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Designing a Variable Control Chart for a Two-Stage Production Process

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Abstract

In this paper, a variable chart has been proposed to study a two-stage serial production process. Measurements of quality characteristics of products processed at each stage are assumed to be independent and follow normal distribution. In order to evaluate performance of the control chart, two cases of equal shifts in both the stages and unequal shifts were considered and results were presented accordingly. The necessary measures are given to calculate the average run length (ARL) for shifted processes. The tables of ARLs are presented for equal and different sample sizes at first- and second- stage. Performance of the proposed control chart is presented using a simulation study.

Key Words: Variable control chart; two-stage process; average run length; process shift.

Mathematical Subject Classification: 86K98

1. Introduction

In the era of competition, manufacturing companies give due importance to statistical quality control (SQC) to monitor and control product quality. While control charts help in checking process quality, acceptance sampling is used in investigating product quality. Considering the process mean and process variability, control limits are derived for the statistic of the desired quality characteristic we wish to monitor and control. If the statistic (of the quality characteristic) is falling outside the control limits or any specific pattern is observed in the subsequent values of the statistic, the process is considered to be out-of-control state. Depending on the nature of the statistic of quality characteristic, a suitable sampling distribution of the statistic would be used to estimate type-1 and type-2 errors. The average run length (ARL) is used to assess the efficiency of control charts. If the control chart is able to detect a shift in the process quickly compared to other control charts, then the chart is considered to be more powerful than the other charts. Based on the nature of the quality characteristic, the control charts are categorized as variable and attribute charts. When the quality characteristic is measurable on a continuous scale, variable control charts such as X-bar, range or standard deviation charts are used. To control high-quality production process, both acceptance sampling and control charts are widely used to monitor quality. Though there are different control charts developed as part of the statistical process control, Shewhart's X-bar control chart has been widely used in the control process due to its simple and the ease of application (W.A. Shewhart 1925). Details pertaining to use of variable control charts can be found in (Al-Oraini HA 2002), (Mohammed M.A. 2001). While attribute control charts (p-chart, np- chart, c-chart, etc) are used for monitoring attribute data (counts) or qualitative data.

A production process usually involves processing of raw/ semi-finished material at several stages to make finished products. When the number of stages is large, it becomes difficult to monitor the quality of the product using simple tools. This poses challenges in monitoring quality of products in multi-stage production processes. Quality of the products processed through several stages would be dependent on the quality control mechanism put in place at different stages (Duffuaa 2009). (Yang 1997) designed an economic X chart and cause-selecting control chart to monitor a two-stage process by considering a cost model. (Yang and Yang 2006) proposed an approach to monitor two-stage processes when data are auto-correlated, while a cause selecting control chart was proposed by (Yang and Yeh 2011) for the two-stage processes with attribute data. (Linn et.al 2002) developed a method to prioritize process variation reduction in multistage processes and thereby to improve the overall process capability index. To monitor the quality characteristic in the second stage under the censored and non-censored reliability data by considering accelerated failure time (AFT) model, (Azam and Amirhossein 2017) used the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control charts on the proposed residuals to monitor the two-stage process. (Dragon et. al 2017) formulated an automatic control of quality in a multi-stage manufacturing process (MMP) to handle inaccurate knowledge of error flow model parameters. Performance of the least-square estimation method was compared with that of Huber's and bi-square robust estimation methods for different degrees of inter-stage auto-correlations and different rates and sizes of outlier observations in two-stage processes by (Farid et.al 2019). (Rasay et.al 2019) developed an integrated model for maintenance planning and statistical process control for a two-stage dependent process, wherein the quality characteristic of the second stage was related to that of the first stage based on a regression formula. Assuming that the process failure mechanism for each stage follows a general continuous distribution, the first stage process was monitored using the Shewhart control chart, while the second stage was monitored using a cause-selecting control chart.

Extensive research was carried out by a number of authors on the design and use of attribute control charts. (Aslam et al. 2018) designed an attribute control chart to assess performance of a two-stage production process in terms of average run lengths (ARLs). Control charts for monitoring the service times of a multi-stage process of a congested system which provides a multi-stage service to its customers were studied by (Mohsen Ebadi 2020). An integrated control chart system was designed by Zhang Wu and Shamsuzzaman 2005) for monitoring process shifts in mean and variance in a multi-stage manufacturing system. For monitoring dependent multi-stage processes, (Shervin 2008) reviewed cause selecting chart (CSC). In another communication, (D. Jearkpaporn 2007) discussed a monitoring scheme for detecting a mean shift in a multistage manufacturing process for a quality characteristic following gamma distribution.

