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Generalized Exponential Ratio Type Estimator for the Finite Population Mean Under Ranked Set Sampling

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Abstract
In this study, we introduce a novel approach for estimating the mean of a finite population using Ranked Set Sampling (RSS), termed the generalized exponential ratio estimator. We derive expressions for the bias and mean squared error (MSE) of the proposed estimator up to the first order of approximation. To assess its performance, we conduct a thorough theoretical and numerical analysis using simulated and real data. Our results demonstrate that the generalized exponential ratio estimator outperforms both the classical ratio estimator and the estimator proposed by Kadilar et al. (2009) under RSS, highlighting its superior efficiency.

Key Words: Exponential type Estimator; ranked set sampling; ratio estimator; mean estimation, MSE.

Mathematical Subject Classification: 62D05, 93C70

1. Introduction

Ranked Set Sampling (RSS) was first introduced by McIntyre (1952) as a method to estimate pasture yield. Since its inception, RSS has attracted sustained interest from researchers who have further developed and refined the technique (Muttlak and Mc Donald, 1990; Kadilar and Cingi, 2005; Ozturk, 2011; Bouza, 2013; Singh et al., 2007; Singh et al., 2014; Kadilar, 2016; Rather and Kadilar, 2021; Rather et al., 2022; Bhushan and Kumar, 2022; Mahdizadeh and Zamanzade, 2022; Alomair and Shahzad, 2023; Bhushan and Kumar, 2023, 2024; Koçyiğit and Rather, 2023; Koçyiğit and Kadılar, 2024a, 2024b). The RSS method involves randomly selecting sets, each comprising n units, from the population. Within each set, the units are visually ranked. The measurement process starts by recording the value of the lowest-ranked unit from the first set of n units. Subsequently, the second lowest-ranked unit is measured from the second set, and this procedure continues until the n-th ranked unit is measured.

McIntyre (1952) demonstrated the advantages of RSS through a computational comparison encompassing five distributions. To provide a concise introduction to the concept of RSS, consider a random variable X with a density function F(x) and (x_1, x_2, ..., x_n) as the unobserved values from n units. These values can be ranked either through visual inspection or based on a concomitant variable. In RSS, one unit is selected from each ranked set, resulting in a total of m units chosen for quantification. For example, the unit with rank 1 is selected from the first set, rank 2 from the second set, and so on, until the m-th ranked unit is chosen from the m-th set. It should be noted that the selected rank order can be any permutation of 1, 2, ..., k. Each cycle consists of m^2 units, out of which only m units are selected for quantification. This cycle can be repeated a certain number of times ($rm = n$). In the case of ranking based on an auxiliary variable, ($y_{[i]}$, $x_{(i)}$) denotes the i-th judgment ordering in the i-th set for both the study and auxiliary variables, respectively.

2. Estimators in Literature

Samawi and Muttlak (1996) introduced an estimator for the population ratio utilizing RSS as follows:

$$\hat{R}_{RSS} = \frac{\bar{y}_{[n]}}{\bar{x}_{(n)}} \tag{1}$$

where $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$. It is worth noting that the estimator presented in equation (1) can also be applied to estimate the population total and mean. Specifically, the estimator for the population mean can be expressed as follows:

$$\bar{y}_{rRSS} = \frac{\bar{y}_{[n]}}{\bar{x}_{(n)}}\bar{X} \tag{2}$$

In Equation (2), we assume that the population mean of an auxiliary variable *x* is known. The expression for the MSE of the estimator is given as follows:

$$MSE(\bar{y}_{rRSS}) \cong \frac{1}{mr}\left(S_y^2 - 2RS_{yx} + R^2S_x^2\right) - \frac{1}{m^2r}\left(\sum_{i=1}^{m}\tau_{y[i]}^2 - 2R\sum_{i=1}^{m}\tau_{yx(i)} + R^2\sum_{i=1}^{m}\tau_{x(i)}^2\right) \tag{3}$$

