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Process Capability Analysis for Simple Linear Profiles in Multistage Processes

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Abstract

When a process is statistically under control, one may be interested in assessing the process performance based on the specification limits provided by the customer. This evaluation is referred to as process capability analysis. Manufacturing operations are often involved with multistage processes, in which the output of a stage is the input of its subsequent stage. This property is known as the cascade property. Existing methods in capability analysis studies are not applicable when a process or product is represented by profiles. This study presents a method to conduct process capability analysis in a multistage process when quality of a product or process is characterized by a simple linear profile. The performance of the proposed method for a two-stage process is evaluated by numerical simulation using an example from the literature. The results indicate that the proposed method eliminates the effect of the cascade property for different shift sizes and autocorrelations.

Key Words: Process capability index, Multistage process, Cascade property, Simple linear profile, Statistical process control.

1 - Introduction

In many applications of statistical process control (SPC), one may use a single or several quality characteristics to monitor the quality of a process or product. However, in manufacturing processes, sometimes the quality of a product is characterized by a functional relationship between a response variable and one or more explanatory variables. This functional relationship or profile can be linear or nonlinear in nature. There are two phases in the application of control charts to monitor a process (Woodall *et al.*, 2004). In Phase I, the aim is to assess the stability of the process and to estimate the in-control values of the process parameters. The objective of Phase II is to monitor the process using online data to detect any change in the process parameters as soon as possible. (Noorossana *et al.*, 2011) addressed the fundamental concepts, methods, and issues related to statistical analysis of profile monitoring.

To evaluate process performance, many quality measures have been proposed. Process capability indices (PCIs) are among popular and widely used measures to assess process performance in recent years. PCIs are used to determine whether a process is capable of producing conforming items within engineering specification limits (SLs). Univariate PCIs involve single quality characteristics while multivariate PCIs deal with the case of simultaneous monitoring of multiple quality characteristics. In profile monitoring, the SLs for PCIs calculations follow the same functional relationship. This functional relationship or profile can be linear or nonlinear in nature.

The first PCI introduced by (Kane, 1986), C_p, measures the capability of a process with no attention to the process mean. It is defined as

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{1}$$

where σ is the process standard deviation and USL and LSL are the upper and lower SLs which reflect the customer's quality requirements.

Many studies are carried out on process capability estimations. (Kotz and Johnson, 2002) outlined 170 studies on PCIs during the years 1992–2000. (Wu *et al.*, 2009) discussed the developments on PCIs between the years 2002 and 2006. (Yum and Kim, 2011) provided a bibliography of the literature on PCIs for the period 2000–2009. A comprehensive study is also performed by (de-Felipe and Benedito, 2017) for univariate and multivariate PCIs. Lately, some authors studied applications of process capability indices. For example, (Bendersky *et al.*, 2020) performed a dual response surface methodology to simultaneously optimize the mean and the variance of a quality characteristic in the field of quality engineering. They suggested using a process capability index - C_{pk} - as the objective function. Also, (Matsuura, 2021) developed a Bayes estimator of C_{pk} is such that the prior mean is set to be equal to a specified value while the prior distribution is weakly informative.

Assessment of process capability in linear profiles was one of the key issues discussed by (Woodall, 2007). Although this research gap has been addressed in recent years, further studies are still needed to investigate this issue for different cases. (Hosseinifard and Abbasi, 2012a) developed a PCI for linear profiles using the proportion of nonconforming items. In another study, (Hosseinifard and Abbasi, 2012b) investigated and compared five methods to estimate non-normal PCIs for linear profiles. (Keshteli *et al.*, 2014) explained a functional approach for measuring PCI for simple linear profiles (SLPs).

(Pakzad *et al.*, 2021) proposed a functional approach for a simple linear profile based on fuzzy set theory for the situations in which the specification limits and target values of the response variable are not precisely specified. (Pakzad and Basiri, 2022) introduced a new functional incapability index for dealing with asymmetric tolerances for simple linear profile. In the study of (Mehri *et al.*, 2021)(Mehri *et al.*, 2021), two robust PCIs for multiple linear profiles are proposed. In their study, the process capability is estimated using the M-estimator and the Fast-τ-estimator. (Ahmad and Cheng, 2022) proposed a new approach to solve the fuzzy $\bar{X} - R$ control charts with fuzzy process capability indices using fuzzy decision parameters using triangular fuzzy numbers. In another study, (Ahmad *et al.*, 2023) introduced a method to deal with obtaining a fuzzy $\bar{X} - S$ control charts using trapezoidal fuzzy number. They conducted a fuzzy process capability analysis to measure the process performance. For more discussion on this issue see references (Ebadi and Shahriari, 2013), (Wang, 2014a), (Wang, 2014b), (Wang and Tamirat, 2015), (Ahmad *et al.*, 2023).

In the area of PCI for multivariate profiles, (Ebadi and Amiri, 2012) proposed three new methods to measure process capability when process output could be modeled by multivariate simple linear profiles (MVSLP). (Wang, 2016) presented a new process yield index to evaluate the process yield for multivariate linear profiles in manufacturing processes. Also, (Wang and Tamirat, 2016) presented two indices to measure the process capability for multivariate linear profiles with one-sided SLs under mutually independent normality. Additionally, they proposed two indices to measure the process capability for multivariate linear profiles with one-sided SLs under multivariate normality assumption. (Guevara G and Alejandra López, 2022) proposed a two-phase methodology based on the concept of depth to measure the capability of processes characterized by the functional relationship of multivariate nonlinear profile data, treated as multivariate functional observations.

In the existing studies, PCIs are generally computed based on response values. However, using predicted values of profile parameters to measure the capability of profiles has received very little attention. (Karimi Ghartemani *et al.*, 2016), (Wu, 2016) and, (Chiang *et al.*, 2017) introduced multivariate PCIs to assess the process capability in SLP based on profile intercept and slope. Despite these few studies, a major gap exists in the proposed PCIs based on profile intercept and slope. One of the main challenges in capability analysis for a SLP is to determine the SLs for the profile parameters. Existing methods in the literature determine the SLs for the intercept and slope of SLP using the SLs for the response variable, i.e., by employing profiles within the SLs of the response variable. However, the mere fact that the profiles fall within the design specifications does not guarantee that they are in control. Since capability analysis is performed when the process is under statistical control, one drawback of previous approaches is that the reported methods to determine SLs for the intercept and slope could involve out-of-control profiles as well. Considering profile SLs and the in-control profiles, we can calculate accurate SLs for the coefficients which enables us to perform capability analysis based on profile parameters.

On the other hand, in practice, manufacturing operations are often involved with multistage processes, in which the output of a stage is the input of its subsequent stage. As stated earlier, this property is known as cascade property. Using common indices to assess the capability may lead to incorrect results as the cascade effect is ignored. To address cascading issue of a process, one can consider approaches such as cause selecting chart (CSC), regression adjusted charts, and state-space modeling of the process.

