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A Novel Robust Class of Estimators for Estimation of Finite Population Mean: A Simulation Study

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Abstract

In the existing survey sampling literature, the ratio-type estimators are an obvious choice to estimate the finite population mean when auxiliary information related to the study variable is readily available. Typically, auxiliary information is incorporated into ratio-type estimators by using conventional measures such as mean, range, coefficient of kurtosis, coefficient of skewness and coefficient of correlation, etc. which are less efficient when extreme observation are present in the data. This study provides a remedy and enhances the efficiency of the ratio-type estimators of population mean in the presence of extreme observations by proposing dual auxiliary variables-based exponential-cum-ratio class of estimators which integrates both conventional and non-conventional measures under simple random sampling without replacement. The expression of the mean squared error and theoretical efficiency conditions for proposed class of estimators have been obtained for comparison purposes. A simulation study was carried out based on contaminated normal distribution and the robustness of the proposed estimators has been assessed in the presence of extreme observations. For practical implementation, six real data sets have been used to compare the performance of the proposed estimators with competing estimators to support the theoretical results. The theoretical and empirical results suggest that the proposed estimators are more precise than usual mean as well as existing estimators' ratio-type considered in this study.

Key Words: Auxiliary information; Exponential-type ratio estimators; Extreme observations; Robustness; Simple random sampling.

Introduction

The utilization of supplementary information related to the variable of interest is extremely popular in estimation of population parameters, such as mean and variance, which results in enhanced efficiency of the estimators. Usually, the auxiliary information is utilized within the framework of ratio, product, and regression type estimators for obtaining more precise estimates of the population parameters. Such gain in efficiency is achieved by utilizing known supplementary conventional and non-conventional measures' information. In literature, several, ratio and regression, estimators are available which utilize the additional information on an auxiliary variable closely related to the study variable when there exists a linear relationship between study and auxiliary variable. On the other hand, Exponential estimators are ideally used when the relationship between both study and auxiliary variable is not strong. Cochran (1940) considered the use of ratio method of estimation for estimating the yields of cereal experiments. Watson (1937) and Yates (1960) introduced regression estimators for estimating the average leaf area of palnt and average volume of timber, respectively. Bahl and Tuteja (1991) suggested the exponential-type ratio and product estimator for population mean utilizing known supplementary information. Murthy (2010) developed the product estimator for situation when auxiliary variable has a negative correlation with the study variable.

In the past decade, several the dual auxiliary information-based estimators were introduced in the survey sampling literature. Lu (2013) suggested chain-ratio and regression estimators that incorporates the dual supplementary information. Using information on two auxiliary variables, Awan and Shabbir (2014) proposed optimum regression estimator for population mean. John and Inyang (2015) introduced exponential ratio estimator based on dual supplementary information. Lu (2017) offered a ratio-cum-product type estimator to estimate the finite population mean that integrates information of two auxiliary variables. Shabbir and Gupta (2017) suggested an improved estimator based on dual supplementary information under simple and stratified random sampling. Irfan et al. (2018) proposed dual supplementary information-based difference-type exponential estimators for estimating the finite population mean. Some improved optimal estimators based on the dual supplementary information were introduced by Javed and Irfan (2020). Robust regression methods were adopted by Grover and Kaur (2021) to provide enhanced regression estimators of population mean. The interested readers may refer to Olkin (1958) , Raj (1965), Abu-Dayyah et al. (2003), Kadilar and Cingi (2005), Singh and Tailor (2005), Lu and Yan (2014), Abid, Abbas and Riaz (2016), Abbas et al. (2018), Niaz et al. (2021), and Singh and Nigam (2021) For more details on dual auxiliary information estimators.

The presence of extreme observations can affect the efficiency of the ratio and regression type estimators as these are usually based on the means of the study and auxiliary variables. In recent years, some robust ratio and regression type estimators have been introduced. For example, Abid et al. (2018) suggested various robust ratio estimators based on non-conventional dual auxiliary information. Zaman (2018) used the robust regression techniques for improving the ratio estimators. Ali et al. (2021) introduced various robust regression type estimators to estimate the finite population mean of sensitive variable. Bulut and Zaman (2022) proposed an enhanced class of robust ratio estimators by incorporating the estimation technique of the minimum covariance determinant. More work related to incorporation of robust techniques and robust measures in ratio method of estimation can be seen in Abid et al. (2020), Zaman and Bulut (2021), Zaman et al. (2021) and the references cited therein.

The development of the exponential-type ratio estimators in the presence of outlier and extreme observations have not yet been considered in the literature, especially in the context of dual auxiliary variables. This study is aimed to suggest a robust class of exponential-type ratio estimators of population mean based on dual auxiliary variables. For this purpose, three robust measures namely the Tri-mean (TM) suggested by Wang et al. (2007), Hodges-Lehmann estimator (HL) discussed by Hettmansperger and McKean (2011) and Deciles-mean (DM) suggested by Rana et al. (2012) were considered for the construction of the exponential-type estimators under simple random sampling without replacement (SRSWOR). The following notations are utilized:

N, Population size

n, Sample size

$f = n/N$ Sampling fraction

Y, X_1, X_2 The Study and auxiliary variable

$\bar{Y}, \bar{X}_1, \bar{X}_2$ Population means

$\bar{y}, \bar{x}_1, \bar{x}_2$ Sample means

C_y, C_{x1}, C_{x2} Coefficient of variations

MSE (.) Mean squared error of an estimator

$\hat{\bar{Y}}_i$ i-th existing estimator

$\hat{\bar{Y}}_{pj}$ j-th proposed estimator

Using the above notations, the traditional ratio estimator for mean of the study variable (Y) based on two auxiliary variables X_1 and X_2, is defined as:

$$\hat{\bar{Y}}_M = \eta_1 \bar{y} \frac{\bar{X}_1}{\bar{x}_1} + \eta_2 \bar{y} \frac{\bar{X}_2}{\bar{x}_2}$$

where η_1 and η_2 are the weights such that $\eta_1 + \eta_2 = 1$. The MSE of $\hat{\bar{Y}}_M$ is defined as:

$$MSE\left(\hat{\bar{Y}}_M\right) \cong \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + \eta_1^2 C_{x1}^2 + \eta_2^2 C_{x2}^2 - 2\eta_1 \rho_{yx1} C_y C_{x1} - 2\eta_2 \rho_{yx2} C_y C_{x2} + 2\eta_1 \eta_2 \rho_{x1x2} C_{x1} C_{x2}\right) \quad (1.1)$$

The optimal values of η_1 and η_2 are:

$$\eta_1^* = \frac{C_{x2}^2 + \rho_{yx1} C_y C_{x1} - \rho_{yx2} C_y C_{x2} - \rho_{x1x2} C_{x1} C_{x2}}{C_{x1}^2 + C_{x2}^2 - 2\rho_{x1x2} C_{x1} C_{x2}}, \eta_2^* = 1 - \eta_1^*$$

So, the minimum MSE is

$$MSE_{min}\left(\hat{\bar{Y}}_M\right) \cong \frac{1-f}{n} \bar{Y}^2 (C_y^2 + \eta_1^{2*} C_{x1}^2 + \eta_2^{2*} C_{x2}^2 - 2\eta_1{}^* \rho_{yx1} C_y C_{x1} - 2\eta_2{}^* \rho_{yx2} C_y C_{x2} + 2\eta_1{}^* \eta_2{}^* \rho_{x1x2} C_{x1} C_{x2} \quad (1.2)$$

The rest of the manuscript is prearranged as: Section 2 presents the structure of the existing estimators. The proposed class of exponential-type estimators are given in Section 3. The efficiency conditions are given in Section 4. The performance of the proposed estimators is evaluated through simulation and empirical studies in Section 5 along with the robustness study. Finally, Section 6 provides a summary and conclusions of the study.

2. Some existing two auxiliary variables-based estimators

Singh's (2003) estimator

Singh (2003) proposed a ratio estimator for estimation of the population mean $\bar{Y}$ based on known population means of the auxiliary variables which is defined as:

$$\hat{\bar{Y}}_s = \bar{y} \left(\frac{\bar{X}_1}{\bar{x}_1}\right) \left(\frac{\bar{X}_2}{\bar{x}_2}\right)$$

The MSE of $\hat{\hat{Y}}_s$ is given by:

$$MSE\left(\hat{\hat{Y}}_s\right) \cong \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + C_{x1}^2 + C_{x2}^2 - 2\rho_{yx1}C_yC_{x1} + 2\rho_{x1x2}C_{x1}C_{x2}\right) \quad (2.1)$$

Singh and Tailor's (2005) estimator

A modified ratio-cum-product type estimator based on known value of correlation coefficient of the auxiliary variables proposed by Singh and Tailor (2005) is defined as:

$$\hat{\hat{Y}}_{ST} = \bar{y}\left(\frac{\bar{X}_1 + \rho_{x1x2}}{\bar{x}_1 + \rho_{x1x2}}\right)\left(\frac{\bar{x}_2 + \rho_{x1x2}}{\bar{X}_2 + \rho_{x1x2}}\right)$$

The MSE of $\hat{\hat{Y}}_{ST}$ is given by:

$$MSE\left(\hat{\hat{Y}}_{ST}\right) \cong \frac{1-f}{n} \bar{Y}^2 \left((C_y^2 + \theta_1^* C_{x1}^2(\theta_1^* - 2k_{yx1}) + \theta_2^* C_{x2}^2(\theta_2^* + 2\left(k_{yx2} - \theta_1^* k_{x1x2}\right)\right) \quad (2.2)$$

where $k_{yx1} = \rho_{yx1}\frac{C_y}{C_{x1}}$, $k_{yx2} = \rho_{yx2}\frac{C_y}{C_{x2}}$, $k_{x1x2} = \rho_{x1x2}\frac{C_{x1}}{C_{x2}}$,$\theta_1^* = \frac{\bar{X}_1}{\bar{X}_1 + \rho_{x1x2}}$ and $\theta_2^* = \frac{\bar{X}_2}{\bar{X}_2 + \rho_{x1x2}}$

Singh et al. (2005) estimator

Singh et al. (2005) proposed a ratio-cum-product estimator on the basis of known population means of the auxiliary variables to estimate the population mean $\bar{Y}$ as:

$$\hat{\hat{Y}}_{ss} = \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)\left(\frac{\bar{X}_2}{\bar{x}_2}\right)$$

The MSE of $\hat{\hat{Y}}_{ss}$ is given as:

$$MSE\left(\hat{\hat{Y}}_{ss}\right) \cong \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + gC_{x1}^2\left(g - 2gk_{x2x1} - 2k_{yx1}\right) + gC_{x2}^2\left(g + 2k_{yx2}\right)\right) \quad (2.3)$$

where $k_{yx1} = \rho_{yx1}\frac{C_y}{C_{x1}}$, $k_{yx2} = \rho_{yx2}\frac{C_y}{C_{x2}}$, $k_{x2x1} = \rho_{x1x2}\frac{C_{x2}}{C_{x1}}$,$g = \frac{n}{N-n}$

Swain's (2012) estimator

Swain (2012) introduced a two auxiliary variables-based difference-cum-ratio estimator for estimation of the population mean, which is defined as:

$$\hat{\hat{Y}}_{sw} = \left(\bar{y} + \eta(\bar{X}_1 - \bar{x}_1)\right)\left(\frac{\bar{X}_2}{\bar{x}_2}\right)$$

The MSE of $\hat{\hat{Y}}_{sw}$ is given by:

$$MSE\left(\hat{\hat{Y}}_{sw}\right) \cong \frac{1-f}{n} \bar{Y}^2 \left[C_y^2 + \left(\frac{\eta}{R}\right)^2 C_{x1}^2 + C_{x2}^2 - 2\left(\frac{\eta}{R}\right)C_{yx1} - C_{yx2} + \frac{2\eta}{R}C_{x1x2}\right] \quad (2.4)$$

The optimum value of η can be obtained as:

$\eta_{opt} = R\left(\frac{C_{yx1} - C_{x1x2}}{C_{x1}^2}\right)$, where $R = \frac{\bar{Y}}{\bar{X}}$.

