Pakistan Journal of Statistics and Operation Research

Integration of 4253HT Smoother with Intuitionistic Fuzzy Time Series Forecasting Model

Nik Muhammad Farhan Hakim Nik Badrul Alam[1*], Nazirah Ramli[2], Adie Safian Ton Mohamed[3], Noor Izyan Mohamad Adnan[4]

* Corresponding Author

1. Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Pahang, 26400 Bandar Tun Abdul Razak Jengka, Pahang, Malaysia, farhanhakim@uitm.edu.my
2. Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Pahang, 26400 Bandar Tun Abdul Razak Jengka, Pahang, Malaysia, nazirahr@utim.edu.my
3. School of Mathematics, Actuarial and Quantitative Studies, Asia Pacific University of Technology and Innovation, Malaysia, adiesafian@gmail.com
4. Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA Pahang, 26400 Bandar Tun Abdul Razak Jengka, Pahang, Malaysia, noorizyan@uitm.edu.my

Abstract

Fuzzy time series is widely used in forecasting time series data in linguistic forms. Implementing the intuitionistic fuzzy sets (IFS) in fuzzy time series can better handle uncertainties and vagueness in the time series data. However, the time series data always fluctuate randomly and cause drastic changes. In this study, the 4253HT smoother is integrated with the intuitionistic fuzzy time series forecasting model to improve the forecasting accuracy. The proposed model is implemented in predicting the Malaysian crude palm oil prices. The data are firstly smoothed, and followed with the fuzzification process. Next are the transformation of fuzzy sets into IFS and the de-i-fuzzification via equal distribution of hesitancy. The forecasted data are calculated based on the defuzzified values considering the new membership degrees of the IFS after de-i-fuzzification. The results show that the integrated model produces a better forecasting performance compared to the common intuitionistic fuzzy time series forecasting model. In the future, the integration of the data smoothing should be considered before the forecasting of data using fuzzy time series could be performed.

Key Words: Crude Palm Oil Prices; Fuzzy Time Series; Intuitionistic Fuzzy Set; Smoothing; 4253HT Smoother.

Mathematical Subject Classification: 03E72; 62A86; 62M10

1. Introduction

Time series data are a collection of observations in a sequence of hours, days, months or even years. However, time series data usually have extreme values which fluctuate and change randomly. The implementation of smoothing techniques in time series analysis can reduce noises in the data series (Azmi et al., 2019). However, the classical forecasting methods such as ARIMA, SARIMA, moving average and exponential smoothing cannot handle data in linguistic variables.

Fuzzy time series was proposed by Song and Chissom (1993b) which allows forecasting of time series data in natural language. The model was used to forecast students' enrollment at the University of Alabama (Song & Chissom, 1993a, 1994). Since then, fuzzy time series received attention from researchers since the implementation of fuzzy sets in forecasting time series data can handle vagueness and produce better forecasting accuracy (Singh, 2017). When compared to the double exponential smoothing technique, fuzzy time series produced better forecasting accuracy (Lesmana et al., 2019).

Many fuzzy time series forecasting model were integrated with the smoothing technique to reduce noises in the data series. Ge et al. (2013) used the integrated model to forecast students' enrollment. In their model, the fuzzy time series was performed in the first place. The exponential smoothing was further used to correct the forecasted values. Higher order fuzzy time series was also integrated with the exponential smoothing by Ye et al. (2016). A hybrid model which employed the combination of fuzzy time series, linear regression and smoothing was proposed by Justo dos Santos and De Arruda Camargo (2015). The exponential smoothing was also employed in the fuzzy time series model to forecast TAIEX data in dos Santos and de Arruda Camargo (2014).

The intuitionistic fuzzy set (IFS) was introduced by Atanassov (1986) as a generalization of the fuzzy set. Many fuzzy time series forecasting models have also been developed based on IFS. Joshi and Kumar (2012) used the hesitation index of IFS to establish fuzzy logical relationships (FLR) of students' enrollment data. Kumar and Gangwar (2015) used fuzzy sets induced from IFS to establish FLR. Further, Abhishekh et al. (2018) used the score function to intuitionistic fuzzify the historical data. Recently, an intuitionistic fuzzy time series forecasting model based on equal distribution of hesitancy de-i-fuzzification was proposed in Alam and Ramli (2021). The intuitionistic fuzzy time series produced better forecasting performance compared to the fuzzy set since the IFS can better handle uncertainties and vagueness.

Since most of time series data are highly associated with extreme values, the implementation of 4253HT smoother can remove spikes in the data series which allows better analysis afterwards (Azmi et al., 2019). It is in fact the most common and established compound smoother (Azmi et al., 2019). Hence, employing the 4253HT smoother in the intuitionistic fuzzy time series forecasting model can improve the forecasting accuracy since the extremely values are removed, thus reducing the uncertainties in the time series data.

This research is conducted to integrate the fuzzy time series based on IFS with the 4253HT smoother. The smoothing is done before the implementation of fuzzy time series to reduce extreme values in the historical data. For the illustration of the proposed model, the Malaysian crude palm oil prices are used. This paper is organized as follows: Section 1 introduces the paper; Section 2 lists some preliminaries related to the fuzzy set, fuzzy time series and IFS; Section 3 proposes the integrated forecasting model; numerical example is illustrated in Section 4; Section 5 presents and discusses the results while Section 6 finally concludes the paper.

2. Preliminaries

In this section, some concepts on the fuzzy sets, fuzzy time series and intuitionistic fuzzy sets (IFS) will be reviewed. The fuzzy sets were introduced by Zadeh (1965) which is very powerful in handling vagueness using linguistic variables. The fuzzy sets are defined as follows:

Definition 1 (Zadeh, 1965)
Let U be the universe of discourse. Then a fuzzy set A in U is a set of ordered pairs written as

$$A = \left\{ \left(x, u_A(x) \right) \mid x \in U \right\} \tag{1}$$

where $u_A(x)$ denotes the membership grade.

