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Gamma Lindley distribution in acceptance sampling plans in terms of truncated life tests with an application to industrial data

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Abstract
Acceptance sampling plans (ASP) are needed in areas where 100% inspection is impractical or expensive. They help ensure product quality meets requirements, reduce inspection costs, and prevent nonconforming goods from reaching customers. Well-planned ASP offer an efficient and dependable way to maintain quality control in manufacturing procedures and supply chains. The Gamma Lindley Distribution (GaLD) is used to design acceptance sampling plans in this study when the life test is truncated at a pre-specified time. The mean is used as the quality parameter. The smallest sample size is required to guarantee that the desired life mean is reached at the consumer's specified risk. In addition to the producer's risk, the operating characteristic values of the sample plans are presented. To evaluate the suggested sampling plans, a real data from the first failure of 20 electric carts utilized for internal transportation and delivery in a big manufacturing facility is provided.

Key Words: Gamma Lindley distribution; Truncated lifetime test; Acceptance sampling plan; Characteristic function; Consumer's risk; Producer's risk

Mathematical Subject Classification: 62A86

1. Introduction

(Zeghdoudi & Nedjar, 2016) introduced a new distribution, named Gamma Lindley Distribution which is based on mixing of two commonly used distributions, the Gamma(2,θ) with probability density function (pdf) $f(x;\theta)=\theta^2 x e^{-\theta x},\quad x>0$ and Lindley (θ) with pdf $f(x;\theta)=\frac{\theta^2}{1+\theta}(1+x)e^{-\theta x},\ x>0,$ with probability $\frac{\beta-1}{\beta}$ and $\frac{1}{\beta}$, respectively. The pdf of the Gamma Lindley distributed random variable X is given by

$$f_{GaLD}(x;\theta,\beta)=\frac{\theta^2}{\beta(1+\theta)}\left[(\beta+\beta\theta-\theta)x+1\right]e^{-\theta x}, x,\theta>0, \beta\geq\frac{\theta}{\theta+1}, \tag{1}$$

with mean $E(X)=\frac{2\beta(\theta+1)-\theta}{\theta(\beta\theta+\beta)}$. (Shafq et al., 2022) provided the condition $\beta\geq\frac{\theta}{\theta+1}$. For the lifetime distribution under examination, the mean serves as a quality level. The rth moment of X is given by

$$E(X^r)=\frac{\Gamma(r+1)}{\theta^r(\beta\theta+\beta)}\left[\beta(r+1)(\theta+1)-r\theta\right],\ r=1,2,3,\ldots. \tag{2}$$

Figure 1 presents some pdf plots of the GaLD for some parameter values. It is obvious that the pdf of the GaLD has many shapes as decreasing and increasing-decreasing, enhancing its adaptability to fit real data sets. In general, the distribution is skewed to the right and the plot peak depends on the parameter values, which is more peak for larger values of the parameters.

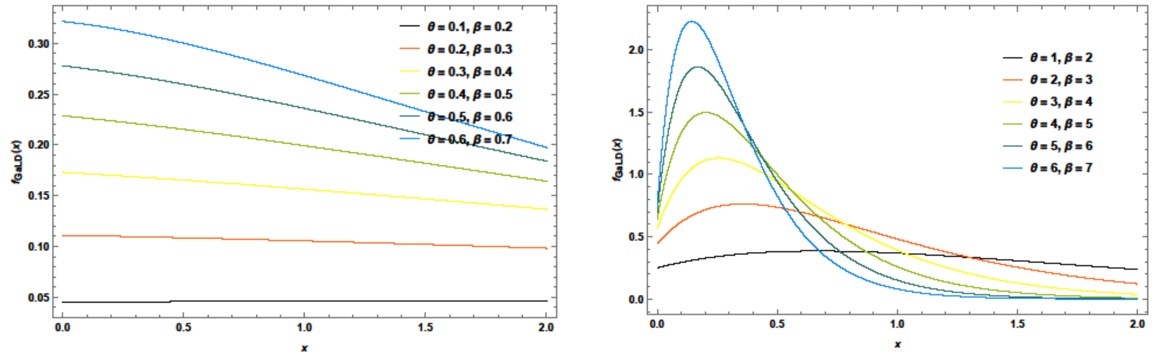

Figure 1. The pdf plots of the GaLD for some parameters

(Zeghdoudi & Nedjar, 2016) derived some properties of the distribution as the mean, variance, stochastic ordering, and maximum likelihood estimators of the distribution parameters. As a modification of GaLD. The corresponding cumulative distribution function (cdf) of the pdf in (1) is

$$F_{GaLD}(x;\theta,\beta)=1-\left(\frac{(\theta\beta+\beta-\theta)(\theta x+1)+\theta}{\beta(\theta+1)}\right)e^{-\theta x}. \quad (3)$$

The mode of the GaLD is given by

$$M_{GaLD}=\begin{cases}\dfrac{\beta\theta+\beta-2\theta}{\theta(\beta+\beta\theta-\theta)}, & \beta\in\left[\dfrac{2\theta}{\theta+1},\infty\right].\\ 0, & \text{otherwise.}\end{cases}$$

(Beghriche & Zeghdoudi, 2019) suggested size-biased Gamma Lindley Distribution as a generalization of the GaLD. Moreover, (Nedjar & Zeghdoudi, 2016) introduced more properties of the GaLD and showed that its quantile function has the form

$$Q_{GaLD}(u)=\frac{\beta(1+\theta)}{\theta(\beta(1+\theta)-\theta)}-\frac{1}{\theta}W_{-1}\left(\frac{\beta(1+\theta)(y-1)}{\beta(1+\theta)-\theta}e^{-\frac{\beta(1+\theta)}{\beta(1+\theta)-\theta}}\right),\ 0<u<1,$$

where W_{-1} denotes negative branch of Lambert W function. The moments estimate of the GaLD parameters are

$$\hat{\theta}=\frac{1}{s^2+m^2}\left(2m+\sqrt{2\left(m^2-s^2\right)}\right) \text{ and } \hat{\beta}=\frac{\hat{\theta}}{\left(1+\hat{\theta}\right)\left(2-\hat{\theta}m\right)},$$

respectively, where m is the first-moment m and s^2 is the variance.

