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A Proposed Method for Finding Initial Solutions to Transportation Problems

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Abstract

The Transportation Model (TM) in the application of Linear Programming (LP) is very useful in optimal distribution of goods. This paper focuses on finding Initial Basic Feasible Solutions (IBFS) to TMs hence, proposing a Demand-Based Allocation Method (DBAM) to solve the problem. This unprecedented proposal goes in contrast to the Cost-Based Resource Allocations (CBRA) associated with existing methods (including North-west Corner Rule, Least Cost Method and Vogel's Approximation Method) which select decision variable before choosing demand and supply constraints. The proposed 'DBAM' on page 66 is implemented in MATLAB and has the ability to solve large-scale transportation problems to meet industrial needs. A sample of five (5) examples are presented to evaluate efficiency of the method. Initial Basic Feasible Solutions drawn from the study are of higher accuracy and will rapidly converge to optima in less iterations. The comparative results also showed that the DBAM outperforms other methods under this study which qualifies it as one of the best methods to solve industrial TMs.

Key Words: Optimization, Basic Feasible Solutions, Logistics, Linear Programming, Vogel's Approximation Method.

Mathematical Subject Classification: 90B06.

1. Introduction

The TM is an Optimization tool for minimizing the shipping cost for transporting units from m sources to various n destinations to provide cost-effective distributing patterns for Logistics and Supply Chain Organizations (LSCO), etc. According to the paper of Kamal et al. (2021), the TM is considered as a logistic (or network) problem to control the delivery of items at minimum costs. This cost minimization is indispensable in maintaining profitability of industries in production and distribution of items. In the general TM; (1) The sum of units at all sources equals the sum of all demands at various destinations, (2) There is a convenient flow of units from all sources to the consumer points, (3) There is a known Total Cost (TC) per unit from all sources to the destinations and (4) The TC for a particular route is linearly proportional to the amount of units sent along it. The LP formulation of TMs (according to Kamal et al. (2021), Prasad and Singh (2020) and Saleh and Shiker (2022)) requires a set of decision variables, an objective function and model constraints.

The broad topic of transportation has gained attention from many research backgrounds as far as production is concerned. This development (of TMs) originated around the years of Monge (1781) who formalized the transport theory for the study of optimal allocation of resources. The works of Leonid Kantrovich (1939) on LP gained advancements through George B. Dantzig (1951) who developed the simplex algorithm for solving LP problems, which he applied to solve TMs. Before Dantzig's work, Frank L. Hitchcock (1941) developed a resource allocation model to mark the actual beginning of TMs as a sub-class of Network Optimization Models (NOM). His proposal on the distribution of units from several sources to different regions is considered as a significant contribution in finding the solutions to transportation models.

Solving a TM requires setting out an IBFS to be improved (using the Stepping-Stone Method (Charnes and Cooper , 1954) or Modified Distribution) in order to become optimal. North-west Corner Rule (NCR), Least Cost Method (LCM) and Vogel's Approximation Method (VAM) are widely used in pursuit of this goal. Methods of Total Differences (TDM1 and TDM2) was developed by Hosseini (2017) as a modification on the VAM, which provides satisfactory solutions to TMs in at least one case. The Total Opportunity Cost Matrix-Minimal Total (TOCM-MT) method (Amalia et al. , 2019) was compared to Hosseini's methods and the VAM for which it was claimed to have performed better than the other methods. Similarly, Ravi et al. (2018) proposed the Direct Sum Cost method in comparison to the Vogel's method (i.e VAM), NCR and the LCM to evaluate its effectiveness in solving the transportation problem. Hanif and Rafi (2018) also suggested a new methodology which requires arithmetic and logical calculations to yield an IBFS in comparison to VAM; even so, the proposed method could not outperform the VAM in such cases.

The Inverse Coefficient of Variation Method (Opara et al. , 2017) finds an IBFS to balanced transportation problems. The variation coefficients in this method are calculated for each row and column as a ratio of means to corresponding standard deviations computed for each row and column. The method selects successive least inverse variation coefficients for allocation. As captured by Abdelati et al. (2020); the Row Minima Method (RMM), Column Minima Method (CMM) and the Russel's Approximation Method (RAM) are three other methods considered in this study

Zangiabadi and Rabie (2012) applied the concept of Fuzzy Goal Programming Problems (FGPPs) to solving a TM with qualitative and quantitative factors. Yeola and Jahav (2016) also proposed a parallel algorithm to solve multi-objective transportation problems with penalties calculated using a fuzzy membership function (Yeola and Jahav , 2016). The algorithm was claimed to yield satisfactory results for multi-objective problems with less complexity.