In many applications of fuzzy sets, fuzzy numbers such as triangular, trapezoidal, and Gaussian fuzzy numbers are used instead of fuzzy sets. The triangular fuzzy number is commonly used since it is the simplest form of fuzzy numbers (Voskoglou, 2019). The triangular fuzzy number is defined as follows:

Definition 2
The triangular fuzzy number, $A = (a, b, c)$ is characterized by the membership function

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & , \ x \in [a,b) \\ 1 & , \ x = b \\ \dfrac{c-x}{c-b} & , \ x \in (b,c] \\ 0 & , \ \text{elsewhere.} \end{cases} \tag{2}$$

Further, Atanassov (1986) generalized the fuzzy set into an IFS by adding the non-membership grade to the set to represent how much the element x does not belong the set. The IFS is defined as follows:

Definition 3 (Atanassov, 1986)
The IFS is a set of elements characterized by the membership and non-membership functions, written in form of

$$I = \left\{ \left(x, u_I\left(x\right), \nu_I\left(x\right) \right) | \, x \in U \right\} \tag{3}$$

where $u_A\left(x\right)$ and $\nu_A\left(x\right)$ denote the membership and non-membership grades, respectively. For any IFS, the degree of non-determinacy is defined as

$$\pi_I(x) = 1 - u_I\left(x\right) - \nu_I\left(x\right) \tag{4}$$

In 1993, Song and Chissom (1993b) introduced the concept of fuzzy time series as follows:

Definition 4 (Song & Chissom, 1993b)
Suppose $Y(t)$ be the universe of discourse with fuzzy sets $A_i(t)$ defined on it. Then the fuzzy time series $Y(t)$ is a collection of $A_i(t)$. If only $Y(t)$ affects $J(t)$, then $J(t) = J\left(t-1\right) \circ R\left(t, t-1\right)$ represents the relation $J\left(t-1\right) \to J(t)$. $R\left(t, t-1\right)$ is a fuzzy relation between $J(t)$ and $J\left(t-1\right)$.

Jurio et al. (2010) proposed a transformation method from fuzzy sets into IFS. The following theorem presents Atanassov's conversion of IFS.

Theorem 1 (Jurio et al., 2010)
Let A_F be in the collection of all fuzzy sets in the universe of discourse U and $\alpha, \beta : U \to \left[0,1\right]$. $f : [0,1]^2 \times [0,1] \to L^*$ where $f\left(x, \alpha, \beta\right) = \left(f_\mu\left(x, \alpha, \beta\right), f_\nu\left(x, \alpha, \beta\right) \right)$ and

$$f_\mu\left(x, \alpha, \beta\right) = x\left(1 - \alpha\beta\right) \tag{5}$$

$$f_\nu\left(x, \alpha, \beta\right) = 1 - \alpha\beta - x\left(1 - \alpha\beta\right). \tag{6}$$

The equal distribution of hesitancy de-i-fuzzification was given in Ansari et al. (2010) and further implemented in the intuitionistic fuzzy time series forecasting model by Alam and Ramli (2021) as follows:

Definition 5 (Alam & Ramli, 2021)
Let the IFS be defined as Eq. (3) and the hesitation index of IFS is defined as Eq. (4), then the new IFS obtained after equal distribution of hesitancy de-i-fuzzification is given by

$$\tilde{I} = \left\{ \left(x, u_I\left(x\right) + 0.5\pi_I(x), \nu_I\left(x\right) + 0.5\pi_I(x) \right) | \, x \in U \right\} \tag{7}$$

Hence, the fuzzy sets can be induced from $\tilde{I}$ as follows:

$$\tilde{A} = \left\{ \left(x, u_I\left(x\right) + 0.5\pi_I(x) \right) | \, x \in U \right\} \tag{8}$$

in which the new membership grade of IFS is considered while the non-membership grade is omitted.

3. Proposed Methodology

In this section, the integrated intuitionistic fuzzy time series forecasting model with the 4253HT smoother is proposed. The proposed model consists of 5 main phases; data smoothing, fuzzification, intuitionistic fuzzification, defuzzification and forecasting.

3.1 Data Smoothing Using 4253HT Smoother

Data smoothing is an exploratory data analysis which is used as the "first touch" before a further analysis is carried out on the data. This method involves single or combined algorithms to capture the important pattern by eliminating noise or unwanted pattern from a raw data. A linear data smoothing works effectively by the presence of normal or Gaussian noise, yet its accuracy is uncertain when it has to deal with a non-normal data. Commonly, a data is interrupted by heavy noise and outlier in most data application. Hence, linear data smoothing is not the best option for this case. Tukey (1977) introduced a non-linear approach to data smoothing, which is also known as a compound smoother. There are many previous studies conducted and proved that non-linear smoother, such as moving median

that could work effectively when a data is interrupted by heavy noise, see Bovik et al. (1983), Hird and McDermid (2009), and Sargent and Bedford (2010). A few years later, Velleman and Hoaglin (1981) then introduced smoother 4253HT as an alternative to linear smoother.
A 4253HT is a compound smoother involves various algorithms in its operation, where includes moving median of even and odd (abbreviated as 4, 2, 5 and 3), Hanning, *H* and Twicing, *T* operations. A sequence *x* is defined as an infinite series of phenomenon, $x_{t-n},...,x_{t-1},x_t,x_{t+1},...,x_{t+n}$ with x_t be as an observed series at the *t*-th time. A smoother *S* is defined as an algorithm that uses *x* to create a new series of smoothed values $S(x_t)$. The procedure in computing 4253HT is as follows:

Step 1: Perform moving median of size four and two:

$$S_4(x_t) = median[x_{t-2}, x_{t-1}, x_t, x_{t+1}] \tag{9}$$

$$S_{42}(x_t) = median[S_4(x_t), S_4(x_{t+1})] \tag{10}$$

Step 2: Perform moving median of size five and three:

$$S_{425}(x_t) = median[S_{42}(x_{t-2}), S_{42}(x_{t-1}), S_{42}(x_t), S_{42}(x_{t+1}), S_{42}(x_{t+2})] \tag{11}$$

$$S_{4253}(x_t) = median[S_{425}(x_{t-1}), S_{425}(x_t), S_{425}(x_{t+1})] \tag{12}$$

Step 3: Perform Hanning, *H* using coefficients $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$:

$$S_{4253H}(x_t) = \frac{1}{4}S_{425}(x_{t-1}) + \frac{1}{2}S_{425}(x_t) + \frac{1}{4}S_{425}(x_{t+1}) \tag{13}$$

Step 4: Smooth the residual, *e* and add it back to the smoothed value:

$$e_t = x_t - S_{4253H}(x_t) \tag{14}$$

$$S_{4253HT}(x_t) = S_{4253H}(x_t) + S_{4253H}(e_t) \tag{15}$$

During the calculation of smoothed value, use the end values, x_1 and x_n without changing them (Velleman & Hoaglin, 1981).

