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Generalized Bayesian Double Group Sampling Plan for Manufacturing Industry

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Abstract

Quality is not simply a goal or a choice for organizations, it is also a need for success in the global market. Acceptance sampling is one of two key strategies for quality assurance in manufacturing industry, along with statistical process control. After inspection the lot is either accepted or rejected based on the acceptance criteria. If historical information about the product is available, then the most effective approach for making the appropriate judgement is the Bayesian approach. To estimate quality regions, this work presents a Bayesian double group sampling plan (BDGSP). Based on acceptance criteria, the binomial distribution is used to build a likelihood function for defective and non-defective items. The beta distribution is utilized as the prior distribution to determine the average probability of acceptance. For some stated values of producer's and consumer's risks, four different quality regions are estimated. The suggested plan estimates variation point values based on various design parameter combinations. Producer's and consumer's risks correlate with acceptable quality levels and limiting quality levels of regions, respectively. Operating characteristic curves are used to monitor the effects of change in the values of specified parameters and for comparison with existing sampling plan. Application based on real data set proves that the proposed plan is applicable for existing manufacturing industry policies.

Key Words: Acceptance sampling; Binomial; Beta distribution; Bayesian; Consumer's risk.

Mathematical Subject Classification: 62D05

1. Introduction

There is fierce competition between businesses nowadays to establish a solid reputation in the industry. Therefore, a well-known corporation is judged on its quality. A high-grade product on the market is necessary to uphold the company's excellent reputation. The entire process from the raw materials to the finished product should be inspected to assure the product's excellent quality (Thomas & Kumar 2024a; 2024b). One option for evaluating the lot of a product is to perform a complete inspection; the alternative is acceptance sampling. Acceptance sampling is a method used for product inspection because it might not be possible to conduct a 100% inspection. In acceptance sampling, we choose a sample from the lot that is being inspected in order to decide whether to accept or reject the lot. Since there are not many items needed for acceptance sampling, the inspection's time and costs can be kept to a minimum (Thomas & Kumar 2022; 2023). Various acceptance sampling processes have been reported in the literature over the years. Epstein (1954) published the first single sampling plan (SSP) that used an exponential distribution to represent the lifetime distribution of a lot. Two approaches, one for a replacement scenario and the other for a non-replacement

situation were examined in order to identify the design parameters. Based on the mean lifetime, the mathematical work was completed for the estimated number of observations, acceptance probability, and testing period.

SSP was later replaced by Dodge (1955) with chain sampling plan-1 (ChSP-1) by carrying into consideration the previous sample in SSP. After that, an attribute single sample plan was proposed to lower average cost (Hald, 1965). If the product life is based on the specified distribution, the required design parameters can be determined by using any acceptance sampling plan. Various distributions are considered in different techniques for various types of data (Chukhrova & Johannssen, 2018). Such techniques are legitimate if they meet the requirements according to pre-specified parameters such as consumer's risk, producer's risk, acceptance number and appropriate testing (Lio, Tsai, & Wu, 2010).

Only sample data is precisely employed in conventional statistics, however older data are frequently relevant to the sample data as well, that effects the choice as well. It is considered prior information known as prior distribution and comes from the similar circumstances as previous lots. It is significant because it represents the data distribution of an unknown parameter, maintaining estimation accuracy while evaluating the chance of a defect. The Bayesian acceptance sampling approach is based on the combination of current lot information and the prior information. The quality that will be examined is expected to be distributed according to the earlier distribution. This distribution is known as prior since it is created before the sample is collected. Sample distribution, sometimes referred to as data distribution, is the empirical information based on a sample that is being studied. A choice is made based on a combination of previous knowledge and empirical data. Bayesian sampling plans require a smaller sample size than a conventional sampling plan with the same producer's and consumer's risks (Suresh & Sangeetha, 2011). Suresh and Latha (2001) proposed a Bayesian single sampling plan (BSSP) with gamma prior for the Poisson distribution. The variation points and tangent at variation point were calculated, and tables for selecting the plan's parameters were provided. They expand their Bayesian chain sampling plan (BChSP-1) for construction and performance measurements by utilizing quality regions (Suresh & Sangeetha, 2011). They use quality regions to enhance their BChSP-1 for construction and performance measures (Suresh & Latha, 2001; Suresh & Sangeetha, 2011). A SSP for Poisson distribution by (Subramani & Haridoss, 2013; Subbiah & Latha, 2017) and Poisson distribution was changed to a BSSP utilizing AQL and LQL by (Raju & Vidya, 2017).

When there are numerous testers available for inspection, Aslam and Jun (2009) developed a group acceptance sampling plan (GASP) to evaluate several items at the identical time. The sample size n is divided into g groups based on the number of testers available. Risk-based sampling plans are described by the operating characteristic (OC) curve, drawing upon the producer and consumer type of risks (Fayomi & Khan, 2024). Hafeez, Aziz and Du (2023), recently presented a Bayesian new two-sided group chain sampling plan (BNTSGChSP) and Hafeez and Aziz (2023), presented a Bayesian two-sided group chain sampling plan (BTSGChSP) to estimated quality regions using different consumer's and producer's risk values. Hafeez, Aziz, Zain, and Kamarudin (2022), consider preceding lots to make a decision about the current lot of one-sided sampling plans and consider preceding with succeeding lots to decide the current lot for two-sided sampling plans by Hafeez, Aziz and Du (2023).

If preceding and succeeding lots are not available or include only current lot for inspection, then the proposed plans by (Hafeez, Aziz, & Du, 2023; Hafeez, Aziz, Zain, & Kamarudin, 2022) cannot be used. In this situation BDGSP can be used for inspection and make a decision about the current lot, this is the main objective of this study. A good plan must use small values of producer's and consumer's risks. This study focuses on the attribute sampling plan and proposes a BDGSP for the inspection of attributes of a product. During inspection BDGSP is sufficient to consider all attributes of a product. In the proposed plan, the producer's risk (α) is associated with an acceptable quality level (AQL), and the consumer's risk (β) is associated with a limiting quality level (LQL). Using AQL and LQL inflection points four different quality regions are estimated for BDGSP. For all quality regions, AQL and LQL are estimated for the specified values design parameters; consider the number of testers (r_1, r_2), and the number of groups (g_1, g_2). Also, for the prior distribution shape parameter (s) numerical illustration is provided for all quality regions. As opposed to the current plans, which base their sampling strategies on a single point-wise description of quality, the new plans base their sample strategies on quality regions, which provide greater coverage and better protection for customers as well as manufacturers.