Following are the reliability and hazard rate functions for the GaLD, respectively as

$$R_{GaLD}(x;\theta,\beta)=1-F_{GaL}(x;\theta,\beta)=\frac{(\beta(\theta+1)-\theta)(\theta x+1)+\theta}{\beta(\theta+1)}e^{-\theta x},$$

and

$$H_{GaLD}(x;\theta,\beta)=\frac{f_{GaL}(x;\theta,\beta)}{S_{GaL}(x;\theta,\beta)}=\frac{\theta^2\left[(\beta+(\beta-1)\theta)x+1\right]}{(\theta(\beta-1)+\beta)(\theta x+1)+\theta}.$$

Figures 2 and 3 show the reliability and hazard function plots of the GaLD for some parameters. It can be seen that the reliability function is decreasing for all parameter values. The hazard function of the distribution is increasing, approaching 5 when $\theta=6,\beta=7$, while it goes to 0.4 for $\theta=0.6,\beta=0.7$.

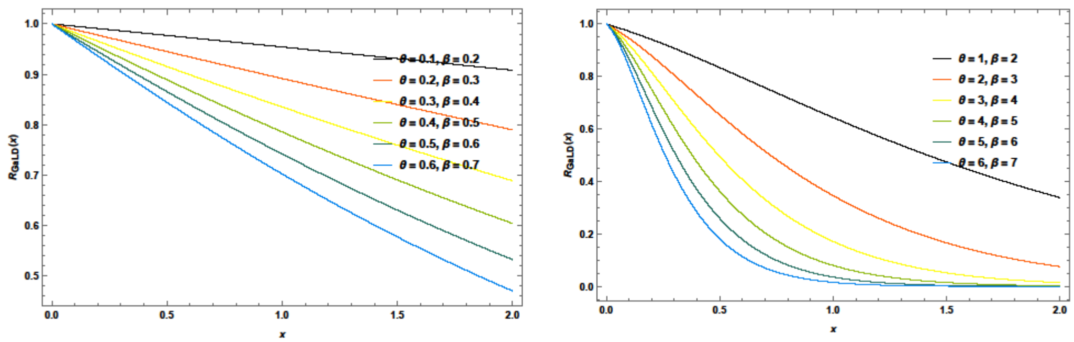

Figure 2. The reliability plots of the GaLD for some parameters

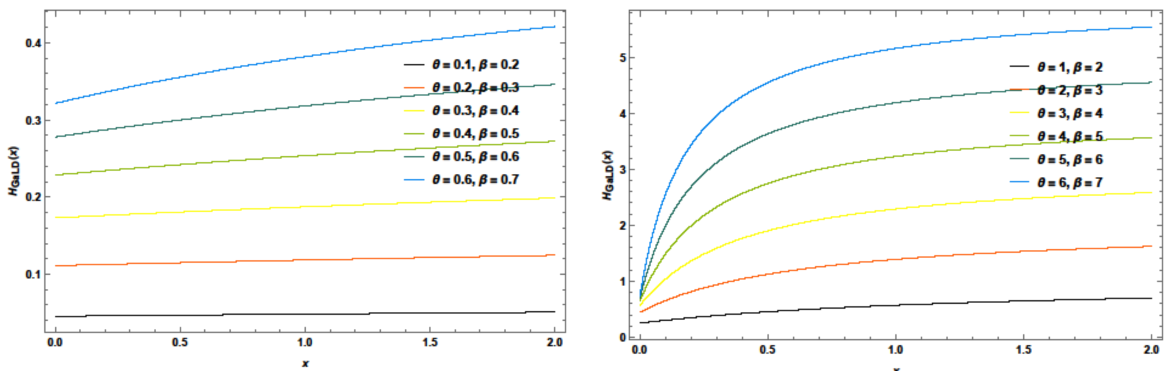

Figure 3. The hazard plots of the GaLD for some parameters

Manufacturers and producers evaluate the quality of a batch or lot of items using acceptance sampling plans as part of their quality control operations. When it is impractical or too expensive to inspect each individual item in the batch, these plans are used. The fundamental concept behind acceptance sampling plans is to check a random sample of the lot's products and base your choice on the sample's quality as to whether or not to accept the full batch. The choice is typically made using pre-established criteria and statistical techniques, with an emphasis on the amount of defective goods discovered in the sample. These strategies have been widely employed to assure product consistency and superior quality while lowering inspection costs in a variety of industries, including factories, medicines, electronic devices, and food production. There are different types of ASP, such as the single ASP, double ASP, Group ASP, repetitive ASP, etc. In the current work, we are interested in a single ASP when the product's lifetime follows the GaLD and the mean is the quality parameter.

For examples about ASP, (Epstein, 1954) was the first to suggest single sampling plans for truncated life tests for mean utilizing exponential distribution. (Baklizi & El Masri, 2004) suggested ASP in terms of truncated life tests for the Birnbaum Saunders model. (M. A. Khan & Islam, 2013) suggested ASP for reliability test for alpha distributed lifetime. (A. I. Al-Omari, 2015) introduced new ASP for generalized inverted exponential distribution. (Aslam et al., 2015) considered ASP for multi-stage process based on time-truncated test for Weibull distribution. (A. I. Al-Omari et al., 2018) offered ASP for the Marshall-Olkin Esscher transformed Laplace model. (A. I. Al-Omari, 2018a) suggested ASP in terms of truncated life tests for the Sushila distribution. (Gogah & Al-Nasser, 2018) considered median ranked ASP for exponential distribution. (A. Al-Omari et al., 2019) proposed ASP for Rama distribution. (Shongwe & Malela-Majika, 2020) investigated a new variable sampling size to monitor the process mean of auto correlated observations. (Adil Hussain et al., 2021) suggested mean ranked ASP under exponential distribution. (Alomani & Al-Omari, 2022) introduced single ASP based on truncated lifetime tests for two-parameter Xgamma distribution with real data application. (Nassr et al., 2022) suggested ASP for the three-parameter inverted Topp–Leone model. (Jayalakshmi & Vijilamery, 2022) considered ASP for truncated life tests based on percentiles using Gompertz Fréchet distribution. (Al-Nasser & Alhroub, 2022) proposed new ASP using hypergeometric theory for finite population under Q-Weibull distribution. (Kaviyarasu & Sivakumar, 2022) considered Bayesian repetitive group sampling plan for quality determination in pharmaceutical products.. New ASP by considering lifetime of products following the half logistic-Marshall Olkin Lomax distribution is studied by (Lishamol Tomy & Meenu Jose, 2022). (M. Z. Khan et al., 2022) proposed fuzzy ASP for transmuted Weibull distribution. (Obulezi et al., 2023) suggested single ASP for Zubair-Exponential model.