3.2 Intuitionistic Fuzzy Time Series Forecasting

Once the data are smoothed using the 4253HT smoother, the minimum and maximum smoothed values are determined. Then, the next steps are described as follows:
Step 1: Define the universe of discourse using the formula

$$U = [s_{\min} - s_1, s_{\max} + s_2] \tag{16}$$

where the $s_{\min}$ and $s_{\max}$ are the smallest and largest smoothed data while s_1 and s_2 are positive real numbers.
Step 2: Using a randomly chosen length, partition the universe of discourse U into several intervals of the same length. Suppose the chosen length is *l*, then the number of intervals is obtained as $n = \frac{U}{l}$.
Step 3: Define the triangular fuzzy numbers for all the intervals. Hence, the smoothed data are converted into fuzzy sets using the triangular membership function in Eq. (2).
Step 4: Using Atanassov's conversion method, the fuzzy sets are then transformed into IFS. Eq. (5) and (6) are used to build the membership and non-membership grades of the IFS, respectively.
Step 5: For each IFS, the hesitation index is calculated using Eq. (4). It is then distributed equally to the membership and non-membership grades. Hence, we obtain fuzzy sets induced from IFS with the new membership and non-membership grades after the de-i-fuzzification.

Step 6: Based on the membership grades of the induced fuzzy sets, the smoothed data are fuzzified. The IFS with higher membership grades are chosen.
Step 7: The fuzzy logical relationships (FLR) are formed and grouped according to the antecedent value.
Step 8: The induced fuzzy sets are then defuzzified using centroid method (Pedrycz, 1993) as follows

$$\tilde{A}^* = \frac{\sum_i \mu(x_i) \cdot x_i}{\sum_i \mu(x_i)} \tag{17}$$

Step 9: Finally, the forecasted outputs are calculated using the following rules:

(i) If the current data is fuzzified as $\tilde{A}_i$ and its FLR is $\tilde{A}_i \rightarrow \tilde{A}_{j1}$, then the forecasted data for the following time is $\tilde{A}_{j1}^*$.

(ii) If the current data is fuzzified as $\tilde{A}_i$ and its FLR is $\tilde{A}_i \rightarrow \tilde{A}_{j1}, \tilde{A}_{j2}, \tilde{A}_{j3}, ..., \tilde{A}_{jn}$, then the forecasted data for the following time is

$$\frac{f_1 \cdot \tilde{A}_{j1}^* + f_2 \cdot \tilde{A}_{j2}^* + f_3 \cdot \tilde{A}_{j3}^* + ... + f_{n-1} \cdot \tilde{A}_{j(n-1)}^* + f_n \cdot \tilde{A}_{jn}^*}{f_1 + f_2 + f_3 + ... + f_{n-1} + f_n},$$

where f_k is the frequency of occurrence of $\tilde{A}_{jk}$ in the FLR group.

(iii) If the current data is fuzzified as $\tilde{A}_i$ and its FLR is $\tilde{A}_i \rightarrow \phi$, then the forecasted data for the following time is $\tilde{A}_i^*$.

The proposed model is summarized as shown in Fig. 1.

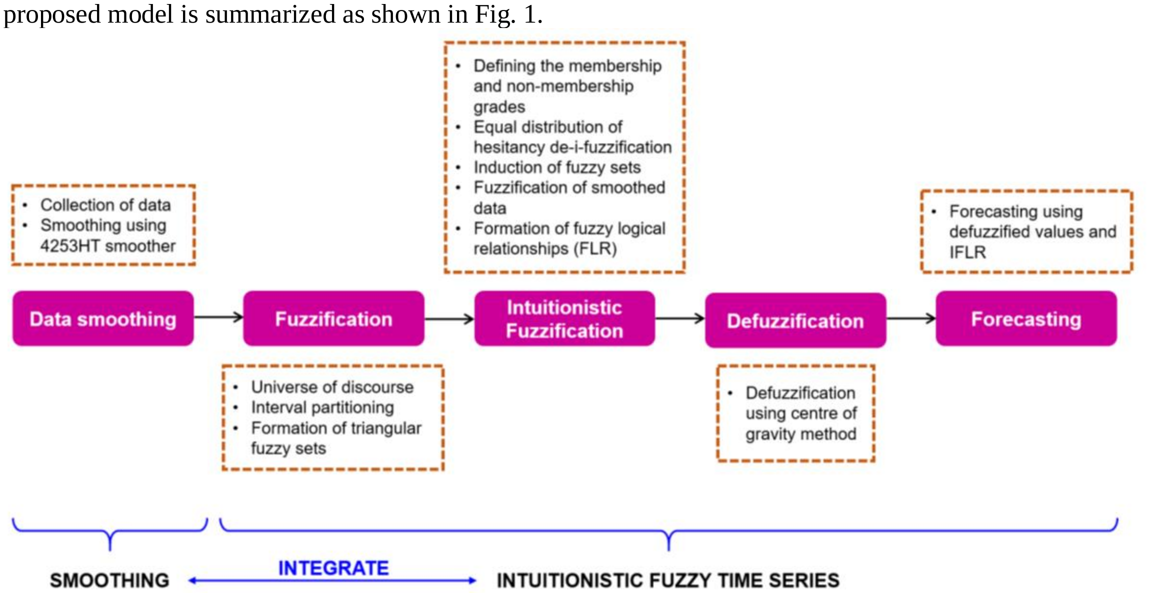

Figure 1: The proposed integrated intuitionistic fuzzy time series forecasting model with the 4253HT smoother.

4. Numerical Example

The integrated forecasting model is implemented in predicting the Malaysian crude palm oil prices (RM/tonne) from January 2017 until December 2021. Using the methodology proposed in the previous section, the raw data are firstly smoothed using the 4253HT smoother.

