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An EPQ Model for Delayed Deteriorating Items with Reliability Consideration, Quadratic Demand and Shortages

Dari Sani[1*]

* Corresponding Author

1. Department of Mathematical Sciences, Kaduna State University, Kaduna, Nigeria. sanisdari@yahoo.com

Abstract

In this paper, an EPQ model for items that exhibit delay in deterioration with partial backordered and reliability consideration is developed. It is assumed that there is no demand and deterioration during production buildup period. It is also supposed that the cost of a unit product is inversely related to the rate of demand and directly related to the process reliability. The demand before deterioration sets in is quadratic time dependent while demand after deterioration sets in is a constant. Shortages are allowed and partially backordered. The model developed will determining the best cycle length, EPQ and the total variable cost of the system. A numerical example will be given to see the applicability of the model and sensitivity analysis will also be carried out to see the effect of changes on some of the system parameters.

Key Words: Delayed deterioration; reliability consideration; quadratic demand; shortages.

Mathematical Subject Classification: 90B05

1.0 INTRODUCTION

Allowing for shortages has always been a standard operating procedure for some stockiest and production managers. The shortages either become lost sales or backordered. In the case of backorders, whenever the inventory runs out, the stockiest would record the backorders and the customer would simply have to wait for the item to be in-stock again. The backorders can either be full or partial, in which case a fraction of the shortages become lost sales.
Many EOQ and EPQ policies were developed on the basis that the demand rate is always constant, but this is not always the case in real life situations. New products in the market and some passion goods are some of the items with their demand as either linear, quadratic or some other polynomial function of time.

Decisions made during the design and manufacture of a product influence the effectiveness of the product (product reliability). Dependence can be viewed as a link to integrate different stages of design, engineering, production, sales and after-sales service into an integrated process Cheng (1989). Hence, it is important to consider reliability consideration in studying inventory models. Many EOQ/EPQ models with deterioration were established on the supposition that the deterioration is instantaneous. However, this is not true in some situations. Apple, orange, cake, grains, vegetables and so on are examples of items with non-instantaneous deterioration. This paper presents an EPQ model for items with delay in deterioration, reliability consideration, quadratic demand and shortages are allowed but partially backorder. It is supposed that the cost of producing a unit item is inversely related to the demand of such item and directly related to the trustworthiness (reliability) of such item (as assumed by Tripathy et al. (2015) and modified by Dari and Sani (2015)). The EPQ model advanced in this paper will determine the optimum total variable cost, the inventory replenishment cycle, the total backorder quantity and the amount of economic production quantity (EPQ). Numerical example will be given and sensitivity analysis will also be carried out to see the effect of changes on some of the system parameters. The following table gives a summary of some related literature from 2012.

2.0 Literature Review

Zhang et al. (2016) invented a new order policy with partial order when ordered customers delayed shopping after the return of stock. They have created a model with the order of the parties, considering the delay in purchasing the ordered customers with the assumption that the number of previous requests is the same as the other orders. Mainly, they model the problem by presenting a new inventory cost section of holding the backordered items. Eduardo and Barron (2001), Chang (2004), Mahapatra et al. (2012), Pal and Chandra (2014), Sahoo and Tripathy (2018) also worked with inventory model with shortages.

Dash (2014) developed a deteriorated EPQ policy for decayed quadratic demand with time value of money and shortages. Osagiede and Osagiede (2007), Mohan and Venkateswarlu (2013), Santosh (2013), Ravish and Amit (2014), Singh and Rathore (2014), Umakanta (2016), Dari and Sani (2017), Malik et al. (2017), Sahoo and Tripathy (2018), Tripathi and Tomar (2018), Palanivel and Gowri (2018) and Dari and Sani (2020) similarly worked with inventory model with quadratic demand.