To the best of authors' knowledge and from the literature review, it is found that there has not been much research done on designing variable charts for a two-stage or multi-stage production process when the quality characteristic follows normal distribution and the process at each stage is independent of the other. We need a decision rule of declaring in-control and out-of-control for a two-stage process. We may use two separate control charts for stage 1 and stage 2, then make a decision rule for the two-stage process. However, when using two separate control charts, we may not derive the run length behavior for the two-stage process. The novelty of this study is to investigate the run-length behavior for the two-stage process for the first time if the mean shifts in stage 1 and/or stage 2 occur. Further, this study may be the basis for developing control charts for the two-stage process when stage 1 and stage 2 are not independent. In this paper, Section 1 outlines introduction and review of literature, The rest of this paper is organized as follows. In Section 2, designing a variable chart is presented. In Section 3, simulation results and discussion are presented. An example is provided in Section 4 to illustrate the process of the chart with simulated data. Section 5 illustrates the model with real data and conclusions are given in Section 6.

2. Designing of a variable control chart for a two-stage production process

Consider a variable control chart for a two-stage serial production system. Let $X_{11}, X_{12}, \ldots\ldots, X_{1n_1}$ and $X_{21}, X_{22}, \ldots\ldots, X_{2n_2}$ be the random samples of sizes n_1 and n_2 measurements of the quality characteristics X_1 and X_2 drawn from the first- and second-stage processes respectively. It is assumed that X_1 and X_2 follow normal distribution with population means of μ_{10} and μ_{20} respectively with a common and known standard deviation σ' for both the stages. The sample means of the first- and second-stage processes are denoted by $\underline{X}_1$ and $\underline{X}_2$. The upper control limit (UCL) and lower control limit (LCL) for the first-stage process are given by:

$$UCL_1 = \mu_{10} + k_1 \frac{\sigma'}{\sqrt{n_1}}$$

$$LCL_1 = \mu_{10} - k_1 \frac{\sigma'}{\sqrt{n_1}} \qquad (1)$$

Similarly, the upper and lower control limits for the second-stage process are given by:

$$UCL_2 = \mu_{20} + k_2 \frac{\sigma'}{\sqrt{n_2}}$$

$$LCL_2 = \mu_{20} - k_2 \frac{\sigma'}{\sqrt{n_2}} \qquad (2)$$

where k_1, k_2 are control constants calculated for first- and second-stage processes respectively. The probability of declaring the first-stage process as in-control when the process is actually in-control is given by

$$P(LCL_1 \leq \underline{X}_1 \leq UCL_1) = \Phi\left(\frac{UCL_1 - \mu_{10}}{\frac{\sigma'}{\sqrt{n_1}}}\right) - \Phi\left(\frac{LCL_1 - \mu_{10}}{\frac{\sigma'}{\sqrt{n_1}}}\right)$$

$$= \Phi(k_1) - \Phi(-k_1) = 2\Phi(k_1) - 1 \qquad (3)$$

where $\Phi(.)$ is the cumulative distribution function of standard normal distribution.
Similarly, the probability of declaring the second-stage process as in-control when the process is actually in-control is given by

$$P\left(LCL_2 \leq \underline{X}_2 \leq UCL_2\right) = \Phi(k_2) - \Phi(-k_2) = 2\Phi(k_2) - 1 \qquad (4)$$

The probability of declaring the two-stage production process as in-control when the process at both the stages is in-control becomes:

$$P_{in}^{0,0} = P\left(LCL_1 \leq \underline{X}_1 \leq UCL_1\right) * P\left(LCL_2 \leq \underline{X}_2 \leq UCL_2\right) \qquad (5)$$

$$P_{in}^{0,0} = [2\Phi(k_1) - 1] * [2\Phi(k_2) - 1] \qquad (6)$$

Probability of declaring the two-stage process as out-of-control when the first-stage process is in-control and the second-stage process is out-of-control is:

$$P_{out}^{0,1} = P\left[LCL_1 \leq \underline{X}_1 \leq UCL_1\right] * P\left[\left(\underline{X}_2 > UCL_2\right) U \left(\underline{X}_2 < LCL_2\right)\right] \qquad (7)$$

Probability of declaring the two-stage process as out-of-control when the first-stage process is out-of-control and the second-stage process is in-control is:

$$P_{out}^{1,0} = P\left[\left(\underline{X}_1 > UCL_1\right) U \left(\underline{X}_1 < LCL_1\right)\right] * P\left[LCL_2 \leq \underline{X}_2 \leq UCL_2\right] \qquad (8)$$

Probability of declaring the two-stage process as out-of-control when the processes at both the stages are out-of-control is:

$$P_{out}^{1,1} = P\left[\left(\underline{X}_1 > UCL_1\right) U \left(\underline{X}_1 < LCL_1\right)\right] * P\left[\left(\underline{X}_2 > UCL_2\right) U \left(\underline{X}_2 < LCL_2\right)\right] \qquad (9)$$