The next section outlines the design structures of the proposed BDGSP (a, b), incorporating mathematical formulations, simulations, and the estimation of quality regions. Section 3 presents the results, including graphical representations and a comparative analysis with existing sampling plans. In Section 4, we demonstrate the application

of our methodology using a real dataset. The final section offers a comprehensive discussion of the key findings and concluding observations from this study.

2. Methodology

2.1 Bayesian Double Group Sampling Plan BDGSP (a, b)

The steps below are the foundation for the BDGSP (a, b) operating procedure.

- Select a random sample of size n_1, and divide into g_1 groups such as r_1 items are assigned to each group, therefore sample size required ($n_1 = g_1 * r_1$).
- Count the number of defectives d_1, that is the sum of defectives in the first stage.
- If $d_1 \leq a$ accept the lot; if $d_1 > b$ reject the lot; if $a < d_1 \leq b$, go for a second random sample.
- Select a random sample of size n_2, and divide into g_2 groups such as r_2 items are assigned to each group, therefore sample size required ($n_2 = g_2 * r_2$).
- Count the number of defectives d_2, that is the sum of defectives in the second stage.
- If $d_1 + d_2 \leq b$ accept the lot, if $d_1 + d_2 > b$ reject the lot.

The above steps in the operating procedure can summarize in a flow chart shown in Figure 1.

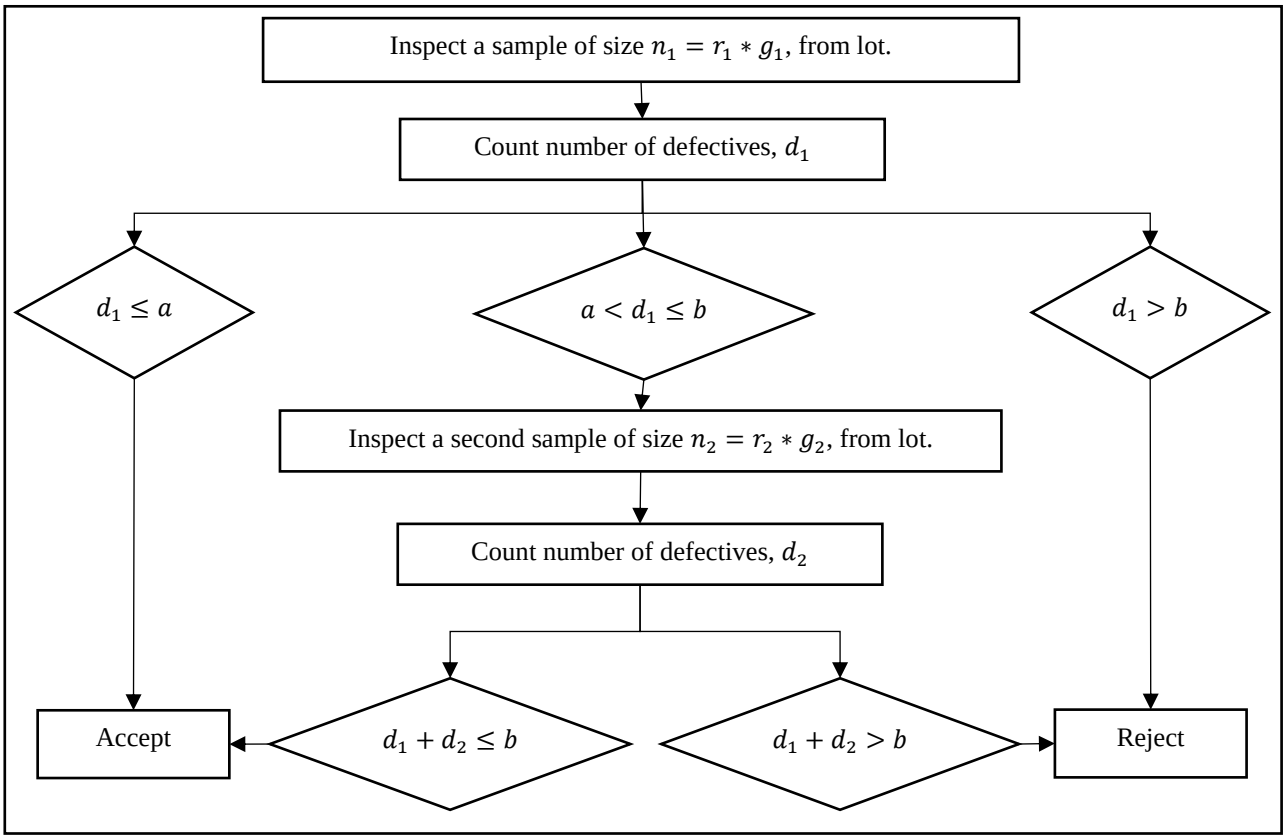

Figure 1: Flow chart for double group sampling plan.

The probabilities of both a defective and non-defective product can be calculated by using binomial distribution. Due to the lot's independent and identical trials, it can be used based on the properties of a binomial experiment. The inspection's findings can be classified as either defective or non-defective, two outcomes that are mutually exclusive. Also, when population size N is large, and the sample fraction is less than 0.10, binomial distribution can be applied (Appaia, Krishnan, & Kalaiselvi, 2014). Hence the probability of the number of defective products d can be estimated by the binomial probability distribution function (PDF).

$$p(d) = \binom{rg}{d} p^d (1-p)^{rg-d} \tag{1}$$

where p, is a parameter represents an unknown probability of defective.