Leung (2007) established a more general results by means of arithmetic-geometric mean inequality in which a general power function is projected to model the connection between production set-up cost and quality assurance. These two were considered as the independent variables while interest and depreciation cost was considered as dependent variable. The objective was to minimize the long-run expected average annual cost function. Porteus (1986), Charles (1987), Cheng (1989), Lee et al. (1996), Mettas (2000), Salameh and Jaber (2000), Huang (2004), Jaber et al. (2008), Jaber et al. (2009), Sakar et al. (2010), Dari and Sani (2013) and Dari and Sani (2020) also worked on inventory model with reliability consideration.

Ouyang et al. (2006) considered EOQ model for items with delay in deterioration, permissible delay in payments and where the demand before and after deterioration starts-in are the same and constant. Monika and Shon (2010), Baraya and Sani (2011), Musa and Sani (2012), Dari and Sani (2013), Dari and Sani (2015), Dari and Sani (2017), Palanivel and Gowri (2018) and Dari and Sani (2020) likewise worked on inventory model with delayed deterioration.

This paper presents an EPQ model for items with delay in deterioration, reliability consideration, quadratic demand and shortages are allowed but partially backorder. It is supposed that the cost of producing a unit item is inversely related to the demand of such item and directly related to the trustworthiness (reliability) of such item (as assumed by Tripathy et al. (2015) and modified by Dari and Sani (2015)).

The following table gives a summary of some related literature from 2012.

Table 1. Summary of selected related literature from the year 2012

Author(s) (Year of Publication)	Model Structure	Demand Pattern	Deterioration	Allowing shortages	Model Optimization	Reliability
Panda et al. (2012)	EPQ	Quadratic	Constant	Yes	Cost	No
Dari and Sani (2013)	EPQ	Linear	Delayed	No	Cost	No
Dash (2014)	EPQ	Quadratic	Weibull	Yes	Profit	No
Singh and Rathore (2014)	EPQ	Exponential	Constant	No	Cost	Yes
Umakanta (2016)	EOQ	Quadratic	Weibull	Yes	Cost	No
Dari and Sani (2017)	EPQ	Quadratic	Delayed	No	Cost	No
Dari and Sani (2017)	EPQ	Quadratic	Delayed	Yes	Cost	No
Malik et al. (2017)	EOQ	Quadratic	Constant	No	Cost	No
Palanivel and Gowri (2018)	EPQ	Quadratic	Delayed	No	Cost	No
Sahoo and Tripathy (2018)	EOQ	Quadratic	Parabolic	Yes	Cost	No
Tripathi and Tomar (2018)	EOQ	Quadratic	Constant	No	Cost	No
Dari and Sani (2020)	EPQ	Quadratic	Delayed	No	Cost	No
Dari and Sani (2020)	EPQ	Quadratic	Delayed	No	Cost	Yes
Present Work	EPQ	Quadratic	Delayed	Yes	Cost	Yes

3.0 Model Formulation

The model is based on the following notation and assumptions:

3.1 Notation

$Y(t)$ The inventory level during the first stage (production build up period)
$Y_1(t)$ The inventory level during the second stage (period before deterioration sets in)
$Y_2(t)$ The inventory level during the third stage (period after deterioration begins)
$b(t)$ The inventory level during shortage time, which is to be backordered

n The inventory cycle period
n_1 The period of first stage
n_2 The period of second stage
n_3 Shortage period
I_o The total maximum inventory level at the end of production build up
I_1 The inventory level at the time deterioration begins
d_1 The rate of demand after production period and before deterioration sets in.
d_2 The rate of demand after deterioration sets in and before shortage period.
λ The deteriorating rate
p Production rate of the items
C_o The cost of unit of item (in Naira)
c_b The cost of unit of backorder (in Naira)
C_B Total backorder cost (Naira per production run)
A_o The set-up cost (in Naira per production run)
H The cost of holding a unit item
i The inventory carrying charge
B Total backorder (per production run)

