Pak.j.stat.oper.res. Vol.20 No. 4 2024 pp 615-632 DOI: http://dx.doi.org/10.18187/pjsor.v20i4.4734

Pakistan Journal of Statistics and Operation Research

Flexible Group Service $MAP/PH/1$ Queueing Model with Working Breakdown, Repair and Balking

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Abstract

In reality, there are many uses of queueing models where services are provided in groups and these types of queueing models are widely studied in the literature. In this paper, we examine a particular queueing model wherein the services are provided in groups ranging from 1 to a pre-defined constant, denoted as K, and the arrival follows a Markovian arrival process. The service time of each customer follows phase-type distribution. The maximum of each customers individual service time within a group is defined as the group's service time. At the service completion moment, if there are fewer customers than K, the server won't begin the subsequent service until the system's customer size reaches K or a randomly assigned admission period expires, whichever happens first. The phase type representation of the service times depends on the group's size. Anytime a server breaks down and it will not proceed to repair, instead, it will serve the affected customer group at a slower pace. After that specific customer group's service is finished, the server will immediately undergo repair to fix any issues. The process of repair and breakdown occurs at an exponential rate. When the server breaks down, the customer might balk. The Markov chain's stability condition is determined and stationary probability vector is computed. Formulas for the primary system performance measures are given. Numerical and graphical representations of the proposed model are illustrated.

Key Words: Markovian Arrival Process; Flexible group service; Phase type Distributions; Working Breakdown; Balking; Repair.

Mathematical Subject Classification: 60K25, 68M20, 90B22.

1. Background

There are numerous practical uses of queueing models with group services which include transportation systems, telecommunication systems, entertainment systems and health care systems. Group service models usually include assumptions about the minimum and maximum of group sizes. In Bailey (1954), Bailey proposed the fixed group size bulk service queueing concept. In Brugno et al. (2017b), D'Arienzo et al. (2020), Dudin et al. (2015) and Haghighi and Mishev (2016), the authors explored on models of group service queueing were arrival follows Markovian arrival process. The authors performed a detailed study of bulk service queues with variations like inventory and retrials in Baba (1996), Chakravarthy et al. (2017), Chaudhry and Templeton (1983) and Neuts (1967).

In Brugno et al. (2017a) and Banerjee et al. (2015), the authors worked on generating a group service queueing model with phase-type service time. A survey on the bulk service queueing models were done by Sasikala and Indhira (2016) is noteworthy. In Niranjan and Indhira (2016) a review on the bulk arrival and batch service queueing models is given. Real life examples of group service queueing models were carried out in detail in Bruneel et al. (2010), Abolnikov and Dukhovny (2003) and Bar-Lev et al. (2007). Using a random service time distribution and random arrivals, waiting time distribution of bulk service queues is obtained in Downton (1955). In Brugno et al. (2017a), Banerjee et al. (2015) the service time for group service queueing models for customer group of size $'i'$ is assumed to be the maximum of $'i'$ identical PH distributions, which in turn a PH distribution, this type of group service model is also examined in D'Arienzo et al. (2020) and Dudin et al. (2015).

A $MAP/PH/1$ queueing model with flexible group service has been studied by the authors in Brugno et al. (2017a). Typically, a fixed number, let's say N, is used in the study of group services. In the situation that there are fewer than N consumers in the queue, the service is not initiated. Upon service completion, the server serves exactly N consumers if there are N or more than N. On the other hand, an admission time commences when there are fewer customers in the queue than N and it follows the PH distribution. If more customers are waiting than N prior to the admission time ending, the admission time is ended and the service is continued with N customers. The group of $'i'$ customers is served continually by the server if the admission time finishes before the Nth customer comes, alternatively, if the admission period ends and no one is in line, a new admissions time begins, and the procedure is repeated. Here, $'i'$ varies from 1 to $N-1$.
Anytime a server may breaks down and it will not proceed to repair, instead, it will serve the affected customer group at a slower pace. After that specific customer group's service is finished, the server will immediately undergo repair to fix any issues, this concept is called working breakdown and it was first proposed by Kalidass and Kasturi in (2012).

In this paper, we study a flexible group service $MAP/PH/1$ queueing model with working breakdown, repair and balking. The following is the order of the sections in the article. Section 2 presents a graphic representation of the mathematical model together with a narrative explaining it and formulating the QBD matrix. In section 3, the steady-state probability vector and the ergodicity (stability) requirement are derived. We calculated a few performance measures for this model in section 4. Section 5 presents a few numerical results together with graphical representations, and Section 6 provides the conclusion.

2. The Narration of the Model

In this paper, The basic operation of the model can be described as:
Arrival process: Markovian arrival process is considered with depiction (D_0, D_1) of order n with the generator matrix $\tilde{D} = D_0 + D_1$. Markovian arrival's fundamental rate is defined as

$$\tau = \varpi D_1 e_m,$$

where ϖ is the vector of stationary probability of $\tilde{D}$.
Service process: We assume that the customers will get service in groups of size K, with $K \geq 2$ a fixed integer. If there are $K-1$ customers in the queue and the server is idle an arriving customer will get a immediate service and its PH representation is denoted as $(\sigma^{(K)}, H^{(K)})$ of order $S^{(K)}$ with

$$H_0^{(K)} + H^{(K)}e = 0 \;\; which \;\; implies \;\; H_0^{(K)} = -H^{(K)}e,$$

otherwise based on the sequence of their arrival, the arriving customers are placed in the buffer and at this moment choosing customers from the buffer at the time a service is complete is defined as follows, when a service is completed and there are K or more customers in the queue, then the server provides a service to the group of exactly K customers with PH representation $(\sigma^{(K)}, H^{(K)})$ of order $S^{(K)}$ and we refer such a group of K customers a *block*. On the other hand, an admission period is initiated if the number of customer is waiting is fewer than K and we refer to this group of customers as a *pool*.
Admission period: The admission period is based on the PH distribution, (γ, G) of order $S^{(0)}$ where

$$G_0 + Ge = 0 \;\; and \, it \, implies \;\; G_0 = -Ge.$$

If the number of customers in the pool reaches K before the admission period expires, the admission period is stopped and a service with K customers starts. If the admission period runs out before the Kth customer turns up, then the server accepts all the waiting customers in the pool and provides a service. So a service of size ranging from 1 to $K-1$ is provided in such a way for a group and the service of k, customers $1 \leq k \leq K-1$ follows the PH distribution $(\sigma^{(k)}, H^{(k)})$ of order $S^{(k)}$ with $H_0^{(k)} + H^{(k)}e = 0$, implying $H_0^{(k)} = -H^{(k)}e$. A fresh admission period begins if there are no customers in the queue. The basic rate of admission period is

$$\lambda = [\gamma(-G)^{-1}e]^{-1}.$$

Fundamental rate of service to k customers where $1 \leq k \leq K$ is defined as

$$\delta_k = [\sigma^{(k)}(-H^k)^{-1}e]^{-1}.$$

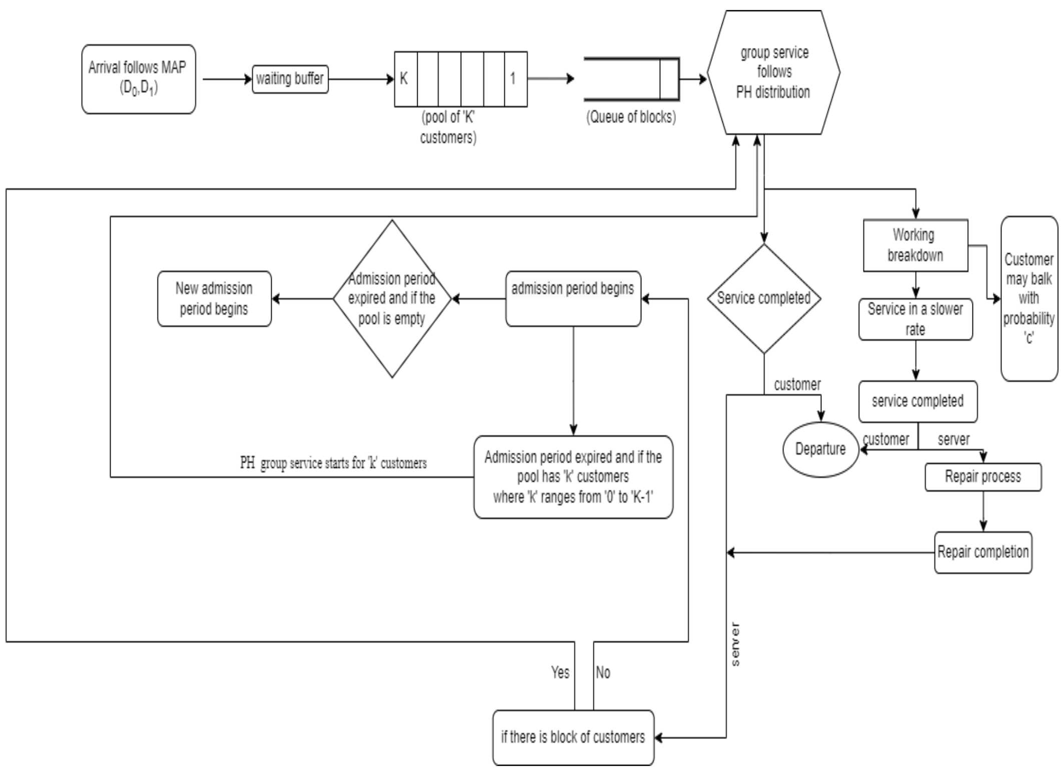


Figure 1: Diagram illustrating the current model

Working breakdown: Anytime a server may breaks down and it will not proceed to repair, instead, it will serve the affected customer group at a slower pace with PH representation $(\sigma^{(k)}, \epsilon H^{(k)})$, where $(0 < \epsilon < 1)$. After that specific customer group's service is finished, the server will immediately undergo repair to fix any issues.

