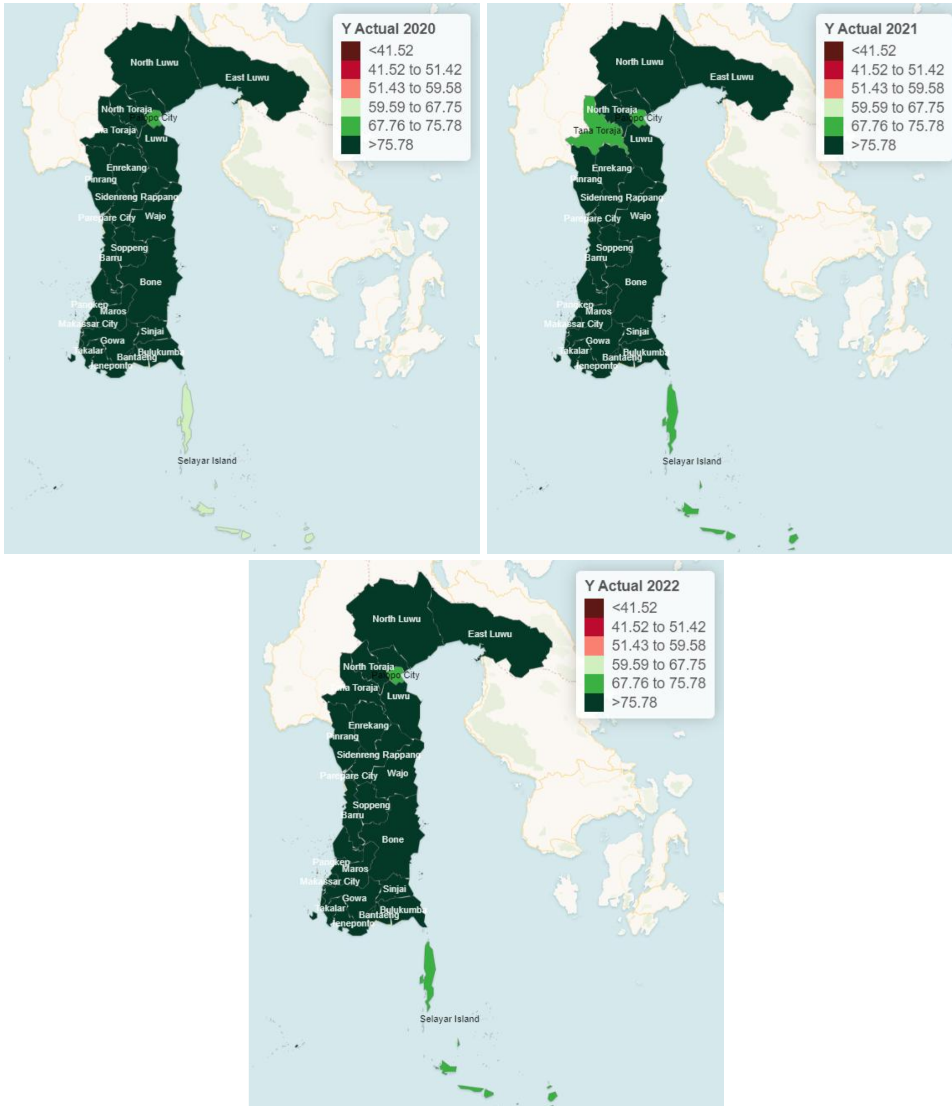


Figure 1: Map of the distribution of food security index values in South Sulawesi Province.

3.2 Spatial and Temporal Heterogeneity Test

The test for the effect of spatial heterogeneity was conducted using the Breusch-Pagan test. This test was carried out simultaneously in 24 districts or cities in South Sulawesi Province in the 2018–2022 period. The outcomes of the spatial heterogeneity test are shown in Table 2.

Table 2: Breusch-Pagan Test.

BP statistic value	***p-value***
19.2000	0.0075

The Breusch-Pagan test results in Table 2 show a p-value of 0.0075, which is below the significance level of $\alpha = 0.05$. This indicates that spatial heterogeneity is present in the data, meaning there are differences across locations. Additionally, temporal heterogeneity was assessed using the boxplot diagram displayed in Figure 2.