Step 1: The data in time series sequence are 3268, 3233, 2955.5, 2752.5, 2803.5, 2686, 2629.5, 2633. 2780.5, 2736, 2689, 2407, 2486.5, 2488, 2426.5, …. We first calculate the moving median of size four and two as follows:

$S_4(3268) = median[3268] = 3268$
$S_4(3233) = median[3233] = 3233$
$S_4(2955.5) = median[3268, 3233, 2955.5, 2752.5] = 3094.25$
$S_4(2752.5) = median[3233, 2955.5, 2752.5, 2803.5] = 2879.5$
$\vdots$

$S_{42}(3268) = median[3268] = 3268$
$S_{42}(3233) = median[3233] = 3233$
$S_{42}(3094.25) = median[3094.25, 2879.5] = 2986.875$
$S_{42}(2879.5) = median[2879.5, 2778] = 2828.75$
$\vdots$

Step 2: Next, we calculate the moving median of size five and three as follows:
$S_{425}(3268) = median[3268] = 3268$
$S_{425}(3233) = median[3233] = 3233$
$S_{425}(2986.875) = median[3268, 3233, 2986.875, 2828.75, 2748.625] = 2986.875$
$S_{425}(2828.75) = median[3233, 2986.875, 2828.75, 2748.625] = 2828.75$
$\vdots$

$S_{4253}(3268) = median[3268] = 3268$
$S_{4253}(3233) = median[3233] = 3233$
$S_{4253}(2986.875) = median[3233, 2986.875, 2828.75] = 2986.875$
$S_{4253}(2828.75) = median[2986.875, 2828.75, 2748.625] = 2828.75$
$\vdots$

Step 3: We then perform the Hanning with coefficients $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$ as follows:
$S_{4253H}(3268) = 3268$
$S_{4253H}(3233) = 0.25(3268) + 0.5(3233) + 0.25(2986.875) = 3180.219$
$S_{4253H}(2986.875) = 0.25(3233) + 0.5(2986.875) + 0.25(2828.75) = 3008.875$
$S_{4253H}(2828.75) = 0.25(2986.875) + 0.5(2828.75) + 0.25(2748.625) = 2848.25$
$\vdots$

Step 4: Smoothed the residual as follows:
$e_{3268} = 3268 - 3268 = 0$
$e_{3233} = 3233 - 3180.219 = 52.781$
$e_{2955.5} = 2955.5 - 3008.875 = -53.375$
$\vdots$

Then, we perform Step 1 to Step 3 for the obtained residual values. We then add the smoothed residual $S_{4253H}(e_t)$ to $S_{4253H}(x_t)$ to get the final smoothed value. In this case, we obtain

$S_{4253H}\left(e_{3268}\right)=0$
$S_{4253H}\left(e_{3233}\right)=-3.568$
$S_{4253H}\left(e_{2955.5}\right)=-11.842$
$\vdots$
and the final smoothed value is obtained as follows:

$S_{4253HT}\left(3268\right)=3268+0=3268$
$S_{4253HT}\left(3233\right)=3180.219-3.568=3176.65$
$S_{4253HT}\left(2986.875\right)=3008.875-11.842=2997.033$
$\vdots$

Hence, the actual and smoothed crude palm oil prices are presented in Fig. 2. Next, we perform the intuitionistic fuzzy time series forecasting procedure on the smoothed crude palm oil prices.

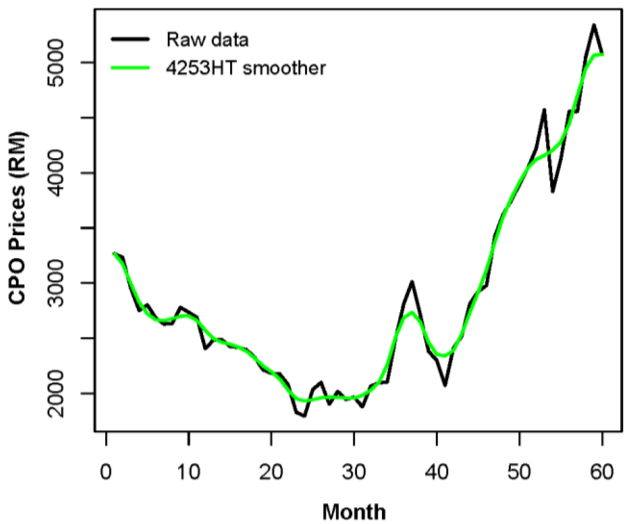

Figure 2: The actual and smoothed crude palm oil prices (January 2017 – December 2021).

Step 1: From the set of smoothed data, the minimum and maximum values are identified as 1932.85 and 5070, respectively. Hence, set $s_1 = 32.85$ and $s_2 = 30$ such that $U = [1900, 5100]$ can be obtained by using Eq. (16).

Step 2: Using an interval length of 100, the universe of discoursed is partitioned into 32 intervals: $u_1 = [1900, 2000]$, $u_2 = [2000, 2100]$, $u_3 = [2100, 2200]$, ..., $u_{31} = [4900, 5000]$ and $u_{32} = [5000, 5100]$.

Step 3: For each of the interval, triangular fuzzy number is defined. Table 1 presents the triangular fuzzy number corresponding to each interval.

Table 1: Triangular fuzzy number corresponding to each interval.

Interval	Triangular Fuzzy Number	Interval	Triangular Fuzzy Number
[1900,2000]	(1900,2000,2100)	[3500,3600]	(3500,3600,3700)
[2000,2100]	(2000,2100,2200)	[3600,3700]	(3600,3700,3800)
[2100,2200]	(2100,2200,2300)	[3700,3800]	(3700,3800,3900)
[2200,2300]	(2200,2300,2400)	[3800,3900]	(3800,3900,4000)
[2300,2400]	(2300,2400,2500)	[3900,4000]	(3900,4000,4100)
[2400,2500]	(2400,2500,2600)	[4000,4100]	(4000,4100,4200)
[2500,2600]	(2500,2600,2700)	[4100,4200]	(4100,4200,4300)

[2600,2700]	(2600,2700,2800)	[4200,4300]	(4200,4300,4400)
[2700,2800]	(2700,2800,2900)	[4300,4400]	(4300,4400,4500)
[2800,2900]	(2800,2900,3000)	[4400,4500]	(4400,4500,4600)
[2900,3000]	(2900,3000,3100)	[4500,4600]	(4500,4600,4700)
[3000,3100]	(3000,3100,3200)	[4600,4700]	(4600,4700,4800)
[3100,3200]	(3100,3200,3300)	[4700,4800]	(4700,4800,4900)
[3200,3300]	(3200,3300,3400)	[4800,4900]	(4800,4900,5000)
[3300,3400]	(3300,3400,3500)	[4900,5000]	(4900,5000,5100)
[3400,3500]	(3400,3500,3600)	[5000,5100]	(5000,5100,5100)