The process of repair and breakdown occurs at an exponential rate with parameter α and β. When the server breaks down, the customer might balk, with probability c.

Notations for our model

- $\otimes$ - the matrix Kronecker product.
- $\oplus$ - the matrix Kronecker sum .
- I_{m} - an identity matrix of m- dimension.

- e- a column matrix with each entry is 1 of an appropriate dimension.
- $diag\{d_1, d_2, \cdots, d_n\}$ is the diagonal matrix, whose entries for the diagonals are enclosed in brackets. For example if $A = diag\{x, y, z\}$ then
$$A = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$$
.
- $A = (a_{gh})$, where A is square matrix of order $K \times K$, that is $1 \le g \le K; 1 \le h \le K$;
- F_k is the row matrix of dimension k each of its entries as 0.

Let
$$\Psi(t) = \left\{B(t), C(t), P(t), M(t), I(t)^{(M(t))}, V(t) : t \ge 0 \right\}$$
where,

- $B(t)$ depicts the number of customer blocks in the system, including one in service, at moment t.
- $C(t)$ depicts the server's position at moment t

$$C(t) = \begin{cases} 0 & while\ server\ is\ not\ in\ use \\ 1 & while\ server\ providing\ service\ in\ normal\ mode \\ 2 & while\ server\ providing\ service\ in\ breakdown \\ 3 & while\ server\ in\ repair. \end{cases}$$

- $P(t)$ depicts number of customers in the pool at moment t, thus $0 \le P(t) \le K - 1$.
- $M(t)$ depicts number of customers receiving the service at moment t. Note that $M(t) = 0$ if $B(t) = 0$, therefore admission period will be taking place and $1 \le M(t) \le K$ if $B(t) \ge 1$.
- If $M(t) = 0$ then $I(t)^{(M(t))}$ depicts the PH process of customer admission with $1 \le I(t)^{(0)} \le S^{(0)}$ and if $1 \le M(t) \le K$ then $I(t)^{(M(t))}$ depicts the PH process of customer service with $1 \le I(t)^{(M(t))} \le S^{(M(t))}$.
- $V(t)$ depicts the Markovian arrival process with $1 \le V(t) \le m$.

$\Psi(t)$ has the below states,
$$\Psi = \psi(0) \bigcup \psi(n)$$
where,
$$\begin{aligned} \psi(0) &= \{(0, 0, p, i, j) : 0 \le p \le K - 1 ; 1 \le i \le S^{(0)} ; 1 \le j \le m \} \\ &\bigcup \{(0, 3, p, i, j) : 0 \le p \le K - 1 ; 1 \le i \le s^{(0)} ; 1 \le j \le m \}. \end{aligned}$$

Moreover, this can be expressed as
$$\psi(0) = \{(0, 0, p) : 0 \le p \le K - 1 \} \bigcup \{(0, 3, p) : 0 \le p \le K - 1 \}$$
phases of Markovian arrival and the PH process of admission period are comprehended.

For $n \ge 1$,
$$\begin{aligned} \psi(n) &= \{(b, 1, p, k, i, j) : b \ge 1 ; 0 \le p \le K - 1 ; 1 \le k \le K ; 1 \le i \le S^{(k)} ; 1 \le j \le m \} \\ &\bigcup \{(b, 2, p, k, i, j) : b \ge 1 ; 0 \le p \le K - 1 ; 1 \le k \le K ; 1 \le i \le S^{(k)} ; 1 \le j \le m \} \\ &\bigcup \{(b, 3, p, k, j) : b \ge 1 ; 0 \le p \le K - 1 ; 1 \le k \le K ; 1 \le j \le m \} \end{aligned}$$

and this can be simply written as

$$\begin{aligned}\psi(n) \;=\; & \{(b,1,p,k) : b \geq 1\,;\, 0 \leq p \leq K-1\,;\, 1 \leq k \leq K\} \\ & \bigcup \{(b,2,p,k) : b \geq 1\,;\, 0 \leq p \leq K-1\,;\, 1 \leq k \leq K\} \\ & \bigcup \{(b,3,p,k) : b \geq 1\,;\, 0 \leq p \leq K-1\,;\, 1 \leq k \leq K\}\end{aligned}$$

phases of Markovian arrival and the PH process of service to k number of customers where $1 \leq k \leq K$ are comprehended.

The rate of transition matrix of the QBD process is given as,

$$Q = \begin{pmatrix} A_{00} & A_{01} & 0 & 0 & 0 & 0 & \cdots & \cdots \\ A_{10} & B^0 & B^+ & 0 & 0 & 0 & \cdots & \cdots \\ 0 & B^- & B^0 & B^+ & 0 & 0 & \cdots & \cdots \\ 0 & 0 & B^- & B^0 & B^+ & 0 & \cdots & \cdots \\ 0 & 0 & 0 & B^- & B^0 & B^+ & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \ddots & \ddots \end{pmatrix}$$

The block matrices of the above matrix are given below

$$A_{00} = \begin{pmatrix} {A_{00}}^{11} & {A_{00}}^{12} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & {A_{00}}^{22} & {A_{00}}^{23} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & & {A_{00}}^{K-1\ K-1} & {A_{00}}^{K-1\ K} & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & {A_{00}}^{K\ K} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & {A_{00}}^{K+1\ K+1} & {A_{00}}^{K+1\ K+2} & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & {A_{00}}^{K+2\ K+2} & {A_{00}}^{K+2\ K+3} & \\ \vdots & \ddots & \cdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \vdots & 0 & & \ddots & \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & {A_{00}}^{2K\ 2K} \end{pmatrix}$$

where

$${A_{00}}^{11} = (G + G_0\gamma) \oplus D_0; {A_{00}}^{12} = D_1 \otimes I_{S^{(0)}},$$

In general for $j, 2 \leq j \leq K-1$,

$${A_{00}}^{j\ j} = G \oplus D_0; {A_{00}}^{j\ j+1} = D_1 \otimes I_{S^{(0)}},$$

and

$${A_{00}}^{K\ K} = G \oplus D_0; {A_{00}}^{K+1\ K+1} = (G + G_0\gamma) \oplus D_0; {A_{00}}^{K+1\ K+2} = D_1 \otimes I_{S^{(0)}},$$

In general for $j, 2 \leq j \leq K-1$,

$${A_{00}}^{K+j\ K+j} = G \oplus D_0; {A_{00}}^{K+j\ K+(j+1)} = D_1 \otimes I_{S^{(0)}}\ and\ {A_{00}}^{2K\ 2K} = G \oplus D_0.$$

$$A_{01} = \begin{pmatrix} A_{01}^{01} & A_{01}^{02} & A_{01}^{03} \\ A_{01}^{31} & A_{01}^{32} & A_{01}^{33} \end{pmatrix},$$

$A_{01}^{01} = ({A_{01}^{01}}_{gh})$, where A_{01}^{01} is square matrix of order $K \times K$, thus $1 \leq g \leq K; 1 \leq h \leq K$ and

$${A_{01}^{01}}_{gh} = \begin{cases} (Q_{01}^0)_{g-1,0}, & \text{if } 2 \leq g \leq K; h = 1 \\ 0\,, & \text{otherwise} \end{cases}$$

where

$$(Q_{01}^{0})_{j,0} = \big(F_{j-1}, G_0 \otimes \sigma^{(j)} \otimes I_m, F_{K-j}\big) \; for \; 1 \leq \; j \; \leq K-2, \;\; and$$
$$(Q_{01}^{0})_{K-1,0} = \big(F_{K-2}, G_0 \otimes \sigma^{(K-1)} \otimes I_m, e_{S^{(0)}} \otimes \sigma^{(K)} \otimes D_1\big).$$

$A_{01}^{33} = (A_{01\,gh}^{33})$, where A_{01}^{33} is square matrix of order $K \times K$, thus $1 \leq g \leq K; 1 \leq h \leq K$;

$$A_{01\,gh}^{33} = \begin{cases} (Q_{01}^{3})_{g-1,0}, & \text{if } 2 \leq g \leq K; h = 1 \\ 0\,, & \text{otherwise} \end{cases}$$

where

$$(Q_{01}^{3})_{j,0} = \big(F_{j-1}, G_0 \otimes I_m, F_{K-j}\big) \; for \; 1 \leq \; j \; \leq K-2, \;\; and$$
$$(Q_{01}^{3})_{K-1,0} = \big(F_{K-2}, G_0 \otimes I_m, e_{S^{(0)}} \otimes D_1\big).$$