Most existing methods of solutions to TMs use objective-based approaches to the solution process. NCR fully satisfies the first destination before the next, to give a quick solution in short times. But it rarely yields near optimal solutions (Mishra , (2017)). LCM and VAM help decision makers to provide best shipping routes and that, they yield best IBFS, due to their potentials in yielding a near optimal solution. However, VAM slows down due to long-time computations (Mishra , (2017)).

In contrast to Cost-Based Resource Allocations (CBRA) associated with existing methods, this study aims to: (1) Present a Demand-Based Allocation Method (DBAM) to find initial basic feasible solutions to TMs, (2) Compare the proposed DBAM to the existing methods, and (3) Determine the effectiveness and efficiency of the DBAM in solving TMs based on the comparative results. The study continues through the Sections 2 to 5.

2. The Transportation Model Design

In the TM, each source i is connected to all destinations (see Figure 1) j by routes $\overline{S_iP_j}$; where $i = 1$ to m; $j = 1$ to n (Ackora Prah et al. , 2022).

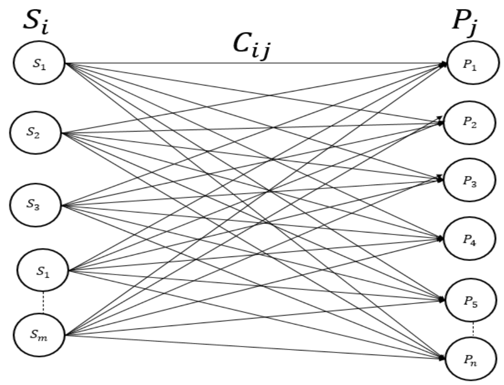

Figure 1: Network representation of the transportation model.

Table 1: The General TST.

S_i \ P_j	1	2	$\cdots$	j	$\cdots$	n	**Supply**
1	C_{11}	C_{12}	$\cdots$	C_{1j}	$\cdots$	C_{1n}	b_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	C_{i1}	C_{i2}	$\cdots$	C_{ij}	$\cdots$	C_{in}	b_i
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	C_{m1}	C_{m2}	$\cdots$	C_{mj}	$\cdots$	C_{mn}	b_m
Demand	a_1	a_2	$\cdots$	a_j	$\cdots$	a_n	

The general LP model formulation of the problem is shown in Equation (1);

$$\begin{aligned} Minimize \quad Z &= \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \\ Subject\ to: &\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} \leq b_i \\ &\sum_{j=1}^{n}\sum_{i=1}^{m} X_{ij} \geq a_j \\ &X_{ij} \geq 0 \; \forall \, i, j, \end{aligned} \tag{1}$$

as it is captured by the Transportation Simplex Tableau (TST) in Table 1, where:

m =the number of sources (rows).
n = the number of destinations (columns).
mn = the number of decision variables (X) in constituting the problem.
b_i =the available supply at a source i, representing capacity constraints.
a_j =the amount of units required at the destination j, representing demand constraints.
P_j =the j^{th} destination (see Figure 1).
S_{ij} =the i^{th} source (see Figure 1).
X_{ij} =the amount of units (of the decision variable X) transported Source (S_i) i Destination (P_j) j.
$C_{ij}X_{ij}$ =the cost of shipping X_{ij} units (of the decision variable X) from the source i to a destination j.

Z =Objective function value.

3. The Proposed Demand-Based Allocation Method (DBAM)

3.1. Algorithm

The first step of the algorithm below (DBAM) states that, a minimum demand must be chosen first before supplies are made via associated least cost cells (representing decision variables).

In this method, the first supplies always go to the first (viable) destination associated with the minimum demand. The first step must be repeated whenever supplies and demands are satisfied concurrently. Otherwise, the next steps of the allocation process continue as outlined in Algorithm 1, until all model constraints are satisfied.