Thus, the minimum MSE of $\hat{\hat{Y}}_{sw}$ can be defined as:

$$MSE_{min}\left(\hat{\hat{Y}}_{sw}\right) \cong \frac{1-f}{n}\,\bar{Y}^2\left[\left(C_y^2 + C_{x2}^2 - 2C_{yx2}\right) - \left(\rho_{yx1}C_y - \rho_{yx2}C_{x2}\right)^2\right] \quad (2.5)$$

Olufadi's (2013) estimator

A new variant of ratio-cum-product estimator based on the known means of the auxiliary variables was proposed by Olufadi (2013), which is given by:

$$\hat{\hat{Y}}_O = \bar{y}\left[\eta\left(\frac{\bar{x}_1}{\bar{X}_1}\frac{\bar{X}_2}{\bar{x}_2}\right) + (1-\eta)\left(\frac{\bar{X}_1}{\bar{x}_1}\frac{\bar{X}_2}{\bar{x}_2}\right)\right]$$

The MSE of $\hat{\hat{Y}}_O$ is defined as:

$$MSE\left(\hat{\hat{Y}}_O\right) \cong \frac{1-f}{n}\,\bar{Y}^2\left(C_y^2 + 2\eta(1-2\eta)\left(\rho_{yx1}C_yC_{x1} - \rho_{yx2}C_yC_{x2}\right) + \eta^2(1-2\eta)^2\left(C_{x1}^2 + C_{x2}^2 - 2\rho_{x1x2}C_{x1}C_{x2}\right)\right) \quad (2.6)$$

where $\eta = \frac{(\rho_{yx1}C_{x1} - \rho_{yx2}C_{x2}) + g\left(C_{x1}^2 + C_{x2}^2 - 2\rho_{x1x2}C_{x1}C_{x2}\right)}{2g\left(C_{x1}^2 + C_{x2}^2 - 2\rho_{x1x2}C_{x1}C_{x2}\right)}$

Lu and Yan's (2014) estimators

Lu and Yan (2014) suggested a class of ratio estimator with utilization the known values of coefficients of variation, kurtosis, and correlation on the basis of dual auxiliary variable. These proposed estimators are as follows:

$$\hat{\hat{Y}}_1 = \eta_1\bar{y}\left(\frac{\bar{X}_1 + C_{x1}}{\bar{x}_1 + C_{x1}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2 + C_{x2}}{\bar{x}_2 + C_{x2}}\right)$$

$$\hat{\hat{Y}}_2 = \eta_1\bar{y}\left(\frac{\bar{X}_1 + \beta_{2(x1)}}{\bar{x}_1 + \beta_{2(x1)}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2 + \beta_{2(x2)}}{\bar{x}_2 + \beta_{2(x2)}}\right)$$

$$\hat{\hat{Y}}_3 = \eta_1\bar{y}\left(\frac{\bar{X}_1\beta_{2(x1)} + C_{x1}}{\bar{x}_1\beta_{2(x1)} + C_{x1}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2\beta_{2(x2)} + C_{x2}}{\bar{x}_2\beta_{2(x2)} + C_{x2}}\right)$$

$$\hat{\hat{Y}}_4 = \eta_1\bar{y}\left(\frac{\bar{X}_1C_{x1} + \beta_{2(x1)}}{\bar{x}_1C_{x1} + \beta_{2(x1)}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2C_{x2} + \beta_{2(x2)}}{\bar{x}_2C_{x2} + \beta_{2(x2)}}\right)$$

$$\hat{\hat{Y}}_5 = \eta_1\bar{y}\left(\frac{\bar{X}_1 + \rho_{yx1}}{\bar{x}_1 + \rho_{yx1}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2 + \rho_{yx2}}{\bar{x}_2 + \rho_{yx2}}\right)$$

$$\hat{\hat{Y}}_6 = \eta_1\bar{y}\left(\frac{\bar{X}_1C_{x1} + \rho_{yx1}}{\bar{x}_1C_{x1} + \rho_{yx1}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2C_{x2} + \rho_{yx2}}{\bar{x}_2C_{x2} + \rho_{yx2}}\right)$$

$$\hat{\hat{Y}}_7 = \eta_1\bar{y}\left(\frac{\bar{X}_1\rho_{yx1} + C_{x1}}{\bar{x}_1\rho_{yx1} + C_{x1}}\right) + \eta_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + C_{x2}}{\bar{x}_2\rho_{yx2} + C_{x2}}\right)$$

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$$\hat{\bar{Y}}_8 = \eta_1 \bar{y} \left(\frac{\bar{X}_1 \beta_{2(x1)} + \rho_{yx1}}{\bar{x}_1 \beta_{2(x1)} + \rho_{yx1}} \right) + \eta_2 \bar{y} \left(\frac{\bar{X}_2 \beta_{2(x2)} + \rho_{yx2}}{\bar{x}_2 \beta_{2(x2)} + \rho_{yx2}} \right)$$

$$\hat{\bar{Y}}_9 = \eta_1 \bar{y} \left(\frac{\bar{X}_1 \rho_{yx1} + \beta_{2(x1)}}{\bar{x}_1 \rho_{yx1} + \beta_{2(x1)}} \right) + \eta_2 \bar{y} \left(\frac{\bar{X}_2 \rho_{yx2} + \beta_{2(x2)}}{\bar{x}_2 \rho_{yx2} + \beta_{2(x2)}} \right)$$

The general expression of the MSE is given as:

$$MSE\left(\hat{\bar{Y}}_i\right) \cong \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + \eta_1^2 R_{1i}^2 C_{x1}^2 + \eta_2^2 R_{2i}^2 C_{x2}^2 - 2\eta_1 R_{1i} \rho_{yx1} C_y C_{x1} - 2\eta_2 R_{2i} \rho_{yx2} C_y C_{x2} + 2\eta_1 \eta_2 R_{1i} R_{2i} \rho_{x1x2} C_{x1} C_{x2} \right. \quad (2.7)$$

where $i = 1,2,3, \ldots ,9.$, and the values of constant R_{1i} *and* R_{2i} are:

$R_{11} = \left(\frac{\bar{X}_1}{\bar{X}_1 + C_{x1}}\right), R_{12} = \left(\frac{\bar{X}_1}{\bar{X}_1 + \beta_{2(x1)}}\right), R_{13} = \left(\frac{\bar{X}_1 \beta_{2(x1)}}{\bar{X}_1 \beta_{2(x1)} + C_{x1}}\right), R_{14} = \left(\frac{\bar{X}_1 C_{x1}}{\bar{X}_1 C_{x1} + \beta_{2(x1)}}\right),$

$R_{15} = \left(\frac{\bar{X}_1}{\bar{X}_1 + \rho_{yx1}}\right), R_{16} = \left(\frac{\bar{X}_1 C_{x1}}{\bar{X}_1 C_{x1} + \rho_{yx1}}\right), R_{17} = \left(\frac{\bar{X}_1 \rho_{yx1}}{\bar{X}_1 \rho_{yx1} + C_{x1}}\right), R_{18} = \left(\frac{\bar{X}_1 \beta_{2(x1)}}{\bar{X}_1 \beta_{2(x1)} + \rho_{yx1}}\right),$

$R_{19} = \left(\frac{\bar{X}_1 \rho_{yx1}}{\bar{X}_1 \rho_{yx1} + \beta_{2(x1)}}\right), R_{21} = \left(\frac{\bar{X}_2}{\bar{X}_2 + C_{x2}}\right), R_{22} = \left(\frac{\bar{X}_2}{\bar{X}_2 + \beta_{2(x2)}}\right), R_{23} = \left(\frac{\bar{X}_2 \beta_{2(x2)}}{\bar{X}_2 \beta_{2(x2)} + C_{x2}}\right),$

$R_{24} = \left(\frac{\bar{X}_2 C_{x2}}{\bar{X}_2 C_{x2} + \beta_{2(x2)}}\right), R_{25} = \left(\frac{\bar{X}_2}{\bar{X}_2 + \rho_{yx2}}\right), R_{26} = \left(\frac{\bar{X}_2 C_{x2}}{\bar{X}_2 C_{x2} + \rho_{yx2}}\right), R_{27} = \left(\frac{\bar{X}_2 \rho_{yx2}}{\bar{X}_2 \rho_{yx2} + C_{x2}}\right),$

$R_{28} = \left(\frac{\bar{X}_2 \beta_{2(x2)}}{\bar{X}_1 \beta_{2(x2)} + \rho_{yx2}}\right), R_{29} = \left(\frac{\bar{X}_2 \rho_{yx2}}{\bar{X}_2 \rho_{yx2} + \beta_{2(x2)}}\right).$

The optimum values of η_1 and η_2 are

$$\eta_1^* = \frac{R_{2i}^2 C_{x2}^2 + \rho_{yx1} R_{1i} C_y C_{x1} - R_{2i} \rho_{yx2} C_y C_{x2} - R_{1i} R_{2i} \rho_{x1x2} C_{x1} C_{x2}}{R_{1i}^2 C_{x1}^2 + R_{2i}^2 C_{x2}^2 - 2R_{1i} R_{2i} \rho_{x1x2} C_{x1} C_{x2}}, \eta_2^* = 1 - \eta_1^*$$

So, the minimum MSE is defined as:

$$MSE_{min}\left(\hat{\bar{Y}}_i\right) = \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + \eta_1^{2*} R_{1i}^2 C_{x1}^2 + \eta_2^{2*} R_{2i}^2 C_{x2}^2 - 2\eta_1^* R_{1i} \rho_{yx1} C_y C_{x1} - 2\eta_2^* R_{2i} \rho_{yx2} C_y C_{x2} + 2\eta_1^* \eta_2^* R_{1i} R_{2i} \rho_{x1x2} C_{x1} C_{x2}\right) \quad (2.8)$$

where $i = 1,2, 3, \ldots ,9$.

Yasmeen et al.'s (2016) estimator

Yasmeen et al. (2016) suggested a ratio-cum-exponential estimator for the estimation of the finite population mean $\bar{Y}$ under simple random scheme that utilize the supplementary information of the means of two auxiliary variables. It is defined as:

$$\hat{\bar{Y}}_{YS} = \bar{y} \left(\frac{\bar{x}_1}{\bar{X}_1}\right) exp \left(\frac{\bar{x}_2 - \bar{X}_2}{\bar{x}_2 + \bar{X}_2}\right)$$

The MSE of $\hat{\bar{Y}}_{YS}$ is give as:

$$MSE\left(\hat{\bar{Y}}_{ss}\right) \cong \frac{1-f}{n} \bar{Y}^2 \left(C_y^2 + g^2 C_{x1}^2 + \frac{1}{4} g^2 C_{x2}^2 - 2g\rho_{yx1} C_y C_{x1} + g^2 \rho_{x1x2} C_{x1} C_{x2} - g\rho_{yx2} C_y C_{x2}\right) \quad (2.9)$$

3. The proposed class of estimators

This section presents a novel class of exponential-cum-ratio type estimators for estimation of finite population mean based on combinations of the conventional and non-conventional measures of the dual auxiliary variables. The robust measures tri-mean (TM), Hodges-Lehmann (HL), and Deciles-mean (DM) are integrated within the framework of the exponential-cum-ratio estimators with an objective to enhance their robustness against possible outliers. The TM estimator was suggested by Wang et al. (2007) and mathematically written as $TM = \frac{Q_{(1)}+2Q_{(2)}+Q_{(3)}}{4}$, where $Q_1, Q_2,$ and Q_3 symbolizes the I^st^, 2^nd^, and 3^rd^ quartiles, respectively. The HL estimator of mean is the median of the pair-wise Walsh averages (Hettmansperger and McKean, 2011). Mathematically, the HL estimator can be determined as $HL = median\left(\frac{X_{(l)}+X_{(k)}}{2}; 1 \le l \le k \le N\right)$, where $X_{(l)}$ and $X_{(k)}$ are the l-th and k-th order statistics, respectively. The DM estimator was proposed by Rana et al. (2012) and mathematically, the DM is defined as $DM = \frac{D_{(1)}+D_{(2)}+\cdots+D_{(9)}}{9}$, where $D_{(j)}$ denotes the j-th decile for $j = 1,2,\ldots,9$. The justification for including TM, HL, and DM within the framework of the proposed exponential-cum-ratio class of estimators lies in their ability to remain robust in the presence of extreme observations. The TM, HL, and DM estimators are integrated with other conventional measures like coefficient of variation, correlation, and kurtosis to enhance the efficiency of the exponential-cum-ratio estimators.