To the best of our knowledge, the process capability of linear profiles in a multistage process has not been addressed in the literature. Therefore, we propose an approach to assess the process capability for SLP in a multistage process. Besides, a method to compute PCIs for profile parameters is developed. Note that this method can be used as a diagnostic aid so that we can specify the state of responses, their parameters, and the corresponding stages when the process is not capable. In summary, the contributions of this study are as follows.

- Process capability evaluation of a SLP in a multistage process
- Handling autocorrelation between response variables
- Developing a method to compute PCI for profile parameters

The structure of the paper is as follows. A brief overview of the SLP and univariate PCIs are presented in Sections 2 and 3, respectively. Multistage modeling and process capability studies in multistage processes are presented in Section 4. The proposed method for evaluating process capability is introduced in Section 5. The PCIs for profile parameters are introduced in Section 6. A simulation study to evaluate the performance of the proposed method is presented in Section 7. Our concluding remarks are provided in the final section.

2 - Simple Linear Profiles

As mentioned earlier, a simple linear profile is defined by a linear relationship between a response variable and an explanatory variable. It is assumed that *m* random samples of size *n* are taken from the process. For the j^{th} random sample, there are *n* fixed values for the explanatory variable. When the process is in statistical control, a SLP can be defined as Equation (2).

$$Y_{ij} = A_0 + A_1X_i + \varepsilon_{ij}, \quad i = 1, 2, \dots, n, \quad j = 1, 2, 3, \dots, m, \tag{2}$$

where A_{0j}, A_{1j} are the slope and intercept, respectively, and the error terms ε_{ij} are assumed to be normally and independently distributed with mean zero and variance σ_j^2. It is considered that in Phase I, the parameters in-control values are unknown and if the process is stable then $A_{0j} = A_0$, $A_{1j} = A_1$ and $\sigma^2_{\varepsilon,j} = \sigma^2_\varepsilon, j = 1,2,\dots,m$. It is assumed that X_i has a fixed value for all the samples. The least-squares estimates of A_0, A_1 and σ^2_ε for profile sample *j*, a_{0j}, a_{1j} and mean square error MSE_j are calculated using Equation (3).

$$a_{0j} = \bar{Y}_j - a_{1j}\bar{X}, \qquad a_{1j} = \frac{S_{XY(j)}}{S_{XX}}, \qquad MSE_j = \frac{\sum_{i=1}^{n} e_{ij}^2}{(n-2)}. \tag{3}$$

where, $\bar{Y}_j = \frac{\sum_{i=1}^{n} Y_{ij}}{n}$, $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, $S_{XY(j)} = \sum_{i=1}^{n}(X_i - \bar{X})Y_{ij}$, $S_{XX} = \sum_{i=1}^{n}(X_i - \bar{X})^2$ and $SSE_j = (Y_{ij} - a_{0j} - a_{1j}X_i)^2$. MSE_j is the unbiased estimator of σ^2 for sample *j* and e_{ij} denotes residuals and is defined as $e_{ij} = Y_{ij} - \hat{Y}_{ij}$.

3 - Univariate Process Capability Indices

Process capability analysis is a statistical method that has been used for decades with the purpose of reducing the variability in industrial processes and products. PCI provides a numerical measure on whether a process conforms to the predefined SLs. The four popular univariate PCIs (UPCI) are C_p, C_{pk} introduced by (Kane, 1986), C_{pm} introducd by (Chan *et al.*, 1988), and C_{pmk} introduced by (Pearn *et al.*, 1992), are commonly used to estimate the capability of a process. In this study, we consider C_{pmk} as the most elaborate index among these four basic UPCIs. This index, defined by Equation (4), considers both process variability and proximity to the target when one is dealing with an in-control normal process.

$$C_{pmk} = min\left\{\frac{USL-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-LSL}{3\sqrt{\sigma^2+(\mu-T)^2}}\right\}, \tag{4}$$

where LSL and USL are the lower and upper SLs, respectively, T is the target value, μ is process mean, and σ^2 is process variance.

4 - Multistage Processes

In many manufacturing systems, one is faced with multistage processes where correlated observations are generated. This implies that the output of a stage is the input of its subsequent stage and a change in a response variable may affect some or all output variables in successive stages. As discussed earlier, this property is referred to as cascade property and is the main feature of multistage processes. Many authors study applications of multistage processes. For example, (Moslemi *et al.*, 2018) proposed a novel methodology for a robust multi-response surface optimization in multistage processes. In recent years, some researchers have focused on presenting control charts in multistage processes. However, capability analysis in the multistage processes has been seldom studied.

(Zhang, 1990) introduced two kinds of PCIs for multistage processes: 1) Total PCI that computes the process capability when the quality variable in the present stage is affected by the quality variables of previous stages. 2) Specific PCI which indicates the capability of a stage when the effects of precedent stages are excluded. (Linn *et al.*, 2002) addressed how to prioritize the process variation reduction to enhance the overall process capability in multistage processes. Based on Taguchi loss function, (Chen *et al.*, 2012) presented a method to calculate PCI for a complex product machining process as a multistage process. (Nikzad *et al.*, 2017) estimated the process capability of the second stage of a two-stage process while the effect of cascade property is removed by using residuals analysis.

While most of the studies in the area of multistage processes deal with univariate or multivariate quality characteristics, in some situations, profiles are streamed through the stages of a multistage process. (Ghahyazi *et al.*, 2014) were the first researchers who considered the quality characteristic in a multistage process as a profile and proposed an approach to monitor SLP in Phase II.

5 - The Proposed Model

To define profile modeling in a multistage process, we assume that *m* samples of size *n* are collected at each of the *k* stages of a multistage process from historical data. At each stage of the process, for sample *j*, the observations $(x_{i,j,s}, y_{i,j,s})$, $i = 1,2,\dots,n$, $j = 1,2,\dots,m$ and $s = 1.2.\dots,k$ are available. (Ghahyazi *et al.*, 2014) introduced the profile model in a multistage process considering the cascade property as

$$\begin{gathered} Y_{i,j,1} = A_{0,1} + A_{1,1}X_{i,j,1} + \varepsilon_{i,j,1}, \\ Y_{i,j,s} = \varphi Y_{i,j,s-1} + A_{0,s} + A_{1,s}X_{i,j,s} + \varepsilon_{i,j,s}, \end{gathered} \tag{5}$$

where $A_{0,s}$ and $A_{1,s}$ are the intercept and slope parameters at stage *s*, $\varepsilon_{i,j,s}$'s are assumed to be i.i.d. $N(0,\sigma_{\varepsilon,s}^2)$ random variables, and φ is the autocorrelation coefficient of the process between stages. It is also assumed that the explanatory variable is fixed from sample to sample for all stages; consequently $X_{i,j,s} = X_i$ for all values of *j* and *s*. Figure 1 presents a graphical display of the proposed multistage model.