In a two-stage production system, the process at each stage is considered to be independent and the system would be out-of-control in the following scenarios: first-stage process is out-of-control; second-stage process is out-of-control or processes at both the stages are out-of-control. Therefore, the probability declaring the two-stage process as out-of-control considering the above three scenarios is given by

$$P_{out} = P_{out}^{0,1} + P_{out}^{1,0} + P_{out}^{1,1} \qquad (10)$$

The out-of-control average run length (ARL) is given by

$$ARL_{00} = \frac{1}{P_{out}} \qquad (11)$$

Now we will derive out-of-control ARL for three different cases: (i) just the first-stage production process shifted, (ii) just the second-stage production process shifted, and (iii) processes at both the stages shifted.

Shift occurred only in the first-stage process

Suppose μ_{10} is shifted to $\mu_{11} = \mu_{10} + c_1\sigma'$, where $c_1 > 0$ is the shift constant for the first-stage process, then the probability of declaring the first-stage process as out-of-control is given by

$$P_{out}^{1',0} = P\left[\left(\underline{X}_1 > UCL_1|\mu_{11}\right)U\left(\underline{X}_1 < LCL_1|\mu_{11}\right)\right] * P\left(\underline{X}_2 > UCL_2|\mu_{20}\right)U\left(\underline{X}_2 < LCL_2|\mu_{20}\right) \quad (12)$$

$$P_{out}^{1',0} = \left[1 - \Phi\left(\frac{UCL_1 - (\mu_{10} + c_1\sigma')}{\frac{\sigma'}{\sqrt{n_1}}}\right) + \Phi\left(\frac{LCL_1 - (\mu_{10} + c_1\sigma')}{\frac{\sigma'}{\sqrt{n_1}}}\right)\right] * 2[1 - \Phi(k_2)] \quad (13)$$

$$P_{out}^{1',0} = \left[1 - \Phi\left(k_1 - c_1\sqrt{n_1}\right) + \Phi\left(-k_1 - c_1\sqrt{n_1}\right)\right] * 2[1 - \Phi(k_2)] \quad (14)$$

The out-of-control ARL when shift occurred only in the first-stage process is:

$$ARL_{1'0} = \frac{1}{P_{out}^{1',0}} \quad (15)$$

Shift occurred only in the second-stage process

Suppose μ_{20} is shifted to $\mu_{21} = \mu_{20} + c_2\sigma'$, where $c_2 > 0$ is the shift constant for the second-stage process, then the probability of declaring the second-stage process as out of control is given by$P_{out}^{0,1'} = P\left[\left(\underline{X}_1 > UCL_1|\mu_{10}\right)U\left(\underline{X}_1 < LCL_1|\mu_{10}\right)\right] * P\left(\underline{X}_2 > UCL_2|\mu_{21}\right)U\left(\underline{X}_2 < LCL_2|\mu_{21}\right)$ (16)

$$P_{out}^{0,1'} = 2[1 - \Phi(k_1)]*\left[1 - \Phi\left(\frac{UCL_2-(\mu_{20}+c_2\sigma')}{\frac{\sigma'}{\sqrt{n_2}}}\right) + \Phi\left(\frac{LCL_2-(\mu_{20}+c_2\sigma')}{\frac{\sigma'}{\sqrt{n_2}}}\right)\right] \quad (17)$$

$$P_{out}^{0,1'} = 2[1 - \Phi(k_1)]*[1 - \Phi(k_2 - c_2\sqrt{n_2}) + \Phi(-k_2 - c_2\sqrt{n_2})] \quad (18)$$

The out-of-control ARL when shift occurred in only in the second-stage process is:

$$ARL_{01'} = \frac{1}{P_{out}^{0,1'}} \quad (19)$$

Shift occurred in both the first- and second-stage processes

In this case, μ_{10} is shifted to $\mu_{11} = \mu_{10} + c_1\sigma'$, μ_{20} is shifted to $\mu_{21} = \mu_{20} + c_2\sigma'$, then the probability of declaring the both the processes as out-of-control is given by

$$P_{out}^{1',1'} = P\left[\left(\underline{X}_1 > UCL_1|\mu_{11}\right)U\left(\underline{X}_1 < LCL_1|\mu_{11}\right)\right] * P\left(\underline{X}_2 > UCL_2|\mu_{21}\right)U\left(\underline{X}_2 < LCL_2|\mu_{21}\right) \quad (20)$$

$$P_{out}^{1',1'} = \left[1 - \Phi\left(\frac{UCL_1-(\mu_{10}+c_1\sigma')}{\frac{\sigma'}{\sqrt{n_1}}}\right) + \Phi\left(\frac{LCL_1-(\mu_{10}+c_1\sigma')}{\frac{\sigma'}{\sqrt{n_1}}}\right)\right]*\left[1 - \Phi\left(\frac{UCL_2-(\mu_{20}+c_2\sigma')}{\frac{\sigma'}{\sqrt{n_2}}}\right) + \Phi\left(\frac{LCL_2-(\mu_{20}+c_2\sigma')}{\frac{\sigma'}{\sqrt{n_2}}}\right)\right]$$