Using various combinations of the conventional and non-conventional statistics of the auxiliary variables, the following estimators are proposed to estimate the population mean:

$$\hat{\bar{Y}}_{p1} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + C_{x1}}{\bar{x}_1 + C_{x1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + C_{x2}}{\bar{x}_2 + C_{x2}}\right)$$

$$\hat{\bar{Y}}_{p2} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + \beta_{2(x1)}}{\bar{x}_1 + \beta_{2(x1)}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + \beta_{2(x2)}}{\bar{x}_2 + \beta_{2(x2)}}\right)$$

$$\hat{\bar{Y}}_{p3} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\beta_{2(x1)} + C_{x1}}{\bar{x}_1\beta_{2(x1)} + C_{x1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\beta_{2(x2)} + C_{x2}}{\bar{x}_2\beta_{2(x2)} + C_{x2}}\right)$$

$$\hat{\bar{Y}}_{p4} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 C_{x1} + \beta_{2(x1)}}{\bar{x}_1 C_{x1} + \beta_{2(x1)}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 C_{x2} + \beta_{2(x2)}}{\bar{x}_2 C_{x2} + \beta_{2(x2)}}\right)$$

$$\hat{\bar{Y}}_{p5} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + \rho_{yx1}}{\bar{x}_1 + \rho_{yx1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + \rho_{yx2}}{\bar{x}_2 + \rho_{yx2}}\right)$$

$$\hat{\bar{Y}}_{p6} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 C_{x1} + \rho_{yx1}}{\bar{x}_1 C_{x1} + \rho_{yx1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 C_{x2} + \rho_{yx2}}{\bar{x}_2 C_{x2} + \rho_{yx2}}\right)$$

$$\hat{\bar{Y}}_{p7} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\rho_{yx1} + C_{x1}}{\bar{x}_1\rho_{yx1} + C_{x1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + C_{x2}}{\bar{x}_2\rho_{yx2} + C_{x2}}\right)$$

$$\hat{\bar{Y}}_{p8} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\beta_{2(x1)} + \rho_{yx1}}{\bar{x}_1\beta_{2(x1)} + \rho_{yx1}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\beta_{2(x2)} + \rho_{yx2}}{\bar{x}_2\beta_{2(x2)} + \rho_{yx2}}\right)$$

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$$\hat{\bar{Y}}_{p9} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\rho_{yx1} + \beta_{2(x1)}}{\bar{x}_1\rho_{yx1} + \beta_{2(x1)}}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + \beta_{2(x2)}}{\bar{x}_2\rho_{yx2} + \beta_{2(x2)}}\right)$$

$$\hat{\bar{Y}}_{p10} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + TM_1}{\bar{x}_1 + TM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + TM_2}{\bar{x}_2 + TM_2}\right)$$

$$\hat{\bar{Y}}_{p11} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + HL_1}{\bar{x}_1 + HL_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + HL_2}{\bar{x}_2 + HL_2}\right)$$

$$\hat{\bar{Y}}_{p12} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 + DM_1}{\bar{x}_1 + DM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 + DM_2}{\bar{x}_2 + DM_2}\right)$$

$$\hat{\bar{Y}}_{p13} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\beta_{2(x1)} + TM_1}{\bar{x}_1\beta_{2(x1)} + TM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\beta_{2(x2)} + TM_2}{\bar{x}_2\beta_{2(x2)} + TM_2}\right)$$

$$\hat{\bar{Y}}_{p14} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 C_{x1} + TM_1}{\bar{x}_1 C_{x1} + TM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 C_{x2} + TM_2}{\bar{x}_2 C_{x2} + TM_2}\right)$$

$$\hat{\bar{Y}}_{p15} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 C_{x1} + HL_1}{\bar{x}_1 C_{x1} + HL_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 C_{x2} + HL_2}{\bar{x}_2 C_{x2} + HL_2}\right)$$

$$\hat{\bar{Y}}_{p16} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1 C_{x1} + DM_1}{\bar{x}_1 C_{x1} + DM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2 C_{x2} + DM_2}{\bar{x}_2 C_{x2} + DM_2}\right)$$

$$\hat{\bar{Y}}_{p17} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\rho_{yx1} + TM_1}{\bar{x}_1\rho_{yx1} + TM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + TM_2}{\bar{x}_2\rho_{yx2} + TM_2}\right)$$

$$\hat{\bar{Y}}_{p18} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\rho_{yx1} + HL_1}{\bar{x}_1\rho_{yx1} + HL_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + HL_2}{\bar{x}_2\rho_{yx2} + HL_2}\right)$$

$$\hat{\bar{Y}}_{p19} = k_1\bar{y}\, exp\left(\frac{\bar{X}_1\rho_{yx1} + DM_1}{\bar{x}_1\rho_{yx1} + DM_1}\right) + k_2\bar{y}\left(\frac{\bar{X}_2\rho_{yx2} + DM_2}{\bar{x}_2\rho_{yx2} + DM_2}\right)$$

The MSE of the proposed estimators is given as follows:

$$MSE\left(\hat{\bar{Y}}_{pj}\right) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{1}{4}k_1^2R_{p1j}^2C_{x1}^2 + k_2^2R_{p2j}^2C_{x2}^2 - k_1R_{p1j}\rho_{yx1}C_yC_{x1} + k_1k_2R_{p1j}Rp_{2j}\rho_{x1x2}C_{x1}C_{x2} - 2k_2R_{p2j}\rho_{yx2}C_yC_{x2}\right) \quad (3.1)$$

where, $j = 1,2,3,\ldots,19$, and the values for constant R_{p1j} *and* R_{p2j} are defined as:

$R_{p11} = \left(\frac{\bar{X}_1}{\bar{X}_1+C_{x1}}\right)$, $R_{p12} = \left(\frac{\bar{X}_1}{\bar{X}_1+\beta_{2(x1)}}\right)$,$R_{p13} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+C_{x1}}\right)$, $R_{p14} = \left(\frac{\bar{X}_1C_{x1}}{\bar{X}_1C_{x1}+\beta_{2(x1)}}\right)$

$R_{p15} = \left(\frac{\bar{X}_1}{\bar{X}_1+\rho_{yx1}}\right)$, $R_{p16} = \left(\frac{\bar{X}_1C_{x1}}{\bar{X}_1C_{x1}+\rho_{yx1}}\right)$,$R_{p17=}\left(\frac{\bar{X}_1\rho_{yx1}}{\bar{X}_1\rho_{yx1}+C_{x1}}\right)$, $R_{p18} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+\rho_{yx1}}\right)$,

$R_{p19} = \left(\frac{\bar{X}_1\rho_{yx1}}{\bar{X}_1\rho_{yx1}+\beta_{2(x1)}}\right)$, $R_{p110} = \left(\frac{\bar{X}_1}{\bar{X}_1+TM_1}\right)$,$R_{p111} = \left(\frac{\bar{X}_1}{\bar{X}_1+HL_1}\right)$, $R_{p112} = \left(\frac{\bar{X}_1}{\bar{X}_1+DM_1}\right)$

$R_{p113} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+TM_1}\right), R_{p114} = \left(\frac{\bar{X}_1C_{x1}}{\bar{X}_1C_{x1}+TM_1}\right), R_{p115} = \left(\frac{\bar{X}_1C_{x1}}{\bar{X}_1C_{x1}+HL_1}\right), R_{p116} = \left(\frac{\bar{X}_1C_{x1}}{\bar{X}_1C_{x1}+DM_1}\right)$

$R_{p117} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+TM_1}\right), R_{p118} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+HL_1}\right), R_{p119} = \left(\frac{\bar{X}_1\beta_{2(x1)}}{\bar{X}_1\beta_{2(x1)}+DM_1}\right), R_{p21} = \left(\frac{\bar{X}_2}{\bar{X}_2+C_{x2}}\right)$

$R_{p22} = \left(\frac{\bar{X}_2}{\bar{X}_2+\beta_{2(x2)}}\right), R_{p23} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_2\beta_{2(x2)}+C_{x2}}\right), R_{p24} = \left(\frac{\bar{X}_2C_{x2}}{\bar{X}_2C_{x2}+\beta_{2(x2)}}\right), R_{p25} = \left(\frac{\bar{X}_2}{\bar{X}_2+\rho_{yx2}}\right)$

$R_{p26} = \left(\frac{\bar{X}_2C_{x2}}{\bar{X}_2C_{x2}+\rho_{yx2}}\right), R_{p27} = \left(\frac{\bar{X}_2\rho_{yx2}}{\bar{X}_2\rho_{yx2}+C_{x2}}\right), R_{p28} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_1\beta_{2(x2)}+\rho_{yx2}}\right), R_{p29} = \left(\frac{\bar{X}_2\rho_{yx2}}{\bar{X}_2\rho_{yx2}+\beta_{2(x2)}}\right)$

$R_{p210} = \left(\frac{\bar{X}_2}{\bar{X}_2+TM_2}\right), R_{p211} = \left(\frac{\bar{X}_2}{\bar{X}_2+HL_2}\right), R_{p212} = \left(\frac{\bar{X}_2}{\bar{X}_2+DM_2}\right), R_{p213} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_2\beta_{2(x2)}+TM_2}\right),$

$R_{p214} = \left(\frac{\bar{X}_2C_{x2}}{\bar{X}_2C_{x2}+TM_2}\right), R_{p215} = \left(\frac{\bar{X}_2C_{x2}}{\bar{X}_2C_{x2}+HL_2}\right), R_{p216} = \left(\frac{\bar{X}_2C_{x2}}{\bar{X}_2C_{x2}+DM_2}\right), R_{p217} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_2\beta_{2(x2)}+TM_2}\right),$

$R_{p218} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_2\beta_{2(x2)}+HL_2}\right), R_{p219} = \left(\frac{\bar{X}_2\beta_{2(x2)}}{\bar{X}_1\beta_{2(x2)}+DM_2}\right)$

The optimum values of k_1 *and* k_2 for the proposed estimators can be obtained by differentiating the MSE expression with respect to k_1 *and* k_2 and equating it to zero. The optimum values of k_1 *and* k_2 are

$$k_1^* = \frac{R_{p2j}^2C_{x2}^2 + \frac{1}{2}Rp_{1j}\rho_{yx1}C_yC_{x1} - \frac{1}{2}R_{p1j}R_{p2j}\rho_{x1x2}C_{x1}C_{x2} - R_{p2j}\rho_{yx2}C_yC_{x2}}{\frac{1}{4}R_{p1j}^2C_{x1}^2 + R_{p2j}^2C_{x2}^2 - R_{p1j}R_{p2j}\rho_{x1x2}C_{x1}C_{x2}}$$

$$k_2^* = 1 - k_1^*$$

So, the minimum MSE of the suggested exponential-type ratio estimators is given as:

$$MSE_{min}\left(\hat{\bar{Y}}_{pj}\right) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{1}{4}k_1^{2^*}R_{p1j}^2C_{x1}^2 + k_2^{2^*}R_{p2j}^2C_{x2}^2 - k_1{}^*R_{p1j}\rho_{yx1}C_yC_{x1} + k_1{}^*k_2{}^*R_{p1j}R_{p2j}\rho_{x1x2}C_{x1}C_{x2} - 2k_2{}^*R_{p2j}\rho_{yx2}C_yC_{x2}\right) \quad (3.2)$$

where $j = 1,2,3,\dots,19$.

The proposed estimators are member of the following general class of estimator

$$\hat{\bar{Y}}_g = K_1\bar{y}\ \exp\left(\frac{T_1(\bar{X}_1 - \bar{x}_1)}{T_1(\bar{X}_1 + \bar{x}_1) + 2P_1}\right) + K_2\bar{y}\ \left(\frac{T_2(\bar{X}_2 - \bar{x}_2)}{T_2(\bar{X}_2 + \bar{x}_2) + 2P_2}\right)$$

where K_1 and K_2 are the weights such that $K_1 + K_2 = 1$. $T_1 \neq 0$, $T_2 \neq 0$, P_1, P_2 are either constants or function of the known parameters of the population. The suitable combinations of T_1, T_2, P_1, and P_2 are reported in Table 1.

Table 1. Some choices of constants for proposed class of estimators.