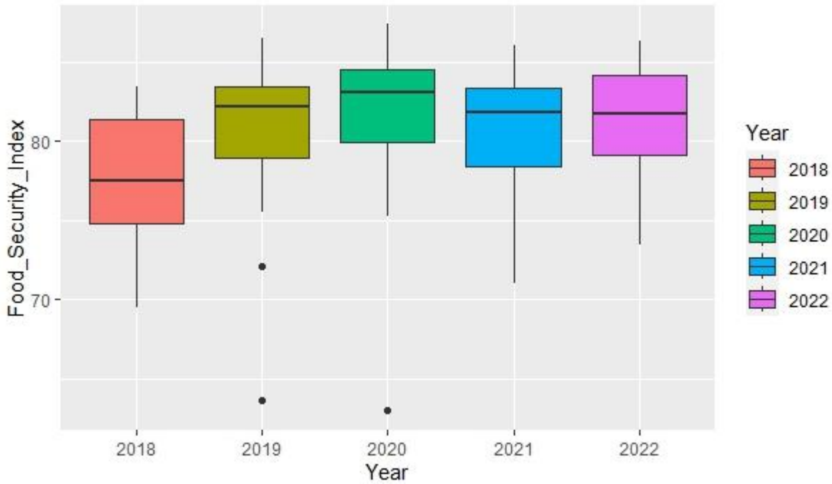


Figure 2: Boxplot of temporal heterogeneity in Each Year.

Figure 2 shows that the IKP value of South Sulawesi Province from 2018 to 2022 has changed, as indicated by the median value of the data over time. In addition, the boxplot diagram also explains that the IKP value in South Sulawesi Province in 2021 has decreased. This is due to the COVID-19 pandemic, which directly affects the global and national economies, including South Sulawesi Province. The sharp increase in food prices and the economic crisis decreased people's purchasing power for food. In addition, restrictions on movement and economic activities during the pandemic also affected food production and distribution.

3.3 Mixed Geographically and Temporally Weighted Regression

The GTWR model analysis yields a local model, leading to varying parameter values across different locations and times. When parameters exhibit a global trend, the MGTWR model is more appropriate. The decision to classify variables as global or local is determined by the proportion of GTWR model parameter estimates that fall within the confidence interval of the parameters from the global regression model (Pongoh et al., 2015). If more than 70% of the GTWR model parameter estimates for a variable are within the confidence interval of the global regression model, the variable is classified as a global variable. Conversely, if it is less than 70%, the variable is considered a local variable. The percentage of GTWR model parameter estimations that fall within the range of the global regression model's parameter confidence interval is displayed in Table 3.

Table 3: Confidence Interval for Global Regression Parameter.

Variable	Interval confidence			Percentage of Parameter Estimates	Information
X_1	-4.6426	$< \beta_1 <$	-3.6951	48%	Local
X_2	-2.2102	$< \beta_2 <$	-1.3433	66%	Local
X_3	-1.6358	$< \beta_3 <$	-0.8629	68%	Local
X_4	-0.7901	$< \beta_4 <$	0.0421	50%	Local
X_5	-2.1503	$< \beta_5 <$	-1.0914	34%	Local
X_6	0.5688	$< \beta_6 <$	1.4051	90%	Global
X_7	-1.0487	$< \beta_7 <$	-0.2177	95%	Global

Table 3 displays the percentage of parameter estimates for variables X_6 and X_7 that are within the more than 70% global regression parameter confidence interval. This indicates that the variables X_6 and X_7 belong in the global variable category. On the other hand, variables X_1, X_2, X_3, X_4, and X_5 are regarded as local variables since they fall within the global regression parameters' confidence interval with a percentage of less than 70%. Thus, the MGTWR model can handle both global and local variables.

Selecting the kernel function to be utilized as a weight is the first step in the MGTWR model analysis. An iterative process is employed to identify the kernel function by finding the minimum cross-validation (CV) value. The most suitable kernel function for the MGTWR model will be determined by looking at which one has the best bandwidth and lowest CV value.

Table 4: Selection of Model Bandwidth value.