Upon simplification of the above equation, we get

$$P_{out}^{1',1'} = [1 - \Phi(k_1 - c_1\sqrt{n_1}) + \Phi(-k_1 - c_1\sqrt{n_1})]* \left[1 - \Phi\left(k_2 - c_2\sqrt{n_2}\right) + \Phi\left(-k_2 - c_2\sqrt{n_2}\right)\right] \quad (21)$$

The total probability of declaring the two-stage process as out-of-control when the process is shifted only in first-stage or process is shifted only in the second stage or process shifted in both the stages, is obtained as:

$$P''_{out} = P_{out}^{1',0} + P_{out}^{0,1'} + P_{out}^{1',1'} \quad (22)$$

The out-of-control ARL when shift occurred in the processes of both the stages is:

$$ARL_{1'1'} = \frac{1}{P''_{out}}$$

The out-of-control ARLs, when the whole process is out-of-control, are derived using the following procedure:

(1) Specify values of n_1, n_2 and the target in-control ARL (r_0).
(2) Find the control constants k_1 and k_2 such that ARL (r_0) $\geq$ specified ARL (r).
(3) Obtain the out-of-control ARLs for various values of shift constants c_1 and c_2.

3. Results and discussion

For the proposed chart, considering different in-control ARLs of 200, 300 and 370, the average run lengths (ARLs) have been derived for two scenarios: (i) when the sample sizes are equal for both the stages (see Table-1), and (ii) when the sample sizes are unequal for the two processes (see Table-2). In both the scenarios, ARLs decrease rapidly with increase in sample sizes, irrespective of sample size being equal or unequal.

Table-1: ARLs when equal sample sizes in stage-1 and stage-2

n_1	5	5	5	10	10	10	20	20	20
n_2	5	5	5	10	10	10	20	20	20
k_1	4.233	5.082	3.604	6.934	3.682	3.000	5.694	2.935	3.000
k_2	2.809	2.935	3.037	2.807	2.957	5.679	2.807	4.800	6.953
Shift		ARL			ARL			ARL	
0	200.00	300.00	370.00	200.00	300.00	370.00	200.00	300.00	370.00
0.05	189.56	283.16	347.03	180.18	266.49	329.00	163.63	241.50	295.45
0.1	163.43	241.53	291.24	137.61	197.03	243.89	102.98	147.35	177.56
0.15	131.90	192.27	227.03	96.63	133.34	165.68	60.30	83.72	99.46
0.2	102.57	147.41	170.26	66.13	88.00	109.87	35.71	48.21	56.55
0.25	78.49	111.28	125.76	45.38	58.29	73.21	21.90	28.79	33.38
0.3	59.85	83.79	92.70	31.60	39.18	49.57	14.00	17.92	20.55
0.35	45.79	63.36	68.63	22.43	26.82	34.24	9.33	11.63	13.21
0.4	35.28	48.28	51.20	16.25	18.71	24.15	6.48	7.86	8.85
0.5	21.52	28.86	29.30	9.08	9.65	12.81	3.52	4.02	4.49
0.6	13.65	17.98	17.44	5.51	5.35	7.39	2.21	2.36	2.66
0.7	9.00	11.68	10.79	3.62	3.17	4.63	1.57	1.53	1.81
0.8	6.16	7.91	6.93	2.56	2.00	3.12	1.23	1.07	1.39
0.9	4.37	5.58	4.61	1.94	1.34	2.26	1.02	0.79	1.18
1	3.20	4.08	3.17	1.57	0.95	1.74	0.86	0.6	1.06

Table-2: ARLs when unequal sample sizes in stage-1 and stage-2

n_1	5	5	5	15	15	15	22	22	22
n_2	10	10	10	20	20	20	25	25	25
k_1	5.883	3.08	4.990	3.087	3.107	2.999	2.807	2.935	6.016
k_2	2.807	3.223	3.000	2.969	3.185	5.411	7.671	7.264	2.999
Shift		ARL			ARL			ARL	
0	200.00	300.00	370.00	200.00	300.00	370.00	200.00	300.00	370.00
0.05	180.18	274.04	328.99	162.85	242.51	311.42	160.64	236.81	280.86
0.1	137.61	215.47	243.88	101.27	148.48	206.21	97.85	139.66	155.08
0.15	96.63	155.16	165.66	58.29	83.93	125.63	55.75	77.18	81.15
0.2	66.13	107.58	109.86	33.76	47.68	76.21	32.36	43.55	43.86
0.25	45.38	73.89	73.20	20.16	27.88	47.28	19.56	25.64	24.94
0.3	31.60	50.96	49.56	12.48	16.88	30.22	12.38	15.81	14.96
0.35	22.43	35.52	34.23	8.01	10.58	19.94	8.20	10.21	9.46
0.4	16.25	25.09	24.15	5.32	6.87	13.58	5.68	6.91	6.30