Estimator	T_1	P_1	T_2	P_2
$\hat{\hat{Y}}_{p1}$	1	C_{x1}	1	C_{x2}
$\hat{\hat{Y}}_{p2}$	1	$B_{2(x1)}$	1	$B_{2(x2)}$
$\hat{\hat{Y}}_{p3}$	$B_{2(x1)}$	C_{x1}	$B_{2(x2)}$	C_{x2}
$\hat{\hat{Y}}_{p4}$	C_{x1}	$B_{2(x1)}$	C_{x2}	$B_{2(x2)}$
$\hat{\hat{Y}}_{p5}$	1	ρ_{yx1}	1	ρ_{yx2}
$\hat{\hat{Y}}_{p6}$	C_{x1}	ρ_{yx1}	C_{x2}	ρ_{yx2}
$\hat{\hat{Y}}_{p7}$	ρ_{yx1}	C_{x1}	ρ_{yx2}	C_{x2}
$\hat{\hat{Y}}_{p8}$	$B_{2(x1)}$	ρ_{yx1}	$B_{2(x2)}$	ρ_{yx2}
$\hat{\hat{Y}}_{p9}$	ρ_{yx1}	$B_{2(x1)}$	ρ_{yx2}	$B_{2(x2)}$
$\hat{\hat{Y}}_{p10}$	1	TM_1	1	TM_2
$\hat{\hat{Y}}_{p11}$	1	HL_1	1	HL_2
$\hat{\hat{Y}}_{p12}$	1	DM_1	1	DM_2
$\hat{\hat{Y}}_{p13}$	$B_{2(x1)}$	TM_1	$B_{2(x2)}$	TM_2
$\hat{\hat{Y}}_{p14}$	C_{x1}	TM_1	C_{x2}	TM_2
$\hat{\hat{Y}}_{p15}$	C_{x1}	HL_1	C_{x2}	HL_2
$\hat{\hat{Y}}_{p16}$	C_{x1}	DM_1	C_{x2}	DM_2
$\hat{\hat{Y}}_{p17}$	ρ_{yx1}	TM_1	ρ_{yx2}	TM_2
$\hat{\hat{Y}}_{p18}$	ρ_{yx1}	HL_1	ρ_{yx2}	HL_2
$\hat{\hat{Y}}_{p19}$	ρ_{yx1}	DM_1	ρ_{yx2}	DM_2

The MSE of general class of exponential-type ratio estimators for $\hat{\hat{Y}}$ can be obtained as:

Let us defines, $e_o = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1 = \frac{\bar{x}_1-\bar{X}_1}{\bar{X}_1}$, $e_2 = \frac{\bar{x}_2-\bar{X}_2}{\bar{X}_2}$,than $\bar{y} = \bar{Y}(1+e_0), \bar{x}_1 = \bar{X}_1(1+e_1),\ and\ \bar{x}_2 = \bar{X}_2(1+e_2)$, From the definition of $e_0, e_1\ and\ e_2$,We get $E(e_0) = E(e_1) = E(e_2) = 0$, where $E(e_0^2) = (\frac{1-f}{n})C_y^2$, $E(e_1^2) = (\frac{1-f}{n})C_{x1}^2$, $E(e_2^2) = (\frac{1-f}{n})C_{x2}^2$, $E(e_o e_1)(\frac{1-f}{n})\rho_{yx1}\ C_y C_{x1}$,

$E(e_o e_2)(\frac{1-f}{n})\rho_{yx2}\ C_y C_{x2}$, $E(e_1 e_2)(\frac{1-f}{n})\rho_{x1x2}\ C_{x1} C_{x2}$,

The general estimator of suggested class $\hat{\hat{Y}}_g$ can be written in the terms of $e_0, e_1\ and\ e_2$ as:

$$\hat{\hat{Y}}_g = K_1\bar{Y}(1+e_0)\ \exp\left(\frac{T_1(\bar{X}_1 - \bar{X}_1(1+e_1))}{T_1(\bar{X}_1 + \bar{X}_1(1+e_1)) + 2P_1}\right) + K_2\bar{Y}(1+e_0)\left(\frac{T_2\bar{X}_2 + P_2}{T_2\bar{X}_2(1+e_2) + P_2}\right)$$

$$\hat{\hat{Y}}_g = K_1\bar{Y}(1+e_0)\ \exp\left(\frac{-T_1\bar{X}_1 e_1}{2(T_1\bar{X}_1 + P_1)\left(1 + \frac{T_1\bar{X}_1 e_1}{2(T_1\bar{X}_1 + P_1)}\right)}\right) + K_2\bar{Y}(1+e_0)\left(\frac{1}{1 + \frac{T_2\bar{X}_2 e_2}{T_2\bar{X}_2 + P_2}}\right)$$

$$\hat{\hat{Y}}_g = K_1\bar{Y}(1+e_0)\ \exp\left(-\frac{\alpha_1 e_1}{2}(1+\frac{\alpha_1}{2})^{-1}\right) + K_2\bar{Y}(1+e_0)(1+\alpha_2 e_2)^{-1}$$

Subtracting $\bar{Y}$ from both sides and neglecting higher order terms, we get

$$\left(\hat{\hat{Y}}_g - \bar{Y}\right) = \bar{Y}\left((K_1+K_2)e_0 - \frac{1}{2}K_1\alpha_1 e_1 - K_2\alpha_2 e_2\right) + \bar{Y}(K_1+K_2-1)$$

$$\left(\hat{\hat{Y}}_g - \bar{Y}\right) = \bar{Y}\left(e_0 - \frac{1}{2}K_1\alpha_1 e_1 - K_2\alpha_2 e_2\right)$$

Squaring and applying expectation on both sides of above equation, the attained MSE is given as:

$$MSE\left(\hat{\hat{Y}}_g\right) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{1}{4}K_1^2\alpha_1^2 C_{x1}^2 + K_2^2\alpha_2^2 C_{x2}^2 - K_1\alpha_1\rho_{yx1}C_y C_{x1} + K_1K_2\alpha_1\alpha_2\rho_{x1x2}C_{x1}C_{x2} - 2K_2\alpha_2\rho_{yx2}C_y C_{x2}\right)$$

For optimum value of $K_1 and\ K_2$, put $K_2 = 1 - K_1$ and minimize the general class of estimators as follows:

$$K_1^* = \frac{\alpha_2^2 C_{x2}^2 + \frac{1}{2}\alpha_1\rho_{yx1}C_y C_{x1} - \frac{1}{2}\alpha_1\alpha_2\rho_{x1x2}C_{x1}C_{x2} - \alpha_2\rho_{yx2}C_y C_{x2}}{\frac{1}{4}\alpha_1^2 C_{x1}^2 + \alpha_2^2 C_{x2}^2 - \alpha_1\alpha_2\rho_{x1x2}C_{x1}C_{x2}}$$

$$K_2^* = 1 - K_1^*$$

$$MSE_{min}\left(\hat{\hat{Y}}_g\right) = \bar{Y}^2\frac{1-f}{n}\left(C_y^2 + \frac{1}{4}K_1^{2^*}\alpha_1^2 C_{x1}^2 + K_2^{2^*}\alpha_2^2 C_{x2}^2 - K_1{}^*\alpha_1\rho_{yx1}C_y C_{x1} + K_1{}^*K_2{}^*\alpha_1\alpha_2\rho_{x1x2}C_{x1}C_{x2} - 2K_2{}^*\alpha_2\rho_{yx2}C_y C_{x2}\right) \quad (3.3)$$

Where $\alpha_1 = \left(\frac{T_1\bar{X}_1}{T_1\bar{X}_1 + P_1}\right), \alpha_2 = \left(\frac{T_2\bar{X}_2}{(T_2\bar{X}_2 + P_2)}\right)$

4. Efficiency comparisons

This section contains the theoretical conditions that support the results of MSEs of new developed class of estimators in the comparison with usual and existing estimators of the study.

The comparison of the MSE of the proposed class of exponential-type ratio estimators and the traditional ratio estimator is:

$$MSE_{min}\left(\hat{\bar{Y}}_{pj}\right) < MSE_{min}\left(\hat{\bar{Y}}_{M}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - \eta_1^{2^*}\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - \eta_2^{2^*}\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\;*}R_{p1j} - \eta_1^{\;*}\right)\rho_{yx1}C_yC_{x1} - 2\left(k_2^{\;*}R_{p2j} - \eta_2^{\;*}\right)\rho_{yx2}C_yC_{x2} + 2\left(\frac{1}{2}k_1^{\;*}k_2^{\;*}R_{p1j}Rp_{2j} - \eta_1^{\;*}\eta_2^{\;*}\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \qquad (4.1)$$

where $j = 1,2,3, \ldots, 19$.

The MSE of the proposed estimator will be minimum than the Singh's ratio estimator (2003) if:

$$MSE_{min}\left(\hat{\bar{Y}}_{pj}\right) < MSE_{min}\left(\hat{\bar{Y}}_{s}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - 1\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - 1\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\;*}R_{p1j} - 1\right)\rho_{yx1}C_yC_{x1} - 2\left(k_2^{\;*}R_{p2j} - 1\right)\rho_{yx2}C_yC_{x2} + 2\left(\frac{1}{2}k_1^{\;*}k_2^{\;*}R_{p1j}Rp_{2j} - 1\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \qquad (4.2)$$

where $j = 1,2,3, \ldots, 19$.

The suggested general class of exponential-type ratio estimator $\hat{\bar{Y}}_{pj}$ will be more efficient than the Singh and Tailor (2005) ratio estimator, i.e., $\hat{\bar{Y}}_{ST}$, if and only if:

$$MSE_{min}\left(\hat{\bar{Y}}_{pj}\right) < MSE_{min}\left(\hat{\bar{Y}}_{ST}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - \theta_1^*(\theta_1^* - 2k_{yx1})\right)C_{x1}^2 + \left(\frac{1}{4}k_2^{2^*}R_{p2j}^2 - \theta_2^*(\theta_2^* + 2\left(k_{yx2} - \theta_1^*k_{x1x2}\right)\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\;*}R_{p1j}\rho_{yx1}C_yC_{x1} + \frac{1}{2}k_2^{\;*}R_{p2j}\rho_{yx2}C_yC_{x2}\right)C_y + \frac{1}{2}k_1^{\;*}k_2^{\;*}R_{p1j}R_{p2j}\rho_{x1x2}C_{x1}C_{x2} < 0 \qquad (4.3)$$

where $j = 1,2,3, \ldots, 19$.

The MSE of the proposed general ratio estimator will be least than the Singh et al. (2005) ratio-cum-product estimator if:

$$MSE_{min}\left(\hat{\bar{Y}}_{pj}\right) < MSE_{min}\left(\hat{\bar{Y}}_{ss}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - g(g - 2gk_{x2x1} - 2k_{yx1})\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - g(g + 2k_{yx2})\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\;*}R_{p1j}\rho_{yx1}C_yC_{x1} + k_2^{\;*}R_{p2j}\rho_{yx2}C_yC_{x2}\right)C_y + k_1^{\;*}k_2^{\;*}R_{p1j}R_{p2j}\rho_{x1x2}C_{x1}C_{x2} < 0 \qquad (4.4)$$

where $j = 1,2,3, \ldots, 19$.

The MSE of the suggested estimator will be minimize than the Swain (2012) ratio estimator if:

$$MSE_{min}\left(\hat{\hat{Y}}_{pj}\right) < MSE_{min}\left(\hat{\hat{Y}}_{sw}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - \left(\frac{\eta}{R}\right)^2\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - 1\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\ *}R_{p1j} - \left(\frac{\eta}{R}\right)\right)\rho_{yx1}C_yC_{x1} - \left(2k_2^{\ *}R_{p2j} - 1\right)\rho_{yx2}C_yC_{x2} + 2\left(\frac{1}{2}k_1^{\ *}k_2^{\ *}R_{p1j}Rp_{2j} - \frac{\eta}{R}\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \quad (4.5)$$

where $j = 1,2,3,\dots,19$.

The MSE of the suggested estimators will be least than the Olufadi (2013) estimator if:

$$MSE_{min}\left(\hat{\hat{Y}}_{pj}\right) < MSE_{min}\left(\hat{\hat{Y}}_{y}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - g^2(1-2\eta)^2\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - g^2(1-2\eta)^2\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\ *}R_{p1j} + g(1-2\eta)\right)\rho_{yx1}C_yC_{x1} - 2\left(k_2^{\ *}R_{p2j} - g(1-2\eta)\right)\rho_{yx2}C_yC_{x2} + 2\left(\frac{1}{2}k_1^{\ *}k_2^{\ *}R_{p1j}Rp_{2j} + g^2(1-2\eta)^2\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \quad (4.6)$$

where $j = 1,2,3,\dots,19$.