$A_{01}^{02}; A_{01}^{03}; A_{01}^{31}; A_{01}^{32}$ $are\; zero\; matrices.$

$$A_{10} = \begin{pmatrix} A_{10}^{10} & 0 \\ 0 & A_{10}^{23} \\ 0 & 0 \end{pmatrix},$$

$A_{10}^{10} = diag\{(Q_{1,0})_{1,1}, (Q_{1,0})_{2,2}, \ldots, (Q_{1,0})_{K,K}\}$,

$$for, \; 1 \leq j \leq K, \; (Q_{1,0})_{j,j} = \begin{pmatrix} {H_0}^{(1)} \otimes \gamma \otimes I_m \\ \vdots \\ {H_0}^{(K)} \otimes \gamma \otimes I_m \end{pmatrix}.$$

$A_{10}^{23} = diag\{(Q_{1,0})_{K+1,K+1}, (Q_{1,0})_{K+2,K+2}, \ldots, (Q_{1,0})_{2K,2K}\}$,

$$for, \; 1 \leq j \leq K, \; (Q_{1,0})_{K+j,K+j} = \begin{pmatrix} \epsilon {H_0}^{(1)} \otimes \gamma \otimes I_m \\ \vdots \\ \epsilon {H_0}^{(K)} \otimes \gamma \otimes I_m \end{pmatrix}.$$

$$B^0 = \begin{pmatrix} B_{11}^0 & B_{12}^0 & 0 \\ 0 & B_{22}^0 & 0 \\ B_{31}^0 & 0 & B_{33}^0 \end{pmatrix},$$

$$B_{11}^0 = \begin{pmatrix} (B_{11}^0)_{0,0} & (B_{11}^0)_{0,1} & 0 & 0 & \cdots & 0 \\ 0 & (B_{11}^0)_{1,1} & (B_{11}^0)_{1,2} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \cdots & 0 \\ \vdots & \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & (B_{11}^0)_{K-2,K-2} & (B_{11}^0)_{K-2,K-1} \\ 0 & 0 & \cdots & & 0 & (B_{11}^0)_{K-1,K-1} \end{pmatrix},$$

$$for \; 0 \leq \; j \; \leq K-1, \; (B_{11}^0)_{j,j} = diag\left\{(D_0 - \alpha I_m) \oplus H^{(k)}; 1 \leq \; k \; \leq K\right\},$$

$$for \; 0 \leq \; j \; \leq K-2, \; (B_{11}^0)_{j,j+1} = diag\left\{D_1 \otimes I_{S^{(k)}}; 1 \leq \; k \; \leq K\right\}.$$

$B_{12}^0 = diag\left\{(B_{12}^0)_{1,K+1}, (B_{12}^0)_{2,K+2}, \ldots, (B_{12}^0)_{K,2K}\right\}$,

$$for \; 1 \leq \; j \; \leq K, \; (B_{12}^0)_{j,K+j} = diag\left\{\alpha I_m \otimes I_{S^{(k)}}; 1 \leq \; k \; \leq K\right\}.$$

$$B_{22}^0 = \begin{pmatrix} (B_{22}^0)_{K+1,K+1} & (B_{22}^0)_{K+1,K+2} & 0 & 0 & \cdots & 0 \\ 0 & (B_{22}^0)_{K+2,K+2} & (B_{22}^0)_{K+2,K+3} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & & 0 \\ 0 & \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & (B_{22}^0)_{2K-1,2K-1} & (B_{22}^0)_{2K-1,2K} \\ 0 & 0 & \cdots & & 0 & (B_{22}^0)_{2K,2K} \end{pmatrix},$$

$$for\ 1 \le\ j\ \le K,\ (B_{22}^0)_{K+j,K+j} = diag\left\{(D_0 +\ cD_1) \oplus \epsilon H^{(k)}; 1 \le\ k\ \le K\right\},$$

$$and\ for\ 1 \le\ k\ \le K-1,\ (B_{22}^0)_{K+j,K+(j+1)} = diag\left\{(1-c)D_1 \otimes I_{S^{(k)}}; 1 \le\ k\ \le K\right\}.$$

$B_{31}^0 = diag\left\{(B_{31}^0)_{2K+1,1}, (B_{31}^0)_{2K+2,2}, \cdots, (B_{31}^0)_{3K,K}\right\}$,

$$for\ 1 \le\ j\ \le K,\ (B_{31}^0)_{2K+j,j} = diag\left\{\beta I_m \otimes \sigma^k; 1 \le\ k\ \le K\right\}.$$

$$B_{33}^0 = \begin{pmatrix} (B_{33}^0)_{2K+1,2K+1} & (B_{33}^0)_{2K+1,2K+2} & 0 & 0 & \cdots & 0 \\ 0 & (B_{33}^0)_{2K+2,2K+2} & (B_{33}^0)_{2K+2,2K+3} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & & 0 \\ 0 & \vdots & & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & (B_{33}^0)_{3K-1,3K-1} & (B_{33}^0)_{3K-1,3K} \\ 0 & 0 & \cdots & & 0 & (B_{33}^0)_{3K,3K} \end{pmatrix},$$

$$for\ 1 \le\ j\ \le K,\ (B_{33}^0)_{2K+j,2K+j} = diag\left\{D_0 - \beta I_m, \ldots, D_0 - \beta I_m\right\},$$

$$for\ 1 \le\ k\ \le K-1,\ (B_{33}^0)_{2K+j,2K+(j+1)} = diag\left\{D_1, \ldots, D_1\right\}.$$

$$B^- = \begin{pmatrix} B_{11}^- & 0 & 0 \\ 0 & 0 & B_{23}^- \\ 0 & 0 & 0 \end{pmatrix}$$

$B_{11}^- = diag\left\{(B_{11}^-)_{0,0}, (B_{11}^-)_{1,1}, \ldots, (B_{11}^-)_{K-1,K-1}\right\}$,

$$for\ 0 \le j \le K-1,\ (B_{11}^-)_{j,j} = \begin{pmatrix} 0 & \cdots & 0 & {H_0}^{(1)} \otimes \sigma^{(K)} \otimes I_m \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & {H_0}^{(K)} \otimes \sigma^{(K)} \otimes I_m \end{pmatrix},$$

$B_{23}^- = diag\left\{(B_{23}^-)_{0,0}, (B_{23}^-)_{1,1}, \ldots, (B_{23}^-)_{K-1,K-1}\right\}$,

$$for\ 0 \le j \le K-1,\ (B_{23}^-)_{j,j} = diag\left\{\epsilon {H_0}^{(1)} \otimes I_m, \epsilon {H_0}^{(2)} \otimes I_m, \ldots, \epsilon {H_0}^{(K)} \otimes I_m\right\}.$$

$$B^+ = \begin{pmatrix} B_{11}^+ & 0 & 0 \\ 0 & B_{22}^+ & 0 \\ 0 & 0 & B_{33}^+ \end{pmatrix}$$

$B_{11}^+ = (B_{11gh}^+)$, where B_{11}^+ is square matrix of order $K \times K$, thus $1 \le g \le K; 1 \le h \le K$;

$$B_{11gh}^+ = \begin{cases} (B_{11}^+)_{g-1,0}, & \text{if } g = K; h = 1 \\ 0\,, & \text{otherwise} \end{cases}$$

$$where\ (B_{11}^+)_{K-1,0} = diag\left\{I_{S^{(k)}} \otimes D_1; 1 \le\ k\ \le K\right\}.$$

$B_{22}^{+} = (B_{22gh}^{+})$, where B_{22}^{+} is square matrix of order $K \times K$, thus $1 \leq g \leq K; 1 \leq h \leq K$;

$$B_{22gh}^{+} = \begin{cases} (B_{22}^{+})_{g-1,0}, & \text{if } g = K; h = 1 \\ 0\,, & \text{otherwise} \end{cases}$$

$$where\ (B_{22}^{+})_{K-1,0} = diag\left\{I_{S^{(k)}} \otimes (1-c)D_1; 1 \leq\ k\ \leq K\right\}.$$