0.5	9.08	13.10	12.81	2.60	3.20	6.91	3.11	3.60	3.24
0.6	5.51	7.29	7.39	1.44	1.70	3.94	1.99	2.21	2.00
0.7	3.62	4.33	4.62	0.90	1.01	2.49	1.46	1.57	1.44
0.8	2.56	2.73	3.12	0.63	0.68	1.69	1.21	1.26	1.16
0.9	1.94	1.83	2.27	0.48	0.51	1.21	1.08	1.10	1.00
1	1.57	1.30	1.76	0.41	0.42	0.89	1.02	1.02	0.86

ARLs have also been derived (see Tables 3-4) for two different scenarios, viz., process shift is the same in both the stages (c = 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1) and process shifts are unequal in the two stages (c_1 = 0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1; c_2=0, 0.02, 0.04, 0.04, 0.06, 0.08, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6, 0.7, 0.75). Considering three cases of r_0 = 200, r_0 = 300 and r_0 = 370 for the target in-control ARL, the results have been presented for $n_1 = n_2$ and $n_1 \neq n_2$. Tables have been prepared for process shifts c_1, c_2, whose values are ranging between 0 to 1. When $c_1 = c_2 = 0$ means the process at both the stages is in-control.

Table-3 and 4 present ARLs for equal and unequal process shifts for a fixed r_0= 370 and different combinations of $n_1, n_2, k_1, and\ k_2$. It is observed that ARLs are decreasing fast with unequal shifts in stage-1 and stage-2 compared to that of equal shifts. Similar trend is seen in ARLs when sample sizes at stage-1 and stage-2 are unequal compared to ARLs with equal sample sizes.

For r_0= 400 and with a common process shift at first- and second-stages, the trend in ARLs is presented for different combinations of sample sizes. As the sample sizes increase, ARLs are observed to be decreasing rapidly (see Figure-2).

Table-3: ARLs when r_0= 370, $n_1 = n_2 = 5$, k_1= 3.8065, k_2= 3.0159

Equal shifts in stage-1 and stage-2	Unequal shifts in stage-1 and stage-2
369.99	369.99
369.05	347.61
338.52	293.00
280.36	229.67
229.67	173.23
173.23	128.65
136.61	95.32
107.44	70.93
84.63	53.17
44.92	30.72
30.72	18.45
18.45	11.52
11.52	7.46
7.46	5.00
5.00	3.47

Table-4: ARLs when r_0= 370, $n_1 = 5, n_2 = 7$, k_1= 3.5398, k_2= 3.0481

Equal shifts in stage-1 and stage-2	Unequal shifts in stage-1 and stage-2
370.04	370.04
358.63	339.83
300.64	271.01
226.60	199.00
162.25	141.13
114.29	99.32
80.60	70.25
57.36	50.23
41.32	36.38

22.35	19.89
12.77	11.50
7.71	7.01
4.89	4.49
3.25	3.02
2.26	2.12

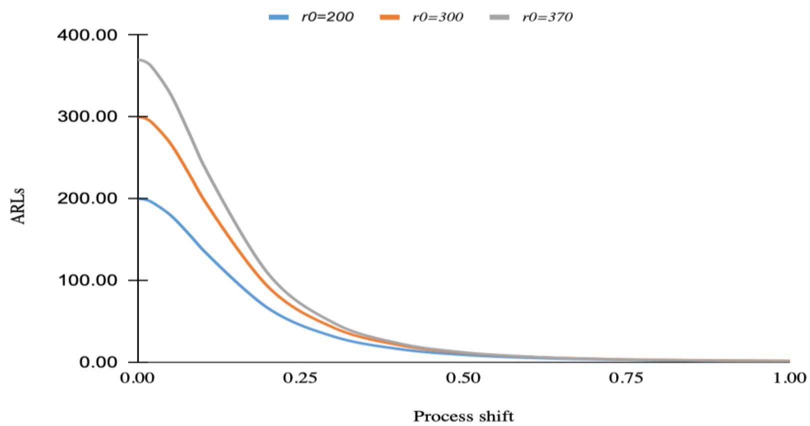

Figure 1. Comparison of ARLs for different process shifts with equal sample size n_1 = n_2=10.