3.2 Assumptions

(i) The cost of production of an item (C_o) is assumed to be varies inversely to the demand rates $(d_1$ & $d_2)$ but directly proportional to the reliability (r). We specifically assume the relationship to be:

$$C_o \propto r\left(d_1 \frac{n_2 - n_1}{n} + d_2 \frac{n_3 - n_2}{n} + \beta d_2 \frac{n - n_3}{n}\right)^{-1}$$

$$\therefore \quad C_o = \frac{a(1-r)^{-1}}{\left(d_1 \frac{n_2 - n_1}{n} + d_2 \frac{n_3 - n_2}{n} + \beta d_2 \frac{n - n_3}{n}\right)} \tag{1}$$

where $a > 0$ (as assumed by Tripathy et al. (2015) and modified by Dari and Sani (2015))

(ii) Shortages are allowed and partially backlogged with a fraction $\beta(0 < \beta < 1)$ of the demand, d_2

(iii) Unconstrained suppliers capital

(iv) All items are of good quality

(v) Demand of product is always less than its supply

(vi) $n_1 \le n_2 \le n$

(vii) Instantaneous production

$(viii)$ Demand before deterioration begins d_1, is assumed to be Quadratic and defined by

$$d_1 = \left(c_1 + c_2 t + c_3 t^2\right), \text{ where } c_2 \,\&\, c_3 > 0$$

(ix) A constant demand, d_2 is considered after deterioration sets in

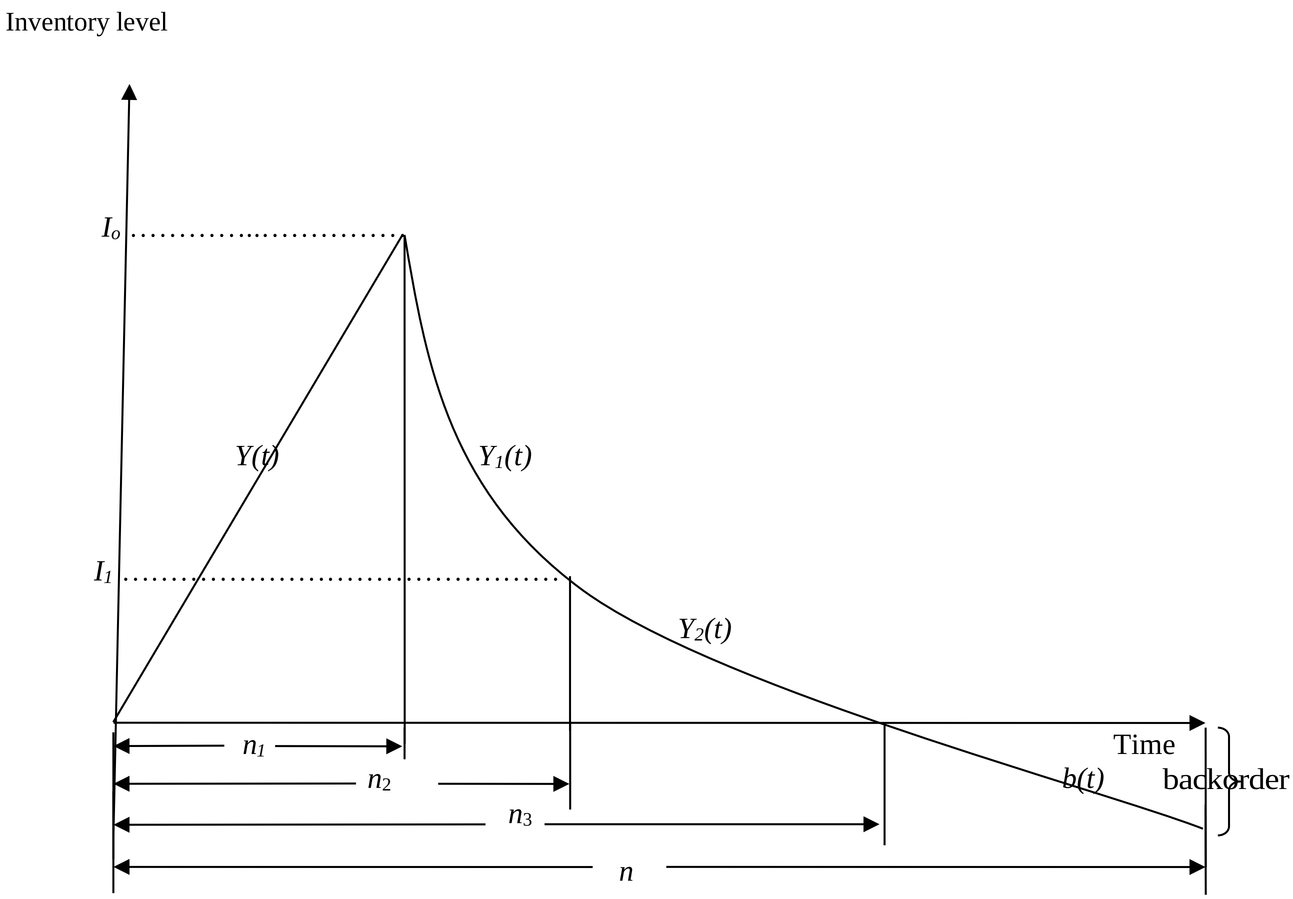

Fig 1: Graphically representation of inventory system with shortages

During the first stage of the production $\left(0 \le t \le n_1\right)$ {i.e. production period}, it is assumed that no demand and no deterioration during this period. Thus, the inventory level depends only on the production rate p and equation (2) describes this situation.

$$\frac{dY(t)}{dt} = p \tag{2}$$

Solving equation (2) by variable separable technics to obtain

$$Y\left(t\right) = pt + L \tag{3}$$

From Fig 1, $Y\left(t\right) = 0,$ at $t = 0$ and substituting into equation (3) to have

$$L = 0$$

Substituting the value of L into equation (3), we have