$B_{33}^{+} = (B_{33gh}^{+})$, where B_{33}^{+} is square matrix of order $K \times K$, thus $1 \leq g \leq K; 1 \leq h \leq K$;

$$B_{33gh}^{+} = \begin{cases} (B_{33}^{+})_{g-1,0}, & \text{if } g = K; h = 1 \\ 0\,, & \text{otherwise} \end{cases}$$

$$where\ (B_{33}^{+})_{K-1,0} = diag\left\{D_1; 1 \leq\ k\ \leq K\right\}.$$

3. Condition for stableness

Let us define the matrix

$$U = B^{-} + B^{0} + B^{+},$$

then

$$U = \begin{pmatrix} F_{11} & F_{12} & 0 \\ 0 & F_{22} & F_{23} \\ F_{31} & & F_{33} \end{pmatrix},$$

$$F_{11} = \begin{pmatrix} E & E_1 & & \\ & \ddots & \ddots & \\ & & E & E_1 \\ E_1 & & & E \end{pmatrix},$$

$$where\ E = \begin{pmatrix} (D_0 - \alpha I_m) \oplus H^{(1)} & & & {H_0}^{(1)}\sigma^{(K)} \otimes I_m \\ & \ddots & & \vdots \\ & & \ddots & {H_0}^{(K-1)}\sigma^{(K)} \otimes I_m \\ & & & ((D_0 - \alpha I_m) \oplus H^{(K)}) + ({H_0}^{(K)}\sigma^{(K)} \otimes I_m) \end{pmatrix},$$

$$and\ E_1 = diag\left\{D_1 \otimes I_{S^{(1)}}, D_1 \otimes I_{S^{(2)}}, \ldots, D_1 \otimes I_{S^{(K)}}\right\}.$$

$$F_{12} = diag\left\{E_6, E_6, \ldots, E_6\right\}$$

$$where\ E_6 = diag\left\{\alpha I_m \otimes I_{S^{(1)}}, \alpha I_m \otimes I_{S^{(2)}}, \ldots, \alpha I_m \otimes I_{S^{(K)}}\right\}.$$

$$F_{22} = \begin{pmatrix} E_2 & E_3 & & \\ & \ddots & \ddots & \\ & & E_2 & E_3 \\ E_3 & & & E_2 \end{pmatrix},$$

$$where\ E_2 = diag\left\{(D_0 + cD_1) \oplus \epsilon H^{(1)}, (D_0 + cD_1) \oplus \epsilon H^{(2)}, \ldots, (D_0 + cD_1) \oplus \epsilon H^{(K)}\right\},$$
$$and\ E_3 = diag\left\{(1-c)D_1 \otimes I_{S^{(1)}}, (1-c)D_1 \otimes I_{S^{(2)}}, \ldots, (1-c)D_1 \otimes I_{S^{(K)}}\right\}.$$

$$F_{23} = diag\left\{E_7, E_7, \ldots, E_7\right\},$$

$$where\ E_7 = diag\left\{\epsilon H^{(1)} \otimes I_m, \epsilon H^{(2)} \otimes I_m, \ldots, \epsilon H^{(K)} \otimes I_m\right\}.$$

$$F_{31} = diag\left\{E_8, E_8, \ldots, E_8\right\},$$

$$where\ E_8 = diag\left\{\beta I_m \otimes \sigma^1, \beta I_m \otimes \sigma^2, \ldots, \beta I_m \otimes \sigma^K\right\}.$$

$$F_{33} = \begin{pmatrix} E_4 & E_5 & & & \\ & \ddots & \ddots & & \\ & & E_4 & E_5 \\ E_4 & & & E_5 \end{pmatrix},$$

$$where\ E_4 = diag\left\{(D_0 - \beta I_m), (D_0 - \beta I_m), \ldots, (D_0 - \beta I_m)\right\},$$

$$and\ E_5 = diag\left\{D_1, D_1, \ldots, D_1\right\}.$$

It is obvious that the generator matrix U is a square matrix of order $2K(S^{(1)}\,m + S^{(2)}\,m + \cdots + S^{(K)}\,m) + K(Km + \cdots + Km)$ or simply $2K\big(S^{(1)} + S^{(2)} + \cdots + S^{(K)}\big)m + K^2 m$. The steady-state probability vector of U is indicated by y. And the vector y is denoted as $y = (y_1, y_2, y_3, \ldots, y_K, y_{K+1}, y_{K+2}, \cdots, y_{2K}, y_{2K+1}, y_{2K+2}, \cdots, y_{3K})$, where $y_i = (y_i^1, y_i^2, \cdots, y_i^K)$, $1 \leq\ i\ \leq 3K$ which satisfies $yU = 0$ and $ye = 1$.

It may be inferred from Neuts(1994) that the fulfillment of the inequality $yB^+e\ <\ yB^-e$, is the criterion for the ergodicity of the Markov chain $\Psi(t)$. It is possible to find the vector y by solving the following equations.

$y_1^1(H^{(1)} \oplus (D_0 - \alpha I_m)) + y_K^1(I_{S^{(1)}} \otimes D_1) + y_{2K+1}^1(\beta I_m \otimes \sigma^1) = 0$

$\vdots$

$y_1^{K-1}(H^{(K-1)} \oplus (D_0 - \alpha I_m)) + y_K^{K-1}(I_{S^{(K-1)}} \otimes D_1) + y_{2K+1}^{K-1}(\beta I_m \otimes \sigma^{K-1}) = 0$

$\sum_{k=1}^{K-1} y_1^k(H_0{}^{(k)} \otimes \sigma^{(K)} \otimes I_m) + y_1^K[(H_0{}^{(K)} \otimes \sigma^{(K)} \otimes I_m) + (H^{(K)} \oplus (D_0 - \alpha I_m))]$

$+\ y_K^K(I_{S^{(K)}} \otimes D_1) + y_{2K+1}^K(\beta I_m \otimes \sigma^K) = 0$

$for\ \ j\ \ , 2\ \leq j \leq K$ we have,

$y_{j-1}^1(I_{S^{(1)}} \otimes D_1) + y_j^1(H^{(1)} \oplus (D_0 - \alpha I_m)) + y_{2K+j}^1(\beta I_m \otimes \sigma^1) = 0$

$\vdots$

$y_{j-1}^{K-1}(I_{S^{(K-1)}} \otimes D_1) + y_j^{K-1}(H^{(K-1)} \oplus (D_0 - \alpha I_m)) + y_{2K+j}^{K-1}(\beta I_m \otimes \sigma^{K-1}) = 0$

$\sum_{k=1}^{K-1} y_j^k(H_0{}^{(k)} \otimes \sigma^{(K)} \otimes I_m) + y_j^K[(H_0{}^{(K)} \otimes \sigma^{(K)} \otimes I_m) + (H^{(K)} \oplus (D_0 - \alpha I_m))]$

$+\ y_{j-1}^K(I_{S^{(K)}} \otimes D_1) + y_{2K+j}^K(\beta I_m \otimes \sigma^K) = 0.$

$y_1^1(I_{S^{(1)}} \otimes \alpha I_m) + y_{K+1}^1((D_0 + cD_1) \oplus \epsilon H^1) + y_{2K}^1(1-c)D_1 \otimes I_{S^{(1)}} = 0$

$y_1^2(I_{S^{(2)}} \otimes \alpha I_m) + y_{K+1}^2((D_0 + cD_1) \oplus \epsilon H^2) + y_{2K}^2(1-c)D_1 \otimes I_{S^{(2)}} = 0$

$\vdots$

$y_1^K(I_{S^{(K)}} \otimes \alpha I_m) + y_{K+1}^K((D_0 + cD_1) \oplus \epsilon H^K) + y_{2K}^K(1-c)D_1 \otimes I_{S^{(K)}} = 0$

$for\ \ j\ \ , 2\ \leq j \leq K$ we have,

$y_j^1(I_{S^{(1)}} \otimes \alpha I_m) + y_{K+(j-1)}^1(1-c)D_1 \otimes I_{S^{(1)}} + y_{K+j}^1((D_0 + cD_1) \oplus \epsilon H^1) = 0$

$y_j^2(I_{S^{(2)}} \otimes \alpha I_m) + y_{K+(j-1)}^2(1-c)D_1 \otimes I_{S^{(2)}} + y_{K+j}^2((D_0 + cD_1) \oplus \epsilon H^2) = 0$

$\vdots$

$y_j^K(I_{S^{(K)}} \otimes \alpha I_m) + y_{K+(j-1)}^K(1-c)D_1 \otimes I_{S^{(K)}} + y_{K+j}^K((D_0 + cD_1) \oplus \epsilon H^K) = 0$

$y_{K+1}^1(\epsilon H_0{}^1 \otimes I_m) + y_{2K+1}^1(D_0 - \beta I_m) + y_{3K}^1 D_1 = 0$

$y_{K+1}^2(\epsilon H_0{}^2 \otimes I_m) + y_{2K+1}^2(D_0 - \beta I_m) + y_{3K}^2 D_1 = 0$

$\vdots$

$y_{K+1}^K(\epsilon H_0{}^K \otimes I_m) + y_{2K+1}^K(D_0 - \beta I_m) + y_{3K}^K D_1 = 0$

$for\ \ j\ \ , 2\ \leq j \leq K$ we have,

$y_{K+j}^1(\epsilon H_0{}^1 \otimes I_m) + y_{2K+(j-1)}^1 D_1 + y_{2K+j}^1(D_0 - \beta I_m) = 0$

$y_{K+j}^2(\epsilon H_0{}^2 \otimes I_m) + y_{2K+(j-1)}^2 D_1 + y_{2K+j}^2(D_0 - \beta I_m) = 0$

$\vdots$

$y^K_{K+j}(\epsilon {H_0}^K \otimes I_m) + y^K_{2K+(j-1)}D_1 + y^K_{2K+j}(D_0 - \beta I_m) = 0$
After performing some computations, the criteria for stability $yB^+e < yB^-e$, that is inferred as

$$\sum_{k=1}^{K} y^k_K(e_{S^{(k)}} \otimes D_1 e_m) + \sum_{k=1}^{K} y^k_{2K}(e_{S^{(k)}} \otimes (1-c)D_1 e_m) + \sum_{k=1}^{K} y^k_{3K}(D_1 e_m) < \sum_{i=1}^{K}\sum_{k=1}^{K} y^k_i({H_0}^{(k)} \otimes e_m) + \sum_{i=K+1}^{2K}\sum_{k=1}^{K} y^k_i(\epsilon {H_0}^{(k)} \otimes e_m).$$