In general, waiting until every product fails before inspecting it can be time-consuming. In order to save money and effort, it is typical to end a life test at a certain time t_0. These tests aim to determine a specific mean life, μ_0 with a probability of at least P^* the consumer's level of confidence, among other things, and to set a confidence limit on that mean life. When the actual mean life of the goods does not fall below the predetermined value μ_0 (consistent with the hypothesis $H_0 : \mu \geq \mu_0$), a lot is deemed to be good. The lot will only be accepted if the observed number of failures falls within a predetermined acceptance number c. If there are more failures than this amount, the test can be stopped at time t_0 and the lot can be rejected. The issue under consideration is obtaining the smallest sample size *m* required to guarantee a particular mean lifetime. The probability that a good lot will be rejected for a specific acceptance sampling plan and an unsatisfactory lot will be accepted are, respectively, the consumer's risk and the producer's risk.

The single ASP for the well-known GaLD based on truncated life tests have not been investigated. In the paper, the problem will be addressed. The rest of this paper is structured as follows: Section 2 comprises a description of acceptance sampling plans based on the GaLD and the minimum sample size, operating characteristic values and the producer's risk are studied. Section 3 contains the numerical results and illustrative examples. In Section 4, we offered two real-world examples to demonstrate the implementation of the proposed time-truncated ASP under the Gal distribution. Section 5 contains the paper's conclusion.

2. Design of the Sampling Plan

This section outlines the proposed single acceptance sampling plan strategy and the parameters that associated along with it. Assume that the lifetimes of the submitted products follow the GaLD shown in (1). A multiple of the specified mean lifetime should be used as the termination time, $t_0 = d\mu_0$, where d is a constant that is positive. The following sums up the ASP:

(1) Choose a size *m* random sample, and test it for the specified amount of time t_0.

(2) A lot may be approved if c or fewer failures are discovered throughout the test time t, according to the acceptance number c.

(3)The ratio t / μ_0, where μ_0 is the previously given average lifetime.

The methods for determining the producer's risk, the operational characteristic values, and the minimum sample size are illustrated in the subsections that follow.

3.1. Minimum sample size

Fix the customer risk first, and the chance of accepting a corrupt lot should not be greater than $1-P^*$. An inadequate lot is one in which the proper mean lifetime is less than the prescribed mean lifetime of μ_0. Thus, P^* is a confidence level in the sense that the probability of rejecting a lot with $\mu < \mu_0$ is at least P^*. For a given P^* value, our sampling plan is distinguished by $(m, c, t/\mu_0)$. In this case, we take a lot of indefinitely large size so that the theory of binomial distribution can be used, and accepting or rejecting the lot is comparable to accepting or rejecting hypothesis $H_0 : \mu \geq \mu_0$. Given P^* $(0 < P^* < 1)$, the t/μ_0 ratio, and an acceptance number c, we must determine the lowest positive integer *m* that allows us to affirm that $\mu \geq \mu_0$ if the number of failures experienced over time *t* does not exceed *c*, with a confidence level of P^*. The smallest positive integer, *m*, that fulfills the following inequality is the necessary sample size,

$$\sum_{i=0}^{c}\binom{m}{i}p_0^i\left(1-p_0\right)^{m-i}\leq 1-p^* \tag{4}$$

according to the design of the suggested sampling plans, where $p=F(t_0;\theta,\beta)$ is the probability of detecting a failure before time t_0, and it is expressed by

$$\begin{aligned} p(t_0;\theta,\beta) &= 1-\left(\frac{(\theta\beta+\beta-\theta)(\theta t_0+1)+\theta}{\beta(\theta+1)}\right)e^{-\theta t_0} \\ &= 1-\left(\frac{(\theta\beta+\beta-\theta)\left(\frac{d\left(2\beta(\theta+1)-\theta\right)}{\theta\left(\mu/\mu_0\right)\left(\beta\theta+\beta\right)}+1\right)+\theta}{\beta(\theta+1)}\right)e^{-\frac{d\left(2\beta(\theta+1)-\theta\right)}{\theta\left(\mu/\mu_0\right)\left(\beta\theta+\beta\right)}}. \end{aligned} \tag{5}$$

If there are c or fewer failures is reported, one can deduce with probability p^* from (4) that $F\left(t;\mu\right)\leq F\left(t;\mu_0\right)$ which infers $\mu\geq\mu_0$. The smallest values of m adequate (4) are obtained for $P^*=0.75$, 0.90, 0.95, 0.99, $t/\mu_0=0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, $c=0,1,...,10$, $\beta=8$, and $\theta=2$ are showed in Table 1. The ratio d used here is in line with the relevant tables of (Kantam et al., 2001), (A. I. Al-Omari, 2018b), and (Baklizi & El Masri, 2004).

3.2. Operating characteristic function

The operating characteristic function (OCF) in acceptance sampling is a mathematical function that reflects the probability of approving a batch or lot of products under a particular sampling plan. The OCF is a fundamental tool for assessing the effectiveness of an acceptance sampling plan. For the acceptance sampling plan $\left(m, c, t/\mu_0\right)$, the OCF is as

$$L(p)=\sum_{i=0}^{c}\binom{m}{i}p^i\left(1-p\right)^{m-i} \tag{6}$$

where the function $p=F\left(t_0;\mu,\theta,\beta\right)$ is considered as a function of μ. For a particular $P^*=0.75$, 0.90, 0.95, 0.99, $t_0/\mu_0=0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712, c and n are selected based on their operational characteristics. Table 2 shows the values of the operational characteristics as a function of μ/μ_0 for a few sample plans with $\beta=8$, and $\theta=2$.