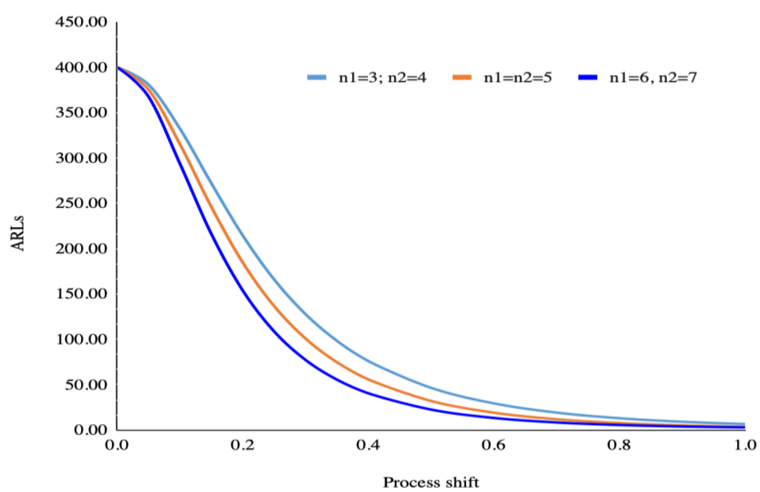

Figure 2. ARLs for for different sample sizes in first-stage and second-stage process when r_0=400 and $c_1 = c_2$

4. Example with simulated data

We illustrate an application of the variable control chart to monitor two quality characteristics - one measured at the stage-1 production process and the other measured at the stage-2 process. The two quality characteristics could be measured either in the same units or in different units. As the process is a two-stage serial production process, it is considered to be in-control when the process in both the stages is in-control. The process is said to be out-of-control,

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when the stage-1 process is in-control and stage-2 process is out-of-control; or stage-1 process is out-of-control and stage-2 process is in-control; or the process in both the stages is out-of-control.

We implemented the variable control chart for the two-stage process using simulated data to detect shifts in the process. For this, we generated random numbers from normal distribution using R-program for the parameters - process mean μ_1 = 5 units and standard deviation σ_1 = 1.5 for the first stage process and process mean μ_2 = 8 units and standard deviation σ_2= 2.3 for the second stage process. The simulated data was used to test it against the structure of the proposed control chart. Actual distribution of process means at stage-1, stage-2 and combined process (of stage-1 and stage-2), was generated with sample sizes of n_1 = 10 for stage-1 process and n_2= 10 for stage-2. All the three scenarios were randomly simulated. In the first case, the first 20 observations were simulated for both stages considering that the process is in-control state, and the next 20 observations were generated from the shifted process with a shift amount of c = 0.5 in the first stage of the process only. In the second case, the first 20 observations were simulated for both stages considering that the process is in control state, after that next 20 observations were generated from the shifted process with shift amount of c = 0.5 in the second stage of the process only. In the third case also, first 20 observations were simulated for both stages considering that the process is in-control state, after that next 20 observations were generated from the shifted process with shift amount of c = 0.5 in the first and second stage of the process. The three cases were plotted in the figures separately (see Figure 3(a), 3(b) and 3(c)) to clearly indicate the shift in all possible scenarios that can be effectively detected. Further, this study may be the basis for developing control charts for the two-stage process when stage 1 and stage 2 are not independent.