3.1. Stationary Probability vector

Let z be the Q 's the steady-state probability vector, then it has the form $z = (z_0, z_1, z_{2,}, \cdots)$, where z_0 is of dimension $2KS^0m$ and $z_1, z_2, z_3, \cdots$ are of dimension $2K\big(S^{(1)} + S^{(2)} + \cdots + S^{(K)}\big)m + K^2m$ respectively. Then z satisfies the condition $zQ = 0$ and $ze = 1$. From the stability of our system, the subvectors z_j of z, for $j \geq 2$ are derived from

$$z_j = z_1 R^{j-1}, \;\; j \geq 2$$

where R refers to the smallest non-negative solution of the matrix quadratic equation $R^2B^- + RB^0 + B^+ = 0$, as described by Neuts in Neuts(1994). The R matrix is a square matrix with order $2K\big(S^{(1)}+S^{(2)}+\cdots+S^{(K)}\big)m+K^2m$ is derived from the above quadratic equation and also fulfils $RB^-e = B^+e$.
From the below two equations

$$z_0A_{00} + z_1A_{10} = 0,$$

$$z_0A_{01} + z_1(B^0 + RB^-) = 0,$$

and with the normalising state equation

$$z_0 e_{2KS^{(0)}m} + z_1(1-R)^{-1} e_{2K\ m\left(S^{(1)}+S^{(2)}+\cdots+S^{(K)}\right)+K^2m} = 1,$$

we can find the sub vectors z_0 and z_1. With the reference of Latouche and Ramaswami in Latouche and Ramaswami(1993), by applying the required stages of the logarithmic reduction procedure for R, the matrix R might theoretically be computed.

4. Performance Measures

- The mean count of blocks of customers, comprising the one being served
  $Mblock = \sum_{k=1}^{\infty} kz_k e = z_1(1-R)^{-2}$.

- The mean count of blocks of customers, not including the one who is being served
  $\tilde{M}block = \sum_{k=1}^{\infty}(k-1)z_k e = Mblock - 1 + z_0e_0$.

- The mean count of customers in the pool
  $Mpool = \sum_{j=0}^{K-1} jz_0\tilde{e}_{0j} + \sum_{k=1}^{\infty}\sum_{j=0}^{K-1}\sum_{n=1}^{K} jz_{kjn}e = \sum_{j=0}^{K-1} jz_0\tilde{e}_{0j} + \sum_{j=0}^{K-1} jx_1(1-R)^{-1}\tilde{e}_j$.

  where $\tilde{e}_{0j}$ is the column vector of order $2KS^{(0)}m$ with $(j(S^{(0)})m\ +1)^{st}$ to $((j+1)S^{(0)}m)^{th}$ entries and $((K+j)(S^{(0)})m\ +1)^{st}$ to $((K+(j+1))S^{(0)}m)^{th}$ entries are 1 and all other entries are zeros; $\tilde{e}_j$ is the column vector of order $(2K(S^{(1)}+\cdots+S^{(K)})m\ +\ K^2m$ with $(j(S^{(1)}+\cdots+S^{(K)})m\ +1)^{st}$ to $((j+1)(S^{(1)}+\cdots+S^{(K)})m)^{th}$ entries, $((K+j)(S^{(1)}+\cdots+S^{(K)})m+1)^{st}$ to $((K+(j+1))(S^{(1)}+\cdots+S^{(K)})m)^{th}$ entries and $((2K(S^{(1)}+\cdots+S^{(K)})m)+jKm+1)^{st}$ to $((2K(S^{(1)}+\cdots+S^{(K)})m)+(j+1)Km)^{th}$ entries are 1 and all other entries are zeros.

- The mean count of customers getting the service
  $Mservice = \sum_{k=1}^{\infty}\sum_{j=0}^{K-1}\sum_{n=1}^{K} nz_{kjn}e$
  $= \sum_{n=1}^{K} n(z_1(1-R)^{-1}e_{0n} + z_1(1-R)^{-1}e_{1n} + \cdots + z_1(1-R)^{-1}e_{K-1n})$

$= \sum_{n=1}^{K} n\big(\sum_{j=0}^{K-1} z_1(1-R)^{-1}e_{jn}\big)$

where e_{jn} are all column vectors of order $(2K(S^{(1)} + \cdots + S^{(K)})m \ + \ K^2m$ defined as for $n = 1$, e_{j1} has $(j\big(\sum_{k=1}^{K} S^{(k)}m\big)+1)^{st}$ to $(j\big(\sum_{k=1}^{K} S^{(k)}m\big)+S^{(1)}m)^{th}$ entries are 1 and all other elements are zeros; for $2 \leq n \leq K-1$, e_{jn} has $(j\big(\sum_{k=1}^{K} S^{(k)}m\big) + \big(\sum_{l=1}^{n-1} S^{(l)}m\big) + 1)^{st}$ to $(j\big(\sum_{k=1}^{K} S^{(k)}m\big) + \big(\sum_{l=1}^{n} S^{(l)}m\big))^{th}$ entries are 1 and all other entries are zeros; and for $n = K$, e_{jK} has $(j\big(\sum_{k=1}^{K} S^{k}m\big) + \big(\sum_{l=1}^{K-1} S^{l}m\big) + 1)^{st}$ to $((K)\big(\sum_{k=1}^{K} S^{(k)}m\big))^{th}$ entries are 1 and all other entries are zeros.

- The mean count of customers in the working breakdown
  $Mworking\,breakdown = \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} n z_{kjn} e$

  $= \sum_{n=1}^{K} n(z_1(1-R)^{-1}\hat{e}_{0n} + z_1(1-R)^{-1}\hat{e}_{1n} + \cdots + z_1(1-R)^{-1}\hat{e}_{K-1n})$

  $= \sum_{n=1}^{K} n\big(\sum_{j=0}^{K-1} z_1(1-R)^{-1}\hat{e}_{jn}\big)$

  where $\hat{e}_{jn}$ are all column vectors of order $(2K(S^{(1)} + \cdots + S^{(K)})m \ + \ K^2m$ defined as for $n = 1$, $\hat{e}_{j1}$ has $((K+j)\big(\sum_{k=1}^{K} S^{(k)}m\big) + 1)^{st}$ to $((K+j)\big(\sum_{k=1}^{K} S^{(k)}m\big) + S^{(1)}m)^{th}$ entries are 1 and all other elements are zeros; for $2 \leq n \leq K-1$, $\hat{e}_{jn}$ has $((K+j)\big(\sum_{k=1}^{K} S^{(k)}m\big) + \big(\sum_{l=1}^{n-1} S^{(l)}m\big) + 1)^{st}$ to $((K+j)\big(\sum_{k=1}^{K} S^{(k)}m\big) + \big(\sum_{l=1}^{n} S^{(l)}m\big))^{th}$ entries are 1 and all other entries are zeros; and for $n = K$, $\hat{e}_{jK}$ has $((K+j)\big(\sum_{k=1}^{K} S^{k}m\big) + \big(\sum_{l=1}^{K-1} S^{j}m\big) + 1)^{st}$ to $((2K)\big(\sum_{k=1}^{K} S^{k}m\big))^{th}$ entries are 1 and all other entries are zeros.

- The mean count of customers in the repair
  $Mrepair = \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} z_{kjn}\hat{e}$ where $\hat{e}$ is column vector of order $(2K(S^{(1)} + \cdots + S^{(K)})m \ + \ K^2m$ with first $(2K(S^{(1)} + \cdots + S^{(K)})m$ entries are zeros and the rest K^2m entries are 1.

- The mean system size at some random time, comprising the customers getting service
  $Msystem = \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} (kK + j + n) z_{kjn} e + \sum_{j=0}^{K-1} j z_0 \tilde{e}_{0j} + \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} z_{kjn}\hat{e}$

  $= K(Mblock) + Mpool + Mservice + Mworking\,breakdown + Mrepair$

- The mean system size at an arbitrary moment not including the customers getting service
  $\tilde{M}system = \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} ((k-1)K + j + n) z_{kjn} e + \sum_{j=0}^{K-1} j z_0 \tilde{e}_{0j} + \sum_{k=1}^{\infty} \sum_{j=0}^{K-1} \sum_{n=1}^{K} z_{kjn}\hat{e}$

  $= K(\tilde{M}block) + Mpool + Mservice + Mworking\,breakdown + Mrepair$

- The probability of the server is not in use at an arbitrary moment
  $Pidle = z_0 e_0$

5. Numerical Results

In this section, we use graphical representations of the numerical values to investigate the model's nature, where the numerical values for arrival process, admission period and service process were referred by Chakravarthy in Chakravarthy(2010).