Algorithm 1 DBAM

function [ ]=DBAM($[C_{ij}],\ [b_i],\ [a_j]$)
  while All constraints are not satisfied **do**
    Step 1:
    Choose $a_j^* = \min\{a_j\}$
    Select $C^* = \min\{A(:,j)\},\ b_i^*$
    Step 2:
    if $a_j^* = b_i^*$ **then**
      $X_{ij} = b_i^*$
      Return to **Step 1**
    else if $a_j^* < b_i^*$ **then**
      $X_{ij} = a_j^*, b_i^* = b_i^* - a_j^*$
      Select $C^* = \min\{A(i,:)\},\ a_j^*$
      Repeat **Step 2**
    else
      $X_{ij} = b_i^*, a_j^* = a_j^* - b_i^*$
      Select $C^* = \min\{A(:,j)\},\ b_i^*$
      Repeat **Step 2**
    end if
  end while
end function

3.2. Ties for choosing Minimum Demand Values (MDVs) and for selecting Decision Variables

(1) Ties for choosing MDVs can be broken arbitrarily, or preference should be given to a column having the least-cost cell.
(2) Ties for selecting decision variables can also be broken arbitrarily, however, preference should be given to the cell where maximum allocations can be made (to a row) or the cell which links to the maximum capacity.

4. Numerical Analysis

In this section, the study presents and analyzes five numerical transportation model examples with results and discussions shown from Sections 4.2 to 4.3 on models' solutions and comparisons. For each Example (I-V), per unit shipping costs (C_{ij}) are in the top-right corner for each cell, supplies (b_i) for each source are arranged correspondingly in the far-right column of each tableau, and demands (a_j) for each destination are arranged correspondingly in the bottom row of each tableau. For example; in Table 2 (i.e. Example I) $b_1 = 350,\ b_2 = 400,\ b_3 = 580;\ a_1 = 300,\ a_2 = 160$ $,\ a_3 = 550,\ a_4 = 50,\ a_5 = 150,\ a_6 = 120;\ C_{11} = 6,\ C_{12} = 14,\ C_{13} = 11,\ C_{14} = 13,\ C_{15} = 2,\ C_{16} = 10,$ $C_{21} = 9, \ldots,\ C_{36} = 20, X_{ij} = 0$ (units shipped initially) $(\forall\ i,\ j)$.

4.1. Illustrative Examples

Consider the following examples:

Table 2: Example I.

S_i \ P_j	1	2	3	4	5	6	**Supply**
1	6	14	11	13	12	10	350
2	9	8	10	15	8	9	400
3	4	7	18	9	7	20	580
Demand	300	160	550	50	150	120	

Table 3: Example II (Hosseini , 2017).

S_i \ P_j	1	2	3	4	**Supply**
1	19	30	50	10	70
2	70	30	40	60	90
3	40	8	70	20	180
Demand	50	80	70	140	

Table 4: Example III (Hasan , 2012).

S_i \ P_j	1	2	3	4	**Supply**
1	13	18	30	8	8
2	55	20	25	40	10
3	30	6	50	10	11
Demand	4	7	6	12	

Table 5: Example IV (Hasan , 2012).

S_i \ P_j	1	2	3	4	5	6	**Supply**
1	9	12	9	6	9	10	5
2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	

Table 6: Example V (Mishra , 2017).

S_i \ P_j	1	2	3	4	**Supply**
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	225	275	250	

4.2. Results and Discussions

4.2.1. Results

Consider the balanced setup in Table 2 with 3 sources; $S_i;\ i = 1, 2, 3$ with supplies (b_i's), 6 destinations; $P_j;\ j = 1, 2, 3, 4, 5, 6$ with demands (a_j's), and shipping costs ($A = [C_{ij}]$). Using the Algorithm 1:

While: All constraints are not satisfied, **do**
Step 1: Choose the MDV;
$a_j^* = \min\{a_1, a_2, \ldots, a_6\} = a_4^* = 50$ units
Select the corresponding least cost in the a_4^*-column ;
$C^* = \min\{A(:, 4)\} = A(3, 4) \quad \therefore C^* = 9$
Select the associated supply from the C^*-row;
$b_i^* = b_3^* = 580$ units.
Step 2: Since $a_4^* < b_3^*$, then allocate 50 units to X_{34} and satisfy the constraints;
$X_{34} = 50,\ b_3^* = b_3^* - a_4^* = 580 - 50 = 530$
Select the next least cost from the b_3^*-row ;
$C^* = \min\{A(3, :)\} = A(3, 1) \quad \therefore C^* = 4$
Choose associated demand from the C^*-column ;
$a_1^* = 300$ units.
Repeat **step 2** since $a_4^* \neq b_3^*$.
Step 2: Since $a_1^* < b_3^*$, then allocate 300 units to X_{31} and satisfy the constraints;
$X_{31} = 300,\ b_3^* = b_3^* - a_1^* = 530 - 300 = 230$
Select the next least cost from the b_3^*-row ;
$C^* = \min\{A(3, :)\} = A(3, 2) \quad \therefore C^* = 7$
Choose associated demand from the C^*-column ;
$a_2^* = 160$ units.
Repeat **step 2** since $a_1^* \neq b_3^*$.
Step 2: Since $a_2^* < b_3^*$, then allocate 160 units to X_{32} and satisfy the constraints;
$X_{32} = 160,\ b_3^* = b_3^* - a_2^* = 230 - 160 = 70$
Select the next least cost from the b_3^*-row ;
$C^* = \min\{A(3, :)\} = A(3, 5) \quad \therefore C^* = 7$
Choose associated demand from the C^*-column ;
$a_5^* = 150$ units.
Repeat **step 2** since $a_2^* \neq b_3^*$.
Step 2: Since $a_5^* > b_3^*$, then allocate 70 units to X_{35} and satisfy the constraints;
$X_{35} = 70,\ a_5^* = a_5^* - b_3^* = 150 - 70 = 80$
Select the next least cost from the a_5^*-column ;
$C^* = \min\{A(:, 5)\} = A(2, 5) \quad \therefore C^* = 8$
Choose associated supply from the C^*-row ;
$b_2^* = 400$ units.
Repeat **step 2** since $a_5^* \neq b_3^*$.
Step 2: Since $a_5^* < b_2^*$, then allocate 80 units to X_{25} and satisfy the constraints;
$X_{25} = 80,\ b_2^* = b_2^* - a_5^* = 400 - 80 = 320$
Select the next least cost from the b_2^*-row ;
$C^* = \min\{A(2, :)\} = A(2, 6) \quad \therefore C^* = 9$
Choose associated demand from the C^*-column ;
$a_6^* = 120$ units.
Repeat **step 2** since $a_5^* \neq b_2^*$.

Step 2: Since $a_6^* < b_2^*$, then allocate 120 units to X_{26} and satisfy the constraints;
$X_{26} = 120,\ b_2^* = b_2^* - a_6^* = 320 - 120 = 200$
Select the next least cost from the b_2^*-row ;
$C^* = \min\{A(2,:)\} = A(2,3) \quad \therefore C^* = 10$
Choose associated demand from the C^*-column ;
$a_3^* = 550$ units.
Repeat **step 2** since $a_6^* \neq b_2^*$.

Step 2: Since $a_3^* > b_2^*$, then allocate 200 units to X_{23} and satisfy the constraints;
$X_{23} = 200,\ a_3^* = a_3^* - b_2^* = 550 - 200 = 350$
Select the next least cost from the a_3^*-column ;
$C^* = \min\{A(:,3)\} = A(1,3) \quad \therefore C^* = 11$
Choose associated supply from the C^*-row ;
$a_5^* = 350$ units.
Repeat **step 2** since $a_3^* \neq b_2^*$.

Step 2: Since $a_3^* = b_1^*$, then allocate 350 units to X_{13} and satisfy the constraints;
$X_{13} = 350,\ b_3^* = b_3^* - a_1^* = 350 - 350 = 0$
Return to **step 1** since $a_3^* = b_1^*$ (supply and demand are satisfied concurrently).