where $R = \frac{\bar{Y}}{\bar{X}}$, S_x^2 and S_y^2 are the population variance of the auxiliary and study variables, S_{yx} is the population covariance between the auxiliary and study variables, $\tau_{x(i)} = \mu_{x(i)} - \bar{X}$, $\tau_{y[i]} = \mu_{y[i]} - \bar{Y}$, and $\tau_{yx(i)} = \left(\mu_{y[i]} - \bar{Y}\right)\left(\mu_{x(i)} - \bar{X}\right)$. Here, $\bar{Y}$ represents the population mean of the study variable. It should be noted that the values of $\mu_{x(i)}$ and $\mu_{y[i]}$ are influenced by the order statistics derived from specific distributions, and these values can be referenced from Arnold et al. (1993). Additionally, it is important to highlight that in the absence of judgment error and under the assumption of identical distributions, the values of $\mu_{x(i)}$ and $\mu_{y[i]}$ can be considered equal (Dell and Clutter, 1972).

Kadilar et al. (2009) introduced an estimator for RSS by adapting the estimator proposed by Prasad (1989). The estimator is defined as follows:

$$\bar{y}_{\kappa RSS} = \frac{\kappa\,\bar{y}_{[n]}}{\bar{x}_{(n)}}\bar{X} = \hat{R}_{\kappa RSS}\bar{X} \tag{4}$$

where κ is a constant that makes the MSE minimum and $\hat{R}_{\kappa RSS} = \kappa\frac{\bar{y}_{[n]}}{\bar{x}_{(n)}} = \kappa\frac{\bar{y}_{RSS}}{\bar{x}_{RSS}}$. The MSE of the estimator in (4) is given as

$$\begin{aligned} MSE_{min}(\bar{y}_{\kappa RSS}) \cong{}& \frac{1}{mr}\left(\kappa^{*2}S_y^2 - 2R\kappa^{*}S_{yx} + R^2S_x^2\right) + \bar{Y}^2(\kappa^{*} - 1)^2 \\ &- \frac{1}{m^2r}\left(\kappa^{*2}\sum_{i=1}^{m}\tau_{y[i]}^2 - 2R\kappa^{*}\sum_{i=1}^{m}\tau_{yx(i)} + R^2\sum_{i=1}^{m}\tau_{x(i)}^2\right) \end{aligned} \tag{5}$$

where

$\kappa^{*} = \frac{1+\gamma\,\rho\,C_y\,C_x - W_{yx(i)}}{1+\gamma\,C_y^2 - W_{y[i]}^2}$,

$W_{yx(i)} = \frac{1}{m^2r\bar{X}\,\bar{Y}}\sum_{i=1}^{m}\tau_{yx(i)}$ and $W_{y[i]}^2 = \frac{1}{m^2r\bar{Y}^2}\sum_{i=1}^{m}\tau_{y[i]}^2$. Here, $\gamma = \frac{1}{mr}$, C_x, and C_y are the population coefficients of variation of the *x* and *y*, ρ is the correlation coefficient between the variables.

3. The Suggested Estimator

Motivated by the work of Yadav and Shukla (2014) and Yadav (2015), we propose the following generalized exponential ratio type estimator of the finite population mean of study variable under RSS,

$$\bar{y}_{pro} = \bar{y}_{[n]}\left[\alpha\left(\frac{\bar{X}}{\bar{x}_{[n]}}\right) + (1-\alpha)\,exp\left(\frac{\bar{X} - \bar{x}_{[n]}}{\bar{X} + \bar{x}_{[n]}}\right)\right] \tag{6}$$

where α is a constant that makes the MSE minimum.