3.3 Producer's risk

The producer's risk is the possibility of rejecting a terrific lot $\left(\mu>\mu_0\right)$. We are curious as to what value of μ/μ_0 will guarantee that the producer's risk is at most λ for the proposed sampling strategy and a given value for the producer's risk λ. Because *p* is a function of μ/μ_0, as demonstrated in (5), μ/μ_0 is the minimum positive number for which *p* supports the inequality:

$$\sum_{i=c+1}^{m}\frac{m!}{i!(m-i)!}p^i(1-p)^{m-i}\leq\lambda. \tag{7}$$

For a specific acceptance sampling plan $\left(m, c, t/\mu_0\right)$ with the stated confidence level P^*, the smallest values of μ/μ_0 accomplishing (7) are displayed in Table 3 for $\lambda=0.05$, $\beta=8$, and $\theta=2$ in the GaLD.

4. Discussion and illustrations

For the purposes of this paper, let us assume that the lifetime follows the GaLD. The OCF values, the lowest difference between the reported and real mean lifespans, and the smallest sample size needed to guarantee that the mean lifespan exceeds a probability P^* are all provided in Tables 1-3.

Take into account a scenario where a researcher needs to verify that the product will have a lifespan average of at least 1000 hours with probability that $P^*=0.75$ (the consumer's risk is $1-P^*=0.25$), and the experimenter would like to end the experiment at $t = 628$ hours. Now, for an acceptance number $c = 6$, from Table 1, the essential sample size matching to the values of $P^* = 0.75$, and $t/\mu_0 = 0.628$ is $m = 22$. As a result, the $m = 22$ units must be tested. If no more than 6 failures out of the 22 units are detected throughout 1000 hours, the experimenter may decide with a confidence level of $P^* = 0.75$ that the mean lifetime is at least 1000 hours. Figure 4 shows the minimum sample size plots for $P^* = 0.75$, 0.90, 0.95, 0.99 based on the proposed ASP for the GaL distribution with parameters $\beta = 8$, and $\theta = 2$. Figure 4 revealed that the minimum sample size plots are decreasing in $t/\mu_0 = 0.628, \ldots, 4.217$.

For a given sampling plan $(m = 22,\ c = 6,\ t/\mu_0 = 0.628)$ under the GaL distribution the OCF values from Table 2 are:

μ/μ_0	2	4	6	8	10	12
$L(p)$	0.95418	0.99982	1	1	1	1
PR	0.04582	0.00018	0	0	0	0

This demonstrates that the producer's risk is around 0.04582 if the true mean life is twice the prescribed mean life $(\mu/\mu_0 = 2)$, and it is zero for $\mu/\mu_0 \geq 6$. Figure 5 presents the OCF plots for $P^* = 0.75$, 90, 95, 0.99, $c = 6$, when $\beta = 8, \theta = 2$ in the GaLD, and it is worth noting that the OCF plots increase as the mean ratio values μ/μ_0 increase.

Table 3 shows the smallest value of μ/μ_0 for numerous values of c and μ/μ_0 so that the producer's risk does not exceed 0.05. As a result of the example mentioned above, we acquire the value $t/\mu_0 = 1.91$. That is, the product must have an average life of 1.91 times the stipulated average life of 1000 hours in order to be accepted with a probability of at least 0.95, which indicates that if $\mu \geq 1.91 \times t_0 / 0.628 = 3.0414 \times t_0 =$ 3041.4 hours, then with sample size $m = 22$ and $c = 6$, the lot will be refused with a probability of 0.05 or less. The true average life needed to transship 95% of the lots is shown in Table 3.

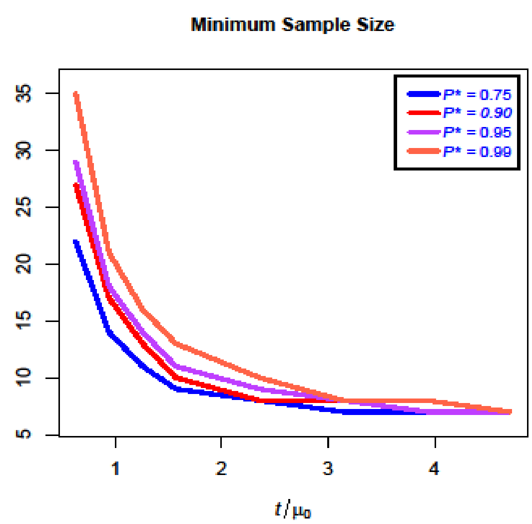

Figure 4: The minimum sample size plots for $P^* = 0.75$, 0.90, 0.95, 0.99 based on the proposed ASP for the GaLD with $\beta = 8$, and $\theta = 2$.

Table 1. Minimum sample sizes of the proposed ASP with $\beta = 8, \theta = 2$ in the GaLD