The comparison of the proposed estimator and the Lu and Yan (2014) ratio estimators in term of MSE is given as below:

$$MSE_{min}\left(\hat{\hat{Y}}_{pj}\right) < MSE_{min}\left(\hat{\hat{Y}}_{i}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - \eta_1^{2^*}R_{1i}^2\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - \eta_2^{2^*}R_{2i}^2\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\ *}R_{p1j} - \eta_1^{\ *}R_{1i}\right)\rho_{yx1}C_yC_{x1} - 2\left(k_2^{\ *}R_{p2j} - \eta_2^{\ *}R_{2i}\right)\rho_{yx2}C_yC_{x2} + 2\left(\frac{1}{2}k_1^{\ *}k_2^{\ *}R_{p1j}Rp_{2j} - \eta_1^{\ *}\eta_2^{\ *}R_{1i}R_{2i}\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \quad (4.7)$$

where $j = 1,2,3,\dots,19$ and $i = 1,2,3,\dots,9$. If the above condition is hold, the proposed class of estimators $\hat{\hat{Y}}_{pj}$ will be more precise than the $\hat{\hat{Y}}_{i}$ ratio estimator.

The MSE of the proposed estimator will be smallest than the Yasmeen *et al.* (2016) ratio estimator if:

$$MSE_{min}\left(\hat{\hat{Y}}_{pj}\right) < MSE_{min}\left(\hat{\hat{Y}}_{YS}\right)$$

$$\left(\frac{1}{4}k_1^{2^*}R_{p1j}^2 - g^2\right)C_{x1}^2 + \left(k_2^{2^*}R_{p2j}^2 - g^2\right)C_{x2}^2 - 2\left(\frac{1}{2}k_1^{\ *}R_{p1j} - g\right)\rho_{yx1}C_yC_{x1} - \left(2k_2^{\ *}R_{p2j} - g\right)\rho_{yx2}C_yC_{x2} + \left(k_1^{\ *}k_2^{\ *}R_{p1j}Rp_{2j} - g^2\right)\rho_{x1x2}C_{x1}C_{x2} < 0 \quad (4.8)$$

where $j = 1,2,3,\dots,19$.

5. Performance evaluation and robustness

This section presents performance of the proposed class of estimators in comparison to the competing existing estimators through simulation and empirical studies. Moreover, it also includes a robustness study of the proposed estimators in presence of outliers.

5.1. Simulation study

A simulation study is also performed to assess the performance of competing and suggested estimators in current section. For this purpose, a contaminated normal (CN) distribution is utilized. The CN distribution is a mixture of two normal components with a common mean such that $(1-\alpha)100\%$ observations belongs to $N(\mu, \sigma^2)$ and $(\alpha)100\%$ observations are taken from $N(\mu, \tau\sigma^2)$, where $0 < \tau < \infty$. The CN distribution is mostly utilized in robustness/outlier studies. The following simulation procedure is used in the R language to compute MSE of proposed estimators and their counterparts:

i. 10,000 Random samples of size $n = 30$ and 40 were generated from CN distribution with $\alpha = 10\%$ level of contamination and the values of the usual mean, existing and proposed estimators were obtained for the each of generated random sample by using their expressions defined in earlier Sections.

ii. The MSE for each of the estimator used in the simulation was obtained.

iii. Step (i) and (ii) were repeated 30000 times to obtain the average values of the MSEs for each of the existing and proposed estimator.

By using the above procedure, the computed MSEs are given in Tables 2-3 and the key findings are described as:

i. The proposed class of estimators turned out to be more efficient than the usual mean and other existing estimators as all members of the proposed class have lower values of the MSEs (cf. Tables 2 and 3).

ii. The MSEs of all the proposed estimators decreases as n increases, and vice versa (cf. Tables 2 and 3).

iii. Among all the proposed estimators, $\hat{\hat{Y}}_{p18}$ which is based on the HL and coefficient of correlation turned out to be most efficient for both choices of n.

Table 2. MSEs of existing estimators for simulated data set.

Estimator	$n = 30$			$n = 40$		
	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE
$\hat{\hat{Y}}_M$	-	-	60143.2	-	-	49210.5
$\hat{\hat{Y}}_S$	-	-	1141686	-	-	689611
$\hat{\hat{Y}}_{ss}$	-	-	68489.9	-	-	24650.2
$\hat{\hat{Y}}_{ST}$	-	-	245902	-	-	132033
$\hat{\hat{Y}}_{sw}$	-	-	60582.7	-	-	55554.9
$\hat{\hat{Y}}_O$	-	-	**46743.7**	-	-	**22183.7**
$\hat{\hat{Y}}_{YS}$	-	-	250825	-	-	136467
$\hat{\hat{Y}}_1$	0.9958	0.9995	59981.2	0.9973	0.9995	49232.9
$\hat{\hat{Y}}_2$	0.9747	0.9959	59088.4	0.9866	0.996	49178.8
$\hat{\hat{Y}}_3$	0.9994	0.9999	60109.1	0.9993	0.9999	49209.8s
$\hat{\hat{Y}}_4$	0.9771	0.9932	58725.7	0.9836	0.9944	48708.1
$\hat{\hat{Y}}_5$	0.9965	0.9993	59942.3	0.9969	0.9993	49182.3
$\hat{\hat{Y}}_6$	0.9968	0.9988	59874.3	0.9962	0.999	49101.2
$\hat{\hat{Y}}_7$	0.9955	0.9994	59969.0	0.9970	0.9994	49230.4
$\hat{\hat{Y}}_8$	0.9995	0.9999	60101.2	0.9992	0.9999	49200.6
$\hat{\hat{Y}}_9$	0.9729	0.9954	59008.4	0.9855	0.9957	49140.8

Table 3. MSEs of proposed estimators for simulated data set.

Estimator $\hat{\hat{Y}}_{pj}$	n=30			n=40		
	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE
$\hat{\hat{Y}}_{p1}$	0.9958	0.9995	19674.55	0.9973	0.9995	11804.73
$\hat{\hat{Y}}_{p2}$	0.9747	0.9959	18291.02	0.9866	0.996	10974.61
$\hat{\hat{Y}}_{p3}$	0.9994	0.9999	19906.92	0.9993	0.9999	11944.15
$\hat{\hat{Y}}_{p4}$	0.9771	0.9932	18419.21	0.9836	0.9944	11051.53
$\hat{\hat{Y}}_{p5}$	0.9965	0.9993	19714.20	0.9969	0.9993	11828.52
$\hat{\hat{Y}}_{p6}$	0.9968	0.9988	19730.42	0.9962	0.999	11838.25
$\hat{\hat{Y}}_{p7}$	0.9955	0.9994	19653.99	0.997	0.9994	11792.39
$\hat{\hat{Y}}_{p8}$	0.9995	0.9999	19912.58	0.9992	0.9999	11947.55
$\hat{\hat{Y}}_{p9}$	0.9729	0.9954	18170.38	0.9855	0.9957	10902.23
$\hat{\hat{Y}}_{p10}$	0.6158	0.8801	8934.82	0.6157	0.8801	5360.88
$\hat{\hat{Y}}_{p11}$	0.5796	0.8679	8734.21	0.5796	0.8678	5240.52
$\hat{\hat{Y}}_{p12}$	0.5403	0.8541	8772.67	0.5403	0.8541	5263.60
$\hat{\hat{Y}}_{p13}$	0.9167	0.9736	14672.90	0.9167	0.9736	8803.73
$\hat{\hat{Y}}_{p14}$	0.6391	0.8143	9144.21	0.6391	0.8143	5486.52
$\hat{\hat{Y}}_{p15}$	0.6037	0.7969	8796.61	0.6037	0.7968	3166.78
$\hat{\hat{Y}}_{p16}$	0.5649	0.7776	8732.48	0.5649	0.7776	3143.69
$\hat{\hat{Y}}_{p17}$	0.5986	0.8665	8804.60	0.5986	0.8665	3169.66
$\hat{\hat{Y}}_{p18}$	0.5619	0.8531	**8722.64**	0.5619	0.8531	**3140.15**
$\hat{\hat{Y}}_{p19}$	0.5223	0.8381	8872.09	0.5223	0.8381	3193.95

5.2. Empirical study

For empirical performance evaluation of the proposed class of estimators and their existing counterparts, 5 real population datasets were considered. These datasets were obtained from Singh and Chaudhary (1986) and Murthy (1967). Table 4 presents various characteristics of these data sets which are assumed to be known.

Table 4. Population characteristic of the datasets for empirical study

Characteristics	*Pop-1*	*Pop-2*	*Pop-3*	*Pop-4*	*Pop-5*
N	34	80	32	31	18
$\bar{Y}$	856.412	5182.637	3733.500	5405.100	3368.100
C_y	0.8561	0.354	0.140	0.140	0.100
n	20	20	20	20	10
$\bar{X}_1$	208.882	285.125	89.530	216.740	71.500
$\bar{X}_2$	119.441	1126.463	515.200	951.900	413.700
C_{x1}	0.721	0.948	0.260	0.460	0.150
C_{x2}	0.753	0.751	0.270	0.350	0.220
ρ_{yx1}	0.449	0.915	0.950	0.940	0.890
ρ_{yx2}	0.445	0.941	0.950	0.920	0.940
ρ_{x1x2}	0.980	0.988	0.960	0.960	0.950

$\boldsymbol{\beta_{2(x1)}}$	2.910	0.698	1.833	2.460	1.157
$\boldsymbol{\beta_{2(x2)}}$	3.732	1.050	2.201	3.188	3.152
$\boldsymbol{TM_1}$	162.250	206.937	85.813	183.375	72.250
$\boldsymbol{TM_2}$	165.562	931.562	525.625	824.625	420.813
$\boldsymbol{HL_1}$	190.000	249.000	86.500	198.500	71.500
$\boldsymbol{HL_2}$	184.000	1040.500	506.000	840.000	418.500
$\boldsymbol{DM_1}$	223.467	276.189	86.511	197.556	71.689
$\boldsymbol{DM_2}$	206.944	1150.700	483.622	834.333	416.678

The MSEs and the percentage relative efficiencies (PREs) have been used as a performance metric to compare the efficiency of the suggested class of estimators over the competing estimators used in the study. The PREs of the suggested estimators (pr) with respect to the existing estimators (e) is calculated as:

$$PRE(e, pr) = \frac{MSE(e)}{MSE(pr)} * 100$$

The values of the constants involved along with the MSEs of the existing estimators and proposed estimators based on dual auxiliary information are presented in the Tables 5 and 6, respectively. The PREs of estimators proposed in this study with respect to their existing counterparts are given in Tables 7-11. Some key results pertaining to the comparison are summarized as:

i. Based on the comparative MSEs and PRESs, it is observed that among all the existing estimators, the estimator $\hat{\bar{Y}}_9$ turned out to be most efficient against population-1, while $\hat{\bar{Y}}_{YS}$ stand superior than others for population-2. Similarly, $\hat{\bar{Y}}_4$ was more efficient for population-3, and $\hat{\bar{Y}}_O$ outperformed the other existing competing estimators for populations 4 and 5 (cf. Table 5).

ii. For all the population datasets used in this study, the proposed family of estimators showed substantially superior precision in terms of smaller MSEs as compared to the traditional and existing estimators of the mean (cf Tables 5 and 6). The proposed estimators $\hat{\bar{Y}}_{p9}$, $\hat{\bar{Y}}_{p16}$, $\hat{\bar{Y}}_{p13}$, $\hat{\bar{Y}}_{p12}$, and $\hat{\bar{Y}}_{p17}$ emerged as the most efficient for populations 1, 2, 3, 4 and 5, respectively (cf. Table 6).

iii. Generally, the PREs of the proposed estimators with respect to the existing estimators remained superior. The minimum gain in efficiency over the existing estimators remained 16% for population-1, 90% for population-2 (except for $\hat{\bar{Y}}_{p1}$ to $\hat{\bar{Y}}_{p9}$ which were 3% less efficient than the existing $\hat{\bar{Y}}_{YS}$ and are based on conventional measures), 358% for population-3, 33% for population-4, and 118% for population-5, while the maximum gain in efficiency turned out to be 298% for population-1, 17081% for population-2, 16818% for population-3, 46579% for population-4 and 9888% for population-5. is also higher than largest gained value of the existing estimator (cf. Tables 7-11).

iv. It is observed that all the proposed estimators that utilize the information of robust statistics like TM, HL, and DM found to be more efficient than all other competing estimators (cf. Tables 7-11).

Table 5. Numerical values of the Constants and MSEs of the existing estimators.