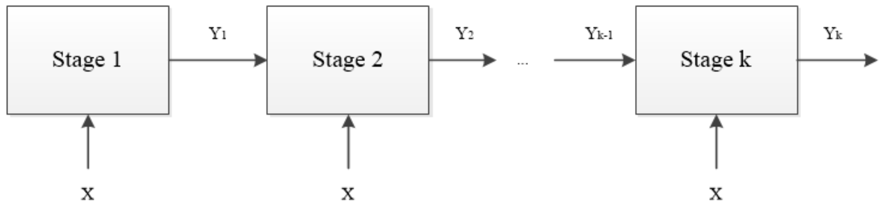

Figure 1. A graphical display of a multistage process

After specifying the profile model in a multistage process, we must define the SLs. Generally, SLs can be considered as fixed values or a function of explanatory variables. We assume that the associated SLs of the response variables ($Y_{i,s}$) in each stage are linear functions of the explanatory variable defined as

$$USL_{i,s} = a_0' + a_1' X_i, \\ LSL_{i,s} = a_0'' + a_1'' X_i, \tag{6}$$

where $i = 1,2,\dots,n$ and $s = 1,2,\dots,k$.

Due to the cascade effect, using common PCIs to assess the capability of intermediate stages ($s > 1$) may lead to misleading results. To deal with this issue, the PCI for the residuals is considered. Residual analysis is the concept of the cause-selecting chart (CSC) proposed by (Zhang, 1984) which is one of the most popular approaches in multistage studies. The idea of CSC is taken from the regression control chart of (Mandel, 1969) in which a variable is dependent on an independent variable. Residuals are not affected by previous processes. Thus, the PCIs for the residuals indicate the specific capability of the process in the preferred stage. For all values of *i*, the residuals are obtained by

$$e_{j,s} = Y_{j,s} - \hat{Y}_{j,s-1}, \tag{7}$$

where $\hat{Y}_{j,s}$ is the predicted value for the response variable $Y_{j,s}$ and can be obtained as follows.

$$\hat{Y}_{j,s} = \hat{\beta}_s + \hat{\alpha}_s Y_{j,s-1}, \tag{8}$$

Based on (Kutner *et al.*, 2005), $\hat{\beta}_s$ and $\hat{\alpha}_s$ are the estimated parameters that are calculated using Equations (9) and (10).

$$\hat{\beta}_s = \bar{Y}_s - \hat{\alpha}_s \bar{Y}_{s-1}, \tag{9}$$

$$\hat{\alpha}_s = \frac{\sum_{j=1}^{m} Y_{j,s}\left(Y_{j,s-1} - \bar{Y}_{s-1}\right)}{\sum_{j=1}^{m}\left(Y_{j,s-1} - \bar{Y}_{s-1}\right)^2}, \tag{10}$$

The variance of the residuals is calculated using

$$\sigma_{\varepsilon,s}^2 = \frac{\sum_{j=1}^{m}\left(Y_{j,s} - \hat{Y}_{j,s}\right)^2}{m-2} = \frac{\sum_{j=1}^{m} {e_{j,s}}^2}{m-2}, \tag{11}$$

To assess the PCI for residuals, SLs of residuals have to be obtained. (Nikzad *et al.*, 2017) proposed a method to calculate SLs for residuals using process yield. According to (Wang, 2014a), process yield has been recognized as a common criterion used in the manufacturing industry for measuring process performance. It measures the performance of the process by computing the percentage of the conforming items based on the process SLs. Under the assumptions that the mean of residuals is equal to zero, as the target value, and the process yield of residuals is 0.9973, the SLs for the residuals are obtained as follows.

$$\text{Process Yield } = P\{USL_e \le \varepsilon \le LSL_e\} = 0.9973, \tag{12}$$

$$\rightarrow P\left\{\frac{LSL_e - \mu_e}{\sigma_e} \le Z \le \frac{USL_e - \mu_e}{\sigma_e}\right\} = 0.9973, \quad \mu_e = 0, \tag{13}$$

$$\rightarrow USL_e = \sigma_e \phi^{-1}(0.99865), \tag{14}$$

$$\rightarrow LSL_e = \sigma_e \phi^{-1}(0.00135), \tag{15}$$

where $\phi^{-1}(.)$ is the inverse cumulative distribution function of the standard normal distribution. By determining the SLs associated with the residuals and the response variable in all stages using C_{pmk}, we can assess the PCI for each stage.

6 - PCIs for Parameters

Alongside the PCIs calculated for each stage based on the response variable, the performance of the stages can be assessed through the PCIs for the parameters. This aids in identifying the source of poor performance among the parameters. To evaluate the PCIs for profile parameters, it is necessary to determine the SLs for the intercept and slope. (Pakzad, 2023) provided a method to measure PCI for a SLP based on its parameters. They considered profile SLs as well as the in-control profile to obtain accurate SLs for the parameters. To assess the in-control profile, control chart limits for monitoring each parameter was considered. Their method is based on (Kim *et al.*, 2003) study which uses coded X-values to make the intercept and the slope of each profile independently.

(Kim *et al.*, 2003) introduced an interpretable method for Phase I profile monitoring by monitoring intercept and slope using separate control charts. In this method, coded X-values are used which make the intercept and slope of each profile independent. Equation (16) presents the coded form of the model given in Equation (2).

$$Y_{ij} = B_{0j} + B_{1j}X_i' + \varepsilon_{ij}, \qquad i = 1,2,\dots,n, \qquad j = 1,2,\dots,m, \tag{16}$$

where $B_{0j} = A_{0j} + A_{1j}\bar{X}$, $B_{1j} = A_{1j}$ and $X_i' = X_i - \bar{X}$. For this model, the least-square estimators of coefficients are obtained using $b_{0j} = \bar{Y}_j$ and $b_{1j} = a_{1j} = \frac{S_{XY(j)}}{S_{XX}}$. It is well known that when the process is in-control, both b_{0j} and b_{1j} are mutually independent and follow a normal distribution as $b_{0j} \sim N(B_0, \frac{\sigma_\varepsilon^2}{n})$ and $b_{1j} \sim N(B_1, \frac{\sigma_\varepsilon^2}{S_{XX}})$. Equations (17) through (20) can be used to construct separate Shewhart control charts for monitoring intercept and slope.

$$LCL_{b_0} = \bar{b}_0 - t_{m(n-2),\frac{\alpha_2}{2}} \sqrt{\frac{(m-1)MSE}{mn}}, \tag{17}$$

$$UCL_{b_0} = \bar{b}_0 + t_{m(n-2),\frac{\alpha_2}{2}} \sqrt{\frac{(m-1)MSE}{mn}}, \tag{18}$$

$$LCL_{b_1} = \bar{b}_1 - t_{m(n-2),\frac{\alpha_2}{2}} \sqrt{\frac{(m-1)MSE}{mS_{XX}}}, \tag{19}$$

$$UCL_{b_1} = \bar{b}_1 + t_{m(n-2),\frac{\alpha_2}{2}} \sqrt{\frac{(m-1)MSE}{mS_{XX}}}, \tag{20}$$

where $\bar{b}_0 = \frac{\sum_{j=1}^{m} b_{0j}}{m}$, $\bar{b}_1 = \frac{\sum_{j=1}^{m} b_{1j}}{m}$, $MSE = \frac{\sum_{j=1}^{m} MSE_j}{m}$ and $t_{m(n-2),\frac{\alpha_2}{2}}$ is a $100(1-\frac{\alpha_2}{2})$ percentile of t distribution with $m(n-2)$ degrees of freedom. Note that $\alpha_2 = \sqrt[m]{(1-\alpha_1)}$ is the marginal probability of signal for each control chart and $\alpha_1 = \sqrt[3]{(1-\alpha)}$ specifies the overall probability of false alarm by each chart.