Step 1: Since all constraints are satisfied,
Stop

End

Table 7 gives the solution where $X_{13} = 350,\ X_{23} = 200,\ X_{25} = 80,\ X_{26} = 120,\ X_{31} = 300,\ X_{32} = 160,\ X_{34} = 50,$ and $X_{35} = 70$.

$$Z = \sum_{i=1}^{3}\sum_{j=1}^{6} C_{ij}X_{ij} = 11(350) + 10(200) + 8(80) + 9(120) + 4(300) + 7(160) + 9(50) + 7(70) = 10830 \quad (2)$$

In the same way algorithm 1 is applied to the Examples II to V. The results are shown respectively in the tables 8 to 11 with $Z_{II} = 7430,\ Z_{III} = 412,\ Z_{IV} = 112,$ and $Z_V = 12075$, where Z_{II} is the value of Z at Example II, and so on.

Table 7: Solution to Example 1

S_i \ P_j	1	2	3	4	5	6	**Supply**
1	6	14	11 350	13	12	10	350
2	9	8	10 200	15	8 80	9 120	400
3	4 300	7 160	18	9 50	7 70	20	580
Demand	300	160	550	50	150	120	

Table 8: Initial Solution to Example II

S_i \ P_j	1	2	3	4	**Supply**
1	19 50	30	50	10 20	70
2	70	30 20	40 70	60	90
3	40	8 60	70	20 120	180
Demand	50	80	70	140	

Table 9: Initial Solution to Example III

S_i \ P_j	1	2	3	4	**Supply**
1	13 4	18	30	8 4	8
2	55	20 4	25 6	40	10
3	30	6 3	50	10 8	11
Demand	4	7	6	12	

Table 10: Initial Solution to Example IV

S_i \ P_j	1	2	3	4	5	6	**Supply**
1	9	12	9 5	6	9	10	5
2	7	3 3	7 1	7	5	5 2	6
3	6 1	5 1	9	11	3	11	2
4	6 3	8	11	2 2	2 4	10	9
Demand	4	4	6	2	4	2	

Table 11: Initial Solution to Example V.

S_i \ P_j	1	2	3	4	**Supply**
1	11 200	13 50	17	14	250
2	16	18 175	14	10 125	300
3	21	24	13 275	10 125	400
Demand	200	225	275	250	

4.2.2. Discussions

In solving the transport network (shown in Figure 1), decision-makers hope to reduce the number (mn) of routes in the end. Hence, methods such as VAM etc., are developed to cover such goals. The problem in 'Example I.' on page 67 has eighteen (18) routes (i.e. $\overline{S_1P_1}$ to $\overline{S_3P_6}$) connecting three (3) sources and six (6) destinations. The study presented in Table 7 the 'Solution to Example 1' on page 69 using 'DBAM' on page 66 (i.e. Algorithm 1), where; (1) 350 units should be sent via $\overline{S_1P_3}$ (i.e. Source (S_i) 1 Destination (P_j) 3), (2) 200 units via $\overline{S_2P_2}$, (3) 80 units via $\overline{S_2P_5}$, (4) 120 units via $\overline{S_2P_6}$, (5) 300 units via $\overline{S_3P_1}$, (6) 160 units should be sent via $\overline{S_3P_2}$, (7) 50 units via $\overline{S_3P_4}$ and (8) 70 units via $\overline{S_3P_5}$. Table 12 shows the 'Distributing pattern for Example I' on page 70, where according to the Stepping-Stone Method (Sharma , 2010), the objective value (Z) is optimal, as compared to results from six other methods captured by the 'Comparative results' on page 72 (Table 17).

Table 12: Distributing pattern for Example I

Source (S_i)	Destination (P_j)	Units (X_{ij})	Cost/Unit	Shipment Cost
1	3	350	11	3850
2	2	80	8	640
2	3	200	10	2000
2	6	120	9	1080
3	1	300	4	1200
3	2	70	7	490
3	4	50	9	450
3	5	160	7	1120
				Z =**10830**

In the same way, the Algorithm 1 is applied to each of the Examples (II to V, shown in the Tables 3 to 6) to provide distributing patterns respectively, as shown in the Tables 13 to 16. All Initial Basic feasible solutions provided in this study represent optimal solution, according to the Stepping-Stone Method and the Modified Distribution. Furthermore, the number of routes has reduced from eighteen (18) to eight (8) for 'Example I'in Table 7, from twelve (12) to six (6) for the Examples (II, III, & V) in the Tables 8 to 11, except Table 10 where the number of straight routes is reduced from twenty-four (24) to nine (9). Comparative results to the study are presented in the following section.