$$Y(t) = p\, t \tag{4}$$

We have from Fig 1 that $Y(t) = I_o$ at $t = n_1$. Replacing this into equation (4), we have

$$p = \frac{I_o}{n_1} \tag{5}$$

Substituting p from equation (5) into equation (4) to obtain

$$Y(t) = \frac{I_o}{n_1} t \tag{6}$$

During the second stage of the replenishment circle, it is assumed that no deterioration at that time and the inventory level at this stage will depend only on demand. Thus, the situation can be describes by:

$$\frac{dY_1(t)}{dt} = -d_1 = -\left(c_1 + c_2 t + c_3 t^2\right) \tag{7}$$

Solving equation (7) by variable separable technic, the solution obtain is

$$Y_1(t) = -\left(c_1 t + c_2 \frac{t^2}{2} + c_3 \frac{t^3}{3}\right) + v \tag{8}$$

Also from Fig 1, we have $Y_1(t) = I_o$ at $t = n_1$. Putting it into equation (8) to have

$$v = I_o + \left(c_1 n_1 + c_2 \frac{n_1^2}{2} + c_3 \frac{n_1^3}{3}\right)$$

Now, substituting v from the above equation into equation (8), we have

$$Y_1(t) = c_1(n_1 - t) + c_2\left(\frac{n_1^2}{2} - \frac{t^2}{2}\right) + c_3\left(\frac{n_1^3}{3} - \frac{t^3}{3}\right) + I_o \tag{9}$$

Now, it can be observed from Fig1 that we have the boundary condition $Y_1(n_2) = I_1$. By substituting this into equation (9), we have

$$I_o = I_1 + c_1(n_2 - n_1) + c_2\left(\frac{n_2^2}{2} - \frac{n_1^2}{2}\right) + c_3\left(\frac{n_2^3}{3} - \frac{n_1^3}{3}\right) \tag{10}$$

Putting equation (10) into equation (9), gives

$$Y_1(t) = c_1(n_1 - t) + c_2\left(\frac{n_1^2}{2} - \frac{t^2}{2}\right) + c_3\left(\frac{n_1^3}{3} - \frac{t^3}{3}\right) + I_1 + c_1(n_2 - n_1) + c_2\left(\frac{n_2^2}{2} - \frac{n_1^2}{2}\right) + c_3\left(\frac{n_2^3}{3} - \frac{n_1^3}{3}\right)$$

$$Y_1(t) = c_1(n_2 - t) + c_2\left(\frac{n_2^2}{2} - \frac{t^2}{2}\right) + c_3\left(\frac{n_2^3}{3} - \frac{t^3}{3}\right) + I_1 \tag{11}$$