Kernel Function	Bandwidth MGWR	CV MGWR	Bandwidth MGTWR	CV MGTWR
Gaussian	0.2749	97.7160	0.2798	98.4849
Exponential	**0.1561**	**86.3682**	**0.1546**	**89.1189**
Bisquare	2.9919	368.4361	3.0762	369.5550
Tricube	0.0036	5481.1980	2.4611	1807.768

When compared to different kernel functions, Table 4 demonstrates that the exponential kernel function with a spatial bandwidth of 0.1561 and a spatial-temporal bandwidth of 0.1546 yields the lowest CV value. Furthermore, balance parameters (whose values are derived from the minimum CV value) are needed for the MGTWR model. These parameters include the spatial distance parameter (λ), temporal distance parameter (μ), and ratio parameter (τ), with their corresponding values listed in Table 5.

Table 5: Values of MGTWR model balancing parameters.

λ	μ	τ
1.0569	0.0009	0.0008

The value of the spatial-temporal bandwidth, along with the obtained λ and μ balancing parameter will next be utilized to compute the spatial-temporal distance ($d_{i,j}^{ST}$). This value will then be employed to compute the spatial-temporal weighting function ($w_{i,j}$). The calculation of the weighting matrix is continued until 120 weighting matrices are obtained for each observation. The next step is the calculation of the global and local parameters of the MGTWR model. After obtaining the MGTWR model, detection is carried out using the Random Walk Bipartite (RWBP) method. RWBP is a technique for identifying outliers in spatial data through the random walk method (X. Liu et al., 2010). The results of outlier detection using the RWBP method identified 29 MGTWR model residuals that were detected as outliers in 5 years of observation.

Table 6: Probability Value of MGTWR model residual outlier.

No.	Location	Year	RWBP MGTWR
1	Takalar	2018	0.7545
2	Sidenreng Rappang	2018	0.6066
3	Maros	2018	0.8971
4	Enrekang	2018	0.6011
5	Makassar City	2018	1.0000
6	Palopo City	2018	0.6071
7	Sinjai	2019	0.5560
8	Enrekang	2019	0.9089

No.	Location	Year	RWBP MGTWR
9	Luwu	2019	0.7459
10	Pare-Pare City	2019	0.8551
11	North Luwu	2019	0.8018
12	Palopo City	2019	1.0000
13	Takalar	2020	0.8388
14	Sinjai	2020	0.6187
15	Tana Toraja	2020	0.8965
16	Luwu	2020	0.8786
17	Pare-Pare City	2020	1.0000
18	North Toraja	2020	0.8891
19	Palopo City	2020	0.8807
20	Selayar Island	2021	1.0000
21	Enrekang	2021	0.9659
22	Luwu	2021	0.9662
23	Pare-Pare City	2021	0.8481
24	Palopo City	2021	0.9487
25	Enrekang	2022	0.7248
26	Tana Toraja	2022	0.7813
27	Luwu	2022	0.7793
28	East Luwu	2022	1.0000
29	Palopo City	2022	0.8755

3.4 Robust Mixed Geographically and Temporally Weighted Regression

The RMGTWR model overcomes outliers in the MGTWR model. Using the previously determined MGTWR model parameters and weights whose values are computed using Tukey weights, the RMGTWR model is calculated. Tukey weights work by giving smaller weight values to observations with large residuals so that outliers in the model have less influence in parameter calculations.

The analysis of the RMGTWR model was carried out with an accuracy level of 0.0001 and obtained a convergent value at the 21st iteration. Convergence achieved at the 21st iteration indicates that the model has reached equilibrium and there is no significant change in the model parameters so that the iteration can be stopped. The results of the global variable parameter estimation calculation in the RMGTWR model are provided in Table 7.

Table 7: Global parameter estimation of RMGTWR model.

Parameter	Estimation Value
β_6	1.2684
β_7	-0.7794

Moreover, Table 8 illustrates how the RMGTWR model local variable parameter estimation was calculated for the Selayar Islands Regency in 2018.

Table 8: Local parameter estimation of the RMGTWR model in Selayar Island in 2018.

Location	Parameter	Estimation Value
Selayar Island	β_0	78.0143
	β_1	-11.5433
	β_2	-1.1933
	β_3	-1.2225
	β_4	0.0902
	β_5	-1.3621

The findings of the parameter estimation indicate that the local parameters vary in value across different locations and times, whereas the global parameters exert a consistent influence regardless of location. Following the estimation of the RMGTWR model, a partial test will be carried out to identify the factors that have a major impact on South Sulawesi Province's food security index between 2018–2022. Partial testing is conducted using a T-test for both global and local parameters.