In order to find the MSE equation of the estimator in (2.6), we use the following notations:

$\bar{y}_{(n)} = \bar{Y}(1+\epsilon_0)$, and $\bar{x}_{(n)} = \bar{X}(1+\epsilon_1)$,

$E(\in_0) = E(\in_1) = 0,$

$E(\in_0)^2 = V\left(\frac{\bar{y}_{(n)}}{\bar{Y}^2}\right) = \frac{1}{mr}\frac{1}{\bar{Y}^2}\left[S_y^2 - \frac{1}{m}\sum t_{y(i)}^2\right] = \left[\theta C_y^2 - w_{y(i)}^2\right],$

$E(\in_1)^2 = V\left(\frac{\bar{x}_{(n)}}{\bar{X}^2}\right) = \frac{1}{mr}\frac{1}{\bar{X}^2}\left[S_x^2 - \frac{1}{m}\sum t_{X(i)}^2\right] = \left[\theta C_x^2 - w_{x(i)}^2\right],$

$E(\in_0\in_1) = \frac{1}{mr}\frac{1}{\bar{Y}\bar{X}}\left[S_{yx} - \frac{1}{m}\sum t_{yx(i)}\right] = \left[\theta\rho_{yx}C_yC_x - w_{yx(i)}\right],$

where $W_{x[i]}^2 = \frac{1}{m^2r\bar{X}^2}\sum_{i=1}^{m}\tau_{x[i]}^2$.

The proposed estimator can be expressed in terms of ϵ_i (i=0, 1) as

$$\bar{y}_{pro} = \bar{y}(1+\in_0)\left[\alpha\left(\frac{\bar{X}}{\bar{X}(1+\in_1)}\right) + (1-\alpha)\,exp\left(\frac{\bar{X}-\bar{X}(1+\in_1)}{\bar{X}+\bar{X}(1+\in_1)}\right)\right] \tag{7}$$

$$\bar{y}_{pro} = \bar{y}(1+\in_0)\left[\alpha(1+\in_1)^{-1} + (1-\alpha)\,exp\left(\frac{-\in_1}{2+\in_1}\right)\right] \tag{8}$$

$$\bar{y}_{pro} = \bar{y}(1+\in_0)\left[\alpha(1-\in_1+\in_1^2) + (1-\alpha)\,exp\left(\frac{-\in_1}{2}\left(1+\frac{\in_1}{2}\right)^{-1}\right)\right] \tag{9}$$

Upto first degree approximation

$$E\left(\bar{y}_{pro}-\bar{y}\right) = \bar{Y}E\left[\epsilon_0 - \frac{1}{2}\epsilon_1 - \frac{1}{2}\alpha\epsilon_1 + \frac{5}{8}\alpha\epsilon_1^2 + \frac{3}{8}\epsilon_1^2 - \frac{1}{2}\alpha\epsilon_0\epsilon_1 - \frac{1}{2}\epsilon_1\epsilon_0\right] \tag{10}$$

The bias of the proposed estimator can be expressed as follows:

$$Bias(\bar{y}_{pro}) \cong \bar{Y}\left[\frac{1}{mr\bar{X}^2}\left(\frac{5}{8}\alpha+\frac{3}{8}\right)\left(S_y^2 - \frac{1}{m}\sum_{i=1}^{m}t_{x(i)}^2\right) - \frac{1}{mr\bar{X}\bar{Y}}\left(\frac{1}{2}\alpha+\frac{1}{2}\right)\left(S_{yx}^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)\right] \tag{11}$$

By squaring both sides and taking the expectation, we can obtain the MSE of up to the first order of approximation as follows:

$$E\left(\bar{y}_{pro}-\bar{y}\right)^2 \cong \bar{Y}^2E\left[\epsilon_0 - \frac{1}{2}\epsilon_1 - \frac{1}{2}\alpha\epsilon_1\right]^2 \tag{12}$$

$$MSE(\bar{y}_{pro}) \cong \bar{Y}^2\left[\epsilon_0^2 + \frac{1}{4}\alpha^2\epsilon_1^2 + \frac{1}{4}\epsilon_1^2 - \alpha\epsilon_0\varepsilon_1 - \epsilon_0\epsilon_1 + \frac{1}{2}\alpha\epsilon_1^2\right] \tag{13}$$