During the third stage of the circle, it is assumed that the inventory level depends on rates demand and spoilage (deterioration). In this case, the following equation represents the situation:

$$\frac{dY_2(t)}{dt} = -\lambda Y_2(t) - d_2 \tag{12}$$

Solution of the above equation [i.e (12)] is

$$Y_2(t) = -\frac{d_2}{\lambda} + fe^{-\lambda t} \tag{13}$$

Also, applying the boundary condition $Y_2(n_2) = I_1$ into equation (13) to have

$$f = \left(\frac{d_2}{\lambda} + I_1\right)e^{\lambda n_2}$$

Substituting f into equation (13) to obtain

$$Y_2(t) = -\frac{d_2}{\lambda} + \left(\frac{d_2}{\lambda} + I_1\right)e^{\lambda(n_2 - t)} \tag{14}$$

From Fig 1, at $t = n_3$, $Y_2(t) = 0$. Substituting this into equation (14), gives

$$I_1 = \frac{d_2}{\lambda}\left(e^{-\lambda(n_2 - n_3)} - 1\right) \tag{15}$$

Substituting I_1 into equation (10), we have

$$I_o = \frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)+c_1\left(n_2-n_1\right)+c_2\left(\frac{n_2^{\,2}}{2}-\frac{n_1^{\,2}}{2}\right)+c_3\left(\frac{n_2^{\,3}}{3}-\frac{n_1^{\,3}}{3}\right) \tag{16}$$

Equation (16) is the maximum inventory produce during the production period

Also, substituting I_1 into equation (11), we have

$$Y_1(t) = c_1\left(n_2-t\right)+c_2\left(\frac{n_2^{\,2}}{2}-\frac{t^2}{2}\right)+c_3\left(\frac{n_2^{\,3}}{3}-\frac{t^3}{3}\right)+\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right) \tag{17}$$

Now, substituting equation (16) into equation (6) to have

$$Y(t) = \left(\frac{d_2}{\lambda n_1}\left(e^{-\lambda(n_2-n_3)}-1\right)+\frac{c_1}{n_1}\left(n_2-n_1\right)+\frac{c_2}{n_1}\left(\frac{n_2^{\,2}}{2}-\frac{n_1^{\,2}}{2}\right)+\frac{c_3}{n_1}\left(\frac{n_2^{\,3}}{3}-\frac{n_1^{\,3}}{3}\right)\right)t \tag{18}$$

Also, substituting equation (15) into equation (14) to have

$$Y_2(t) = \frac{d_2}{\lambda}\left(e^{\lambda(n_3-t)}-1\right) \tag{19}$$

3.3 Backorders

During the shortage period $[n_3, n]$, the demand is partially backlogged with a fraction $\beta(0<\beta<1)$ of the demand d_2. Therefore, the following equation represents the case:

$$\frac{db(t)}{dt} = -\beta d_2 \tag{20}$$

Solution of (19) is obtained as follows:

$$b(t) = -\beta d_2 t + g \tag{21}$$

Also, at $t = n_3,\ b(t) = 0,$ so that equation(21) becomes

$$g = \beta d_2 n_3$$

Substituting g back into equation (21) gives

$$b(t) = \beta d_2\left(n_3 - t\right)$$

3.4 Number of Deteriorated Items

The total items that deteriorate $d(n_2)$ is as follows:

$$d(n_2) = I_1 - d_2\left(n_3-n_2\right)$$

$$= \frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)-d_2\left(n_3-n_2\right) \tag{22}$$

3.5 The Items Sold out During the Second Stage (Period Before Deterioration Sets-In)

The items sold out during the second stage is given by

$$d_1 = \int_{n_1}^{n_2}\left(c_1+c_2t+c_3t^2\right)dt = c_1\left(n_2-n_1\right)+\frac{c_2\left(n_2^{\,2}-n_1^{\,2}\right)}{2}+\frac{c_3\left(n_2^{\,3}-n_1^{\,3}\right)}{3} \tag{23}$$