Now we consider a special case for this study, where BDGSP ($a = 0, b = 1$) and the mathematical work will be done for zero and one defective product. After replacing zero and one defective in equation (1), Figure 1 and the operating procedure of BDGSP can be summarized as in the mathematical equation:

$$L(p) = \binom{r_1 g_1}{0} p^0 (1-p)^{r_1 g_1 - 0} + \binom{r_1 g_1}{1} p^1 (1-p)^{r_1 g_1 - 1} \left[\binom{r_2 g_2}{0} p^0 (1-p)^{r_2 g_2 - 0} \right] \tag{2}$$

$$L(p) = (1-p)^{r_1 g_1} + r_1 g_1 p (1-p)^{r_1 g_1 - 1} (1-p)^{r_2 g_2} \tag{3}$$

$$L(p) = (1-p)^{r_1 g_1} + r_1 g_1 p (1-p)^{r_1 g_1 + r_2 g_2 - 1} \tag{4}$$

In Bayesian beta distribution is an appropriate prior distribution for the binomial distribution (Latha & Arivazhagan, 2015). This means that in a binomial PDF p unknown parameter has the beta distribution with PDF.

$$f(p) = \frac{1}{\beta(s,t)} p^{s-1} (1-p)^{t-1} \tag{5}$$

where $\mu = \frac{s}{s+t}$ is the beta distribution's mean and $s, t > 0$ both serve as shape parameters.
In Bayesian, the general equation for the average probability of acceptance is.

$$P = \int_0^1 L(p) f(p)\, dp \tag{6}$$

After replacing Equation (4) and Equation (5) in Equation (6), we get the equation of the average probability of acceptance for BDGSP based on beta and binomial distributions.

$$P = \int_0^1 ((1-p)^{r_1 g_1} + r_1 g_1 p (1-p)^{r_1 g_1 + r_2 g_2 - 1}) \frac{1}{\beta(s,t)} p^{s-1} (1-p)^{t-1}\, dp \tag{7}$$

$$P = \frac{1}{\beta(s,t)} [\beta(s, r_1 g_1 + t) + r_1 g_1 \beta(s+1, r_1 g_1 + r_2 g_2 + t - 1)] \tag{8}$$

$$P = \frac{\Gamma(s+t)\Gamma(r_1 g_1 + t)}{\Gamma(t)\Gamma(s + r_1 g_1 + t)} + r_1 g_1 s \frac{\Gamma(s+t)\Gamma(r_1 g_1 + r_2 g_2 + t - 1)}{\Gamma(t)\, \Gamma(s + r_1 g_1 + r_2 g_2 + t)} \tag{9}$$

Equation (9) is a mixed distribution of beta and binomial distributions. As the beta distribution has mean $\mu = \frac{s}{s+t}$ that gives $s + t = \frac{s}{\mu}$, and $t = \frac{s}{\mu} - s$; by replacing these terms in Equation (9) and solving it,
for $s = 1$:

$$P = \frac{1-\mu}{r_1 g_1 \mu + 1 - \mu} + \frac{r_1 g_1 \mu (1-\mu)}{(r_1 g_1 \mu + r_2 g_2 \mu + 1 - \mu)(r_1 g_1 \mu + r_2 g_2 \mu + 1 - 2\mu)} \tag{10}$$

for $s = 2$:

$$P = \frac{(2-\mu)(2-2\mu)}{(r_1 g_1 \mu + 2 - \mu)(r_1 g_1 \mu + 2 - 2\mu)} + \frac{2 r_1 g_1 \mu (2-\mu)(2-2\mu)}{(r_1 g_1 \mu + r_2 g_2 \mu + 2 - \mu)(r_1 g_1 \mu + r_2 g_2 \mu + 2 - 2\mu)(r_1 g_1 \mu + r_2 g_2 \mu + 2 - 3\mu)} \tag{11}$$

for $s = 3$:

$$P = \frac{(3-\mu)(3-2\mu)(3-3\mu)}{(r_1 g_1 \mu + 3 - \mu)(r_1 g_1 \mu + 3 - 2\mu)(r_1 g_1 \mu + 3 - 3\mu)} + \frac{3 r_1 g_1 \mu (3-\mu)(3-2\mu)(3-3\mu)}{(r_1 g_1 \mu + r_2 g_2 \mu + 3 - \mu)(r_1 g_1 \mu + r_2 g_2 \mu + 3 - 2\mu)(r_1 g_1 \mu + r_2 g_2 \mu + 3 - 3\mu)(r_1 g_1 \mu + r_2 g_2 \mu + 3 - 4\mu)} \tag{12}$$

By using Newton's approximation, Equations. (10, 11 & 12) are used to find the quality regions for BDGSP, where μ is used as the point of control by reducing P. When five testers are used ($r_1 = r_2 = 5$), Table 1 represents the average proportion of defectives that are generated for all prespecified design parameters.

Table 1: Certain μ values in BDGSP for specified values of P, g_1 and g_2, when $r_1 = r_2 = 5$.