Table 13: Distributing pattern for Example II

Source (S_i)	**Destination** (P_j)	**Units** (X_{ij})	**Cost/Unit**	**Shipment Cost**
1	1	50	19	950
1	4	20	10	200
2	2	20	30	600
2	3	70	40	2800
3	2	60	8	480
3	4	120	20	2400
				$Z =$ **7430**

Table 14: Distributing pattern for Example III

Source (S_i)	**Destination** (P_j)	**Units** (X_{ij})	**Cost/Unit**	**Shipment Cost**
1	1	4	13	52
1	4	4	8	32
2	2	4	20	80
2	3	6	25	150
3	2	3	6	18
3	4	8	10	80
				$Z =$ **412**

Table 15: Distributing pattern for Example IV

Source (S_i)	**Destination** (P_j)	**Units** (X_{ij})	**Cost/Unit**	**Shipment Cost**
1	3	5	9	45
2	2	3	3	9
2	3	1	7	7
2	6	2	5	10
3	1	1	6	6
3	2	1	5	5
4	1	3	6	18
4	4	2	2	4
4	5	4	2	8
				$Z =$**112**

Table 16: Distributing pattern for Example V

Source (S_i)	**Destination** (P_j)	**Units** (X_{ij})	**Cost/Unit**	**Shipment Cost**
1	1	200	11	2200
1	2	50	13	650
2	2	175	18	3150
2	4	125	10	1250
3	3	275	13	3575
3	4	125	10	1250
				$Z =$**12075**

4.3. Comparing the proposed DBAM to the existing methods.

The study is supported with a sample of five (5) TM examples to evaluate efficiency of the DBAM. The 'Comparative results' on page 72 show Initial Basic Solutions (IBS) to the five examples, represented by their respective objective values, Z (in the Table 17) which are presented six methods (i.e. NCR, LCM, VAM, RAM, RMM and CMM) in comparison to the proposed method.

Table 17: Comparative results

	Objective value (Z)						
Problem	NCR	LCM	VAM	RAM	RMM	CMM	**DBAM**
Example I	14670	10930	12810	10830	15600	11230	**10830**
Example II	10150	8140	7790	8070	11100	7790	**7430**
Example III	484	516	476	454	457	476	**412**
Example IV	139	112	112	115	128	118	**112**
Example V	12200	12200	12075	12200	13175	12075	**12075**

The proposed method outperforms the other methods in Table 17.

5. Conclusions

This study has presented the Demand-Based Allocation Method (DBAM) as a means to find Initial Basic Feasible Solutions (IBFS) to Transportation Models (TMs). Using MATLAB (See 'Computer Solutions' on page 73, Appendix A, Figures 2 to 5) as well as hand calculation, the DBAM Algorithm 1 has been implemented with five models examples in which the study results qualify the proposed method as efficient and effective in solving the models. The DBAM yields solutions that can converge rapidly to optima (through testing) in less iterations. Moreover, it outperforms the other methods (including VAM and LCM) with reference to the comparative results in Table 17. This makes it one of the best methods applicable to solving large-scale TMs for Logistics and Supply-Chain Systems.

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Appendices

A. Computer Solutions

```
Command Window
Enter cost matrix:[6 14 11 13 12 10;9 8 10 15 8 9;4 7 18 9 7 20]
Enter supplies in a column:[350;400;580]
Enter demands in a row:[300 160 550 50 150 120]
SOLUTION:

SHIPMENTS =

           0           0         350           0           0           0         350
           0           0         200           0          80         120         400
         300         160           0          50          70           0         580
         300         160         550          50         150         120        1330


SHIPPING_COST =

           0           0        3850           0           0           0         350
           0           0        2000           0         640        1080         400
        1200        1120           0         450         490           0         580
         300         160         550          50         150         120        1330

FUNC(Z*) =10830
```

Figure 2: IBFS - Example I

Note. $X_{13} = 350,\ X_{23} = 200,\ X_{25} = 80,\ X_{26} = 120,\ X_{31} = 300,\ X_{32} = 160,\ X_{34} = 50,\ X_{35} = 70$