3.6 The Holding Cost (Inventory Carrying Cost)

The total cost of keeping the whole inventory is given by:

$$H = iC_0\left[\int_0^{n_1} Y(t)dt + \int_{n_1}^{n_2} Y_1(t)dt + \int_{n_2}^{n_3} Y_2(t)dt\right] \text{[where } i \text{ is the inventory carrying charge]} \tag{24}$$

Substituting equation (1), equation (17) to equation (19) into equation (24) to have

$$H = \frac{ia(1-r)^{-1}}{\left(d_1\frac{n_2-n_1}{n} + d_2\frac{n_3-n_2}{n} + \beta d_2\frac{n-n_3}{n}\right)}\left[\begin{array}{l}\int_0^{n_1}\left(\left(\frac{d_2}{\lambda n_1}\left(e^{-\lambda(n_2-n_3)}-1\right)+\frac{c_1}{n_1}(n_2-n_1)+\frac{c_2}{n_1}\left(\frac{n_2^2}{2}-\frac{n_1^2}{2}\right)+\frac{c_3}{n_1}\left(\frac{n_2^3}{3}-\frac{n_1^3}{3}\right)\right)t\right)dt \\ +\int_{n_1}^{n_2}\left(c_1(n_2-t)+c_2\left(\frac{n_2^2}{2}-\frac{t^2}{2}\right)+c_3\left(\frac{n_2^3}{3}-\frac{t^3}{3}\right)+\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)\right)dt \\ +i\int_{n_2}^{n_3}\frac{d_2}{\lambda}\left(e^{\lambda(n_3-t)}-1\right)dt\end{array}\right]$$

$$\therefore\ H = \frac{ia(1-r)^{-1}}{\left(d_1\frac{n_2-n_1}{n} + d_2\frac{n_3-n_2}{n} + \beta d_2\frac{n-n_3}{n}\right)}\left[\begin{array}{l}-\frac{d_2n_1e^{-\lambda(n_2-n_3)}}{2\lambda}+\frac{d_2n_1}{2\lambda}+\frac{d_2n_2e^{-\lambda(n_2-n_3)}}{\lambda}-\frac{d_2n_3}{\lambda}+\frac{d_2e^{-\lambda(n_2-n_3)}}{\lambda^2}-\frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^2-n_1n_2}{2}\right)+c_2\left(\frac{4n_2^3-3n_1n_2^2-n_1^3}{12}\right)+c_3\left(\frac{3n_2^4-2n_1n_2^3-n_1^4}{12}\right)\end{array}\right] \tag{25}$$

3.7 The Backorder Cost

The backorder cost per cycle C_B is calculated as:

$$C_B = c_b\int_{n_3}^{n}(-b(t))dt = c_b\beta d_2\int_{n_3}^{n}(t-n_3)dt = c_b\beta d_2\left(\frac{t^2}{2}-n_3t\right)\Bigg|_{n_3}^{n}$$

$$= c_b\beta d_2\left(\left(\frac{n^2}{2}-nn_3\right)-\left(\frac{n_3^2}{2}-n_3^2\right)\right) = c_b\beta d_2\left(\frac{n^2}{2}-nn_3+\frac{n_3^2}{2}\right)$$

$$= \frac{c_b\beta d_2}{2}\left(n^2-2nn_3+n_3^2\right) = \frac{c_b\beta d_2}{2}(n-n_3)^2 \tag{26}$$