Next, the fuzzy sets are defined using Eq. (1) as follows:

$$A_1 = 0.328/1932.85 + 0.447/1944.65 + 0.533/1953.34 + 0.601/1960.07 + 0.602/1960.21 + 0.615/1961.49$$
$$+0.661/1966.14 + 0.688/1968.79 + 0.837/1983.67 + 0.697/2030.33 + 0.674/2032.64$$
$$A_2 = 0.303/2030.33 + 0.326/2032.64 + 0.991/2100.94 + 0.675/2132.54 + 0.029/2197.05$$
$$A_3 = 0.009/2100.94 + 0.325/2132.54 + 0.971/2197.05 + 0.501/2249.92 + 0.389/2261.08$$
$$\vdots$$
$$A_{30} = 0.543/4945.74$$
$$A_{31} = 0.457/4945.74 + 0.327/5067.28 + 0.300/5070.00$$
$$A_{32} = 0.673/5067.28 + 0.700/5070.00$$

Step 4: Using Atanassov's conversion method, the fuzzy sets are transformed into IFS. Hence, the following IFS are built:

$$I_1 = \{(1932.85, 0.238, 0.487), (1944.65, 0.324, 0.401), (1953.34, 0.387, 0.338), (1960.07, 0.436, 0.290),$$
$$(1961.49, 0.446, 0.279), (1966.14, 0.480, 0.246), (1968.79, 0.499, 0.226), (1983.67, 0.607, 0.118),$$
$$(2030.33, 0.505, 0.220), (2032.64, 0.488, 0.237)\}$$
$$I_2 = \{(2030.33, 0.294, 0.676), (2032.64, 0.317, 0.654), (2100.94, 0.962, 0.009), (2132.54, 0.655, 0.316),$$
$$(2197.05, 0.029, 0.942)\}$$
$$I_3 = \{(2100.94, 0.009, 0.982), (2132.54, 0.322, 0.668), (2197.05, 0.962, 0.029), (2249.92, 0.496, 0.495),$$
$$(2261.08, 0.386, 0.605)\}$$
$$\vdots$$
$$I_{30} = \{(4945.74, 0.383, 0.323)\}$$
$$I_{31} = \{(4945.74, 0.395, 0.468), (5067.28, 0.282, 0.580), (5070.00, 0.259, 0.604)\}$$
$$I_{32} = \{(5067.28, 0.356, 0.173), (5070.00, 0.370, 0.159)\}$$

Step 5: The IFS are then de-i-fuzzified. Hence, the following are fuzzy sets induced from the IFS after equal distribution of hesitancy de-i-fuzzification:

$$\tilde{A}_1 = 0.376/1932.85 + 0.461/1944.65 + 0.524/1953.34 + 0.573/1960.07 + 0.574/1960.21 + 0.583/1961.49$$
$$+0.617/1966.14 + 0.636/1968.79 + 0.744/1983.67 + 0.643/2030.33 + 0.626/2032.64$$
$$\tilde{A}_2 = 0.309/2030.33 + 0.332/2032.64 + 0.976/2100.94 + 0.669/2132.54 + 0.043/2197.05$$
$$\tilde{A}_3 = 0.014/2100.94 + 0.327/2132.54 + 0.966/2197.05 + 0.501/2249.92 + 0.390/2261.08$$
$$\vdots$$
$$\tilde{A}_{30} = 0.530/4945.74$$
$$\tilde{A}_{31} = 0.463/4945.74 + 0.351/5067.28 + 0.327/5070.00$$
$$\tilde{A}_{32} = 0.591/5067.28 + 0.606/5070.00$$

Step 6: The smoothed data are then fuzzified. For example, the price RM 2030.33 is contained in both $\tilde{A}_1$ and $\tilde{A}_2$. However, its membership grades are 0.643 and 0.309 in $\tilde{A}_1$ and $\tilde{A}_2$, respectively. Hence, it is fuzzified as $\tilde{A}_1$ because the membership grade is higher. The rest of the prices are fuzzified analogously and Table 2 presents the fuzzified prices.

Table 2: Fuzzified crude palm oil prices.