$$\alpha_{(opt)} = \frac{2\left(\epsilon_0\epsilon_1 - \frac{1}{2}\epsilon_1^2\right)}{\epsilon_1^2} \tag{14}$$

$$MSE_{min}(\bar{y}_{pro}) \cong \bar{Y}^2E\left[\epsilon_0^2 - \frac{(\epsilon_0\epsilon_1)^2}{\epsilon_1^2}\right] \tag{15}$$

$$MSE_{min}(\bar{y}_{pro}) \cong \frac{1}{mr}\left[\left(S_y^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{y(i)}^2\right) - \frac{\left(S_{yx} - \frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2 - \frac{1}{m}\sum_{i=1}^{m}\tau_{x(i)}^2\right)}\right] \tag{16}$$

4. Efficiency Comparisons

This section aims to evaluate and compare the performance of the proposed estimator with both the traditional ratio estimator in RSS and the estimator proposed by Kadilar et al. (2009). The results of the performance analysis are presented as follows:

(i) $\text{MSE}(\bar{y}_{rRSS}) - \text{MSE}_{min}(\bar{y}_{Pro}) > 0$

$$\frac{1}{mr}\left(S_y^2 - 2RS_{yx} + R^2S_x^2\right) - \frac{1}{m^2r}\left(\sum_{i=1}^m \tau_{y(i)}^2 - 2R\sum_{i=1}^m \tau_{yx(i)} + R^2\sum_{i=1}^m \tau_{x(i)}^2\right) - \frac{1}{mr}\left[\left(S_y^2 - \frac{1}{m}\sum_{i=1}^m t_{y(i)}^2\right) - \frac{\left(S_{yx}^2 - \frac{1}{m}\sum_{i=1}^m t_{yx(i)}\right)^2}{\left(S_y^2 - \frac{1}{m}\sum_{i=1}^m t_{x(i)}^2\right)}\right] > 0 \quad (17)$$

(ii) $\text{MSE}\left(\bar{y}_{\kappa RSS}\right) - \text{MSE}_{min}(\bar{y}_{Pro}) > 0,$

$$\frac{1}{mr}\left(\kappa^{*2}S_y^2 - 2R\kappa^*S_{yx} + R^2S_x^2\right) + \bar{Y}^2(\kappa^* - 1)^2 - \frac{1}{m^2r}\left(\kappa^{*2}\sum_{i=1}^m \tau_{y(i)}^2 - 2R\kappa^*\sum_{i=1}^m \tau_{yx(i)} + R^2\sum_{i=1}^m \tau_{x(i)}^2\right) - \frac{1}{mr}\left[\left(S_y^2 - \frac{1}{m}\sum_{i=1}^m t_{y(i)}^2\right) - \frac{\left(S_{yx}^2 - \frac{1}{m}\sum_{i=1}^m t_{yx(i)}\right)^2}{\left(S_y^2 - \frac{1}{m}\sum_{i=1}^m t_{x(i)}^2\right)}\right] > 0 \quad (18)$$

When the efficiency conditions specified in equations (17) and (18) are satisfied, we can conclude that the proposed estimator demonstrates superior efficiency when compared to both the traditional ratio estimator in RSS and the estimator proposed by Kadilar et al. (2009).

5. Numerical Analysis

This section presents numerical studies using both real and simulated data to compare the performance of the proposed estimator with other existing estimators at different values of the correlation coefficients, *m*, and *r*.

5.1. Simulation study

A finite population of size 400 is generated from a bivariate normal distribution with specified means and covariances for both the auxiliary and study variables.

(i) $\mu = \binom{5}{1}$, $\Sigma = \begin{pmatrix} 100 & 35 \\ 35 & 25 \end{pmatrix}$, and $\rho = (0.3, 0.6, and\ 0.9)$

The following steps summarize the procedure for finding the average MSE and relative efficiency (PRE) of the estimators under study.