Estimator ↓	Population-1			Population-2			Population-3			Population-4			Population-5		
	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE
$\hat{\hat{Y}}_M$	-		10543.0	-	-	67683.0	-	-	4606.7	-	-	21202.0	-	-	2782.2
$\hat{\hat{Y}}_S$	-	-	35177.3	-	-	2397933	-	-	58994.5	-	-	280806	-	-	58938.9
$\hat{\hat{Y}}_{ss}$	-	-	12187.3	-	-	94272.2	-	-	10431.0	-	-	23830.3	-	-	20740.8
$\hat{\hat{Y}}_{ST}$	-	-	11697.4	-	-	68492.6	-	-	7463.4	-	-	43858.8	-	-	10384.6
$\hat{\hat{Y}}_{sw}$	-	-	10965.7	-	-	190348	-	-	5404.8	-	-	26922.2	-	-	8591.2
$\hat{\hat{Y}}_O$	-	-	11004.3	-	-	67885.6	-	-	5041.71	-	-	5202.42	-	-	1738.3
$\hat{\hat{Y}}_{YS}$	-	-	29706.4	-	-	31117.8	-	-	70588.8	-	-	533463	-	-	27136.0
$\hat{\hat{Y}}_1$	0.997	0.996	10518.1	0.997	0.999	68397.1	0.997	0.999	4567.6	0.998	0.999	21248.0	0.998	0.999	2764.2
$\hat{\hat{Y}}_2$	0.986	0.982	10446.7	0.998	0.999	67967.9	0.980	0.996	4327.2	0.989	0.997	21312.8	0.984	0.992	2652.7
$\hat{\hat{Y}}_3$	0.999	0.999	10534.0	0.995	0.999	68885.9	0.998	0.999	4585.6	0.999	0.999	21223.8	0.998	0.999	2766.2
$\hat{\hat{Y}}_4$	0.981	0.976	10410.1	0.997	0.998	67843.8	0.927	0.984	3548.5	0.976	0.991	21214.1	0.903	0.966	2066.8
$\hat{\hat{Y}}_5$	0.998	0.998	10527.3	0.997	0.999	68267.2	0.990	0.998	4463.5	0.996	0.999	21275.3	0.988	0.998	2677.1
$\hat{\hat{Y}}_6$	0.997	0.997	10521.2	0.997	0.998	68173.4	0.960	0.993	4052.1	0.991	0.997	21296.5	0.923	0.989	2188.5
$\hat{\hat{Y}}_7$	0.992	0.992	10488.2	0.996	0.999	68475.0	0.997	0.999	4565.6	0.998	0.999	21250.1	0.998	0.999	2762.0
$\hat{\hat{Y}}_8$	0.999	0.999	10537.3	0.995	0.998	68742.7	0.994	0.999	4529.5	0.998	0.999	21240.3	0.989	0.999	2689.3
$\hat{\hat{Y}}_9$	0.970	0.960	10333.4	0.997	0.999	68009.7	0.979	0.995	4312.4	0.988	0.996	21312.2	0.982	0.992	2636.8

Table 6. Numerical values of the Constants and MSEs of the proposed estimators $\hat{\hat{Y}}_{pj}$.

Estimator $\hat{\hat{Y}}_{pj}$	Population-1			Population-2			Population-3			Population-4			Population-5		
	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE	R_{1i}	R_{2i}	MSE
$\hat{\hat{Y}}_{p1}$	0.997	0.996	8830.73	0.997	0.999	32105.87	0.997	0.999	461.63	0.998	0.999	3889.90	0.998	0.999	797.92
$\hat{\hat{Y}}_{p2}$	0.986	0.985	8830.31	0.998	0.999	32171.61	0.979	0.996	453.91	0.989	0.997	3752.40	0.984	0.992	789.13
$\hat{\hat{Y}}_{p3}$	0.999	0.999	8830.83	0.995	0.999	32006.13	0.998	0.999	462.25	0.999	0.999	3910.12	0.998	0.999	798.13
$\hat{\hat{Y}}_{p4}$	0.981	0.976	8830.12	0.997	0.999	32167.87	0.927	0.984	434.40	0.976	0.991	3575.81	0.903	0.966	744.17
$\hat{\hat{Y}}_{p5}$	0.998	0.998	8830.79	0.997	0.999	32116.94	0.989	0.998	458.14	0.996	0.999	3855.57	0.988	0.998	791.83
$\hat{\hat{Y}}_{p6}$	0.997	0.997	8830.76	0.997	0.999	32109.74	0.961	0.993	446.20	0.991	0.997	3780.67	0.923	0.990	756.83
$\hat{\hat{Y}}_{p7}$	0.992	0.992	8830.56	0.996	0.999	32085.25	0.997	0.999	461.55	0.998	0.999	3887.87	0.998	0.999	797.77
$\hat{\hat{Y}}_{p8}$	0.999	0.999	8830.85	0.995	0.999	32020.37	0.994	0.999	460.31	0.998	0.998	3895.60	0.989	0.999	792.97
$\hat{\hat{Y}}_{p9}$	0.970	0.960	**8829.76**	0.997	0.999	32156.83	0.979	0.996	453.46	0.988	0.996	3742.21	0.982	0.992	787.97
$\hat{\hat{Y}}_{p10}$	0.563	0.546	8911.18	0.579	0.547	16346.69	0.511	0.495	491.64	0.542	0.536	1143.68	0.497	0.496	591.90
$\hat{\hat{Y}}_{p11}$	0.523	0.520	8865.04	0.534	0.519	15615.21	0.509	0.505	486.24	0.522	0.531	1143.04	0.500	0.497	592.09
$\hat{\hat{Y}}_{p12}$	0.483	0.491	8870.95	0.508	0.495	15152.78	0.509	0.516	480.37	0.523	0.533	**1142.84**	0.499	0.498	592.18
$\hat{\hat{Y}}_{p13}$	0.789	0.818	8832.80	0.490	0.559	15751.85	0.657	0.683	**417.24**	0.744	0.786	1673.03	0.534	0.756	618.43

$\hat{\hat{Y}}_{p14}$	0.481	0.476	8881.06	0.566	0.476	14980.01	0.213	0.209	775.37	0.352	0.288	1969.29	0.129	0.178	596.93
$\hat{\hat{Y}}_{p15}$	0.442	0.449	8887.03	0.521	0.448	14379.75	0.212	0.216	746.14	0.334	0.284	1999.47	0.130	0.179	597.00
$\hat{\hat{Y}}_{p16}$	0.403	0.421	8893.33	0.495	0.424	**13957.13**	0.212	0.223	719.47	0.335	0.285	1985.90	0.130	0.179	596.74
$\hat{\hat{Y}}_{p17}$	0.366	0.349	8919.24	0.558	0.532	15946.71	0.498	0.482	500.29	0.526	0.515	1142.96	0.468	0.480	**590.07**
$\hat{\hat{Y}}_{p18}$	0.331	0.325	8923.94	0.512	0.505	15301.49	0.496	0.496	494.29	0.507	0.510	1149.31	0.471	0.482	590.22
$\hat{\hat{Y}}_{p19}$	0.296	0.300	8928.84	0.486	0.480	14885.75	0.496	0.503	487.84	0.508	0.512	1148.57	0.470	0.483	590.29

Table 7. PREs of proposed estimators $\hat{\hat{Y}}_{pj}$ with respect to existing estimators using dataset of population-1.

Existing Estimator	Proposed Estimators								
	$\hat{\hat{Y}}_{p1}$	$\hat{\hat{Y}}_{p2}$	$\hat{\hat{Y}}_{p3}$	$\hat{\hat{Y}}_{p4}$	$\hat{\hat{Y}}_{p5}$	$\hat{\hat{Y}}_{p6}$	$\hat{\hat{Y}}_{p7}$	$\hat{\hat{Y}}_{p8}$	$\hat{\hat{Y}}_{p9}$
$\hat{\hat{Y}}_{M}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{S}$	398	398	398	398	398	398	398	398	398
$\hat{\hat{Y}}_{ss}$	138	138	138	138	138	138	138	138	138
$\hat{\hat{Y}}_{ST}$	132	132	132	132	132	132	132	132	132
$\hat{\hat{Y}}_{sw}$	124	124	124	124	124	124	124	124	124
$\hat{\hat{Y}}_{O}$	125	125	125	125	125	125	125	125	125
$\hat{\hat{Y}}_{YS}$	336	336	336	336	336	336	336	336	336
$\hat{\hat{Y}}_{1}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{2}$	118	118	118	118	118	118	118	118	118
$\hat{\hat{Y}}_{3}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{4}$	118	118	118	118	118	118	118	118	118
$\hat{\hat{Y}}_{5}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{6}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{7}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{8}$	119	119	119	119	119	119	119	119	119
$\hat{\hat{Y}}_{9}$	117	117	117	117	117	117	117	117	117

Existing Estimator	Proposed Estimators									
	$\hat{\hat{Y}}_{p10}$	$\hat{\hat{Y}}_{p11}$	$\hat{\hat{Y}}_{p12}$	$\hat{\hat{Y}}_{p13}$	$\hat{\hat{Y}}_{p14}$	$\hat{\hat{Y}}_{p15}$	$\hat{\hat{Y}}_{p16}$	$\hat{\hat{Y}}_{p17}$	$\hat{\hat{Y}}_{p18}$	$\hat{\hat{Y}}_{p19}$
$\hat{\hat{Y}}_{M}$	118	119	119	119	119	119	119	118	118	118
$\hat{\hat{Y}}_{S}$	395	397	397	398	396	396	396	394	394	394
$\hat{\hat{Y}}_{ss}$	137	137	137	138	137	137	137	137	137	136
$\hat{\hat{Y}}_{ST}$	131	132	132	132	132	132	132	131	131	131
$\hat{\hat{Y}}_{sw}$	123	124	124	124	123	123	123	123	123	123
$\hat{\hat{Y}}_{O}$	123	124	124	125	124	124	124	123	123	123
$\hat{\hat{Y}}_{YS}$	333	335	335	336	334	334	334	333	333	333
$\hat{\hat{Y}}_{1}$	118	119	119	119	118	118	118	118	118	118
$\hat{\hat{Y}}_{2}$	117	118	118	118	118	118	117	117	117	117

$\hat{\hat{Y}}_{3}$	118	119	119	119	119	119	118	118	118	118
$\hat{\hat{Y}}_{4}$	117	117	117	118	117	117	117	117	117	117
$\hat{\hat{Y}}_{5}$	118	119	119	119	119	118	118	118	118	118
$\hat{\hat{Y}}_{6}$	118	119	119	119	118	118	118	118	118	118
$\hat{\hat{Y}}_{7}$	118	118	118	119	118	118	118	118	118	117
$\hat{\hat{Y}}_{8}$	118	119	119	119	119	119	118	118	118	118
$\hat{\hat{Y}}_{9}$	116	117	116	117	116	116	116	116	116	116

Table 8. PREs of proposed estimators $\hat{\hat{Y}}_{pj}$ with respect to existing estimators using dataset population-2.