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	2	2	1	1	1	1	1
	1	7	4	3	3	2	2	2	2
	2	10	6	5	4	3	3	3	3
	3	13	8	6	5	4	4	4	4
	4	16	10	8	7	6	5	5	5
	5	19	12	9	8	7	6	6	6
	6	22	14	11	9	8	7	7	7
	7	25	16	12	11	9	8	8	8
	8	28	18	14	12	10	9	9	9
	9	31	20	15	13	11	10	10	10
	10	34	22	17	15	12	11	11	11
0.90	0	6	3	2	2	1	1	1	1
	1	9	6	4	3	3	2	2	2
	2	13	8	6	5	4	3	3	3
	3	17	10	8	6	5	4	4	4
	4	20	12	9	8	6	5	5	5
	5	23	14	11	9	7	6	6	6
	6	27	17	13	10	8	8	7	7
	7	30	19	14	12	9	9	8	8
	8	33	21	16	13	11	10	9	9
	9	36	23	17	15	12	11	10	10
	10	39	25	19	16	13	12	11	11
0.95	0	7	4	3	2	2	1	1	1
	1	11	7	5	4	3	2	2	2
	2	15	9	7	5	4	3	3	3
	3	19	12	8	7	5	5	4	4
	4	23	14	10	8	6	6	5	5
	5	26	16	12	10	8	7	6	6
	6	29	18	14	11	9	8	7	7
	7	33	20	15	13	10	9	8	8
	8	36	22	17	14	11	10	9	9
	9	39	24	18	15	12	11	10	10
	10	43	27	20	17	13	12	11	11
0.99	0	11	6	4	3	2	2	1	1
	1	15	9	6	5	3	3	2	2
	2	20	12	8	7	5	4	4	3
	3	24	14	10	8	6	5	5	4
	4	28	17	12	10	7	6	6	5
	5	32	19	14	11	8	7	7	6
	6	35	21	16	13	10	8	8	7
	7	39	24	17	14	11	9	9	8
	8	43	26	19	16	12	11	10	9
	9	46	28	21	17	13	12	11	11
	10	50	30	23	19	14	13	12	12

Table 2. OCF values of the proposed ASP with $c = 6$, $\beta = 8, \theta = 2$ in the GaLD

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	22	0.628	0.96318	0.99987	1	1	1	1
	14	0.942	0.95418	0.99982	1	1	1	1
	11	1.257	0.93377	0.99968	0.99999	1	1	1
	9	1.571	0.93485	0.99964	0.99999	1	1	1
	8	2.356	0.78397	0.99693	0.99990	0.99999	1	1
	7	3.141	0.76173	0.99480	0.99980	0.99999	1	1
	7	3.927	0.53369	0.97658	0.99871	0.99989	0.99999	1
	7	4.712	0.33298	0.93347	0.99480	0.99945	0.99992	0.99999
0.90	27	0.628	0.90148	0.99945	0.99999	1	1	1
	17	0.942	0.87521	0.99923	0.99998	1	1	1
	13	1.257	0.83625	0.99872	0.99997	1	1	1
	10	1.571	0.87040	0.99902	0.99998	1	1	1
	8	2.356	0.78397	0.99693	0.99990	0.99999	1	1
	8	3.141	0.45271	0.97559	0.99882	0.99990	0.99999	1
	7	3.927	0.53369	0.97658	0.99871	0.99989	0.99999	1
	7	4.712	0.33298	0.93347	0.99480	0.99945	0.99992	0.99999
0.95	29	0.628	0.86681	0.99913	0.99998	1	1	1
	18	0.942	0.83942	0.99884	0.99997	1	1	1
	14	1.257	0.77252	0.99778	0.99994	1	1	1
	11	1.571	0.78530	0.99779	0.99994	1	1	1
	9	2.356	0.59201	0.99030	0.99964	0.99997	1	1
	8	3.141	0.45271	0.97559	0.99882	0.99990	0.99999	1
	7	3.927	0.53369	0.97658	0.99871	0.99989	0.99999	1
	7	4.712	0.33298	0.93347	0.99480	0.99945	0.99992	0.99999
0.99	35	0.628	0.73496	0.99719	0.99991	0.99999	1	1
	21	0.942	0.71094	0.99677	0.99990	0.99999	1	1
	16	1.257	0.62817	0.99438	0.99982	0.99999	1	1
	13	1.571	0.58198	0.99218	0.99974	0.99998	1	1
	10	2.356	0.40713	0.97721	0.99902	0.99993	0.99999	1
	8	3.141	0.45271	0.97559	0.99882	0.99990	0.99999	1
	8	3.927	0.19664	0.90852	0.99315	0.99931	0.99990	0.99998
	7	4.712	0.33298	0.93347	0.99480	0.99945	0.99992	0.99999

Table 3. The μ/μ_0 for a lot's acceptance with 0.05 PR and $\beta=8,\theta=2$ in the GaLD

		t/μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	11.655	10.482	13.987	10.789	16.180	21.571	26.969	32.360
	1	4.121	4.062	4.308	5.384	5.599	7.464	9.331	11.197
	2	2.928	2.956	3.382	3.444	3.756	5.007	6.259	7.511
	3	2.450	2.499	2.574	2.664	2.994	3.991	4.990	5.987
	4	2.190	2.248	2.447	2.672	3.358	3.428	4.286	5.143
	5	2.025	2.086	2.119	2.329	2.955	3.067	3.834	4.600
	6	1.910	1.974	2.094	2.091	2.674	2.812	3.516	4.218
	7	1.825	1.890	1.903	2.157	2.466	2.622	3.278	3.933
	8	1.760	1.824	1.905	1.997	2.305	2.473	3.092	3.710
	9	1.708	1.772	1.775	1.870	2.177	2.353	2.942	3.529
	10	1.665	1.729	1.786	1.931	2.071	2.254	2.817	3.381
0.90	0	15.992	14.080	13.987	17.481	16.180	21.571	26.969	32.360
	1	4.955	5.517	5.421	5.384	8.073	7.464	9.331	11.197
	2	3.554	3.711	3.944	4.227	5.164	5.007	6.259	7.511
	3	2.991	3.000	3.335	3.217	3.994	3.991	4.990	5.987
	4	2.590	2.619	2.732	3.059	3.358	3.428	4.286	5.143
	5	2.341	2.380	2.575	2.648	2.955	3.067	3.834	4.600
	6	2.233	2.331	2.462	2.365	2.674	3.565	3.516	4.218
	7	2.099	2.194	2.227	2.379	2.466	3.288	3.278	3.933
	8	1.998	2.088	2.180	2.196	2.671	3.073	3.092	3.710
	9	1.917	2.005	2.025	2.218	2.510	2.902	2.942	3.529
	10	1.852	1.938	2.005	2.086	2.377	2.761	2.817	3.381
0.95	0	18.100	17.483	18.788	17.481	26.215	21.571	26.969	32.360
	1	5.739	6.181	6.426	6.775	8.073	7.464	9.331	11.197
	2	3.944	4.059	4.463	4.227	5.164	5.007	6.259	7.511
	3	3.245	3.457	3.335	3.712	3.994	5.325	4.990	5.987
	4	2.870	2.962	2.999	3.059	3.358	4.477	4.286	5.143
	5	2.564	2.654	2.784	2.942	3.493	3.939	3.834	4.600
	6	2.355	2.443	2.633	2.617	3.136	3.565	3.516	4.218
	7	2.255	2.289	2.377	2.586	2.874	3.288	3.278	3.933
	8	2.133	2.172	2.310	2.381	2.671	3.073	3.092	3.710
	9	2.037	2.079	2.142	2.218	2.510	2.902	2.942	3.529
	10	1.995	2.069	2.109	2.232	2.377	2.761	2.817	3.381
0.99	0	26.334	23.987	23.329	23.481	26.215	34.950	26.969	32.360
	1	7.216	7.432	7.362	8.031	8.073	10.763	9.331	11.197
	2	4.860	5.026	4.951	5.578	6.339	6.885	8.607	7.511
	3	3.843	3.885	4.003	4.168	4.824	5.325	6.658	5.987
	4	3.311	3.439	3.494	3.748	4.007	4.477	5.597	5.143
	5	2.984	3.037	3.176	3.218	3.493	3.939	4.925	4.600
	6	2.704	2.763	2.956	3.077	3.547	3.565	4.457	4.218
	7	2.550	2.652	2.661	2.783	3.235	3.288	4.111	3.933
	8	2.433	2.489	2.555	2.725	2.995	3.561	3.842	3.710
	9	2.304	2.362	2.471	2.530	2.804	3.345	3.628	4.353
	10	2.232	2.258	2.402	2.506	2.648	3.169	3.451	4.141