s	g_1	g_2	0.99	0.95	0.90	0.75	0.5	0.25	0.1	0.05	0.01
1	5	5	0.0026	0.0068	0.0108	0.0234	0.0557	0.1381	0.3159	0.4908	0.8328
		10	0.0020	0.0053	0.0087	0.0194	0.0477	0.1222	0.2889	0.4599	0.8152
		15	0.0017	0.0046	0.0077	0.0175	0.0443	0.1161	0.2787	0.4480	0.8082
		20	0.0015	0.0042	0.0070	0.0164	0.0425	0.1131	0.2738	0.4423	0.8048
	10	5	0.0016	0.0041	0.0066	0.0141	0.0336	0.0855	0.2112	0.3582	0.7422
		10	0.0013	0.0034	0.0054	0.0118	0.0286	0.0740	0.1873	0.3248	0.7131
		15	0.0011	0.0029	0.0048	0.0106	0.026	0.0684	0.1754	0.3080	0.6973
		20	0.0010	0.0027	0.0044	0.0098	0.0244	0.0650	0.1688	0.2984	0.6879
	15	5	0.0011	0.003	0.0048	0.0103	0.0246	0.0632	0.1619	0.2869	0.6748
		10	0.0009	0.0025	0.0041	0.0088	0.0212	0.0551	0.1435	0.2589	0.6433
		15	0.0008	0.0022	0.0036	0.0079	0.0192	0.0506	0.1330	0.2427	0.6234
		20	0.0008	0.002	0.0033	0.0073	0.0179	0.0477	0.1265	0.2325	0.6104
	20	5	0.0009	0.0023	0.0038	0.0081	0.0194	0.0505	0.1319	0.2404	0.6202
		10	0.0008	0.002	0.0033	0.0071	0.017	0.0445	0.1179	0.2178	0.5896
		15	0.0007	0.0018	0.003	0.0064	0.0155	0.0408	0.1091	0.2034	0.5686
		20	0.0006	0.0017	0.0028	0.0059	0.0145	0.0384	0.1032	0.1937	0.5538
2	5	5	0.0029	0.0073	0.0114	0.0228	0.0470	0.0952	0.1792	0.2605	0.5012
		10	0.0023	0.0058	0.0091	0.0186	0.0398	0.0839	0.1628	0.2404	0.4752
		15	0.0019	0.005	0.0079	0.0166	0.0368	0.0798	0.1576	0.2344	0.4680
		20	0.0017	0.0045	0.0072	0.0155	0.0353	0.078	0.1555	0.2320	0.4653
	10	5	0.0018	0.0045	0.0069	0.0138	0.0282	0.0571	0.1095	0.1630	0.3437
		10	0.0015	0.0037	0.0057	0.0114	0.0238	0.0493	0.096	0.1445	0.3121
		15	0.0012	0.0032	0.005	0.0102	0.0216	0.0455	0.0899	0.1364	0.2988
		20	0.0011	0.0029	0.0046	0.0094	0.0202	0.0433	0.0867	0.1324	0.2923
	15	5	0.0013	0.0032	0.005	0.01	0.0205	0.0418	0.0807	0.1215	0.2669
		10	0.0011	0.0027	0.0043	0.0086	0.0177	0.0363	0.0708	0.1074	0.2401
		15	0.0010	0.0024	0.0038	0.0076	0.016	0.0333	0.0655	0.0999	0.2262
		20	0.0009	0.0022	0.0035	0.007	0.0149	0.0313	0.0624	0.0957	0.2184
	20	5	0.0010	0.0026	0.004	0.0079	0.0162	0.0331	0.0643	0.0975	0.2193
		10	0.0009	0.0022	0.0034	0.0069	0.0142	0.0291	0.057	0.0867	0.1978
		15	0.0008	0.002	0.0031	0.0062	0.0129	0.0267	0.0526	0.0804	0.1852
		20	0.0007	0.0019	0.0029	0.0057	0.012	0.0251	0.0498	0.0764	0.1773
3	5	5	0.0031	0.0076	0.0117	0.0226	0.0444	0.0838	0.1455	0.2010	0.3597
		10	0.0024	0.006	0.0093	0.0184	0.0375	0.0739	0.1328	0.1865	0.3420
		15	0.0020	0.0051	0.008	0.0164	0.0346	0.0704	0.1292	0.1828	0.3382
		20	0.0018	0.0046	0.0073	0.0152	0.0331	0.0689	0.1279	0.1816	0.3371
	10	5	0.0019	0.0046	0.0071	0.0136	0.0266	0.0500	0.0871	0.1215	0.2269
		10	0.0015	0.0038	0.0058	0.0113	0.0225	0.0430	0.0764	0.1077	0.2054
		15	0.0013	0.0033	0.0051	0.0101	0.0203	0.0396	0.0718	0.1021	0.1975
		20	0.0012	0.0030	0.0046	0.0092	0.0190	0.0378	0.0695	0.0995	0.1941
	15	5	0.0014	0.0034	0.0051	0.0099	0.0194	0.0364	0.0637	0.0893	0.1700
		10	0.0012	0.0028	0.0044	0.0085	0.0167	0.0316	0.0559	0.0788	0.1522
		15	0.0010	0.0025	0.0039	0.0076	0.0151	0.0289	0.0518	0.0736	0.1436
		20	0.0009	0.0023	0.0035	0.007	0.014	0.0273	0.0494	0.0706	0.1392
	20	5	0.0011	0.0027	0.0041	0.0079	0.0153	0.0288	0.0506	0.0711	0.1369
		10	0.0009	0.0023	0.0035	0.0069	0.0134	0.0254	0.0448	0.0632	0.1228
		15	0.0008	0.002	0.0032	0.0062	0.0122	0.0232	0.0413	0.0587	0.1150
		20	0.0008	0.0019	0.0029	0.0057	0.0113	0.02180	0.0392	0.0558	0.1104

It can be observed in Table 1, that the average proportion of defectives decreased as the value of s, g_1 and g_2 increased.

2.2 Designing of Quality Regions for BDGSP

i. Probabilistic Quality Region (PQR)

In this quality region, the product is accepted with a maximum probability of 0.95 and the minimum probability of 0.05, where 0.95 corresponds to AQL $(1-\alpha)$ and 0.05 corresponds to LQL (β). In other words, PQR (R_1) is exactly the conventional setting of AQL $=\mu_1$ and LQL $=\mu_2$. In Figure 2, it is showed that the PQR lies between $\mu_1 \leq \mu \leq \mu_2$.

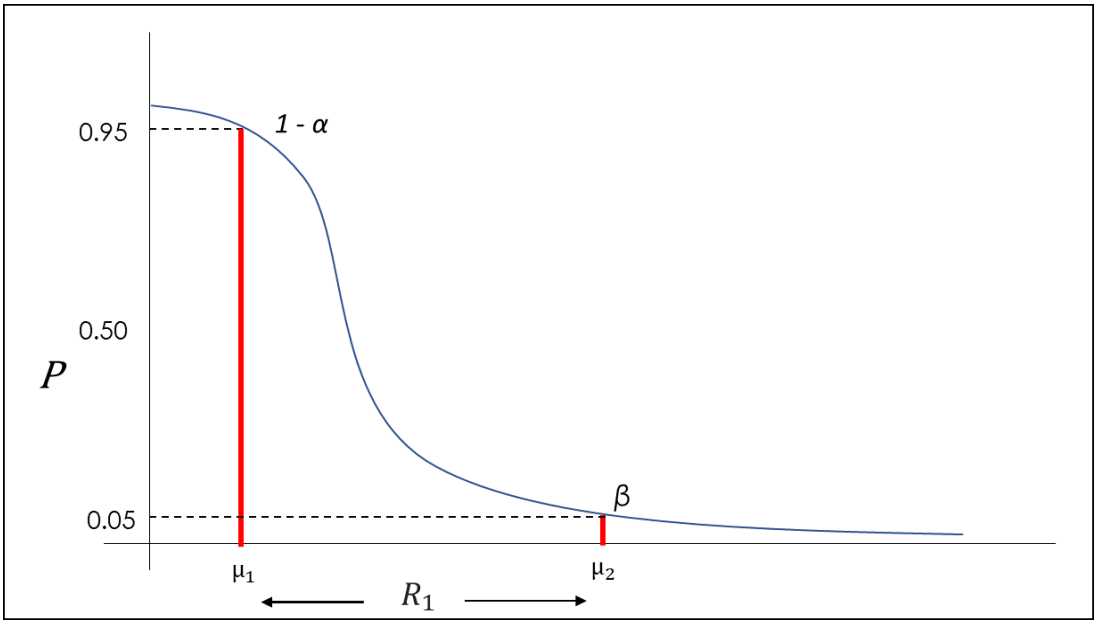

Figure 2: OC curve with pair of coordinates for PQR (Hafeez, Aziz, & Shabbir, 2024).