Numerical values for Markovian arrival process are,

- (Exp-A)

$$D_0 = \begin{pmatrix}-1\end{pmatrix}, \quad D_1 = \begin{pmatrix}1\end{pmatrix}$$

- (Erl-A)

$$D_0 = \begin{pmatrix}-2 & 2\\ 0 & -2\end{pmatrix}, \quad D_1 = \begin{pmatrix}0 & 0\\ 2 & 0\end{pmatrix}$$

- (Hyp-A)

$$D_0 = \begin{pmatrix}-1.90 & 0\\ 0 & -0.19\end{pmatrix}, \quad D_1 = \begin{pmatrix}1.710 & 0.190\\ 0.171 & 0.019\end{pmatrix}$$

- (Neg-A)

$$D_0 = \begin{pmatrix} -1.00243 & 1.00243 & 0 \\ 0 & -1.00243 & 0 \\ 0 & 0 & -225.797 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0.01002 & 0 & 0.99241 \\ 223.539 & 0 & 2.258 \end{pmatrix}$$

- (Pos-A)

$$D_0 = \begin{pmatrix} -1.00243 & 1.00243 & 0 \\ 0 & -1.00243 & 0 \\ 0 & 0 & -225.797 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0.99241 & 0 & 0.01002 \\ 2.258 & 0 & 223.539 \end{pmatrix}$$

where $(Exp - A)$ is Exponential Arrival, $(Erl - A)$ is Erlang Arrival, $(Hyp - A)$ is Hyper-Exponential Arrival, $(Neg - A)$ is MAP-Negative Correlation Arrival and $(Pos - A)$ is MAP-Positive Correlation Arrival and numerical values for Phase type admission period are

- Exponential admission period (Exp-AP)

$$\gamma = \begin{pmatrix} 1 \end{pmatrix}, \quad G = \begin{pmatrix} -1 \end{pmatrix}$$

- Erlang admission period (Erl-AP)

$$\gamma = \begin{pmatrix} -1, & 0 \end{pmatrix}, \quad G = \begin{pmatrix} -2 & 2 \\ 0 & -2 \end{pmatrix}$$

- Hyper-Exponential admission period (Hyp-AP)

$$\gamma = \begin{pmatrix} 0.8, & 0.2 \end{pmatrix}, \quad G = \begin{pmatrix} -2.80 & 0 \\ 0 & -0.28 \end{pmatrix}$$

We assume that the numerical values for Phase type distributions $(\sigma^k, H^{(k)})$ of service times to k customers for $1 \leq k \leq K$ where all either exponential distributions or Erlang distributions, regardless of size.

- Exponential service (E-S)

$$\sigma^k = \begin{pmatrix} 1 \end{pmatrix}, \quad H^{(k)} = \begin{pmatrix} -1 \end{pmatrix} \; \forall \, 1 \; \leq \; k \; \leq \; K$$

- Erlang service (Erl-S)

$$\sigma^k = \begin{pmatrix} -1, & 0 \end{pmatrix}, \quad H^{(k)} = \begin{pmatrix} -2 & 2 \\ 0 & -2 \end{pmatrix}$$

Illustration: 5.1 We have illustrated the effect of the rate of admission period in counter to the mean size of the system, where service follows both exponential and Erlang distribution in the figure 3. We assume $\tau = 1$, $\lambda = 1$, $\delta = 6$, $\alpha = 1$, $\beta = 2$, $c = 0.6$ and $\epsilon = 0.4$ and we increase the pace of admission so that the values does not affect the stability of the system. We execute the example for batch size $N = 2$.

In figures 2(a) and 2(b) we fixed the arrival to follow exponential distribution and we assume the admission period to follow exponential, Erlang and hyper-exponential distribution respectively. We observed that by accelerating the admission period rate, the mean system size drops steadily both in exponential service and Erlang service, in all three admission period distributions.

In figures 2(c) and 2(d) we fixed the arrival to follow Erlang distribution and we assume the admission period to follow exponential, Erlang and hyper-exponential distribution respectively. We observed that by accelerating the admission period rate in exponential service, the mean system size attains the higher values in hyper-exponential admission period and lower values in exponential admission period. Also, we observed that in Erlang service, when the admission period rate goes up, the mean system size attains the higher values in exponential admission period and lower values in hyper-exponential admission period.

In figures 2(e) and 2(f) we fixed the arrival to follow hyper-exponential distribution and we assume the admission period to follow exponential, Erlang and hyper-exponential distribution respectively. We observed that by accelerating the admission period rate in both exponential and Erlang service, the mean system size attains the higher values in Erlang admission period and lower values in exponential admission period.

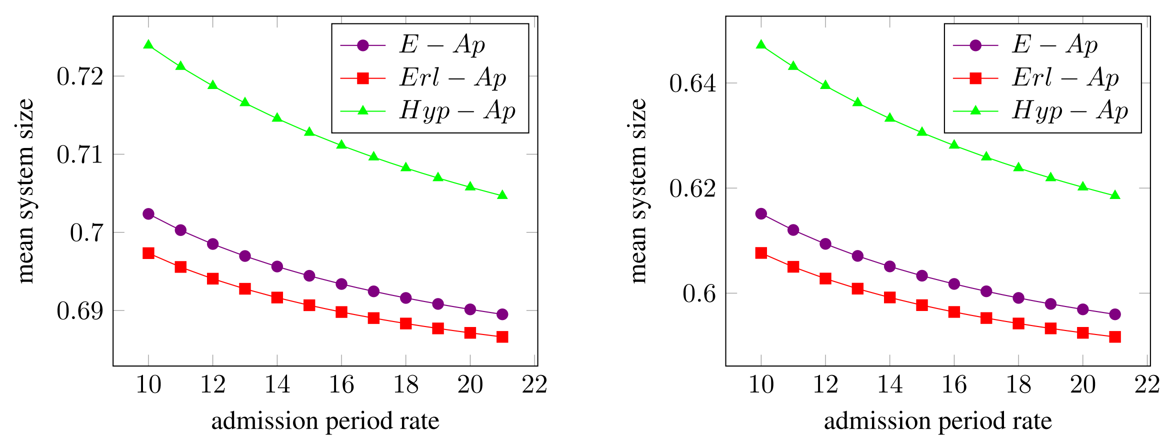
In figures 2(g) we fixed the arrival to follow MAP-positive correlation distribution and we assume the admission period to follow exponential, Erlang and hyper-exponential distribution respectively. We observed that by accelerating the admission period rate, the mean system size attains huge values for all admission periods and attains its maximum values in exponential admission period distribution.

Illustration: 5.2. We assume $\tau = 1$, $\lambda = 1$, $\delta = 6$, $\alpha = 1$, $\beta = 2$, $c = 0.6$ and $\epsilon = 0.4$ and we execute the example for batch size $N = 2$.

In Figure 3, we fixed the service to follow Erlang distribution and we have analysed the mean system size against the repair rate and service rate, under various admission period rates and arrival rates. We increase the pace of respective rates, so that the values does not affect the stability of the system. we have shown that the mean system size decreases steadily for Erlang arrival with hyper-exponential admission period, exponential arrival with Erlang admission period, hyper-exponential arrival with exponential admission period and MAP-negative correlation arrival with exponential admission period.

In Figure 4, we fixed the service to follow exponential distribution and we have analysed the mean system size against the repair rate and service rate, under various admission period rates and arrival rates. We increase the pace of respective rates, so that the values does not affect the stability of the system. we have shown that the mean system size decreases gradually for MAP-positive correlation arrival with Erlang admission period, MAP-negative correlation arrival with hyper-exponential admission period, Erlang arrival with exponential admission period, hyper-exponential arrival with exponential admission period and exponential arrival with exponential admission period.

In Figure 5, we fixed the service to follow exponential or Erlang distribution and we have analysed the mean system size against the service rate and breakdown rate, under various admission period rates and arrival rates. We increase the pace of respective rates, so that the values does not affect the stability of the system. we have shown that the mean system size decreases steadily in all the cases.