Month	Actual Price	Smoothed Price	Fuzzy Set	Month	Actual Price	Smoothed Price	Fuzzy Set
1/2017	3268	3268.00	$\tilde{A}_{14}$	7/2019	1879	1983.67	$\tilde{A}_1$
2/2017	3233	3176.65	$\tilde{A}_{13}$	8/2019	2066.5	2030.33	$\tilde{A}_1$
3/2017	2955.5	2997.03	$\tilde{A}_{11}$	9/2019	2097	2100.94	$\tilde{A}_2$
4/2017	2752.5	2826.49	$\tilde{A}_9$	10/2019	2104	2261.08	$\tilde{A}_4$
5/2017	2803.5	2722.81	$\tilde{A}_8$	11/2019	2493.5	2508.71	$\tilde{A}_6$
6/2017	2686	2669.08	$\tilde{A}_8$	12/2019	2813	2687.78	$\tilde{A}_8$
7/2017	2629.5	2659.74	$\tilde{A}_8$	1/2020	3013.5	2735.52	$\tilde{A}_8$
8/2017	2633	2678.94	$\tilde{A}_8$	2/2020	2714.5	2648.86	$\tilde{A}_7$
9/2017	2780.5	2700.88	$\tilde{A}_8$	3/2020	2382	2463.71	$\tilde{A}_6$
10/2017	2736	2703.65	$\tilde{A}_8$	4/2020	2299	2353.38	$\tilde{A}_5$
11/2017	2689	2661.38	$\tilde{A}_8$	5/2020	2074	2341.53	$\tilde{A}_4$
12/2017	2407	2572.85	$\tilde{A}_7$	6/2020	2411.5	2389.77	$\tilde{A}_5$
1/2018	2486.5	2496.90	$\tilde{A}_6$	7/2020	2519	2539.61	$\tilde{A}_6$
2/2018	2488	2464.54	$\tilde{A}_6$	8/2020	2815	2736.38	$\tilde{A}_8$
3/2018	2426.5	2445.95	$\tilde{A}_5$	9/2020	2924	2918.35	$\tilde{A}_{10}$
4/2018	2418	2421.51	$\tilde{A}_5$	10/2020	2979.5	3121.36	$\tilde{A}_{12}$
5/2018	2396	2382.22	$\tilde{A}_5$	11/2020	3422	3369.20	$\tilde{A}_{15}$
6/2018	2324	2319.26	$\tilde{A}_4$	12/2020	3620.5	3598.92	$\tilde{A}_{17}$
7/2018	2215	2249.92	$\tilde{A}_3$	1/2021	3748.5	3770.08	$\tilde{A}_{19}$
8/2018	2183.5	2197.05	$\tilde{A}_3$	2/2021	3895.5	3915.37	$\tilde{A}_{20}$
9/2018	2177.5	2132.54	$\tilde{A}_2$	3/2021	4041.5	4042.66	$\tilde{A}_{21}$
10/2018	2082	2032.64	$\tilde{A}_1$	4/2021	4220	4120.58	$\tilde{A}_{22}$
11/2018	1830.5	1953.34	$\tilde{A}_1$	5/2021	4572	4158.13	$\tilde{A}_{23}$
12/2018	1794.5	1932.85	$\tilde{A}_1$	6/2021	3830.5	4204.04	$\tilde{A}_{23}$
1/2019	2037	1944.65	$\tilde{A}_1$	7/2021	4128.5	4282.88	$\tilde{A}_{24}$
2/2019	2100.5	1961.49	$\tilde{A}_1$	8/2021	4555	4441.73	$\tilde{A}_{25}$
3/2019	1903.5	1968.79	$\tilde{A}_1$	9/2021	4556	4700.52	$\tilde{A}_{28}$
4/2019	2018.5	1966.14	$\tilde{A}_1$	10/2021	5051	4945.74	$\tilde{A}_{30}$
5/2019	1946.5	1960.07	$\tilde{A}_1$	11/2021	5341	5067.28	$\tilde{A}_{32}$
6/2019	1968	1960.21	$\tilde{A}_1$	12/2021	5070	5070.00	$\tilde{A}_{32}$

Step 7: In reference to Table 2, the FLR are formed and grouped as shown in Table 3.

Table 3: Fuzzy logical relationship groups.

Group	Fuzzy logical relationship	Group	Fuzzy logical relationship
1	$\tilde{A}_1 \to \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_1, \tilde{A}_2$	15	$\tilde{A}_{15} \to \tilde{A}_{17}$
2	$\tilde{A}_2 \to \tilde{A}_1, \tilde{A}_4$	16	$\tilde{A}_{17} \to \tilde{A}_{19}$
3	$\tilde{A}_3 \to \tilde{A}_2, \tilde{A}_3$	17	$\tilde{A}_{19} \to \tilde{A}_{20}$
4	$\tilde{A}_4 \to \tilde{A}_3, \tilde{A}_5, \tilde{A}_6$	18	$\tilde{A}_{20} \to \tilde{A}_{21}$
5	$\tilde{A}_5 \to \tilde{A}_4, \tilde{A}_4, \tilde{A}_5, \tilde{A}_5, \tilde{A}_6$	19	$\tilde{A}_{21} \to \tilde{A}_{22}$
6	$\tilde{A}_6 \to \tilde{A}_5, \tilde{A}_5, \tilde{A}_6, \tilde{A}_8, \tilde{A}_8$	20	$\tilde{A}_{22} \to \tilde{A}_{23}$
7	$\tilde{A}_7 \to \tilde{A}_6, \tilde{A}_6$	21	$\tilde{A}_{23} \to \tilde{A}_{23}, \tilde{A}_{24}$
8	$\tilde{A}_8 \to \tilde{A}_7, \tilde{A}_7, \tilde{A}_8, \tilde{A}_8, \tilde{A}_8, \tilde{A}_8, \tilde{A}_8, \tilde{A}_8, \tilde{A}_8, \tilde{A}_{10}$	22	$\tilde{A}_{24} \to \tilde{A}_{25}$
9	$\tilde{A}_9 \to \tilde{A}_8$	23	$\tilde{A}_{25} \to \tilde{A}_{28}$
10	$\tilde{A}_{10} \to \tilde{A}_{12}$	24	$\tilde{A}_{28} \to \tilde{A}_{30}$
11	$\tilde{A}_{11} \to \tilde{A}_9$	25	$\tilde{A}_{30} \to \tilde{A}_{32}$
12	$\tilde{A}_{12} \to \tilde{A}_{15}$	26	$\tilde{A}_{32} \to \tilde{A}_{32}$
13	$\tilde{A}_{13} \to \tilde{A}_{11}$	27	$\tilde{A}_{32} \to \phi$
14	$\tilde{A}_{14} \to \tilde{A}_{13}$		

Step 8: The induced fuzzy sets are then defuzzified using centre of gravity defuzzification. Hence, the defuzzified values are $\tilde{A}_1^* = 1975.40$, $\tilde{A}_2^* = 2092.72$, $\tilde{A}_3^* = 2210.26$, $\tilde{A}_4^* = 2313.65$, $\tilde{A}_5^* = 2400.66$, $\tilde{A}_6^* = 2491.26$, $\tilde{A}_7^* = 2622.41$, $\tilde{A}_8^* = 2692.59$, $\tilde{A}_9^* = 2771.50$, $\tilde{A}_{10}^* = 2899.05$, $\tilde{A}_{11}^* = 2980.28$, $\tilde{A}_{12}^* = 3136.72$, $\tilde{A}_{13}^* = 3189.80$, $\tilde{A}_{14}^* = 3303.58$, $\tilde{A}_{15}^* = 3369.20$, $\tilde{A}_{16}^* = 3598.92$, $\tilde{A}_{17}^* = 3598.92$, $\tilde{A}_{18}^* = 3770.08$, $\tilde{A}_{19}^* = 3770.08$, $\tilde{A}_{20}^* = 3915.37$, $\tilde{A}_{21}^* = 4011.44$, $\tilde{A}_{22}^* = 4109.03$, $\tilde{A}_{23}^* = 4188.75$, $\tilde{A}_{24}^* = 4277.85$, $\tilde{A}_{25}^* = 4441.73$, $\tilde{A}_{26}^* = 4441.73$, $\tilde{A}_{28}^* = 4700.52$, $\tilde{A}_{29}^* = 4700.52$, $\tilde{A}_{30}^* = 4945.74$, $\tilde{A}_{31}^* = 5018.74$ and $\tilde{A}_{32}^* = 5068.65$.