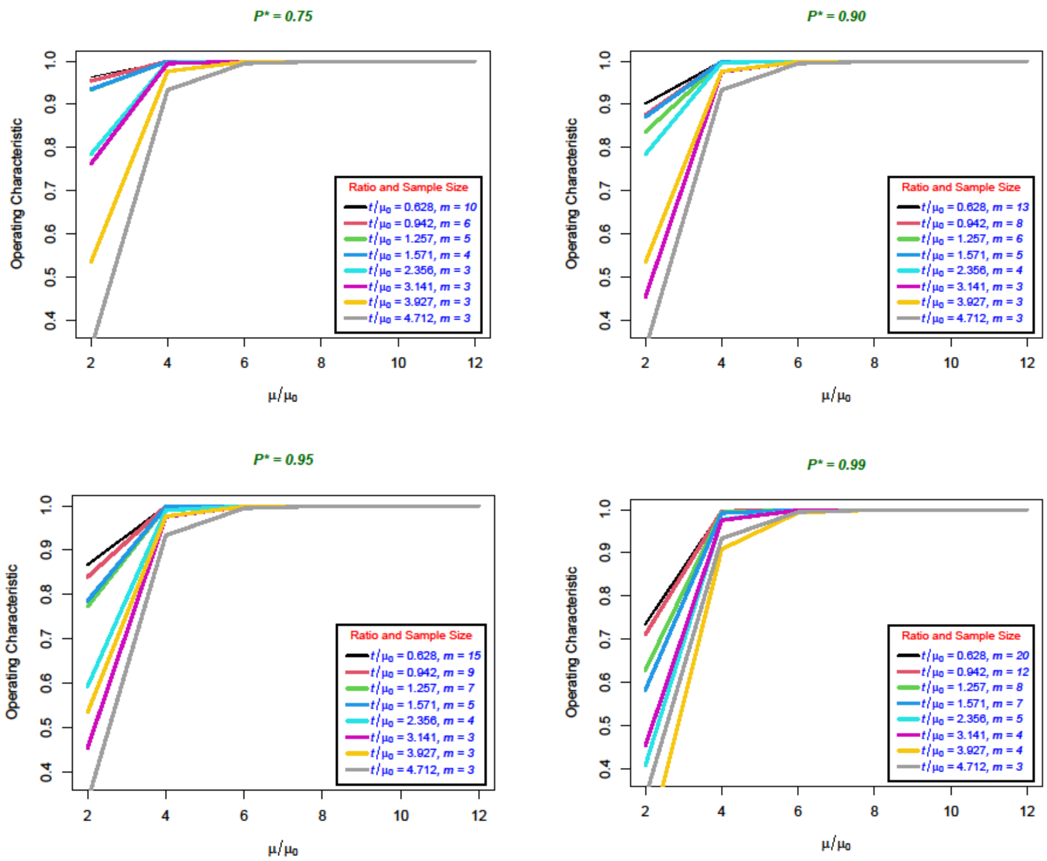

Figure 5. The OCF plots for $P^* = 0.75$, 90, 95, 0.99, $c = 6$ for $\beta = 8, \theta = 2$ in the GaLD

5. Application of real data

In this section, we will provide an example to demonstrate our recommended plan for industrial usage. The data set relates to the lifetime (measured in months) of 20 electric carts used for delivery and internal movement within a sizable industrial plant, where the values are 0.9, 1.5, 2.3, 6.2, 15, 16.3, 7.5, 8.3, 3.2, 3.9, 11.1, 12.6, 5.0, 10.4, 19.3, 22.6, 24.8, 31.5, 38.1, 53. This data is considered by (Aslam et al., 2011) for an improved group acceptance sampling plan for Dagum distribution under percentiles lifetime. (Lio et al., 2010) utilizing the Burr type XII percentiles for constructing ASP from truncated life tests. Table 4 provides the descriptive statistics for the data. Also, Figure 6 presents the box, density, histogram and TTT plots based on the real data.