This region considers the same values for consumer's and producer's risk, that is $\alpha = \beta = 0.05$ and the range of PQR is $R_1 = \mu_2 - \mu_1$ as given in Table 2.

ii. Quality Decision Region (QDR)

The product is accepted with a maximum probability of 0.95 and a minimum probability of 0.25 in this quality region, where 0.95 equates to AQL $(1-\alpha)$ and 0.25 equates to LQL (β). In other words, QDR (R_2) exactly matches the standard setup of AQL $=\mu_1$ and LQL $=\mu_\beta$, and it can be shown that the QDR is located between these two positions: $\mu_1 \leq \mu \leq \mu_\beta$. The levels for consumer's and producer's risk in this area are $\alpha = 0.05$ and $\beta = 0.25$ respectively. $R_2 = \mu_\beta - \mu_1$ is the QDR range, as seen in Table 2.

iii. Limiting Quality Region (LQR)

The product is accepted with a maximum probability of 0.75 and a minimum probability of 0.05 in this quality region, where 0.75 equates to AQL $(1-\alpha)$ and 0.05 equates to LQL (β). In other words, LQR (R_3) exactly matches the standard setup of AQL $=\mu_\alpha$ and LQL $=\mu_2$, and it can be shown that the LQR is located between these two positions: $\mu_\alpha \leq \mu \leq \mu_2$. The levels for consumer's and producer's risk in this area are $\alpha = 0.25$ and $\beta = 0.05$ respectively. $R_3 = \mu_2 - \mu_\alpha$ is the LQR range, as seen in Table 2.

iv. Indifference Quality Region (IQR)

The product is accepted with a maximum probability of 0.5 and a minimum probability of 0.05 in this quality region, where 0.5 equates to AQL $(1-\alpha)$ and 0.05 equates to LQL (β). In other words, IQR (R_4) exactly matches the standard setup of AQL $=\mu_*$ and LQL $=\mu_2$, and it can be shown that the IQR is located between these two positions: $\mu_* \leq \mu \leq \mu_2$. The levels for consumer's and producer's risk in this area are $\alpha = 0.5$ and $\beta = 0.05$ respectively. $R_4 = \mu_2 - \mu_*$ is the IQR range, as seen in Table 2.

2.3 Selection of Sampling Plan

When $r_1 = r_2 = 5$, Table 2 lists the quality regions PQR (R_1), QDR (R_2), LQR (R_3), and IQR (R_4) together with the related design parameters s, g_1 and g_2. We can determine the operating ratios $T = \frac{R_1}{R_2}$, $T_1 = \frac{R_1}{R_3}$ and $T_2 = \frac{R_1}{R_4}$ for any

given quality region value (Hafeez et al., 2024). Columns T, T_1 and T_2 in Table 2 can be used to determine the value for the design parameters s, g_1 and g_2 that is equal to or less than the given ratio. The parameters for the BDGSP can be calculated from this ratio.

Table 2: Certain values of QDR, PQR, LQR, IQR and operating characteristic ratio under BDGSP for specified values of g_1 and g_2, when $r_1 = r_2 = 5$.

s	g_1	g_2	R_1	R_2	R_3	R_4	T	T_1	T_2
1	5	5	0.484	0.1313	0.4674	0.4351	3.686	1.0357	1.1125
		10	0.4545	0.1169	0.4405	0.4122	3.8885	1.0319	1.1027
		15	0.4434	0.1115	0.4305	0.4037	3.9773	1.0299	1.0983
		20	0.4381	0.1089	0.4259	0.3998	4.0241	1.0286	1.0958
	10	5	0.354	0.0814	0.344	0.3246	4.3512	1.0291	1.0908
		10	0.3214	0.0707	0.3129	0.2962	4.549	1.0271	1.0851
		15	0.305	0.0654	0.2974	0.282	4.6633	1.0257	1.0817
		20	0.2958	0.0624	0.2887	0.274	4.7426	1.0247	1.0794
	15	5	0.2839	0.0602	0.2766	0.2623	4.714	1.0265	1.0824
		10	0.2564	0.0526	0.2501	0.2377	4.8709	1.0253	1.0786
		15	0.2404	0.0483	0.2347	0.2234	4.975	1.0242	1.0761
		20	0.2304	0.0456	0.2252	0.2145	5.0488	1.0234	1.0742
	20	5	0.2381	0.0481	0.2323	0.221	4.9479	1.025	1.0774
		10	0.2157	0.0425	0.2107	0.2007	5.0727	1.0241	1.0747
		15	0.2015	0.039	0.197	0.1879	5.166	1.0233	1.0728
		20	0.192	0.0367	0.1877	0.1792	5.2262	1.0227	1.0715
2	5	5	0.2532	0.0879	0.2377	0.2135	2.8812	1.0651	1.1858
		10	0.2346	0.0781	0.2218	0.2005	3.0031	1.0579	1.1698
		15	0.2294	0.0749	0.2178	0.1976	3.0644	1.0536	1.1613
		20	0.2276	0.0736	0.2165	0.1968	3.0933	1.051	1.1565
	10	5	0.1586	0.0527	0.1493	0.1349	3.0109	1.0622	1.1757
		10	0.1408	0.0456	0.1331	0.1207	3.0886	1.0585	1.1672
		15	0.1332	0.0422	0.1262	0.1148	3.1531	1.0553	1.1599
		20	0.1295	0.0404	0.123	0.1122	3.2032	1.0527	1.1544
	15	5	0.1183	0.0385	0.1115	0.101	3.0704	1.0604	1.1711
		10	0.1046	0.0336	0.0988	0.0897	3.1176	1.0588	1.1661
		15	0.0975	0.0308	0.0923	0.0839	3.1624	1.0566	1.1615
		20	0.0935	0.0291	0.0886	0.0808	3.211	1.0546	1.157
	20	5	0.0949	0.0306	0.0896	0.0812	3.1043	1.0598	1.1684
		10	0.0845	0.0269	0.0798	0.0725	3.1392	1.0586	1.1658
		15	0.0784	0.0247	0.0742	0.0675	3.1712	1.0574	1.1623
		20	0.0745	0.0232	0.0706	0.0643	3.2107	1.0551	1.1582
3	5	5	0.1934	0.0762	0.1783	0.1566	2.5381	1.0844	1.2352
		10	0.1805	0.0679	0.168	0.149	2.6593	1.0741	1.2115
		15	0.1777	0.0653	0.1664	0.1482	2.7231	1.0677	1.1986
		20	0.177	0.0643	0.1664	0.1485	2.751	1.0638	1.192
	10	5	0.1168	0.0453	0.1078	0.0949	2.5771	1.0835	1.2318
		10	0.104	0.0393	0.0964	0.0853	2.6477	1.0787	1.2195
		15	0.0988	0.0363	0.092	0.0818	2.7186	1.0739	1.2077
		20	0.0965	0.0348	0.0903	0.0806	2.7717	1.0689	1.198
	15	5	0.086	0.0331	0.0794	0.0699	2.5981	1.083	1.2291
		10	0.076	0.0288	0.0703	0.0622	2.6427	1.0806	1.2223
		15	0.0711	0.0264	0.066	0.0585	2.6909	1.0764	1.2141
		20	0.0684	0.025	0.0637	0.0567	2.7358	1.0737	1.2059
	20	5	0.0685	0.0262	0.0632	0.0558	2.6174	1.0826	1.2268
		10	0.0609	0.0231	0.0563	0.0498	2.6405	1.0812	1.223
		15	0.0566	0.0212	0.0525	0.0465	2.6741	1.0787	1.2179
		20	0.0539	0.0199	0.0501	0.0446	2.7122	1.0753	1.2102