Assuming the response variable SLs for each level of the explanatory variable ($i = 1, 2, \dots, n$) are linear functions of the explanatory variable, two regression lines can be fitted as was stated in Equation (6). According to separate control chart method, the transformed model of the SLs can be written as Equation (21).

$$\begin{aligned} USL_i &= b_0' + b_1' X_i', \\ LSL_i &= b_0'' + b_1'' X_i', \end{aligned} \tag{21}$$

where b_0', b_1', b_0'', and b_1'' are the intercepts and slopes for USL_i and LSL_i, respectively. Note that the SLs are not necessarily parallel to each other or the profile line. However, in this study, we assume that SLs are parallel, so $b_1'' = b_1' = b$. A process is called "capable" if the response variable falls within the profile SLs. Hence,

$$b_0'' + bX_i' \le b_0 + b_1 X_i' \le b_0' + bX_i', \tag{22}$$

According to (Pakzad, 2023), SLs for the intercept and slope can be calculated using Equations (23) and (24).

$$b_0'' + (b - b_1)X_i' \le b_0 \le b_0' + (b - b_1)X_i', \tag{23}$$

$$\begin{cases} b + \frac{(b_0'' - b_0)}{X_i'} \le b_1 \le b + \frac{(b_0' - b_0)}{X_i'}, & X_i' > 0, \\ b + \frac{(b_0' - b_0)}{X_i'} \le b_1 \le b + \frac{(b_0'' - b_0)}{X_i'}, & X_i' < 0, \end{cases} \tag{24}$$

It is worth mentioning that although all profiles in Equation (22) are within the SLs of the response variable, they are not necessarily in-control. Thus, all the intercepts and slopes in Equations (23) and (24) are not necessarily in-control either. To define accurate SLs for profile parameters, (Pakzad, 2023) considered both conforming and statistically in-control profiles in Equation (22). As a result, the SLs for the intercept and the slope parameters are given by Equations (25) and (26).

$$b_0'' + (b - LCL_{b_1})X_L' \le b_0 \le b_0' + (b - UCL_{b_1})X_L', \tag{25}$$

$$\begin{aligned} &Min\,\{conforming\ and\ in-control\ slopes\} \le b_1 \le \\ &Max\,\{conforming\ and\ in-control\ slopes\}, \end{aligned} \tag{26}$$

where X_L' is the minimum value of X', X_U' is the maximum value of X', and $b + \frac{(b_0'' - LCL_{b_0})}{X_U'}, b + \frac{(b_0' - UCL_{b_0})}{X_U'}, b + \frac{(b_0' - UCL_{b_0})}{X_L'}$ and $b + \frac{(b_0'' - LCL_{b_0})}{X_L'}$ are all conforming and in-control slopes.

Hence, using the SLs associated with the response variable and considering the in-control values of intercept and slope, Equations (25) and (26) are the SLs for intercept and slope parameters, respectively and C_{pmk} for b_0 and b_1 can be obtained. It is worth mentioning that both indices $C_{pmk_{b_0}}$ and $C_{pmk_{b_1}}$ are used simultaneously and process is deemed “incapable” if at least one of the indices is indicative of a low process performance.

7 - Simulation Study

In this section, we considered the example discussed by (Khedmati and Niaki, 2016) and used MATLAB (ver. R2018a) to conduct simulation analyses to evaluate the performance of our proposed method. We utilized the example provided by. They considered a SLP in a two-stage process where the profiles for the first and second stages are given as

$$\begin{aligned} Y_{i,j,1} &= 3 + 2X_i + \varepsilon_{i,j,1}, \\ Y_{i,j,2} &= \varphi Y_{i,j,1} + 2 + X_i + \varepsilon_{i,j,2}, \end{aligned} \tag{27}$$

where $\varepsilon_{i,j,1} \sim N(0, \sigma_1^2)$, $\varepsilon_{i,j,2} \sim N(0, \sigma_2^2)$ and $\sigma_1^2 = \sigma_2^2 = 1$. The explanatory variable with four fixed X_i-values of 2, 4, 6, and 8 are used in the simulation study. In our proposed method, by coding X_i-values, we obtain the transformed model as $Y_{i,j,1} = 13 + 2X_i' + \varepsilon_{ij1}$ and $Y_{i,j,2} = \varphi Y_{i,j,1} + 7 + X_i' + \varepsilon_{ij2}$ with X_i'-values as -3, -1, 0, 1 and 3. We considered the SLs regression lines in each stage for both the original and transformed models in Table 1.

Table 1. Response variable SLs in each stage

Stage	Original model SLs		Transformed model SLs
Stage 1	$LSL_1 = 0.0012 + 2X_i$	$\rightarrow$	$LSL_1 = 10.0012 + 2X_i'$
	$USL_1 = 6.02 + 2X_i$		$USL_1 = 16.02 + 2X_i'$
Stage 2	$LSL_2 = 0.683 + 2.8X_i$	$\rightarrow$	$LSL_2 = 14.683 + 2.8X_i'$
	$USL_2 = 8.73 + 2.8X_i$		$USL_2 = 22.73 + 2.8X_i'$

The values of C_{pmk} for all response and parameters are obtained based on 10,000 simulation replications where $\sigma_1^2 = \sigma_2^2 = 1$ and $m = 25$.

In the following tables, we investigate the PCIs for both response variables and parameters. It must be noted again that the specified PCI of stage 2 is calculated based on residuals. The performance of stage 1 is related to the parameters of the profile in the first stage, while total performance is associated with the parameters of the profile in all stages. The effect of different values of sample size on C_{pmk} of each stage and parameter for both weak and strong autocorrelation coefficients ($\varphi = 0.1, 0.9$) is presented in Table 2. In the following tables, b_{0-1} and b_{1-1} refer to intercept and slope of the profile in stage 1 and b_{0-2} and b_{1-2} refer to intercept and slope of the profile in stage 2, respectively.