Step 9: Finally, the forecasted Malaysian crude palm oil prices are calculated. The results are presented in the next section.

5. Results and Discussion

Table 4 presents the the actual, smoothed and forecasted Malaysian crude palm oil prices (RM/tonne) using the integrated intuitionistic fuzzy time series forecasting model with the 4253HT smoother.

Table 4: Actual, smoothed and forecasted Malaysian crude palm oil prices.

Month	Actual Price	Smoothed Price	Forecasted Price	Month	Actual Price	Smoothed Price	Forecasted Price
1/2017	3268	3268.00	-	8/2019	2066.5	2030.33	1986.07
2/2017	3233	3176.65	3189.80	9/2019	2097	2100.94	1986.07
3/2017	2955.5	2997.03	2980.28	10/2019	2104	2261.08	2144.52
4/2017	2752.5	2826.49	2771.50	11/2019	2493.5	2508.71	2367.39
5/2017	2803.5	2722.81	2692.59	12/2019	2813	2687.78	2535.55
6/2017	2686	2669.08	2699.20	1/2020	3013.5	2735.52	2699.20
7/2017	2629.5	2659.74	2699.20	2/2020	2714.5	2648.86	2699.20
8/2017	2633	2678.94	2699.20	3/2020	2382	2463.71	2491.26

9/2017	2780.5	2700.88	2699.20	4/2020	2299	2353.38	2535.55
10/2017	2736	2703.65	2699.20	5/2020	2074	2341.53	2383.97
11/2017	2689	2661.38	2699.20	6/2020	2411.5	2389.77	2367.39
12/2017	2407	2572.85	2699.20	7/2020	2519	2539.61	2383.97
1/2018	2486.5	2496.90	2491.26	8/2020	2815	2736.38	2535.55
2/2018	2488	2464.54	2535.55	9/2020	2924	2918.35	2699.20
3/2018	2426.5	2445.95	2535.55	10/2020	2979.5	3121.36	3136.72
4/2018	2418	2421.51	2383.97	11/2020	3422	3369.20	3369.20
5/2018	2396	2382.22	2383.97	12/2020	3620.5	3598.92	3598.92
6/2018	2324	2319.26	2383.97	1/2021	3748.5	3770.08	3770.08
7/2018	2215	2249.92	2367.39	2/2021	3895.5	3915.37	3915.37
8/2018	2183.5	2197.05	2151.49	3/2021	4041.5	4042.66	4011.44
9/2018	2177.5	2132.54	2151.49	4/2021	4220	4120.58	4109.03
10/2018	2082	2032.64	2144.52	5/2021	4572	4158.13	4188.75
11/2018	1830.5	1953.34	1986.07	6/2021	3830.5	4204.04	4233.30
12/2018	1794.5	1932.85	1986.07	7/2021	4128.5	4282.88	4233.30
1/2019	2037	1944.65	1986.07	8/2021	4555	4441.73	4441.73
2/2019	2100.5	1961.49	1986.07	9/2021	4556	4700.52	4700.52
3/2019	1903.5	1968.79	1986.07	10/2021	5051	4945.74	4945.74
4/2019	2018.5	1966.14	1986.07	11/2021	5341	5067.28	5068.65
5/2019	1946.5	1960.07	1986.07	12/2021	5070	5070.00	5068.65
6/2019	1968	1960.21	1986.07	1/2022	-	-	5068.65
7/2019	1879	1983.67	1986.07				

The performance of the proposed model is then measured using mean square error (MSE), root mean square error (RMSE) and mean absolute error (MAE). Table 5 presents the comparison of the forecasting accuracy produced by the proposed model with the existing models.

Table 5: Comparison of MSE, RMSE and MAE of the proposed model with the existing models.

Models	MSE	RMSE	MAE
Alam and Ramli (2021)	31081.06	176.30	115.92
Alam et al. (2022)	31226.64	176.71	119.20
Proposed Model	21524.73	146.71	107.56

The proposed model outperforms the existing intuitionistic fuzzy time series forecasting models. The model in Alam & Ramli (2021) used equal distribution of hesitancy de-i-fuzzification. However, the model was not integrated with any smoothing method, which used 37 intervals of length 100 when the Malaysian crude palm oil prices data were implemented. On the other hand, the proposed model only divided the universe of discourse into 32 intervals since the smoothing process has reduced the length of the universe of discourse. Hence, the proposed model simplified the calculation and produced a better forecasting performance.

Using the intuitionistic fuzzy time series forecasting model from Alam et al. (2022), 36 intervals were defined since the average-based length method (Huarng, 2001) was used to partition the universe of discourse. The score functions of IFS were used to intuitionistic fuzzify the historical data and their absolute values were utilized for the defuzzification. However, predicting Malaysian crude palm oil prices using this model could not be as efficient as the proposed model which was integrated with the 4253HT smoother that has lessened the number of intervals defined from the universe of discourse.

6. Conclusion

The integration of the 4253HT smoother with the intuitionistic fuzzy time series forecasting model has directed the fuzzy time series into the development of a better forecasting method. The integrated model was implemented in the Malaysian crude palm oil prices data from January 2017 until December 2021 and has resulted in a better forecasting performance as compared to the existing fuzzy time series models. The smoother managed to cater randomly fluctuating data and thus reduced the uncertainties which were then processed by the intuitionistic fuzzy set in the proposed model. In fact, the smoother reduced the length of the universe of discourse, hence lessened the number of intervals for fuzzification. The advantages of the smoother were adopted to improve the intuitionistic fuzzy time series forecasting model to become a more efficient one. In the future, the integration of the smoother should be implemented in the development of fuzzy time series forecasting model such that the vagueness of the data can be reduced.

Acknowledgement

The authors would like to thank Universiti Teknologi MARA Pahang for funding this research under Geran Lestari Khas 600-TNCPI 5/3/DDN (06) (009/2020).