Partial test results with a significance level (α) of 5% indicate that the global parameter β_6 related to life expectancy at birth does not significantly impact the food security index in South Sulawesi Province for the years 2018–2022. Conversely, the global parameter β_7, representing the percentage of stunted toddlers significantly affects the food security index in South Sulawesi Province during the same period. The partial test results for local parameters are illustrated using a scatter map, as depicted in Figure 3.

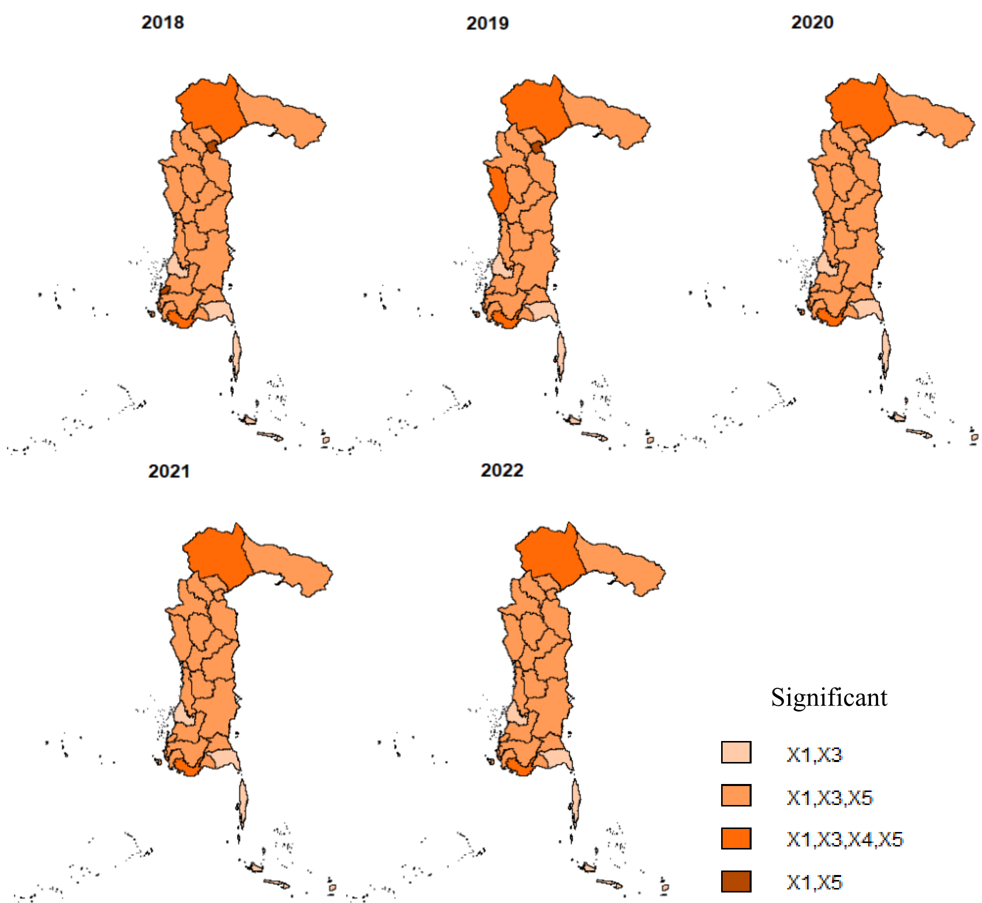


Figure 3: Influence map of predictor variables on response variables of RMGTWR model.