Existing Estimator	Proposed Estimators								
	$\hat{\hat{Y}}_{p1}$	$\hat{\hat{Y}}_{p2}$	$\hat{\hat{Y}}_{p3}$	$\hat{\hat{Y}}_{p4}$	$\hat{\hat{Y}}_{p5}$	$\hat{\hat{Y}}_{p6}$	$\hat{\hat{Y}}_{p7}$	$\hat{\hat{Y}}_{p8}$	$\hat{\hat{Y}}_{p9}$
$\hat{\hat{Y}}_{M}$	211	210	211	210	211	211	211	211	210
$\hat{\hat{Y}}_{S}$	7469	7454	7492	7454	7466	7468	7474	7489	7457
$\hat{\hat{Y}}_{ss}$	294	293	295	293	294	294	294	294	293
$\hat{\hat{Y}}_{ST}$	213	213	214	213	213	213	213	214	213
$\hat{\hat{Y}}_{sw}$	593	592	595	592	593	593	593	594	592
$\hat{\hat{Y}}_{O}$	211	211	212	211	211	211	212	212	211
$\hat{\hat{Y}}_{YS}$	97	97	97	97	97	97	97	97	97
$\hat{\hat{Y}}_{1}$	213	213	214	213	213	213	213	214	213
$\hat{\hat{Y}}_{2}$	212	211	212	211	212	212	212	212	211
$\hat{\hat{Y}}_{3}$	215	214	215	214	214	215	215	215	214
$\hat{\hat{Y}}_{4}$	211	211	212	211	211	211	211	212	211
$\hat{\hat{Y}}_{5}$	213	212	213	212	213	213	213	213	212
$\hat{\hat{Y}}_{6}$	212	212	213	212	212	212	212	213	212
$\hat{\hat{Y}}_{7}$	213	213	214	213	213	213	213	214	213
$\hat{\hat{Y}}_{8}$	214	214	215	214	214	214	214	215	214
$\hat{\hat{Y}}_{9}$	212	211	212	211	212	212	212	212	211

Existing Estimator	Proposed Estimators									
	$\hat{\hat{Y}}_{p10}$	$\hat{\hat{Y}}_{p11}$	$\hat{\hat{Y}}_{p12}$	$\hat{\hat{Y}}_{p13}$	$\hat{\hat{Y}}_{p14}$	$\hat{\hat{Y}}_{p15}$	$\hat{\hat{Y}}_{p16}$	$\hat{\hat{Y}}_{p17}$	$\hat{\hat{Y}}_{p18}$	$\hat{\hat{Y}}_{p19}$
$\hat{\hat{Y}}_{M}$	414	433	447	430	452	471	485	424	442	455
$\hat{\hat{Y}}_{S}$	14669	15356	15825	15223	16008	16676	17181	15037	15671	16109
$\hat{\hat{Y}}_{ss}$	577	604	622	598	629	656	675	591	616	633
$\hat{\hat{Y}}_{ST}$	419	439	452	435	457	476	491	430	448	460
$\hat{\hat{Y}}_{sw}$	1164	1219	1256	1208	1271	1324	1364	1194	1244	1279
$\hat{\hat{Y}}_{O}$	415	435	448	431	453	472	486	426	444	456
$\hat{\hat{Y}}_{YS}$	190	199	205	198	208	216	223	195	203	209
$\hat{\hat{Y}}_{1}$	418	438	451	434	457	476	490	429	447	459
$\hat{\hat{Y}}_{2}$	416	435	449	431	454	473	487	426	444	457

$\hat{\hat{Y}}_3$	421	441	455	437	460	479	494	432	450	463
$\hat{\hat{Y}}_4$	415	434	448	431	453	472	486	425	443	456
$\hat{\hat{Y}}_5$	418	437	451	433	456	475	489	428	446	459
$\hat{\hat{Y}}_6$	417	437	450	433	455	474	488	428	446	458
$\hat{\hat{Y}}_7$	419	439	452	435	457	476	491	429	448	460
$\hat{\hat{Y}}_8$	421	440	454	436	459	478	493	431	449	462
$\hat{\hat{Y}}_9$	416	436	449	432	454	473	487	426	444	457

Table 9. PREs of proposed estimators $\hat{\hat{Y}}_{pj}$ with respect to existing estimators using dataset population-3.

Existing Estimator	Proposed Estimators								
	$\hat{\hat{Y}}_{p1}$	$\hat{\hat{Y}}_{p2}$	$\hat{\hat{Y}}_{p3}$	$\hat{\hat{Y}}_{p4}$	$\hat{\hat{Y}}_{p5}$	$\hat{\hat{Y}}_{p6}$	$\hat{\hat{Y}}_{p7}$	$\hat{\hat{Y}}_{p8}$	$\hat{\hat{Y}}_{p9}$
$\hat{\hat{Y}}_M$	998	1015	997	1060	1006	1032	998	1001	1016
$\hat{\hat{Y}}_S$	12780	12997	12762	13581	12877	13222	12782	12816	13010
$\hat{\hat{Y}}_{ss}$	2260	2298	2257	2401	2277	2338	2260	2266	2300
$\hat{\hat{Y}}_{ST}$	1617	1644	1615	1718	1629	1673	1617	1621	1646
$\hat{\hat{Y}}_{sw}$	1171	1191	1169	1244	1180	1211	1171	1174	1192
$\hat{\hat{Y}}_O$	1092	1111	1091	1161	1100	1130	1092	1095	1112
$\hat{\hat{Y}}_{YS}$	15291	15551	15271	16250	15408	15820	15294	15335	15567
$\hat{\hat{Y}}_1$	989	1006	988	1051	997	1024	990	992	1007
$\hat{\hat{Y}}_2$	937	953	936	996	945	970	938	940	954
$\hat{\hat{Y}}_3$	993	1010	992	1056	1001	1028	994	996	1011
$\hat{\hat{Y}}_4$	769	782	768	817	775	795	769	771	783
$\hat{\hat{Y}}_5$	967	983	966	1028	974	1000	967	970	984
$\hat{\hat{Y}}_6$	878	893	877	933	884	908	878	880	894
$\hat{\hat{Y}}_7$	989	1006	988	1051	997	1023	989	992	1007
$\hat{\hat{Y}}_8$	981	998	980	1043	989	1015	981	984	999
$\hat{\hat{Y}}_9$	934	950	933	993	941	966	934	937	951

Existing Estimator	Proposed Estimators									
	$\hat{\hat{Y}}_{p10}$	$\hat{\hat{Y}}_{p11}$	$\hat{\hat{Y}}_{p12}$	$\hat{\hat{Y}}_{p13}$	$\hat{\hat{Y}}_{p14}$	$\hat{\hat{Y}}_{p15}$	$\hat{\hat{Y}}_{p16}$	$\hat{\hat{Y}}_{p17}$	$\hat{\hat{Y}}_{p18}$	$\hat{\hat{Y}}_{p19}$
$\hat{\hat{Y}}_M$	937	947	959	1104	594	617	640	921	932	944
$\hat{\hat{Y}}_S$	12000	12133	12281	14139	7609	7907	8200	11792	11935	12093
$\hat{\hat{Y}}_{ss}$	2122	2145	2171	2500	1345	1398	1450	2085	2110	2138
$\hat{\hat{Y}}_{ST}$	1518	1535	1554	1789	963	1000	1037	1492	1510	1530
$\hat{\hat{Y}}_{sw}$	1099	1112	1125	1295	697	724	751	1080	1093	1108
$\hat{\hat{Y}}_O$	1025	1037	1050	1208	650	676	701	1008	1020	1033
$\hat{\hat{Y}}_{YS}$	14358	14517	14695	16918	9104	9461	9811	14110	14281	14470
$\hat{\hat{Y}}_1$	929	939	951	1095	589	612	635	913	924	936
$\hat{\hat{Y}}_2$	880	890	901	1037	558	580	601	865	875	887

$\hat{\bar{Y}}_3$	933	943	955	1099	591	615	637	917	928	940
$\hat{\bar{Y}}_4$	722	730	739	850	458	476	493	709	718	727
$\hat{\bar{Y}}_5$	908	918	929	1070	576	598	620	892	903	915
$\hat{\bar{Y}}_6$	824	833	844	971	523	543	563	810	820	831
$\hat{\bar{Y}}_7$	929	939	950	1094	589	612	635	913	924	936
$\hat{\bar{Y}}_8$	921	932	943	1086	584	607	630	905	916	928
$\hat{\bar{Y}}_9$	877	887	898	1034	556	578	599	862	872	884

Table 10. PREs of proposed estimators $\hat{\bar{Y}}_{pj}$ with respect to existing estimators using dataset population-4.

Existing Estimator	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_M$	545	565	542	593	550	561	545	544	567
$\hat{\bar{Y}}_S$	7219	7483	7182	7853	7283	7427	7223	7208	7504
$\hat{\bar{Y}}_{ss}$	613	635	609	666	618	630	613	612	637
$\hat{\bar{Y}}_{ST}$	1128	1169	1122	1227	1138	1160	1128	1126	1172
$\hat{\bar{Y}}_{sw}$	692	717	689	753	698	712	692	691	719
$\hat{\bar{Y}}_O$	134	139	133	145	135	138	134	134	139
$\hat{\bar{Y}}_{YS}$	13714	14217	13643	14919	13836	14110	13721	13694	14255
$\hat{\bar{Y}}_1$	546	566	543	594	551	562	547	545	568
$\hat{\bar{Y}}_2$	548	568	545	596	553	564	548	547	570
$\hat{\bar{Y}}_3$	546	566	543	594	550	561	546	545	567
$\hat{\bar{Y}}_4$	545	565	543	593	550	561	546	545	567
$\hat{\bar{Y}}_5$	547	567	544	595	552	563	547	546	569
$\hat{\bar{Y}}_6$	547	568	545	596	552	563	548	547	569
$\hat{\bar{Y}}_7$	546	566	543	594	551	562	547	545	568
$\hat{\bar{Y}}_8$	546	566	543	594	551	562	546	545	568
$\hat{\bar{Y}}_9$	548	568	545	596	553	564	548	547	570

Existing Estimator	Proposed Estimators									
	$\hat{\bar{Y}}_{p10}$	$\hat{\bar{Y}}_{p11}$	$\hat{\bar{Y}}_{p12}$	$\hat{\bar{Y}}_{p13}$	$\hat{\bar{Y}}_{p14}$	$\hat{\bar{Y}}_{p15}$	$\hat{\bar{Y}}_{p16}$	$\hat{\bar{Y}}_{p17}$	$\hat{\bar{Y}}_{p18}$	$\hat{\bar{Y}}_{p19}$
$\hat{\bar{Y}}_M$	1854	1855	1855	1267	1077	1060	1068	1855	1845	1846
$\hat{\bar{Y}}_S$	24553	24567	24571	16784	14259	14044	14140	24568	24433	24448
$\hat{\bar{Y}}_{ss}$	2084	2085	2085	1424	1210	1192	1200	2085	2073	2075
$\hat{\bar{Y}}_{ST}$	3835	3837	3838	2622	2227	2194	2209	3837	3816	3819
$\hat{\bar{Y}}_{sw}$	2354	2355	2356	1609	1367	1346	1356	2355	2342	2344
$\hat{\bar{Y}}_O$	455	455	455	311	264	260	262	455	453	453
$\hat{\bar{Y}}_{YS}$	46644	46671	46679	31886	27089	26680	26863	46674	46416	46446
$\hat{\bar{Y}}_1$	1858	1859	1859	1270	1079	1063	1070	1859	1849	1850
$\hat{\bar{Y}}_2$	1864	1865	1865	1274	1082	1066	1073	1865	1854	1856

$\hat{\bar{Y}}_3$	1856	1857	1857	1269	1078	1061	1069	1857	1847	1848
$\hat{\bar{Y}}_4$	1855	1856	1856	1268	1077	1061	1068	1856	1846	1847
$\hat{\bar{Y}}_5$	1860	1861	1862	1272	1080	1064	1071	1861	1851	1852
$\hat{\bar{Y}}_6$	1862	1863	1863	1273	1081	1065	1072	1863	1853	1854
$\hat{\bar{Y}}_7$	1858	1859	1859	1270	1079	1063	1070	1859	1849	1850
$\hat{\bar{Y}}_8$	1857	1858	1859	1270	1079	1062	1070	1858	1848	1849
$\hat{\bar{Y}}_9$	1863	1865	1865	1274	1082	1066	1073	1865	1854	1856

Table 11. PREs of proposed estimators $\hat{\bar{Y}}_{pj}$ with respect to existing estimators using dataset population-5.