Table 2. C_{pmk} for each stage under different values of φ and m

	m			
	25	**50**	**100**	**200**

$\boldsymbol{\varphi = 0.1}$	$C_{pmk-\text{Stage1}}$	0.9569	0.9657	0.9776	0.9843
	$C_{pmk-\text{Stage2}}$	0.9654	0.9673	0.9703	0.9722
	$C_{pmk-\text{Total}}$	0.1729	0.1731	0.1731	0.1732
	$C_{pmk-b_{0-1}}$	3.5852	3.6404	3.7029	3.7690
	$C_{pmk-b_{1-1}}$	0.6191	0.5884	0.5538	0.5170
	$C_{pmk-b_{0-2}}$	0.0479	0.0526	0.0584	0.0643
	$C_{pmk-b_{1-2}}$	0.5406	0.5359	0.5306	0.5254
$\boldsymbol{\varphi = 0.9}$	$C_{pmk-\text{Stage1}}$	0.9546	0.9648	0.9788	0.9844
	$C_{pmk-\text{Stage2}}$	0.9633	0.9669	0.9705	0.9723
	$C_{pmk-\text{Total}}$	0.9479	0.9598	0.9709	0.9792
	$C_{pmk-b_{0-1}}$	3.5852	3.6404	3.7029	3.7690
	$C_{pmk-b_{1-1}}$	0.6191	0.5884	0.5537	0.5170
	$C_{pmk-b_{0-2}}$	3.5772	3.6324	3.6949	3.7610
	$C_{pmk-b_{1-2}}$	0.6069	0.5762	0.5415	0.5048

From Table 2 we infer that different values of the autocorrelation coefficient do not affect the process performance in stage 1 and stage 2 (when the cascade property is removed). However, the total index is strongly correlated to this coefficient. Higher values of autocorrelation coefficients result in higher values of the total C_{pmk}. The performance of the parameters in stage 1 is not correlated to the autocorrelation coefficient either. As we see in Table 2, the values of C_{pmk} for b_{0-1} and $b_{1-1}$are not affected by different values of φ; while the capability of parameters in stage 2 (b_{0-2} and b_{1-2}) are highly correlated to φ. On the other hand, it is clear from Table 2 that as sample size increases, the values of C_{pmk} for all stages and parameters increase except for the slope parameters. Also, capability values for both b_{1-1} and b_{1-2} decrease slightly when sample size increases.

The effect of different variances of error terms (σ_1^2 and σ_2^2) on C_{pmk} in each stage and related parameters, while φ = 0.9 and the sample size equals 25 is presented in Table 3.

Table 3. C_{pmk} for each stage and parameters under different values of σ_1^2 and σ_2^2

			$\boldsymbol{\sigma_2^2 = 1}$			
$\boldsymbol{\sigma_1^2}$	**0.7**	**0.8**	**0.9**	**1**	**1.1**	**1.2**
$C_{pmk-\text{Stage1}}$	1.3838	1.2104	1.0622	0.9546	0.8590	0.7868
$C_{pmk-\text{Stage2}}$	0.9989	0.9876	0.9758	0.9633	0.9438	0.9320
$C_{pmk-\text{Total}}$	1.0827	1.0393	0.9894	0.9479	0.9016	0.8650
$C_{pmk-b_{0-1}}$	3.9751	3.8208	3.6933	3.5852	3.4923	3.4112
$C_{pmk-b_{1-1}}$	0.9120	0.7962	0.7002	0.6191	0.5492	0.4883
$C_{pmk-b_{0-2}}$	3.7260	3.6727	3.6233	3.5772	3.5341	3.4937
$C_{pmk-b_{1-2}}$	0.7181	0.6783	0.6414	0.6069	0.5747	0.5444
			$\boldsymbol{\sigma_1^2 = 1}$			
$\boldsymbol{\sigma_2^2}$	**0.7**	**0.8**	**0.9**	**1**	**1.1**	**1.2**

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$C_{pmk-\text{Stage1}}$	0.9546	0.9546	0.9546	0.9546	0.9546	0.9546
$C_{pmk-\text{Stage2}}$	1.0620	0.9970	0.9825	0.9633	0.9456	0.9221
$C_{pmk-\text{Total}}$	1.1295	1.0671	1.0052	0.9479	0.8941	0.8414
$C_{pmk-b_{0-1}}$	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852
$C_{pmk-b_{1-1}}$	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191
$C_{pmk-b_{0-2}}$	3.7660	3.6972	3.6346	3.5772	3.5244	3.4756
$C_{pmk-b_{1-2}}$	0.7481	0.6966	0.6498	0.6069	0.5674	0.5309