3.8 Total Variable Cost Per Unit Time

The total variable cost per unit time is given as

$$Z(n) = \frac{A}{n} + \frac{c_b\beta d_2(n-n_3)^2}{2n} + a(1-r)^{-1}\left(d_1(n_2-n_1)+d_2(n_3-n_2)+\beta d_2(n-n_3)\right)^{-1}\left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)-d_2(n_3-n_2)\right]$$

$$+ia(1-r)^{-1}\left(d_1(n_2-n_1)+d_2(n_3-n_2)+\beta d_2(n-n_3)\right)^{-1}\left[\begin{array}{l}-\frac{d_2n_1e^{-\lambda(n_2-n_3)}}{2\lambda}+\frac{d_2n_1}{2\lambda}+\frac{d_2n_2e^{-\lambda(n_2-n_3)}}{\lambda}-\frac{d_2n_3}{\lambda}+\frac{d_2e^{-\lambda(n_2-n_3)}}{\lambda^2}-\frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^2-n_1n_2}{2}\right)+c_2\left(\frac{4n_2^3-3n_1n_2^2-n_1^3}{12}\right)+c_3\left(\frac{3n_2^4-2n_1n_2^3-n_1^4}{12}\right)\end{array}\right] \tag{27}$$

3.9 Optimality Condition

To obtain the best circle length L which will optimize the total cost function per unit time $Z(n)$, the necessary condition for optimality need to be taken we taken (i.e $\frac{dZ(n)}{dn}=0$). The root of that equation gives the minimum provided $\frac{d^2Z(n)}{dn^2}>0$ (sufficient condition). The derivative is given as:

$$\frac{dZ(n)}{dn}=-\frac{A}{n^2}+\frac{c_b\beta d_2\left(n^2-n_3^{\,2}\right)}{2n^2}$$
$$-\frac{a\beta d_2(1-r)^{-1}}{\left(d_1(n_2-n_1)+d_2(n_3-n_2)+\beta d_2(n-n_3)\right)^2}\left\{\begin{array}{l}\left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)-d_2(n_3-n_2)\right]\\+i\left[\begin{array}{l}-\frac{d_2n_1e^{-\lambda(n_2-n_3)}}{2\lambda}+\frac{d_2n_1}{2\lambda}+\frac{d_2n_2e^{-\lambda(n_2-n_3)}}{\lambda}-\frac{d_2n_3}{\lambda}+\frac{d_2e^{-\lambda(n_2-n_3)}}{\lambda^2}-\frac{d_2}{\lambda^2}\\+c_1\left(\frac{n_2^{\,2}-n_1n_2}{2}\right)+c_2\left(\frac{4n_2^{\,3}-3n_1n_2^{\,2}-n_1^{\,3}}{12}\right)+c_3\left(\frac{3n_2^{\,4}-2n_1n_2^{\,3}-n_1^{\,4}}{12}\right)\end{array}\right]\end{array}\right\} \quad (28)$$

Setting the derivative in (28) to zero to have

$$-\left[c_b(\beta d_2)^2\right]n^4-\left[2c_b(\beta d_2)^2\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}\right]n^3$$
$$+\left[\beta d_2\left(\begin{array}{l}-c_b\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}^2+c_b(\beta d_2)^2n_3^{\,2}+2A(\beta d_2)\\+2a(1-r)^{-1}\left\{\begin{array}{l}\left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)-d_2(n_3-n_2)\right]\\+i\left[\begin{array}{l}-\frac{d_2n_1e^{-\lambda(n_2-n_3)}}{2\lambda}+\frac{d_2n_1}{2\lambda}+\frac{d_2n_2e^{-\lambda(n_2-n_3)}}{\lambda}-\frac{d_2n_3}{\lambda}+\frac{d_2e^{-\lambda(n_2-n_3)}}{\lambda^2}-\frac{d_2}{\lambda^2}\\+c_1\left(\frac{n_2^{\,2}-n_1n_2}{2}\right)+c_2\left(\frac{4n_2^{\,3}-3n_1n_2^{\,2}-n_1^{\,3}}{12}\right)+c_3\left(\frac{3n_2^{\,4}-2n_1n_2^{\,3}-n_1^{\,4}}{12}\right)\end{array}\right]\end{array}\right\}\end{array}\right)\right]n^2$$
$$+\left[2\beta d_2\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}\left[2A+c_b(\beta d_2)n_3^{\,2}\right]\right]n$$
$$+\left[\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}^2\left[2A+c_b(\beta d_2)n_3^{\,2}\right]\right]=0 \quad (29)$$