Existing Estimator	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_M$	349	353	349	374	351	368	349	351	353
$\hat{\bar{Y}}_S$	7387	7469	7385	7920	7443	7788	7388	7433	7480
$\hat{\bar{Y}}_{ss}$	2599	2628	2599	2787	2619	2740	2600	2616	2632
$\hat{\bar{Y}}_{ST}$	1301	1316	1301	1395	1311	1372	1302	1310	1318
$\hat{\bar{Y}}_{sw}$	1077	1089	1076	1154	1085	1135	1077	1083	1090
$\hat{\bar{Y}}_O$	218	220	218	234	220	230	218	219	221
$\hat{\bar{Y}}_{YS}$	3401	3439	3400	3646	3427	3585	3401	3422	3444
$\hat{\bar{Y}}_1$	346	350	346	371	349	365	346	349	351
$\hat{\bar{Y}}_2$	332	336	332	356	335	351	333	335	337
$\hat{\bar{Y}}_3$	347	351	347	372	349	365	347	349	351
$\hat{\bar{Y}}_4$	259	262	259	278	261	273	259	261	262
$\hat{\bar{Y}}_5$	336	339	335	360	338	354	336	338	340
$\hat{\bar{Y}}_6$	274	277	274	294	276	289	274	276	278
$\hat{\bar{Y}}_7$	346	350	346	371	349	365	346	348	351
$\hat{\bar{Y}}_8$	337	341	337	361	340	355	337	339	341
$\hat{\bar{Y}}_9$	330	334	330	354	333	348	331	333	335

Existing Estimator	Proposed Estimators									
	$\hat{\bar{Y}}_{p10}$	$\hat{\bar{Y}}_{p11}$	$\hat{\bar{Y}}_{p12}$	$\hat{\bar{Y}}_{p13}$	$\hat{\bar{Y}}_{p14}$	$\hat{\bar{Y}}_{p15}$	$\hat{\bar{Y}}_{p16}$	$\hat{\bar{Y}}_{p17}$	$\hat{\bar{Y}}_{p18}$	$\hat{\bar{Y}}_{p19}$
$\hat{\bar{Y}}_M$	470	470	470	450	466	466	466	472	471	471
$\hat{\bar{Y}}_S$	9958	9954	9953	9530	9874	9873	9877	9988	9986	9985
$\hat{\bar{Y}}_{ss}$	3504	3503	3502	3354	3475	3474	3476	3515	3514	3514
$\hat{\bar{Y}}_{ST}$	1754	1754	1754	1679	1740	1739	1740	1760	1759	1759
$\hat{\bar{Y}}_{sw}$	1451	1451	1451	1389	1439	1439	1440	1456	1456	1455
$\hat{\bar{Y}}_O$	294	294	294	281	291	291	291	295	295	294
$\hat{\bar{Y}}_{YS}$	4585	4583	4582	4388	4546	4545	4547	4599	4598	4597
$\hat{\bar{Y}}_1$	467	467	467	447	463	463	463	468	468	468
$\hat{\bar{Y}}_2$	448	448	448	429	444	444	445	450	449	449

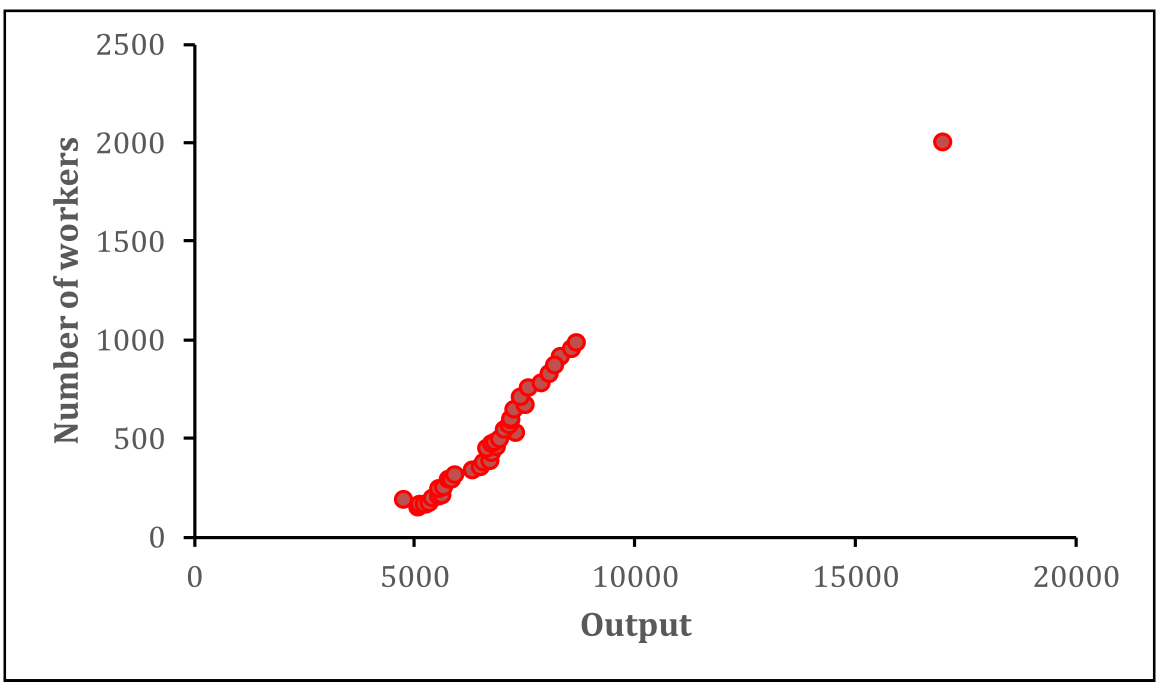
$\hat{\hat{Y}}_3$	467	467	467	447	463	463	464	469	469	469
$\hat{\hat{Y}}_4$	349	349	349	334	346	346	346	350	350	350
$\hat{\hat{Y}}_5$	452	452	452	433	448	448	449	454	454	454
$\hat{\hat{Y}}_6$	370	370	370	354	367	367	367	371	371	371
$\hat{\hat{Y}}_7$	467	466	466	447	463	463	463	468	468	468
$\hat{\hat{Y}}_8$	454	454	454	435	451	450	451	456	456	456
$\hat{\hat{Y}}_9$	445	445	445	426	442	442	442	447	447	447

5.3. Robustness of proposed class of estimators

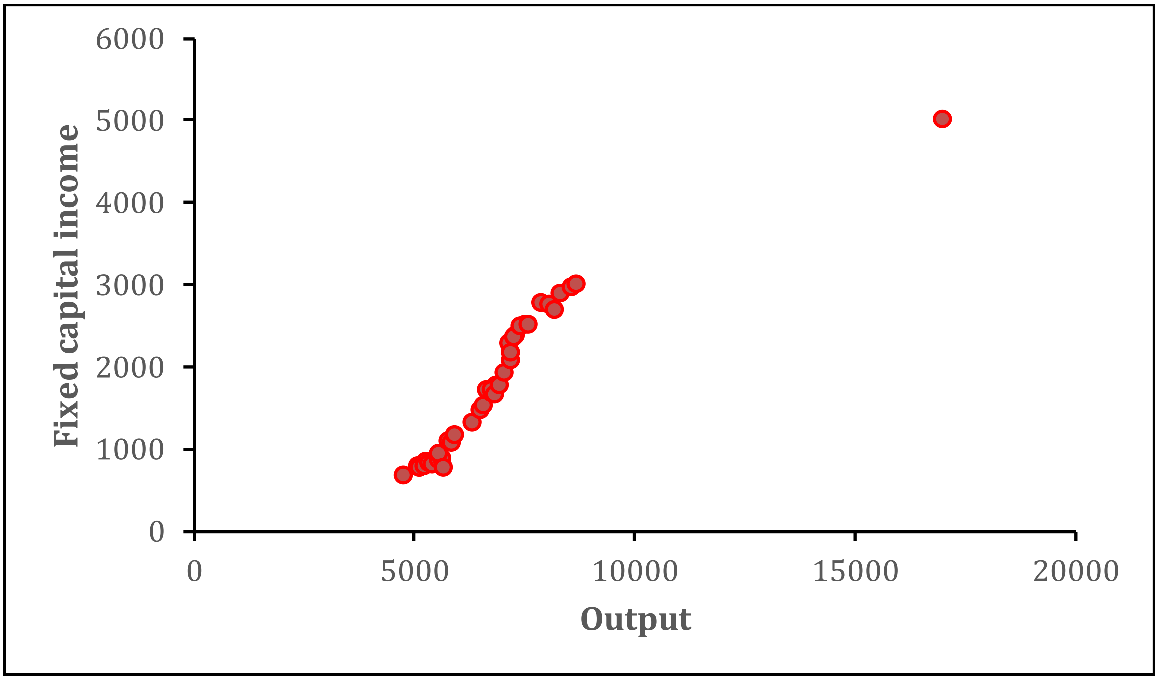
Since the robust measures TM, HL, and DM are likely to remain stable in the presence of outliers, therefore to gauge their impact a robustness study was carried out by using the real population dataset taken from Murthy (1967) that contain an outlier. Various characteristics of this dataset are given as:

$N = 41, \overline{\mathrm{Y}} = 6658.400$, $\mathrm{C_y} = 0.170$, $n = 20$, $\overline{\mathrm{X}}_1 = 47.410$, $\overline{\mathrm{X}}_2 = 1721.400$, $\mathrm{C_{x1}} = 0.560$, $\mathrm{C_{x2}} = 0.460$, $\rho_{\mathrm{yx1}} = 0.980$, $\rho_{\mathrm{yx2}} = 0.990$, $\beta_{2(\mathrm{x1})} = 2.355$, $\beta_{2(\mathrm{x2})} = 1.915$, $\mathrm{TM_1} = 443.00$, $\mathrm{TM_2} = 1658.750$, $\mathrm{HL_1} = 457.750$, $\mathrm{HL_2} = 1712.000$, $\mathrm{DM_1} = 455.444$, and $\mathrm{DM_2} = 1688.444$.

The scatter diagram presented in Figure 1(a) and 1(b) clearly shows the presence of outlier in the data. For comparison, the MSEs of both the existing and proposed estimators were computed and given in Table 12. The results show the superior efficiency of the suggested class of estimators as compared to their competitors in terms of smaller values of the MSEs. These results suggest that the new proposed class of estimators is more efficient when outliers are present in the data.



(a)



(b)

Figure 1. Scatter diagram between **(a)** X_1 = Number of workers, Y = Output **(b)** X_2 = Fixed capital income, Y = Output.

Table 12. The values of MSEs of the existing and suggested estimators for outlier data.

Existing	R_{1i}	R_{2i}	MSE	Proposed	R_{1i}	R_{2i}	MSE
$\hat{\bar{Y}}_M$	-	-	62396.20	$\hat{\bar{Y}}_{p1}$	0.999	0.999	7069.99
$\hat{\bar{Y}}_S$	-	-	990519.1	$\hat{\bar{Y}}_{p2}$	0.995	0.999	6954.82
$\hat{\bar{Y}}_{ss}$	-	-	19383.97	$\hat{\bar{Y}}_{p3}$	0.999	0.999	7091.20
$\hat{\bar{Y}}_{ST}$	-	-	146974.3	$\hat{\bar{Y}}_{p4}$	0.991	0.998	6843.09
$\hat{\bar{Y}}_{sw}$	-	-	97262.12	$\hat{\bar{Y}}_{p5}$	0.998	0.999	7043.92
$\hat{\bar{Y}}_O$	-	-	**18714.86**	$\hat{\bar{Y}}_{p6}$	0.996	0.999	6997.59
$\hat{\bar{Y}}_{YS}$	-	-	384745.2	$\hat{\bar{Y}}_{p7}$	0.999	0.999	7069.22
$\hat{\bar{Y}}_1$	0.999	0.999	62561.13	$\hat{\bar{Y}}_{p8}$	0.999	0.999	7080.38
$\hat{\bar{Y}}_2$	0.995	0.999	63095.09	$\hat{\bar{Y}}_{p9}$	0.995	0.999	6951.66
$\hat{\bar{Y}}_3$	0.999	0.999	62455.13	$\hat{\bar{Y}}_{p10}$	0.515	0.509	723.74
$\hat{\bar{Y}}_4$	0.991	0.998	63458.37	$\hat{\bar{Y}}_{p11}$	0.507	0.501	696.59
$\hat{\bar{Y}}_5$	0.998	0.999	62639.03	$\hat{\bar{Y}}_{p12}$	0.509	0.505	703.13
$\hat{\bar{Y}}_6$	0.996	0.999	62734.35	$\hat{\bar{Y}}_{p13}$	0.715	0.665	2305.41
$\hat{\bar{Y}}_7$	0.999	0.999	62565.67	$\hat{\bar{Y}}_{p14}$	0.373	0.323	853.64
$\hat{\bar{Y}}_8$	0.999	0.999	62475.45	$\hat{\bar{Y}}_{p15}$	0.366	0.316	902.56
$\hat{\bar{Y}}_9$	0.995	0.999	63114.29	$\hat{\bar{Y}}_{p16}$	0.367	0.319	876.45
-	-	-	-	$\hat{\bar{Y}}_{p17}$	0.5106	0.507	709.36
-	-		-	$\hat{\bar{Y}}_{p18}$	0.502	0.499	**683.80**
-	-	-	-	$\hat{\bar{Y}}_{p19}$	0.504	0.502	690.02

6. Summary and conclusion

In this study, a new class of exponential-type ratio estimators is developed based on dual auxiliary variables which utilize information on robust measures like TM, HL, and DM. The theoretical, simulated, and empirical studies were carried out for performance evaluation which showed that the proposed class of estimators are more efficient in term of lower MSE and higher PREs as compared to usual and existing estimators considered in this study. To evaluate the performance in presence of outliers, a robustness study was carried out that confirmed the superior efficiency of the proposed estimators over competing estimators when data contain outliers.

The current work can be extended to other sampling techniques such as ranked set sampling and probability proportional to size sampling, etc.

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