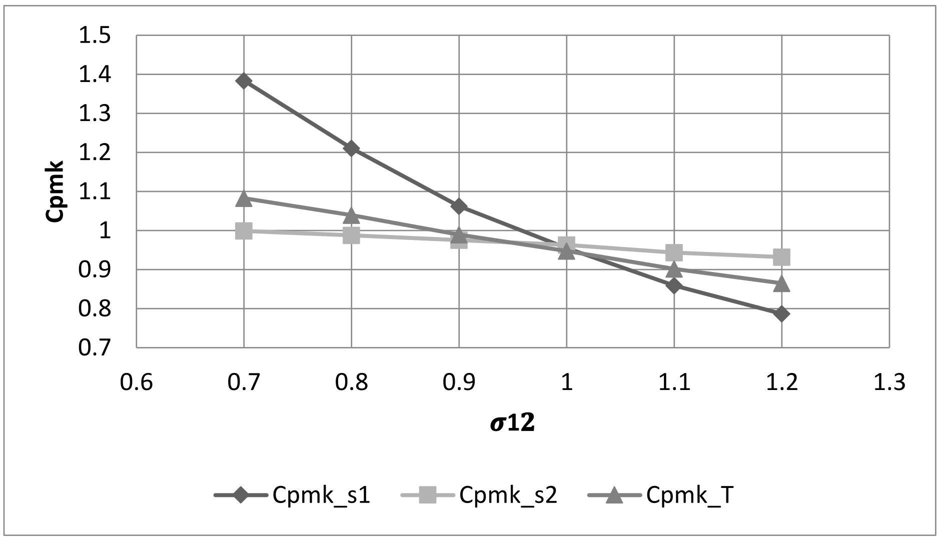

Figure 2. C_{pmk} values under different values of σ_1^2 where $\sigma_2^2 = 1$

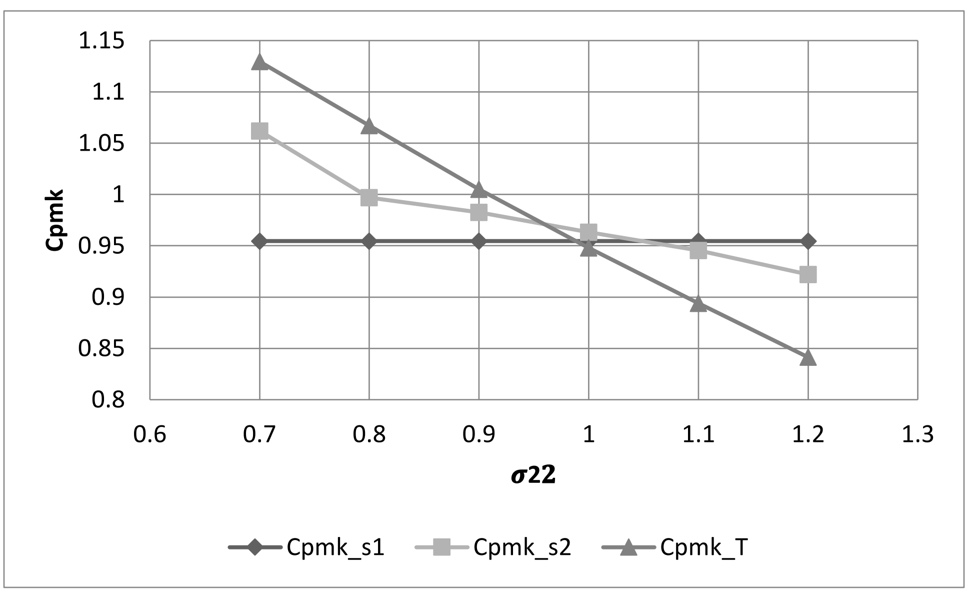

Figure 3. C_{pmk} values under different values of σ_2^2 where $\sigma_1^2 = 1$

From Table 3 we found that different values of σ_1^2 affect the performance of all stages but changes in σ_2^2 have no particular effect on the first stage performance. One can see that as σ_1^2 decreases, C_{pmk} of stage 1, stage 2, and total capability and also C_{pmk} for all parameters increase. For example, when $\sigma_1^2 = 0.7$ and $\sigma_2^2 = 1$, PCIs for stage1 (1.3838) and its related parameters (3.9751 and 0.9120) show better capability in comparison to the cases in which σ_1^2 takes higher values. As mentioned, the variation of σ_2^2 does not affect the performance of the first stage. As σ_2^2 decreases, total C_{pmk} and also C_{pmk} for profile parameters of stage 2 increase. For example, when $\sigma_1^2 = 1$ and $\sigma_2^2 = 0.7$, total PCI (1.1295) and the PCI for stage2 (1.0620) and its related parameters (3.7660 and 0.7481) show better capability in comparison to the cases in which σ_2^2 takes higher values. Figures 2 and 3 show how different values of σ_1^2 and σ_2^2 affect the performance of stage 1, stage 2, and total performance.

Similarly, for different values of σ_1^2 and σ_2^2, the capability of all stages and parameters is interpretable. Also, simultaneous changes of σ_1^2 and σ_2^2 can be explained. For example, when both σ_1^2 and σ_2^2 are equal to 0.6, the C_{pmk} for stage1, stage2, total, b_{0-1}, b_{1-1}, b_{0-2}, and b_{1-2} is obtained as 1.6232, 1.3680, 1.6085, 4.1662, 1.0558, 4.1563, and 1.0400, respectively which shows acceptable performance. Obviously, the lower values of σ_1^2 and σ_2^2 result in better performance of the process.

In Tables 4 and 5, we studied the effect of shifts in intercept and slope of the profile in stage 1 on the process performance when φ = 0.9 and the sample size equals 25.

TABLE 4. PCIs values under stage 1 intercept shifts from b_{0-1} to $b_{0-1} + \lambda\sigma$

	λ						
PCI	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$C_{pmk-\text{Stage1}}$	0.8590	0.9075	0.9371	0.9546	0.9455	0.9177	0.8717
$C_{pmk-\text{Stage2}}$	0.9670	0.9645	0.9665	0.9633	0.9655	0.9643	0.9665
$C_{pmk-\text{Total}}$	0.7988	0.8746	0.9214	0.9479	0.9324	0.8783	0.8075
$C_{pmk-b_{0-1}}$	2.8763	3.1820	3.4363	3.5852	3.4783	3.2420	2.9428
$C_{pmk-b_{1-1}}$	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191
$C_{pmk-b_{0-2}}$	3.1850	3.3614	3.4970	3.5772	3.5123	3.3839	3.2124
$C_{pmk-b_{1-2}}$	0.6069	0.6069	0.6069	0.6069	0.6069	0.6069	0.6069

TABLE 5. PCIs values under stage 1 slope shifts from b_{1-1} to $b_{1-1} + \beta\sigma$

	β						
PCI	-0.15	-0.1	-0.05	0	0.05	0.1	0.15
$C_{pmk-\text{Stage1}}$	0.8535	0.9002	0.9431	0.9546	0.9423	0.9037	0.8496
$C_{pmk-\text{Stage2}}$	0.9642	0.9646	0.9651	0.9633	0.9640	0.9640	0.9653
$C_{pmk-\text{Total}}$	0.7766	0.8545	0.9196	0.9479	0.9242	0.8553	0.7721
$C_{pmk-b_{0-1}}$	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852
$C_{pmk-b_{1-1}}$	0.3124	0.4090	0.5121	0.6191	0.5314	0.4290	0.3284
$C_{pmk-b_{0-2}}$	3.5772	3.5772	3.5772	3.5772	3.5772	3.5772	3.5772
$C_{pmk-b_{1-2}}$	0.3352	0.4269	0.5014	0.6069	0.5509	0.4859	0.4172

Table 4 is related to PCIs values under small intercept shifts in stage 1. It is assumed that the values for the slope and variance are known with no shift. From Table 4, we conclude that as intercept shift increases, the performance of parameters deteriorates. Table 5 shows the effect of different shifts in the slope in stage 1 on the performance of parameters. It is clear that as the shift size in slope increases, the process performance deteriorates.

According to Tables 4 and 5, stage 1 parameters shifts are considered to investigate how PCIs values are affected. In the Figures 4 and 5, the performance of stage 1, stage 2, and total performance is demonstrated.

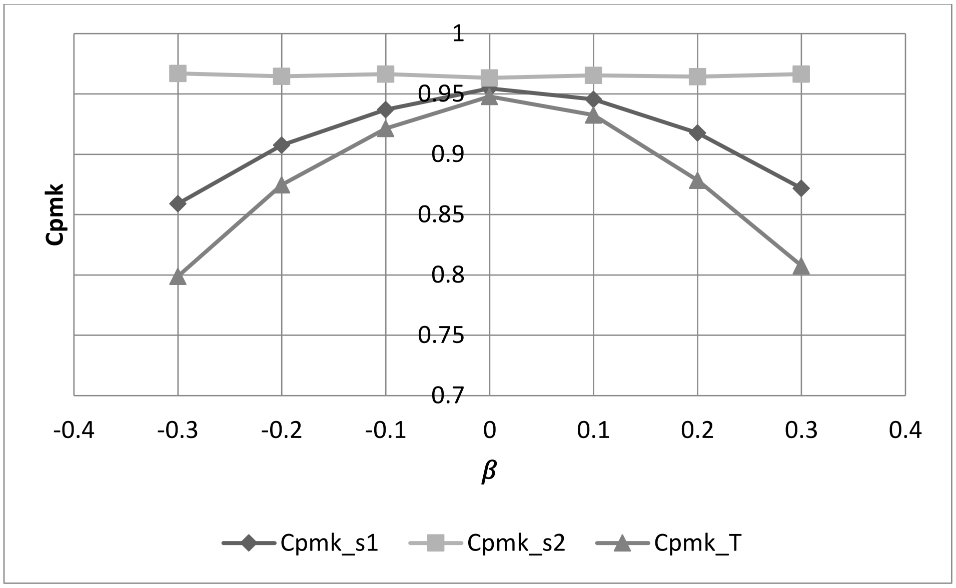

Figure 4. C_{pmk} values under stage 1 intercept shifts from b_{0-1} to $b_{0-1}+\lambda\sigma$