Table 4: Descriptive statistics of the electric carts data set

n	**Mean**	**SD**	**Median**	**Min**	**Max**	**Range**	**Skew**	**Kurtosis**	**SE**
20	14.68	13.66	10.75	0.9	53	52.1	1.25	0.86	3.06

We evaluate the GaLD, Darna Distribution (DD) offered by (Shraa & Al-Omari, 2019), Length Biased Two Parameters Mirra Distribution (LBMD), and Power Length-Biased Suja Distribution (PLBSD) suggested by (A. I. Al-Omari et al., 2019) distributions for this data. The pdfs of these competitor models are defined as follows:

$$f_{DD}(x;\alpha,\theta) = \frac{\theta}{2\alpha^2+\theta^2}\left(2\alpha + \frac{\theta^4 x^2}{2\alpha^3}\right)e^{-\frac{\theta x}{\alpha}}; x>0, \alpha,\theta>0, \theta>\alpha, \theta\neq\alpha,$$

$$f_{LBMD}(x;\alpha,\theta) = \frac{\theta^4}{3\alpha+\theta^2}x\left(1+\frac{x^2\alpha}{2}\right)e^{-x\theta}, x\geq 0,\ \alpha\geq 0, \theta>0.$$

$$f_{PLBSD}(x;\alpha,\beta)=\frac{\alpha^{6}\beta}{\alpha^{4}+120}x^{2\beta-1}\left(x^{4\beta}+1\right)e^{-\alpha x^{\beta}};x\geq 0,\alpha,\beta>0.$$

The MLE approach is used to estimate parameters for all models based on voltage data. We calculate the MLEs of the models parameters with their standard errors (SE), and use various goodness of fit measures to determine which competitor has the best fit for the data set under consideration as Anderson-Darling statistics (AD), Cramér von Mises statistic (CVM), Akaike information criterion (AIC), Bayesian information criterion (BIC), consistent AIC (CAIC), Hannan-Quinn information criterion (HQIC), Kolmogorov-Smirnov statistic (KS), with its p-value. These measures can be defined as:

$$CVM=\frac{1}{12n}-+\sum_{j=1}^{n}\left[\frac{2j-1}{2n}-F\left(x_i\right)\right]^2,\quad AD=-n-\frac{1}{n}\sum_{j=1}^{n}\frac{2j-1}{n}\left[\ln\left(F\left(x_i\right)\right)+\ln\left(1-F\left(x_{n+1-i}\right)\right)\right],$$

$K-S=Sup_n\left|F_n(x)-F(x)\right|$, where $F_n(x)=\frac{1}{n}\sum_{i=1}^{n}I_{x_i\leq x}$ is empirical distribution function and F(x) is cumulative distribution. Also, $\text{BIC}=-2\ell+\eta\ln(n)$, $\text{AIC}=-2\ell+2\eta$, $\text{HQIC}=2\eta\ln[\ln(n)]-2\ell$, $\text{CAIC}=-2\ell+\frac{2\eta n}{n-\eta-1}$,

where ℓ is the negative maximized log-likelihood, n is the sample size and η is the number of parameters. The results are displayed in Table 5. The best distribution is indicated by lower AD, CVM, K-S, AIC, BIC, CAIC, HQIC for goodness of statistics. Among all other competitive models, it is to be noted that the GaLD has the lowest values of AD, CVM, AIC, BIC, CAIC, HQIC and K–S statistics with largest p-value. As a result, it might be identified as the best model to fit the provided data set.

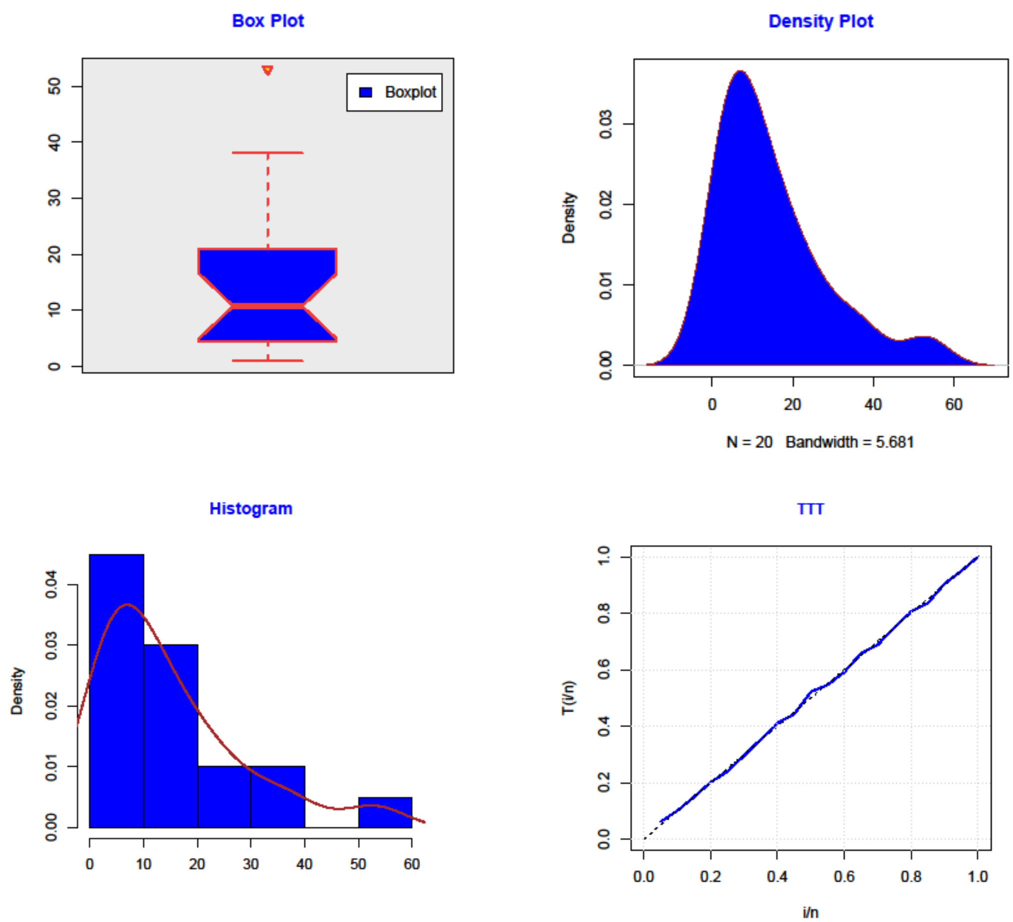

Figure6: The box (a), density (b), histogram (c) and TTT (d) plots based on the real data