Set $m = 3, 5, 7$ and cycle $r = 5, 10, 15$ to obtain a sample of size $n = mr$.

(ii) Use the sample data from (i) to obtain the MSE of all the estimators under study.

(iii) Repeat steps (i) and (ii) 100 times to obtain 100 values for MSEs.

(iv) The average of the 100 values obtained in (iii) are the MSE of each estimator of population mean.

(v) The values of MSEs obtained in (iv) are used in calculating the values of percent relative efficiencies (PREs) defined as

$$PRE = \frac{MSE(\bar{y}_{rRSS})}{MSE(\bar{y}_j)} \times 100,$$

where $j = \bar{y}_{\kappa RSS}$ and $\bar{y}_{Pro}$.

Simulation studies show that the proposed estimator outperforms other existing estimators for different values of *m* and *r*. It is observed that the value of PRE for $\bar{y}_{\kappa RSS}$ increases with increasing correlation coefficient. However, the value of PRE for $\bar{y}_{Pro}$ decreased with increasing correlation coefficient. Moreover, the values of PRE for all estimators decreased as the value of r increased.

Figure 1 displays plots of the PRE for the finite population mean estimator at various values of *m* and *r*. The plot clearly illustrates that the proposed estimator consistently exhibits higher PRE compared to the Samawi and Mutlak et al. (1996) estimator and the Kadilar et al. (2009) estimator. Notably, as the correlation coefficient increases, the PRE values of the proposed estimators gradually decrease. These findings emphasize the advantageous performance of the proposed estimator, particularly in scenarios with higher correlation coefficients, where it demonstrates superior efficiency compared to the alternative estimators.

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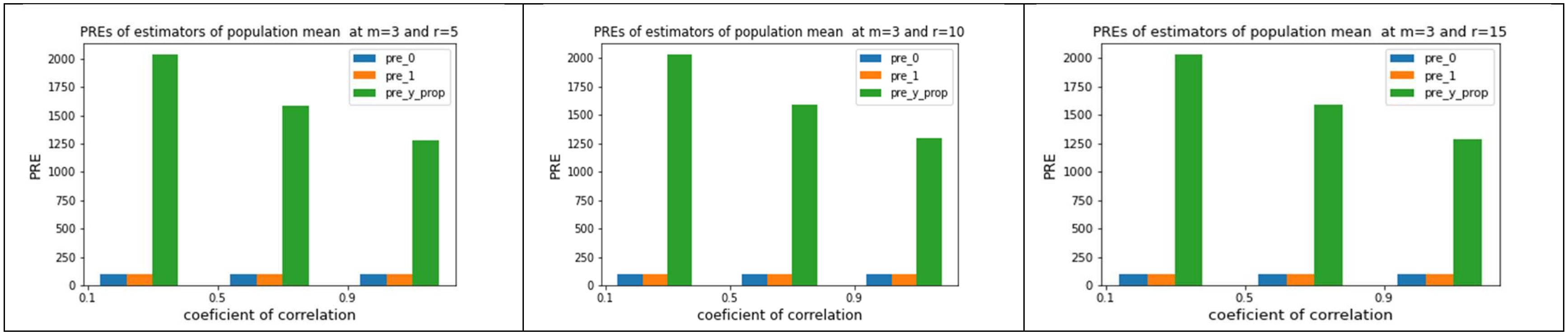
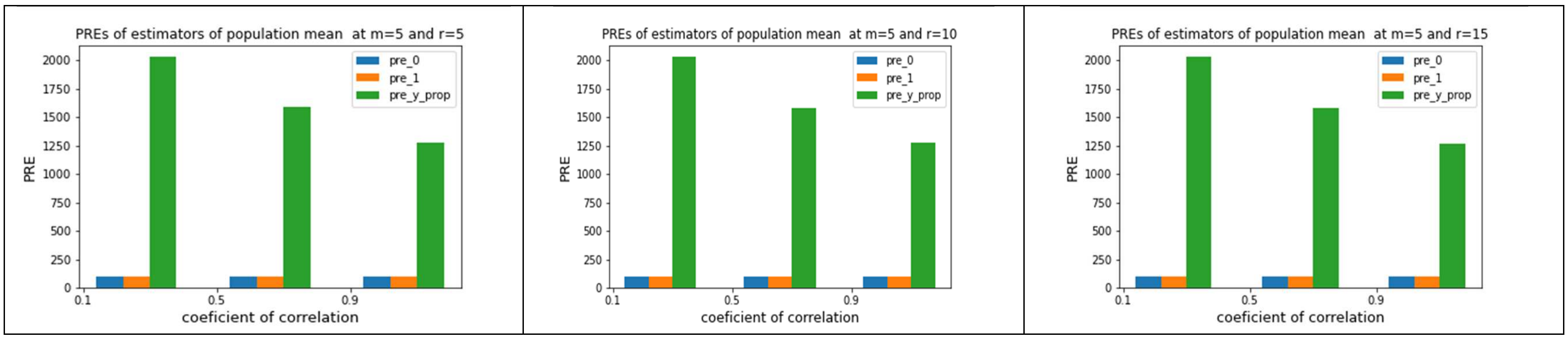
Table 1: PREs of different estimators for simulated data