All AQL and LQL values for limited available number of testers $r_1 = r_2 = 5$ are shown in Table 3, with the specified values of other design parameters $s,\ \alpha, \beta,\ g_1$ and g_2.

Table 3: AQL and LQL for some specified values of s, α, β, g_1 and g_2, when $r_1 = r_2 = 5$.

s	α	g_1, g_2	$\beta = 0.01$	$\beta = 0.05$	$\beta = 0.10$	$\beta = 0.25$
1	0.01	5	0.0026-0.8328	0.0026-0.4908	0.0026-0.3159	0.0026-0.1381
		10	0.0013-0.7131	0.0013-0.3248	0.0013-0.1873	0.0013-0.0740
		15	0.0008-0.6234	0.0008-0.2427	0.0008-0.1330	0.0008-0.0506
		20	0.0006-0.5538	0.0006-0.1937	0.0006-0.1032	0.0006-0.0384
	0.05	5	0.0068-0.8328	0.0068-0.4908	0.0068-0.3159	0.0068-0.1381
		10	0.0034-0.7131	0.0034-0.3248	0.0034-0.1873	0.0034-0.0740
		15	0.0022-0.6234	0.0022-0.2427	0.0022-0.1330	0.0022-0.0506
		20	0.0017-0.5538	0.0017-0.1937	0.0017-0.1032	0.0017-0.0384
	0.10	5	0.0108-0.8328	0.0108-0.4908	0.0108-0.3159	0.0108-0.1381
		10	0.0054-0.7131	0.0054-0.3248	0.0054-0.1873	0.0054-0.0740
		15	0.0036-0.6234	0.0036-0.2427	0.0036-0.1330	0.0036-0.0506
		20	0.0028-0.5538	0.0028-0.1937	0.0028-0.1032	0.0028-0.0384
	0.25	5	0.0234-0.8328	0.0234-0.4908	0.0234-0.3159	0.0234-0.1381
		10	0.0118-0.7131	0.0118-0.3248	0.0118-0.1873	0.0118-0.0740
		15	0.0079-0.6234	0.0079-0.2427	0.0079-0.1330	0.0079-0.0506
		20	0.0059-0.5538	0.0059-0.1937	0.0059-0.1032	0.0059-0.0384
2	0.01	5	0.0029-0.5012	0.0029-0.2605	0.0029-0.1792	0.0029-0.0952
		10	0.0015-0.3121	0.0015-0.1445	0.0015-0.0960	0.0015-0.0493
		15	0.0010-0.2262	0.0010-0.0999	0.0010-0.0655	0.0010-0.0333
		20	0.0007-0.1773	0.0007-0.0764	0.0007-0.0498	0.0007-0.0251
	0.05	5	0.0073-0.5012	0.0073-0.2605	0.0073-0.1792	0.0073-0.0952
		10	0.0037-0.3121	0.0037-0.1445	0.0037-0.0960	0.0037-0.0493
		15	0.0024-0.2262	0.0024-0.0999	0.0024-0.0655	0.0024-0.0333
		20	0.0019-0.1773	0.0019-0.0764	0.0019-0.0498	0.0019-0.0251
	0.10	5	0.0114-0.5012	0.0114-0.2605	0.0114-0.1792	0.0114-0.0952
		10	0.0057-0.3121	0.0057-0.1445	0.0057-0.0960	0.0057-0.0493
		15	0.0038-0.2262	0.0038-0.0999	0.0038-0.0655	0.0038-0.0333
		20	0.0029-0.1773	0.0029-0.0764	0.0029-0.0498	0.0029-0.0251
	0.25	5	0.0228-0.5012	0.0228-0.2605	0.0228-0.1792	0.0228-0.0952
		10	0.0114-0.3121	0.0114-0.1445	0.0114-0.0960	0.0114-0.0493
		15	0.0076-0.2262	0.0076-0.0999	0.0076-0.0655	0.0076-0.0333
		20	0.0057-0.1773	0.0057-0.0764	0.0057-0.0498	0.0057-0.0251
3	0.01	5	0.0031-0.3597	0.0031-0.2010	0.0031-0.1455	0.0031-0.0838
		10	0.0015-0.2054	0.0015-0.1077	0.0015-0.0764	0.0015-0.0430
		15	0.0010-0.1436	0.0010-0.0736	0.0010-0.0518	0.0010-0.0289
		20	0.0008-0.1104	0.0008-0.0558	0.0008-0.0392	0.0008-0.0218
	0.05	5	0.0076-0.3597	0.0076-0.2010	0.0076-0.1455	0.0076-0.0838
		10	0.0038-0.2054	0.0038-0.1077	0.0038-0.0764	0.0038-0.0430
		15	0.0025-0.1436	0.0025-0.0736	0.0025-0.0518	0.0025-0.0289
		20	0.0019-0.1104	0.0019-0.0558	0.0019-0.0392	0.0019-0.0218
	0.10	5	0.0117-0.3597	0.0117-0.2010	0.0117-0.1455	0.0117-0.0838
		10	0.0058-0.2054	0.0058-0.1077	0.0058-0.0764	0.0058-0.0430
		15	0.0039-0.1436	0.0039-0.0736	0.0039-0.0518	0.0039-0.0289
		20	0.0029-0.1104	0.0029-0.0558	0.0029-0.0392	0.0029-0.0218
	0.25	5	0.0226-0.3597	0.0226-0.2010	0.0226-0.1455	0.0226-0.0838
		10	0.0113-0.2054	0.0113-0.1077	0.0113-0.0764	0.0113-0.0430
		15	0.0076-0.1436	0.0076-0.0736	0.0076-0.0518	0.0076-0.0289
		20	0.0057-0.1104	0.0057-0.0558	0.0057-0.0392	0.0057-0.0218