 DOI: http://dx.doi.org/10.18187/pjsor.v19i1.4196

```
Command Window
Enter cost matrix:[19 30 50 10;70 30 40 60;40 8 70 20]
Enter supplies in a column:[70;90;180]
Enter demands in a row:[50 80 70 140]
SOLUTION:

SHIPMENTS =

    50     0     0    20    70
     0    20    70     0    90
     0    60     0   120   180
    50    80    70   140   340


SHIPPING_COST =

         950           0           0         200          70
           0         600        2800           0          90
           0         480           0        2400         180
          50          80          70         140         340

FUNC(Z*) =7430
```

Figure 3: IBFS - Example II

Note. $X_{11} = 50, \;\; X_{14} = 20, \;\; X_{22} = 20, \;\; X_{23} = 70, \;\; X_{32} = 60, \;\; X_{34} = 120$

```
Command Window
Enter cost matrix:[13 18 30 8;55 20 25 40;30 6 50 10]
Enter supplies in a column:[8;10;11]
Enter demands in a row:[4 7 6 12]
SOLUTION:

SHIPMENTS =

     4     0     0     4     8
     0     4     6     0    10
     0     3     0     8    11
     4     7     6    12    29


SHIPPING_COST =

    52     0     0    32     8
     0    80   150     0    10
     0    18     0    80    11
     4     7     6    12    29

FUNC(Z*) =412
```

Figure 4: IBFS - Example III

Note. $X_{11} = 4, \;\; X_{14} = 4, \;\; X_{22} = 4, \;\; X_{23} = 6, \;\; X_{32} = 3, \;\; X_{34} = 8$

```
Command Window
Enter cost matrix:[9 12 9 6 9 10;7 3 7 7 5 5;6 8 11 2 2 10;5 6 9 11 3 11]
Enter supplies in a column:[5;6;9;2]
Enter demands in a row:[4 4 6 2 4 2]
SOLUTION:

SHIPMENTS =

     0     0     5     0     0     0     5
     0     3     1     0     0     2     6
     3     0     0     2     4     0     9
     1     1     0     0     0     0     2
     4     4     6     2     4     2    22


SHIPPING_COST =

     0     0    45     0     0     0     5
     0     9     7     0     0    10     6
    18     0     0     4     8     0     9
     5     6     0     0     0     0     2
     4     4     6     2     4     2    22

FUNC(Z*) =112
```

Figure 5: IBFS - Example IV

Note. $X_{13} = 5,\ \ X_{22} = 3,\ \ X_{23} = 1,\ \ X_{26} = 2,\ \ X_{31} = 3,\ \ X_{34} = 2,\ \ X_{35} = 4,\ \ X_{41} = 1,\ \ X_{42} = 1$

Pseudocoode

Algorithm 2 DBAM - MATLAB

function [ ]=DBAM($[C_{ij}],\ [b_i],\ [a_j]$)
  Input: $A = [C_{ij}];\ \ B = [b_i]^T\ \ D = [a_j]$, and Initialize: $OS = zeros(m, n);\ \ T = \min(D);\ \ J = D$
  if $\sum B \neq \sum D$ **then**
    Display('Enter a balanced problem.')
  else
    while constraints are not satisfied, **do**
      for $j = 1 : n$ **do**
        if $T = J(j) = True$ **then**
          $K = min(A(:, j));\ s = j;\ \ J(:, :) = \infty$
        end if
      end for
      for $j = s : n$ **do**
        for $i = 1 : m$ **do**
          if $K = A(i, j) = True$ **then**
            if $C(j) < B(i) = True$ **then**
              Set $B(i) = \ B(i) - D(j);\ OS(i, j) = D(j); T = [\ \ ]\ K = \min(A(i, :)); s = 1$
              Satisfy Rim conditions.
            else if $D(j) = B(i) = True$ **then**
              Set , $OS(i, j) = D(j);$, satisfy RC and set $T = \min(D;\ J = D$
            else
              Set $D(j) = D(j) - B(i); OS(i, j) = B(i); T = [\ \ ]; K = \min(A(:, j)); s = j$
              Satisfy Rim conditions.
            end if
          end if
        end for
      end for
    end while
  end if
end function