References

1. Abhishekh, Gautam, S. S., & Singh, S. R. (2018). A Score Function-Based Method of Forecasting Using Intuitionistic Fuzzy Time Series. *New Mathematics and Natural Computation*, *14*(1), 91–111. https://doi.org/10.1142/S1793005718500072
2. Alam, N. M. F. H. N. B., & Ramli, N. (2021). Time Series Forecasting Model Based on Intuitionistic Fuzzy Set via Equal Distribution of Hesitancy De-I-Fuzzification. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *29*(06), 1015–1029. https://doi.org/10.1142/s0218488521500458
3. Alam, N. M. F. H. N. B., Ramli, N., & Nassir, A. A. (2022). Predicting Malaysian Crude Palm Oil Prices Using Intuitionistic Fuzzy Time Series Forecasting Model. *ESTEEM Academic Journal*, *18,* 61–70.
4. Ansari, A. Q., Philip, J., Siddiqui, S. A., & Alvi, J. A. (2010). *Fuzzification of Intuitionistic Fuzzy Sets*. *8*(3), 94–95.
5. Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3
6. Azmi, N. N. K., Adam, M. B., Ali, N., & Mustafa, M. S. (2019). Adaptive adjusted compound smoother in recovering signal from noise. *ASM Science Journal*, *12*(Special Issue 1), 256–264.
7. Bovik, A., Huang, T., & Munson, D. (1983). A generalization of median filtering using linear combinations of order statistics. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, *31*(6), 1342–1350. https://doi.org/10.1109/TASSP.1983.1164247
8. dos Santos, F. J. J., & de Arruda Camargo, H. (2014). Forecasting in fuzzy time series by an extension of simple exponential smoothing. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, *8864*, 257–268. https://doi.org/10.1007/978-3-319-12027-0_21
9. Ge, P., Wang, J., Ren, P., Gao, H., & Luo, Y. (2013). A new improved forecasting method integrated fuzzy time series with the exponential smoothing method. *International Journal of Environment and Pollution*, *51*(3–4), 206–221. https://doi.org/10.1504/IJEP.2013.054030
10. Hird, J. N., & McDermid, G. J. (2009). Noise reduction of NDVI time series: An empirical comparison of selected techniques. *Remote Sensing of Environment*, *113*(1), 248–258. https://doi.org/10.1016/j.rse.2008.09.003
11. Huarng, K. (2001). Effective lengths of intervals to improve forecasting in fuzzy time series. *Fuzzy Sets and Systems*, *123*(3), 387–394. https://doi.org/10.1016/S0165-0114(00)00057-9
12. Joshi, B. P., & Kumar, S. (2012). Intuitionistic fuzzy sets based method for fuzzy time series forecasting. *Cybernetics and Systems*, *43*(1), 34–47. https://doi.org/10.1080/01969722.2012.637014
13. Jurio, A., Paternain, D., Bustince, H., Guerra, C., & Beliakov, G. (2010). A construction method of Atanassov's intuitionistic fuzzy sets for image processing. *2010 IEEE International Conference on Intelligent Systems, IS 2010 - Proceedings*, 337–342. https://doi.org/10.1109/IS.2010.5548390
14. Justo dos Santos, F. J., & De Arruda Camargo, H. (2015). A hybrid forecast model combining fuzzy time series, linear regression and a new smoothing technique. *Proceedings of the 2015 Conference of the International Fuzzy Systems Association and the European Society for Fuzzy Logic and Technology*, *89*(February 2018). https://doi.org/10.2991/ifsa-eusflat-15.2015.192
15. Kumar, S., & Gangwar, S. S. (2015). A fuzzy time series forecasting method induced by intuitionistic fuzzy sets. *International Journal of Modeling, Simulation, and Scientific Computing*, *6*(4). https://doi.org/10.1142/S1793962315500415
16. Lesmana, E., Anggriani, N., Sukono, Fatimah, & Bon, A. T. (2019). Comparison of double exponential smoothing holt and fuzzy time series methods in forecasting stock prices (case study: PT bank central Asia Tbk). *Proceedings of the International Conference on Industrial Engineering and Operations Management*, *July*, 1615–1625.

17. Pedrycz, W. (1993). *Fuzzy Control and Fuzzy Systems* (Issue June). Research Studies Press Ltd.
18. Sargent, J., & Bedford, A. (2010). Improving Australian Football League player performance forecasts using optimized nonlinear smoothing. *International Journal of Forecasting*, *26*(3), 489–497. https://doi.org/10.1016/j.ijforecast.2009.10.003
19. Singh, P. (2017). A brief review of modeling approaches based on fuzzy time series. *International Journal of Machine Learning and Cybernetics*, *8*(2), 397–420. https://doi.org/10.1007/s13042-015-0332-y
20. Song, Q., & Chissom, B. S. (1993a). Forecasting enrollments with fuzzy time series - Part I. *Fuzzy Sets and Systems*, *54*(1), 1–9. https://doi.org/10.1016/0165-0114(93)90355-L
21. Song, Q., & Chissom, B. S. (1993b). Fuzzy Time Series and Its Models. *Fuzzy Sets and Systems*, *54*, 269–277.
22. Song, Q., & Chissom, B. S. (1994). Forecasting enrollments with fuzzy time series - Part II. *Fuzzy Sets and Systems*, *62*(1), 1–8. https://doi.org/10.1016/0165-0114(93)90355-L
23. Tukey, J. W. (1977). *Exploratory Data Analysis*. Addison-Wesley Publishing Company Reading. https://doi.org/10.1525/9780520338210
24. Velleman, P. F., & Hoaglin, D. C. (1981). *Applications, Basics, and Computing of Exploratory Data Analysis*. Duxbury Press.
25. Voskoglou, M. (2019). Application of Fuzzy Numbers to Assessment Processes. In K.-P. Mehdi (Ed.), *Encyclopedia of Information Science and Technology* (pp. 407–420). IGI Global. https://doi.org/10.4018/978-1-5225-7368-5.ch030
26. Ye, F., Zhang, L., Zhang, D., Fujita, H., & Gong, Z. (2016). A novel forecasting method based on multi-order fuzzy time series and technical analysis. *Information Sciences*, *367–368*, 41–57. https://doi.org/10.1016/j.ins.2016.05.038
27. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X