Table 3 shows that the values of AQL and LQL for the given parameters $s = 1, g_1 = g_2 = 10$ are 0.0013 and 0.7131, respectively, for $\alpha = 0.01$ and $\beta = 0.01$. That results in the range of quality region being 0.7118, which implies that with a 1% risk for the consumer's and producer's, 71.18% of the products will fulfill the requirements for quality. If we merely modify $\alpha = 0.10$ and $\beta = 0.10$ for the identical values of the other design parameters, the AQL and LQL values are 0.0054 and 0.1873, respectively. This results in a range of quality region of 0.1819, which implies that with a 10% risk for the consumer's and producer's, only 18.19% of items will meet requirements for quality.

Table 3 also present that the value of AQL rises as the value of α rises, but LQL is unaffected because LQL is independent of α. Since AQL is the left-hand boundary of the quality area, a rise in AQL value actually represents a decline in the quality region range. LQL value decreases as the value of β rises, but this has no effect on AQL value because AQL is independent of β.

3. Results and discussion

In Figures 3, 4, and 5, for $s = 1,2$, and 3, respectively, the average proportion of defectives is plotted for a range of group size values to track the impact of design parameters. In this case, $r = 5$ testers are taken into account, with $\alpha = 0.05$ and $\beta = 0.05$, risks.

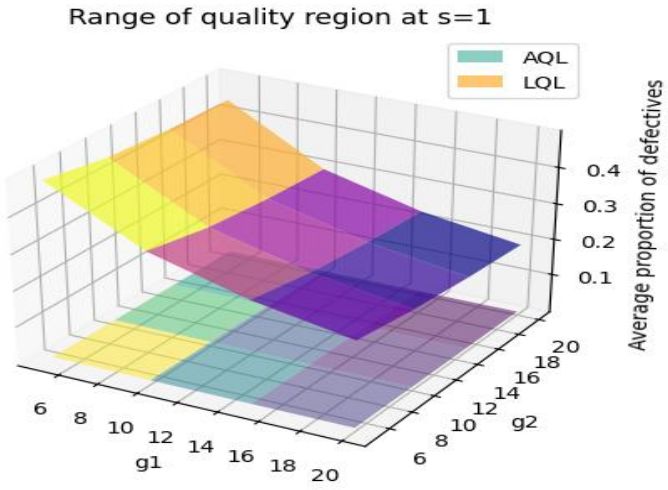

Figure 3: Range of quality region with proportion of defective at $s = 1$.

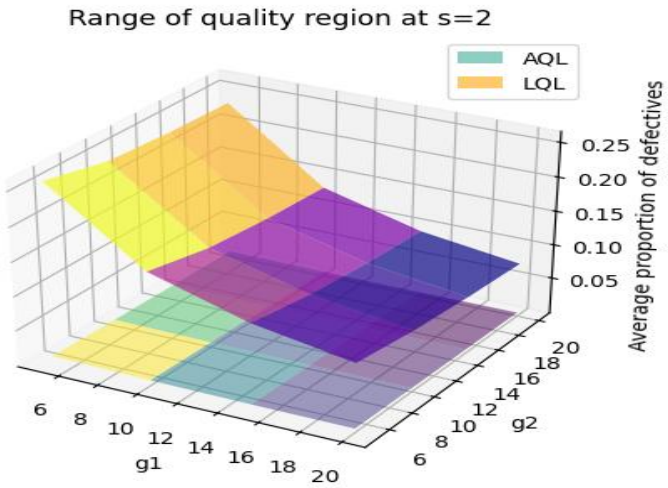

Figure 4: Range of quality region with proportion of defective at $s = 2$.

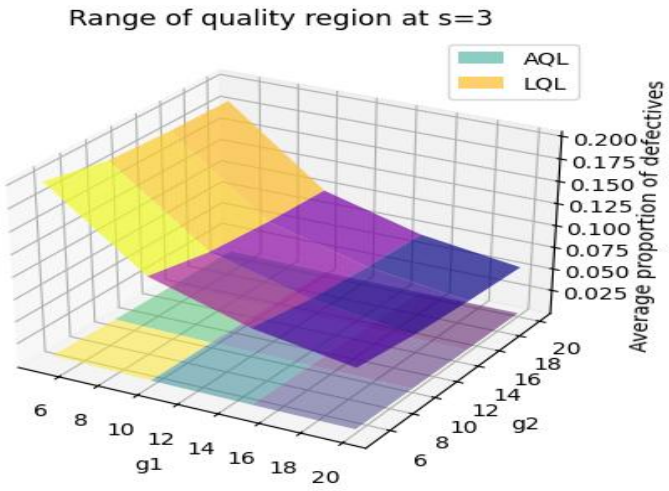

Figure 5: Range of quality region with proportion of defective at $s = 3$.

It is visible in Figures 3, 4, and 5 that the average proportion of defective decreases as the values of g_1, g_2 and s increase. This also means that the lot will be accepted with a greater probability of acceptance and a lower proportion of defects that are acceptable to both the consumer and the producer.

In Figure 6, BDGSP and BGChSP proposed by Hafeez and Aziz (2019) are compared for the sake of comparison. OC curves of both plans are shown with the same values of the design parameters $s = 1, r_1 = r_2 = 5$, and $g_1 = g_2 = 5$.