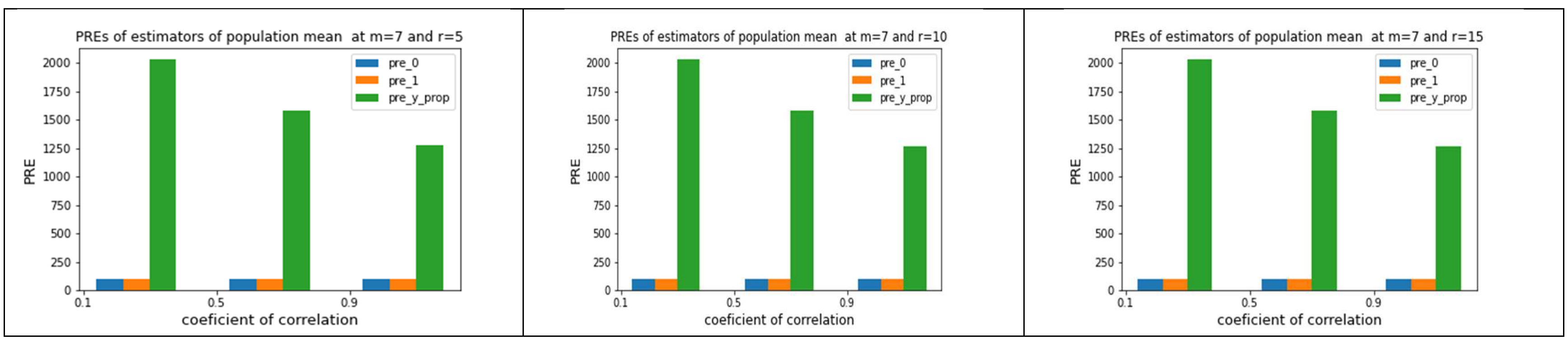
	ρ	0.1			0.5			0.9		
	r	5	10	15	5	10	15	5	10	15
m=3	PRE_0	100	100	100	100	100	100	100	100	100
	PRE_1	100.21	100.09	100.07	100.23	100.165	100.10	102.85	101.66	101.12
	PRE_2	2037.07	2034.51	2035.22	1583.27	1588.93	1587.67	1275.75	1293.73	1288.88
m=5	PRE_0	100	100	100	100	100	100	100	100	100
	PRE_1	100.12	100.07	100.05	100.18	100.09	100.06	101.82	101.95	100.64
	PRE_2	2031.32	2032.53	2033.23	1584.29	1582.67	1582.53	1275.12	1272.33	1271.19
m=7	PRE_0	100	100	100	100	100	100	100	100	100
	PRE_1	100.10	100.05	100.03	100.13	100.06	100.04	101.36	100.68	100.44
	PRE_2	2033.26	2034.55	2034.56	1584.03	1582.66	1580.81	1278.13	1270.94	1264.36



m=3



m=5



m=7

Figure 1. PRE of estimators of the finite population mean at different values of *m* and *r*