Table 5: The AIC, CAIC, BIC, HQIC, -2LL, KS, P-value based on the electric carts

	Model			
Measures	**GaLD**	**DD**	**LBTMD**	**PLBSD**
AIC	151.2821	151.4540	153.9274	240.9003
CAIC	151.9880	152.1598	154.6332	241.6062
BIC	153.2736	153.4454	155.9188	242.8917
HQIC	151.6708	151.8427	154.3161	241.2890
CVM	0.0096	0.0076	0.0073	0.0110
AD	0.0867	0.0724	0.0698	0.0993
K-S	0.0517	0.0596	0.1403	0.0656
P-Value	0.9999	0.9999	0.7756	0.9999
- ℓ	73.6411	73.7270	74.9637	118.4501

Table 6: The MLEs of the model parameters with the corresponding Std. Dev. As well as the 95% CI for the electric carts data

Model	MLE	Std. Dev.	Inf.95%CI	Sup.95%CI
GaLD	$\hat{\beta}=0.1375$	0.1689	-0.1935	0.4685
	$\hat{\theta}=0.0938$	0.0403	0.0147	0.1728
DD	$\hat{\alpha}=1.4070$	21.0631	-39.8759	42.6899
	$\hat{\theta}=0.0962$	1.4407	-2.7275	2.9200
LBTMD	$\hat{\alpha}=0.0025$	0.0034	-0.0040	0.0091
	$\hat{\theta}=0.1667$	0.0320	0.1038	0.2295
PLBSD	$\hat{\alpha}=1.7860$	0.2741	1.2487	2.3233
	$\hat{\beta}=0.4703$	0.0571	0.3583	0.5821

Based on this data set, the MLEs of the GaLD parameters as $\hat{\beta}=0.1375$ and $\hat{\theta}=0.0938$. Tables 7-8 present the minimum sample sizes of the ASP using the electric carts data with $\hat{\beta}=0.1375$ and $\hat{\theta}=0.0938$, for $P^*=0.99$. The estimated mean of the data is $\hat{\mu}_{GaLD}=\dfrac{2\hat{\beta}(\hat{\theta}+1)-\hat{\theta}}{\hat{\theta}\hat{\beta}(\hat{\theta}+1)}=14.6729$.

Assume that $\mu_0=14.6729$ months represents the identified average lifetime, with testing time $t_0=13.8219$ months and it is required to accept or reject the lot. For the probability level of $P^*=0.99$, ratio $t_0/\mu_0=0.942$, minimum sample size $n=20$ from Table 7 , the acceptance number is $c=6$. Now, we can accept the lot with mean lifetime of 14.6729 with probability of 0.99 if there are 6 failures or less before $t_0=13.8219$. New, as 12 failures occurred before the stipulated time, therefore, the lot will be rejected. The producers' risk for the ASP $\left(n=20,\ c=6,\ t/\mu_0=0.942\right)$ are 0.58139, 0.06667, 0.00985, 0.00209, 0.00058, 0.00019, which are decreasing in the μ/μ_0 .

Table 7. Minimum sample sizes of the sampling plans based on electric carts data with $\hat{\beta}=0.1375$ and $\hat{\theta}=0.0938$ in the GaLD for $P^{*}=0.99$

c	t/μ_0							
	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0	8	6	4	3	2	2	2	1
1	12	8	6	5	4	3	3	3
2	16	11	8	7	5	4	4	4
3	19	13	10	9	7	6	5	5
4	23	16	12	10	8	7	6	6
5	26	18	14	12	9	8	7	7
6	29	20	16	13	10	9	8	8
7	32	22	18	15	12	10	9	9
8	35	24	19	16	13	11	10	10
9	38	26	21	18	14	12	11	11
10	40	28	23	19	15	13	13	12

Table 8. OCF and PR values of the GaLD ASP based on electric carts data with $\hat{\beta}=$ 0.1375, $\hat{\theta}=0.0938$ for $c=6$ and $P^{*}=0.99$

m	t/μ_0	μ/μ_0					
		2	4	6	8	10	12
29	0.628	0.40932	0.92917	0.98926	0.99768	0.99935	0.99978
	PR	0.59068	0.07083	0.01074	0.00232	0.00065	0.00022
20	0.942	0.41861	0.93333	0.99015	0.99791	0.99942	0.99981
	PR	0.58139	0.06667	0.00985	0.00209	0.00058	0.00019
16	1.257	0.39114	0.92686	0.98907	0.99767	0.99935	0.99978
	PR	0.60886	0.07314	0.01093	0.00233	0.00065	0.00022
13	1.571	0.43350	0.93855	0.99123	0.99818	0.99950	0.99984
	PR	0.56650	0.06145	0.00877	0.00182	0.00050	0.00016
10	2.356	0.39954	0.92901	0.98961	0.99782	0.99940	0.99980
	PR	0.60046	0.07099	0.01039	0.00218	0.00060	0.00020
9	3.141	0.28534	0.88578	0.98111	0.99580	0.99881	0.99959
	PR	0.71466	0.11422	0.01889	0.00420	0.00119	0.00041
8	3.927	0.29597	0.88344	0.98028	0.99557	0.99874	0.99957
	PR	0.70403	0.11656	0.01972	0.00443	0.00126	0.00043
8	4.712	0.14838	0.77902	0.95373	0.98837	0.99646	0.99874
	PR	0.85162	0.22098	0.04627	0.01163	0.00354	0.00126

5. Conclusions

The Gamma-Lindley distribution is used in this article to identify new ASPs for life experiments that are truncated at a predetermined time. The tables provide the minimal sample sizes required to determine a certain mean life of the test units. Calculated together with the operational characteristic function values are the associated producer risks. Using actual data from the initial failure of 20 electric carts used for delivery and internal movement within a sizable manufacturing facility, the benefit of the suggested ASPs is examined. The usefulness of the suggested GaLD acceptance sampling plans is proved in the study by the use of a real data set. Therefore, it is suggested that researchers utilize the new ASPs. Other ASPs, such as group ASP and double sampling plans for the mean as a quality parameter of other parameters can be considered as a future work.

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