Figure 3 shows the location groups formed based on the similarity of influencing factors. Four location groups were formed in 2018 and 2019. However, only three location groups were formed in 2020, 2021, and 2022. Scatter map partial test of local parameters the RMGTWR model shows that there is no change or diversity that is too large from the factors that influence the food security index in South Sulawesi Province annually. The following are the RMGTWR models that were based on important variables that influence the food security index and that formed with the maximum R^2_{local} value:

Selayar Island Regency in 2018 with R^2_{local} of 99.6312%:

$$\hat{y}_1 = 78.0143 - 11.5433X_1 - 1.2225X_3 - 0.7794X_7$$

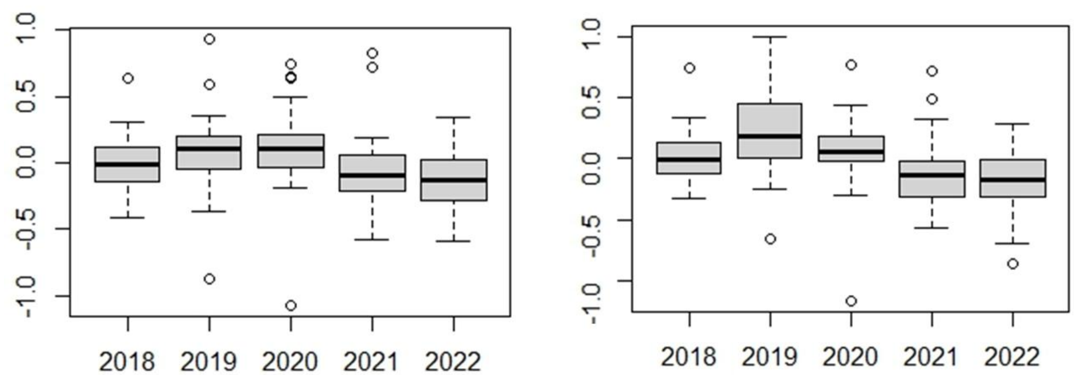
Bulukumba Regency in 2022 with R^2_{local} of 99.6319%:

$$\hat{y}_{98} = 77.8459 - 11.8856X_1 - 1.2286X_3 - 0.7794X_7$$

North Luwu Regency in 2022 with R^2_{local} of 99.2965%:

$$\hat{y}_{116} = 80.6445 - 4.0995X_1 - 1.2681X_3 - 0.3487X_4 - 2.4973X_5 - 0.7794X_7$$

The RWBP approach is then used to detect outliers once the RMGTWR model has been obtained. The results of outlier detection using the RWBP method show that there are 21 model residuals detected as outliers. The number of outliers is less than the outliers in the MGTWR model. In addition, outlier detection is also done using boxplots. The outlier comparison of the MGTWR model and the RGTWR model is shown in Figure 4.



Boxplot of MGTWR residuals Boxplot of RMGTWR residuals

Figure 4: Boxplot of residuals comparison of MGTWR and RMGTWR models.

The number of outliers using the boxplot is less than the RWBP method. The RWBP method considers spatial elements in the outlier detection process. Figure 4 shows that in 2019 and 2020, outliers are reduced. Therefore, the RMGTWR model effectively reduces outliers in the MGTWR model. Additionally, the optimal model selection is conducted to determine which model is most suitable for the food security index data in South Sulawesi Province from 2018 to 2022. The selection criteria involve comparing the R^2 and Akaike Information Criterion (AIC) values, as shown in Table 9.

Table 9: Model Fit Comparison.

Model	R^2	AIC
MGTWR	99.5564%	430.7742
RMGTWR	99.4815%	414.9719

The results of the model goodness test show that the MGTWR model has the highest R^2 value, but it is not much different from the R^2 value of the RMGTWR model. Meanwhile, the RMGTWR model with an M-Estimator has a lower AIC value compared to the AIC value of the MGTWR model. Therefore, the RMGTWR model with M-Estimator is still said to be better at explaining IKP in South Sulawesi in 2018–2022, because it succeeded in reducing the number of outliers in the residuals of the MGTWR model and producing a lower AIC value.

3.5 Conclusion

The RMGTWR model with the M-Estimator proved to be the most effective approach for explaining the Food Security Index in South Sulawesi during 2018–2022, as reflected in the reduced AIC value (414.9719) and the high R^2 (99.48%). The main influencing factors identified include the ratio of per capita normative consumption to net production, high household food expenditure burden, limited access to electricity and clean water, and the prevalence of stunting among toddlers. These findings highlight priority areas for policymakers to strengthen food security through improved infrastructure, better resource allocation, and targeted health interventions. Further research is recommended to explore the dynamic interactions among these factors and to assess the long-term effectiveness of policy measures across districts and cities in South Sulawesi Province.

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