5.2. Real life applicability

The real data used are from Kadilar and Cingi (2003), especially from the Marmara region (Source: Institute of Statistics of the Republic of Turkey). In the dataset, the survey and auxiliary variables are apple yield and number of apple trees. The population parameters are:

N=106, n=12, m=3, r=4, $\rho_{XY} = 0.82$, $\bar{y}_1 = -0.846$. $\bar{y}_2 = 0$, $\bar{y}_3 = 0.846$,

$\bar{x}_1 = -0.846$, $\bar{x}_2 = 0$, $\bar{x}_3 = 0.846$, $\bar{X} = 6.97 \times 10^{-17}$, $\bar{Y} = 1.87 \times 10^{-16}$

R=-2.398, $S_y^2 = 1$, $S_x^2 = 1$, k=-1.55

Table 2 presents the MSE and PRE values for various estimators of finite population means. The numerical analysis consistently shows that the proposed estimator achieves the smallest MSE among all compared estimators. Furthermore, the generalized class of estimators proposed in this study demonstrates superior performance compared to the Samawi and Mutlak et al. (1996) estimator as well as the Kadilar et al. (2009) estimator. Figure 2 visually illustrates the higher efficiency of the proposed generalized exponential ratio type estimator compared to other existing estimators for estimating finite population means under RSS. These findings provide compelling evidence for the superiority of the proposed estimator and its potential to significantly enhance estimation accuracy in RSS applications.

Table 2. MSEs and PREs of different estimators of the population mean

Estimator	MSE	PRE
Samawi and Mutlak (t_{rRSS})	0.43115	100.00
Kadilar et al. (t_{kRSS})	0.14283	301.84
Proposed (t_{prop})	0.02483	1735.99

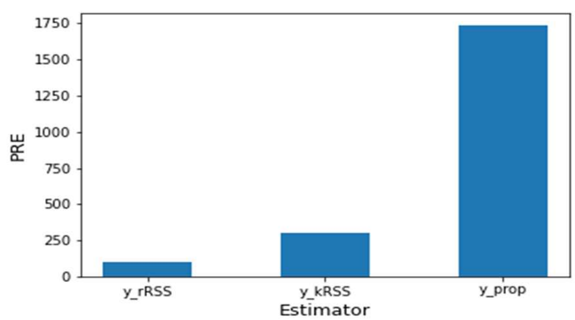


Figure 2. Plot of PREs of different estimators for real data

6. Conclusion

In conclusion, this study introduces a novel generalized exponential ratio estimator for estimating the finite population mean under RSS. By deriving bias and MSE expressions of the proposed estimator up to the first order of approximation using Taylor's series, we establish a robust framework for population mean estimation in RSS. Our findings highlight the effectiveness and superiority of the generalized exponential estimator compared to existing RSS estimators in the literature.

Through extensive numerical analysis and comparisons, we demonstrate that our proposed estimator significantly enhances the efficiency of RSS estimators. These results not only validate the utility of the generalized exponential ratio estimator but also underscore its practical value in accurately estimating population means in real-world applications.

In particular, future work could explore the adaptation of the estimators presented in this article to stratified random samples. Drawing upon the methodologies established by Samawi and Siam (2003) and Kadilar and Cingi (2005), we anticipate that our estimators can be effectively extended to accommodate more intricate sampling designs. This advancement would cater to diverse research contexts and further enhance the applicability of our proposed methods.

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