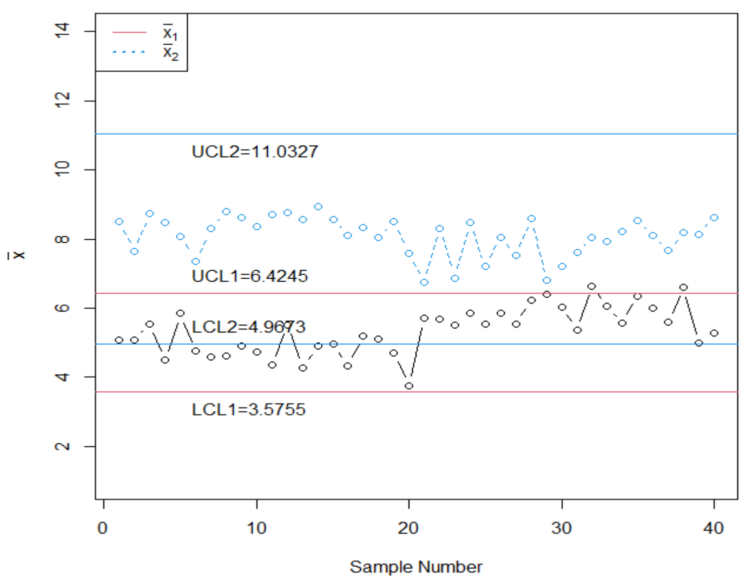

Figure 3(a): Control chart for the process when shift occurred only in stage-1

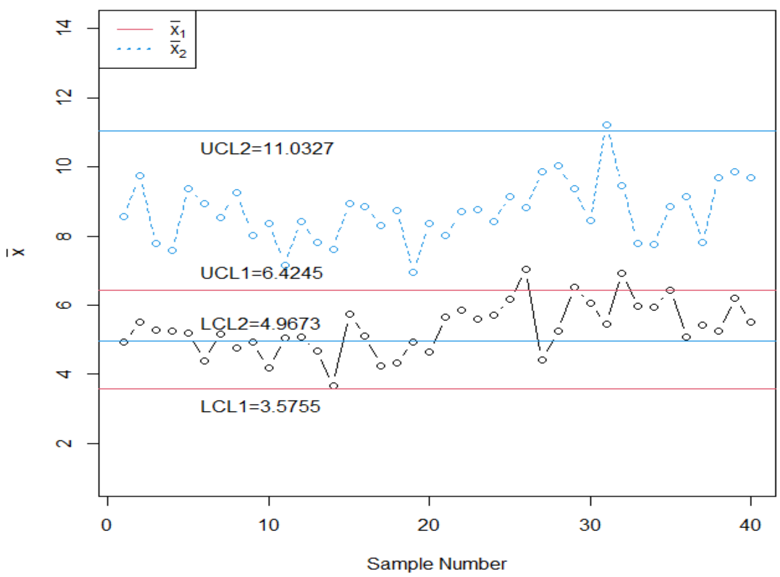

Figure 3(b): Control chart for the process when shift occurred only in stage-2

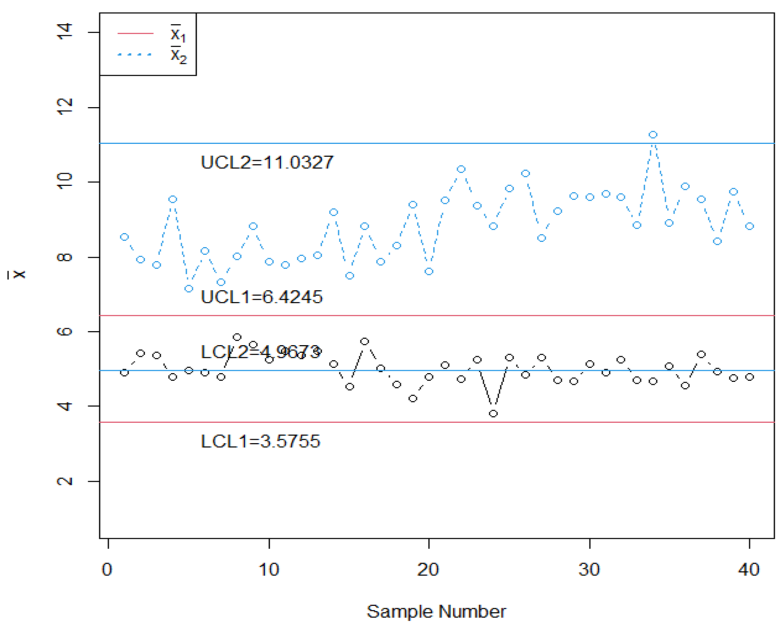

Figure 3(c): Control chart for the process when shift occurred in stage-1 and stage-2

5. Real-life Example

In this section, a real-life example data set of inside diameter measurements for automobile engine piston rings from multiple samples used by Montgomery (2000) have been to illustrate the application of a two-stage sampling process where quality characteristic follows normal distribution.

Table-5: Sample data set of inside diameter measurements (mm) for Automobile Engine Piston Rings

74.030, 74.002, 74.019, 73.992, 74.008
73.995, 73.992, 74.001, 74.011, 74.004

73.988, 74.024, 74.021, 74.005, 74.002
74.002, 73.996, 73.993, 74.015, 74.009
73.992, 74.007, 74.015, 73.989, 74.014
74.009, 73.994, 73.997, 73.985, 73.993
73.995, 74.006, 73.994, 74.000, 74.005
73.985, 74.003, 73.993, 74.015, 73.988
74.008, 73.995, 74.009, 74.005, 74.004
73.998, 74.000, 73.990, 74.007, 73.995
73.994, 73.998, 73.994, 73.995, 73.990
74.004, 74.000, 74.007, 74.000, 73.996
73.983, 74.002, 73.998, 73.997, 74.012
74.006, 73.967, 73.994, 74.000, 73.984
74.012, 74.014, 73.998, 73.999, 74.007

Manufacturing of automobile engine piston rings involves a multi-stage production process. For validating our variable control chart we consider the piston rings production process involving two-stages. In stage-1, piston rings are cut from the cylinder of round cross-section and inner diameter of semi-finished piston rings are measured. Then in stage-2, the semi-finished piston rings are to be grounded or machined to make it balanced and homogeneous (that is, finished piston rings) and inner diameter is again measured to monitor the process.