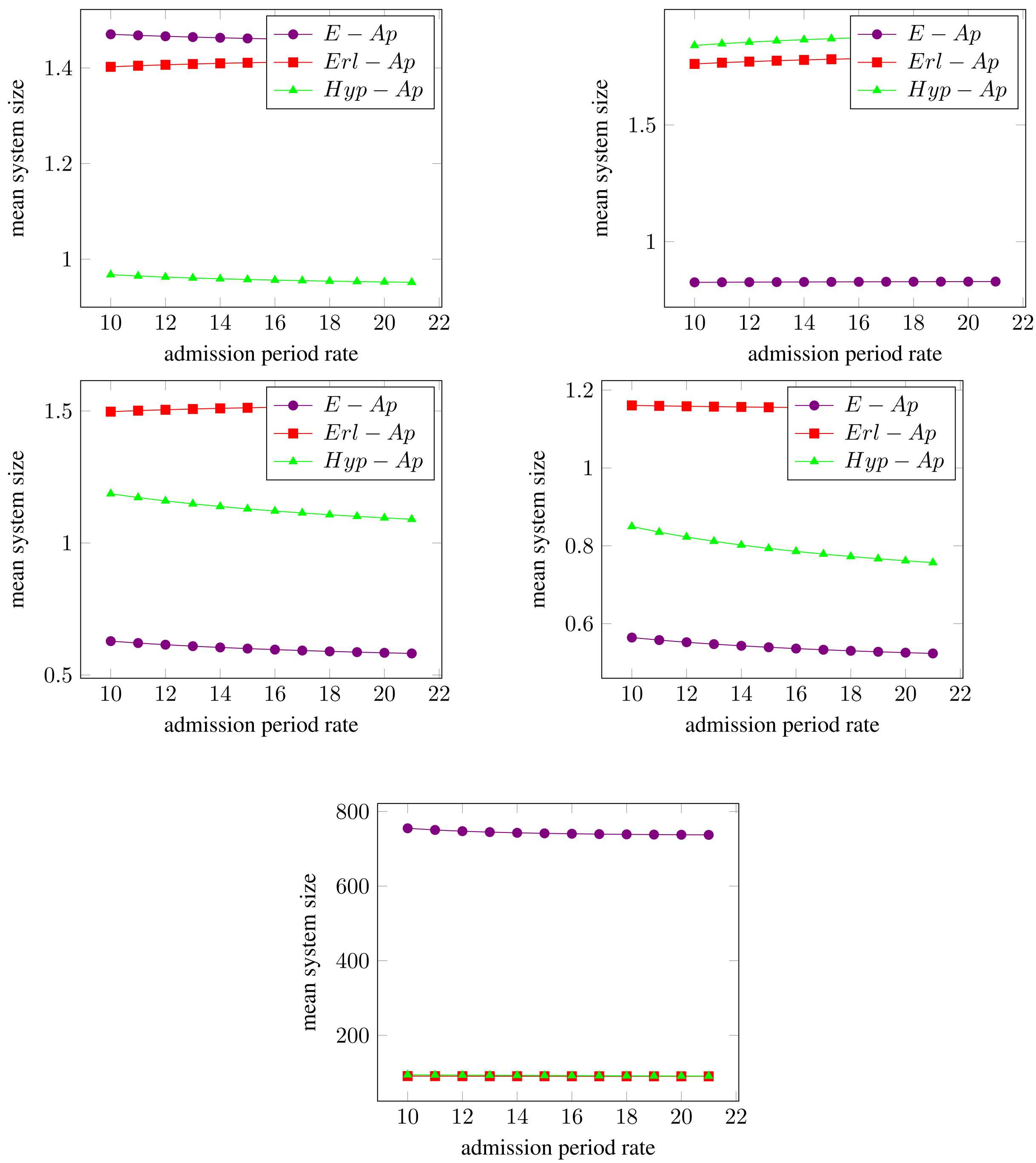


Figure 2: Admission Period Rate $Exp - Ap,\ Erl - Ap,\ Hyp - Ap$ (vs) Msystem.

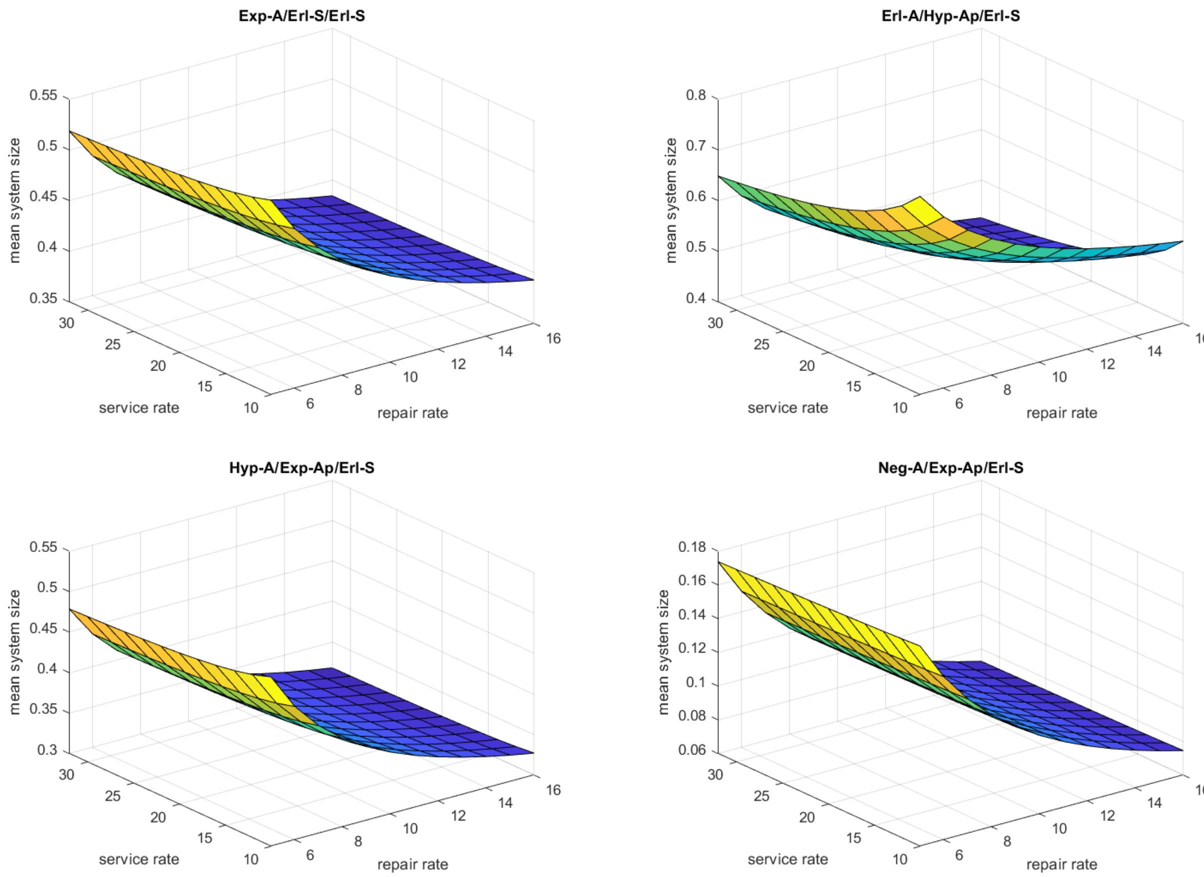
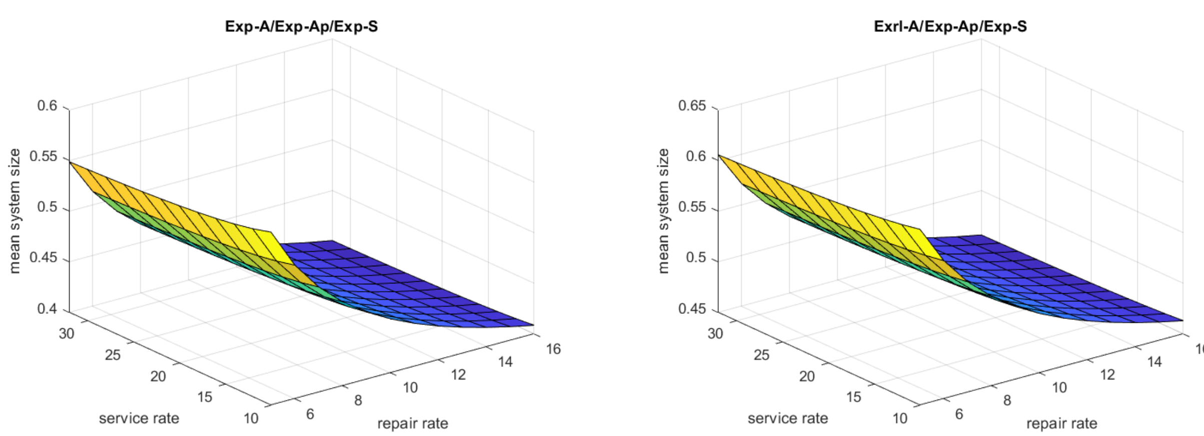