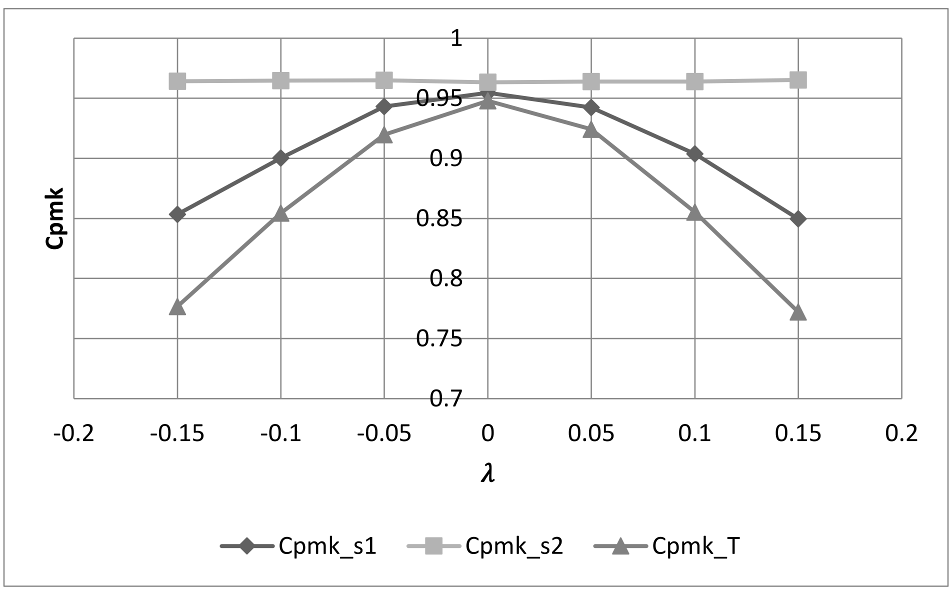

Figure 5. C_{pmk} values under stage 1 slope shifts from b_{1-1} to $b_{1-1}+\beta\sigma$

As expected, shifts in the parameters in stage 1 affect the performance of its own stage and the total performance. Besides, larger shifts cause poorer performance. On the other hand, these shifts have insignificant effect on stage 2 performance.

TABLE 6. PCIs values under stage 2 intercept shifts from b_{0-2} to $b_{0-2}+\lambda'\sigma$

				λ'			
PCI	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
$C_{pmk-\text{Stage1}}$	0.9500	0.9512	0.9537	0.9546	0.9531	0.9511	0.9547
$C_{pmk-\text{Stage2}}$	0.8943	0.9152	0.9440	0.9633	0.9437	0.9216	0.9040
$C_{pmk-\text{Total}}$	0.8908	0.9195	0.9393	0.9479	0.9453	0.9244	0.8970

$C_{pmk-b_{0-1}}$	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852
$C_{pmk-b_{1-1}}$	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191
$C_{pmk-b_{0-2}}$	3.1201	3.3251	3.4844	3.5772	3.5006	3.3489	3.1485
$C_{pmk-b_{1-2}}$	0.6069	0.6069	0.6069	0.6069	0.6069	0.6069	0.6069

TABLE 7. PCIs values under stage 1 slope shifts from $\boldsymbol{b_{1-2}}$ to $\boldsymbol{b_{1-2}} + \beta'\boldsymbol{\sigma}$

	β'						
PCI	-0.15	-0.1	-0.05	0	0.05	0.1	0.15
$C_{pmk-\text{Stage1}}$	0.9525	0.9508	0.9547	0.9546	0.9490	0.9564	0.9529
$C_{pmk-\text{Stage2}}$	0.8892	0.9202	0.9458	0.9633	0.9347	0.9151	0.8961
$C_{pmk-\text{Total}}$	0.8855	0.9170	0.9377	0.9479	0.9355	0.9174	0.8832
$C_{pmk-b_{0-1}}$	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852	3.5852
$C_{pmk-b_{1-1}}$	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191	0.6191
$C_{pmk-b_{0-2}}$	3.5772	3.5772	3.5772	3.5772	3.5772	3.5772	3.5772
$C_{pmk-b_{1-2}}$	0.3827	0.4654	0.5241	0.6069	0.5440	0.4708	0.3944

The effect of shifts in parameters in stage 2 on the process performance are evaluated in Tables 6 and 7. The results indicate that as parameters shift increases, the performance of stage 2, its parameters, and total performance deteriorate. According to Table 6, we notice that as intercept shift increases, the performance of intercept, stage 2, and total performance deteriorate. Table 7 shows the effect of different shifts in the slope parameter in stage 2 on the performance. It is clear that as the shift size in slope increases, the process performance deteriorates.

According to Tables 6 and 7, PCIs values under shifts in the parameters in stage 2 are discussed. In Figures 6 and 7, the performance of stage 1, stage 2, and total performance is illustrated.

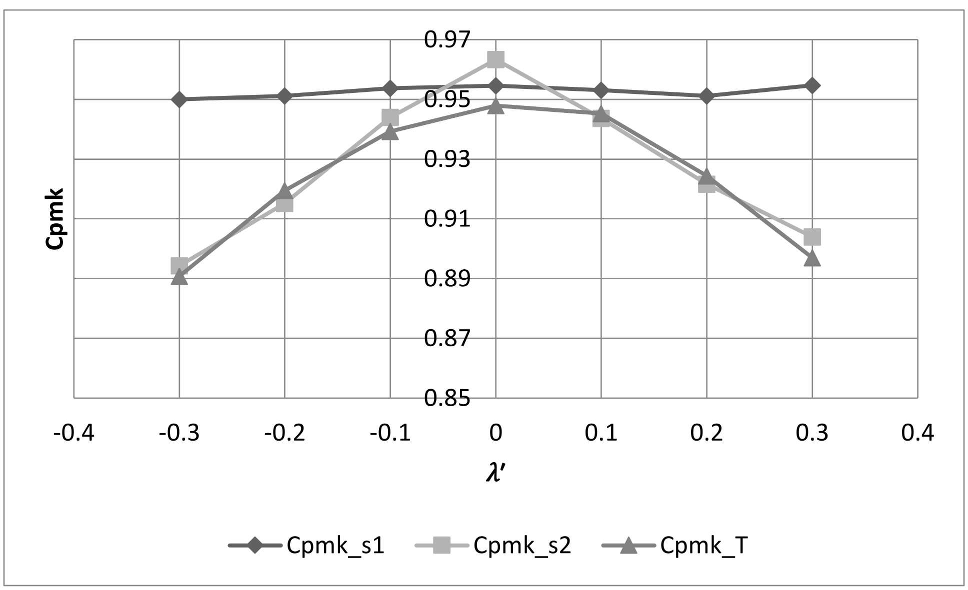

Figure 6. $\boldsymbol{C_{pmk}}$ values under stage 2 intercept shifts from $\boldsymbol{b_{0-2}}$ to $\boldsymbol{b_{0-2}} + \lambda'\boldsymbol{\sigma}$