For generating the control chart for stage-1, it uses a dataset containing inside diameter measurements of semi-finished piston rings from multiple samples. The grand mean is set at 74.001, and the average range is 0.0094, with a sample size of 5. The control limits are computed using an estimated standard deviation (σ'). These limits are adjusted by a coefficient (k_1), and the resulting upper and lower control limits (UCL and LCL) are plotted along with the data on a control chart in Fig 4(a) depicting stage-1 process. In stage-2, the semi-finished piston rings undergo further processing to make them balanced and homogeneous, simulated data of inside diameter is generated using a normal distribution with the same mean as the grand mean from stage-1, and a standard deviation based on σ' and the sample size. The UCL and LCL for Stage 2 are calculated with a different coefficient (k_2, set at 3.037) and are displayed on a separate control chart in Fig 4(b). All the inside measurements of piston rings are within the control limits of the control chart for stage-2 process, which indicate that the two-stage process is in control.

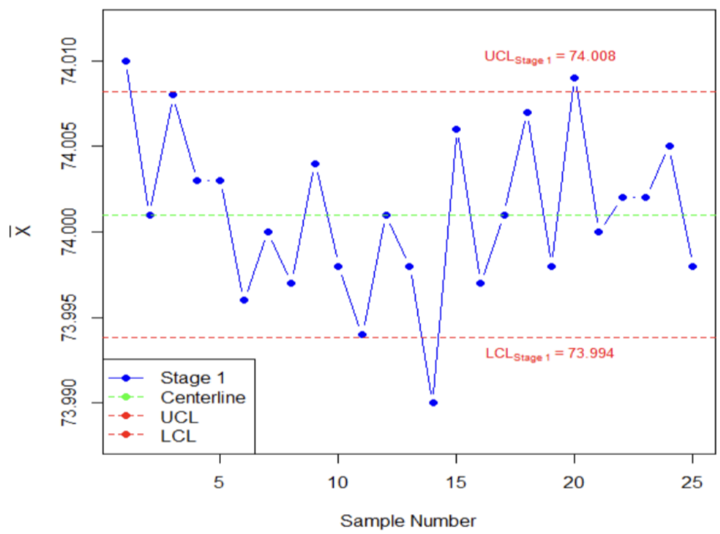

Figure 4(a): Control chart for the stage-1 process

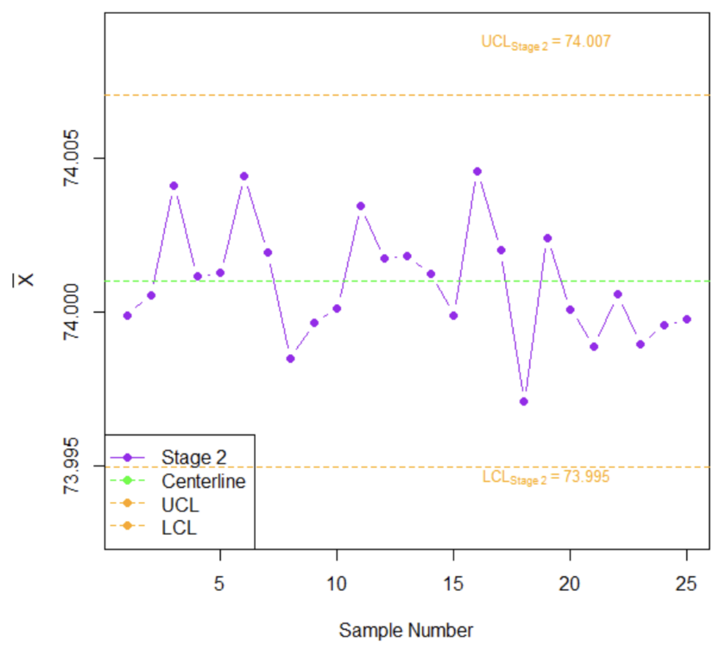

Fig 4(b): Control chart for the stage-2 process

6. Conclusion

In this paper, a variable control chart was developed for monitoring a two-stage production process. The simulation study has also been conducted using analytical expressions developed for the proposed control chart. The proposed chart can be applied effectively for monitoring quality characteristics at each stage of a serial production system and detecting shifts in the process. Considering process shift in each stage as a random variable following a certain probability distribution, a variable chart may be developed for a serial production process as a future research.

Data Availability: The data is available with the corresponding author upon a reasonable request.

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