Figure 3: Repair Rate and Service Rate (vs) Msystem with $Erlang\ Service$



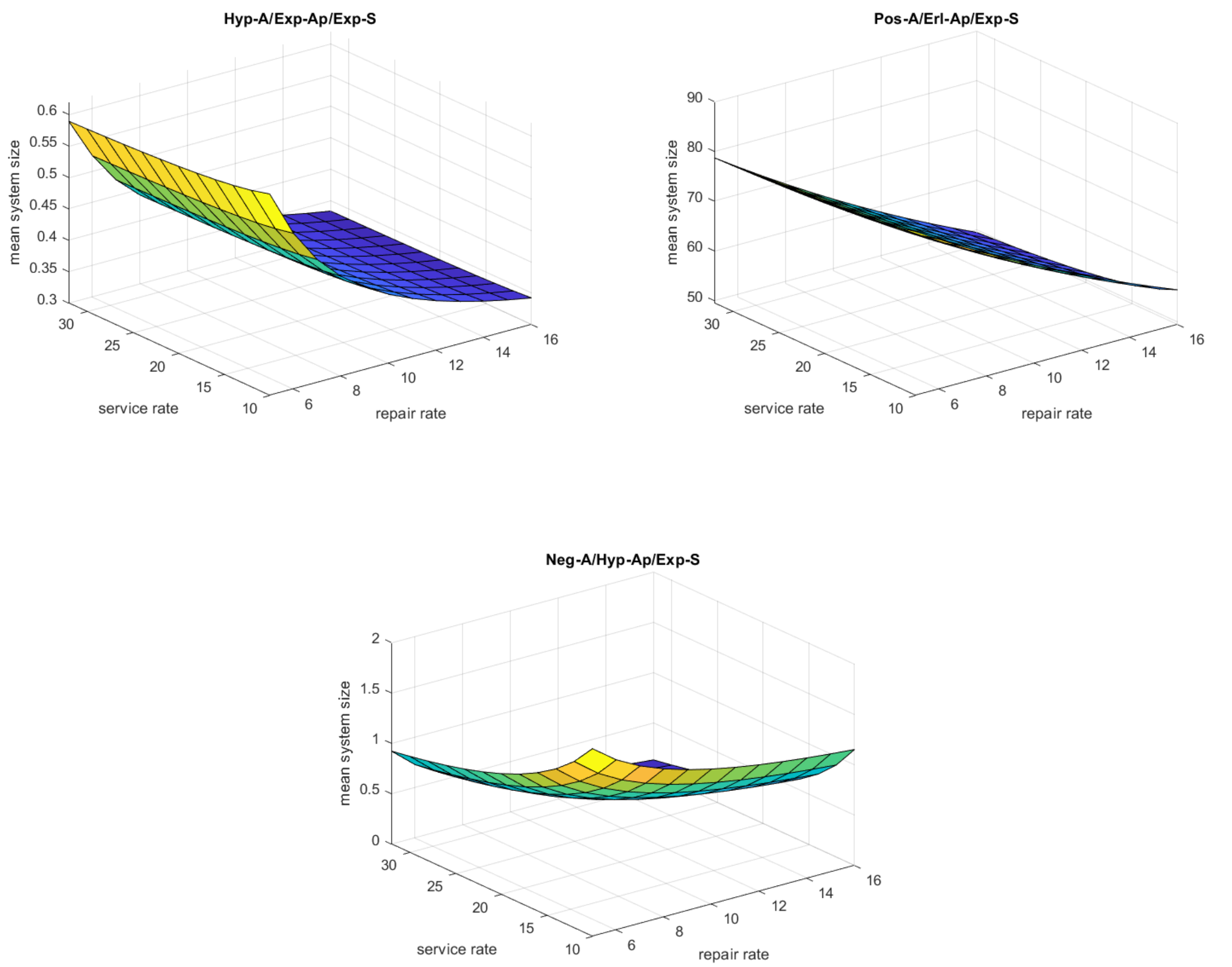
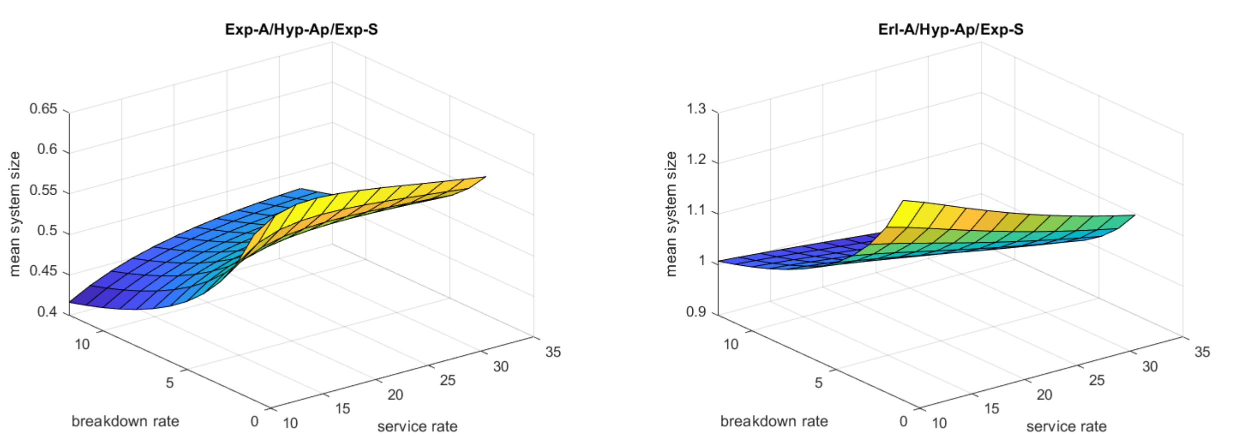


Figure 4: Repair Rate and Service Rate (vs) Msystem with $Exponential\ Service$



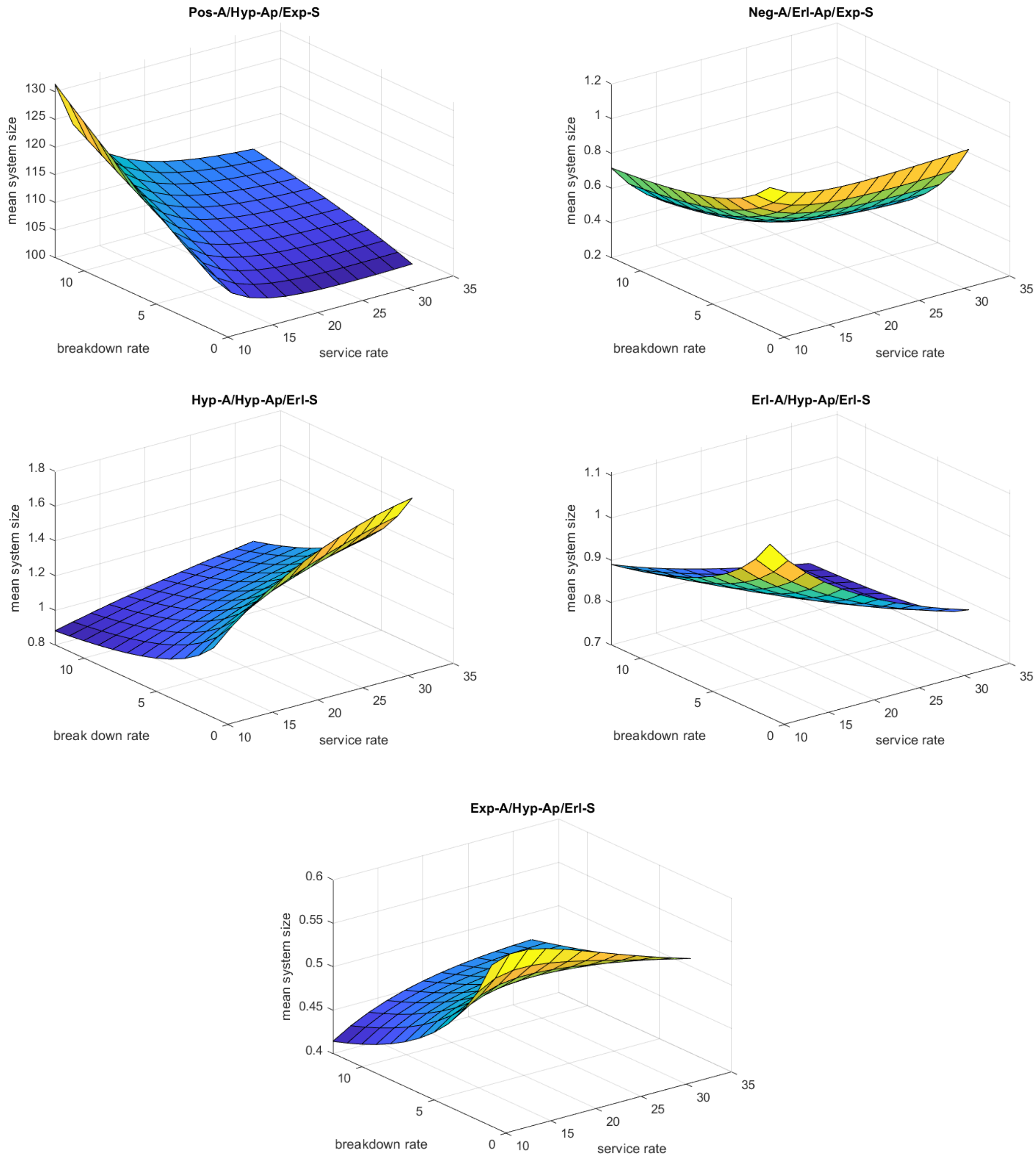


Figure 5: Service Rate and Breakdown Rate (vs) Msystem with *Exponential and Erlang Service*

6. The conclusion

In this research, we examined a group service queueing model where arrivals follow a Markovian arrival process. When a server breaks down, arrivals may balk the system. The service uses phase-type distributions, where the group size might change. Depending on the group size, (that is, the total number of customers receiving service) different Phase-type distribution representations for the service time exist. The server could break down at any time, in which case it would not proceed to repair but would instead continue to serve the affected customer group at a slower pace. After the service for that specific customer group is finished, the server will then promptly undergo repair to get it fixed. The rate of breakdown and the process of repair have an exponential distribution. We compared the mean size of the system counter to service, admission period, balking, and repair rates, respectively, using the numerical values of arrival and service periods of time. The results are shown through images. We are currently investigating how this model might be expanded to retrial and bulk arrival queueing models with different server limitations.

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