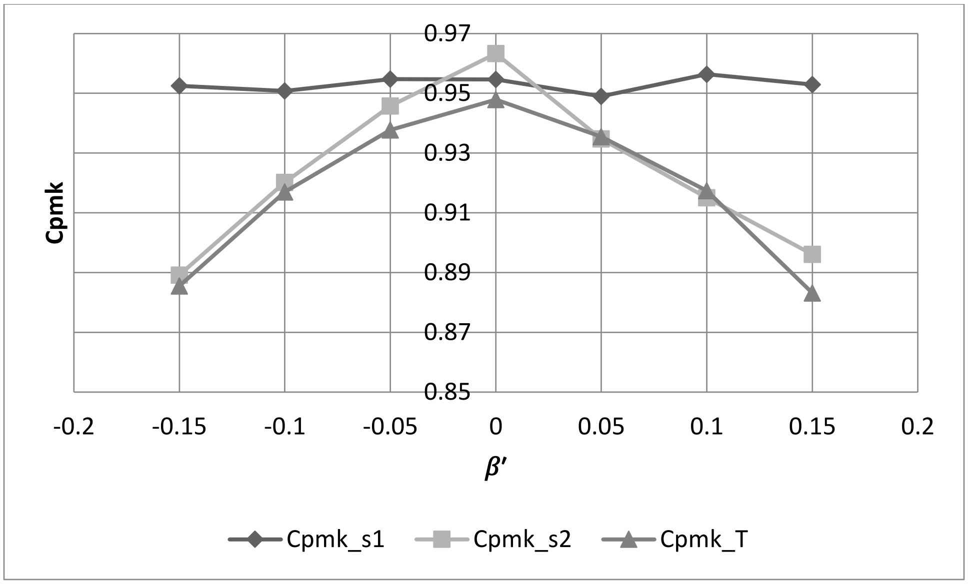

Figure 7. C_{pmk} **values under stage 2 slope shifts from** b_{1-2} **to** $b_{1-2} + \beta'\sigma$

As shown in Figures 6 and 7, shifts in the parameters in stage 2 affect the performance of its own stage and the total performance. As one could expect, larger shifts result in poorer performance. On the other hand, these shifts do not have considerable effect on stage 1 performance.

According to above-mentioned tables and figures, the results show that the proposed method removes the cascade property effect for different shift sizes and autocorrelations. As shown in Figures 4 and 5, different shifts in the parameters of the first-stage profile do not affect the performance of the second stage, where its PCI remains almost unchanged. The results in Figures 6 and 7 show that as we expected, different shift sizes affect the performance of second stage and total performance.

8 - Case study

In this section a piston manufacturing process, initially considered by (Fong and Lawless, 1998), is used to conduct a process capability analysis using the proposed method. In this case study, a piston is produced in a four-stage machining process, where in each stage the diameters of a piston are inspected in microns at heights 4 mm, 10 mm, 36.7 mm, and 58.7 mm from the bottom of the part. The functional relationship between the diameter and the height of each piston can be stated as a profile for each stage of the process. In this study, 25 profiles for each of the four heights were considered. Without loss of generality and for the sake of simplicity, we used the first two stages of this process for the analyses.

In stage1 and stage2, the underlying in-control process with a SLP is regarded as $Y_{i,j,1} = 89.11867 - 0.01322\, X_i + \varepsilon_{i,j,1}$ and $Y_{i,j,2} = 89.12032 - 0.01328\, X_i + \varepsilon_{i,j,2}$, where $\varepsilon_{i,j,1} \sim N(0, \sigma_1^2)$, $\varepsilon_{i,j,2} \sim N(0, \sigma_2^2)$. Models were fitted with $\sigma_1^2 = \sigma_2^2 = 0.04167$ microns^2 and $\sigma_1^2 = \sigma_2^2 = 0.1$ microns^2. The SLs of each stage are presented in Table 8.

Table 8. Estimated SLs for the two-stage piston manufacturing process

	Stage 1	**Stage 2**
SLs	$USL_1 = 95.23 - 0.011X_i$	$USL_2 = 96.32 - 0.012X_i$
	$LSL_1 = 81.73 - 0.011X_i$	$LSL_2 = 82.37 - 0.012X_i$

When $\sigma_1^2 = \sigma_2^2 = 0.04167$, the intercept and slope for each stage, b_{0-1}, b_{1-1}, b_{0-2}, and b_{1-2}, are calculated as 2.1672, 1.2568, 2.1653, and 1.3040, and the C_{pmk} for stage1, stage2, and total are obtained as 1.7542, 1.2512, and 1.8021, respectively which shows acceptable performance. On the other hand, when $\sigma_1^2 = \sigma_2^2 = 0.1$, the intercept and slope for each stage, b_{0-1}, b_{1-1}, b_{0-2}, and b_{1-2}, are calculated as 2.1626, 0.7018, 2.0168, and 0.7121 and C_{pmk} for stage1, stage2, and total are obtained as 1.5232, 1.1680, 1.6085, respectively. Thus, since ${C_{pmk}}_{b_1}$ index is less than one, it is concluded that the process is incapable.

9 - Conclusions

In this study, we introduced an approach to assess process capability in a multistage process when quality characteristics are represented by a SLP. Moreover, a method was applied to specify the performance of a profile based on its parameters. The capability of an in-control process was evaluated in terms of two new independent univariate PCIs for profile parameters. The SLs for profile parameters were obtained based on SLs of the response variable, as well as considering the in-control profile. Also, the effect of parameters shifts in stage 1 and 2 on process performance was investigated. Generally, as one expects, shift in the parameters results in a less efficient performance of the process. In addition, the results show that the proposed method removes the effect of the cascade property for different shift sizes and autocorrelations. This study focused on PCI evaluation of a SLP based on its parameters. The proposed approach can be extended to other types of profile models such as polynomial, nonlinear, and multivariate responses. In addition, future studies may include calculation of PCIs in the presence of contamination, where robust estimation methods can be applied to handle the challenge.

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