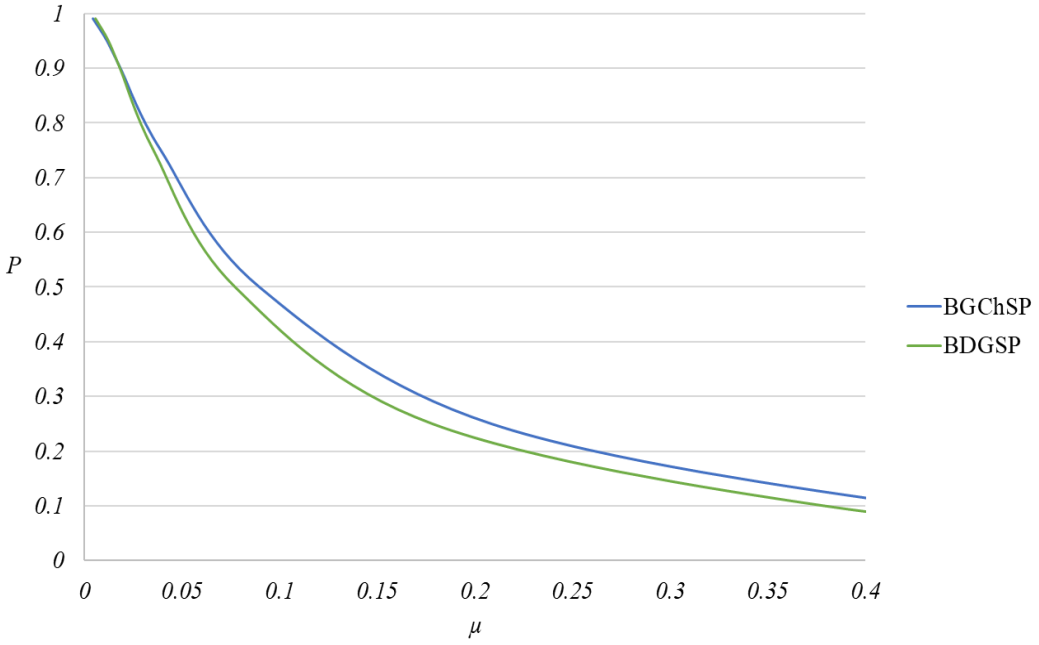

Figure 6: OC curves of BDGSP and BGChSP for $s = 1, r_1 = r_2 = 5$, and $g_1 = g_2 = 5$.

It is clear from Figure 6 that the suggested BDGSP will result in fewer defects than the current BGChSP if we employ both sampling plans under the same circumstances. Hence, we can conclude that BDGSP is more efficient plan to reduce the cost and time of inspection. By having a lower probability of acceptance, BDGSP manage to reduce the consumer risk from receiving defective items compare to BGChSP.

4. Application of BDGSP

This section details how the suggested idea was put into practice using information from Montgomery (2009), where rubber belts are produces in lots. Inspection records on the last 20 lots each of size 2500 and the number of defective items in each lot are as follows: 230, 435, 221, 346, 230, 327, 285, 311, 342, 308, 456, 394, 285, 331, 198, 414, 131, 269, 221 and 407.

Here, for the beta distribution the estimated mean square error (MSE) and Bayesian information criterion (BIC) values are 0.000039 and -469.4965, respectively. The shape parameters are predicted to have values of $s = 1$ and $t = 10$ using the maximum likelihood estimate (MLE). The BDGSP is applied by taking into account the following values for the design parameters based on the number of testers available: $r_1 = r_2 = 10$. As a result, the entire sample will be divided into $g_1 = 250$ groups for the first sample and $g_2 = 250$ groups for the second sample, or $n_1 = r_1 * g_1$ and $n_2 = r_2 * g_2$.

When the experimenter sets up the BDGSP plan, they do it in accordance with the aforementioned specifications. Assume that the producer and consumer risks for PQR are both equal to 0.05. The process will now be repeated using Equation (10), just as it was for Table 1. Then the estimated values for PQR are, AQL ($\mu_1 = 0.000061$), LQL ($\mu_2 = 0.0095$), and the range $R_1 = \mu_2 - \mu_1 = 0.00944$.

The same procedure will be repeated for QDR, then values are estimated as: $\mu_1 = 0.000061$, $\mu_\beta = 0.00160$, $R_2 = 0.00154$ and $T = 6.1432$.

Similarly identical procedure will be repeated for LQR, then values are estimated as: $\mu_\alpha = 0.00026$, $\mu_2 = 0.0095$, $R_3 = 0.00924$ and $T_1 = 1.0216$.

Equally same process will be repeated for IQR, then values are estimated as: $\mu_* = 0.00056$, $\mu_2 = 0.0095$, $R_4 = 0.00894$ and $T_2 = 1.0561$.

5. Conclusion

There are several ways to set up an acceptance sampling plan. Some sampling plans consider the risks to the producer's and the consumer's, while others focus on non-economic criteria. The proposed BDGSP can be utilised to decide regarding the lot under inspection based on present lot and prior information about the product. BDGSP is recommended in this study to estimate quality regions for the average probability to accept a lot. To convince both the consumer and the industrialist, this study can take into account both customer and producer risks. Four quality regions are accessed for all possible combinations of design parameters s, g_1, g_2, r_1, r_2 and different values of consumer's risk and producer's risk. Based on tables and OC curves, we can conclude that if the value of pre-specified design parameters s, g_1, g_2, r_1 and r_2 increase, then proportion of defective decreases. This means that, by increasing the value of any design parameter the probability of acceptance of a defective item can be reduced. As the values of α and β increase, AQL and LQL become close to each other, and the range of quality region decrease. For future research, incorporating lifetime distributions such as Weibull, lognormal, exponential, and other variants could significantly enhance the robustness of the proposed plan. Additionally, tailoring the selection of prior distributions to specific contexts could further refine the methodology. For instance, using a Poisson distribution with a gamma prior to estimate the average number of nonconformities may provide valuable insights into the quality assessment process. Based on the manufacturer's option, quality regions will be calculated for various consumer's and producer's risks values. The proposed BDGSP is distinguished by its capacity to effectively manage risks for both producers and consumers, thereby ensuring comprehensive quality control. Its flexible design parameters accommodate a range of quality standards and manufacturing needs, while the incorporation of historical data significantly enhances the accuracy of quality assessments. Also suggested plan can be utilized to build the design parameters utilizing AQL and LQL, using the minimum angle or relative slope methods.

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