Equation (29) can be viewed as:

$$-Pn^4-Qn^3+Rn^2+Sn+T=0 \quad (30)$$

where

$$P=\left[c_b(\beta d_2)^3\right],\ Q=\left[2c_b(\beta d_2)^2\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}\right]$$

$$R=\left[\beta d_2\left(\begin{array}{l}-c_b\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}^2+c_b(\beta d_2)^2n_3^{\,2}+2A(\beta d_2)\\+2a(1-r)^{-1}\left\{\begin{array}{l}\left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)}-1\right)-d_2(n_3-n_2)\right]\\+i\left[\begin{array}{l}-\frac{d_2n_1e^{-\lambda(n_2-n_3)}}{2\lambda}+\frac{d_2n_1}{2\lambda}+\frac{d_2n_2e^{-\lambda(n_2-n_3)}}{\lambda}-\frac{d_2n_3}{\lambda}+\frac{d_2e^{-\lambda(n_2-n_3)}}{\lambda^2}-\frac{d_2}{\lambda^2}\\+c_1\left(\frac{n_2^{\,2}-n_1n_2}{2}\right)+c_2\left(\frac{4n_2^{\,3}-3n_1n_2^{\,2}-n_1^{\,3}}{12}\right)+c_3\left(\frac{3n_2^{\,4}-2n_1n_2^{\,3}-n_1^{\,4}}{12}\right)\end{array}\right]\end{array}\right\}\end{array}\right)\right]$$

$$S=\left[2\beta d_2\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}\left[2A+c_b(\beta d_2)n_3^{\,2}\right]\right] \text{ and } T=\left[\left\{d_1(n_2-n_1)+d_2\left(n_3[1-\beta]-n_2\right)\right\}^2\left[2A+c_b(\beta d_2)n_3^{\,2}\right]\right]$$

Lemma 1: *If* $d_1 = \frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}$, then $Q = S = T = 0$, and $P > 0$

Proof: Since $d_1 = \frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}$, then

$$Q = \left[2c_b\left(\beta d_2\right)^2\left\{d_1\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\right] = \left[2c_b\left(\beta d_2\right)^2\left\{\frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\right]$$

$$= \left[2c_b\left(\beta d_2\right)^2\left\{d_2\left(n_2 - n_3\left[1-\beta\right]\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\right]$$

$$= 0$$

Also,

$$S = \left[2\beta d_2\left\{d_1\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\left[2A + c_b\left(\beta d_2\right)n_3^{\,2}\right]\right] = \left[2\beta d_2\left\{\frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\left[2A + c_b\left(\beta d_2\right)n_3^{\,2}\right]\right]$$

$$= \left[2\beta d_2\left\{d_2\left(n_2 - n_3\left[1-\beta\right]\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}\left[2A + c_b\left(\beta d_2\right)n_3^{\,2}\right]\right]$$

$$= 0$$

Similarly,

$$T = \left[\left\{d_1\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}^2\left[2A + c_b\left(\beta d_2\right)n_3^{\,2}\right]\right] = \left[\left\{d_1 = \frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}^2\left[2A + c_b\left(\beta d_2\right)n_3^{\,2}\right]\right]$$

$$= 0$$

Finally, for

$$R = \left[\beta d_2\left(\begin{array}{l} -c_b\left\{d_1\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}^2 + c_b\left(\beta d_2\right)^2 n_3^{\,2} + 2A\left(\beta d_2\right) \\ +2a\left(1-r\right)^{-1}\left\{\begin{array}{l}\left[\frac{d_2}{\lambda}\left(e^{-\lambda\left(n_2 - n_3\right)} - 1\right) - d_2\left(n_3 - n_2\right)\right] \\ +i\left[\begin{array}{l} -\frac{d_2 n_1 e^{-\lambda\left(n_2 - n_3\right)}}{2\lambda} + \frac{d_2 n_1}{2\lambda} + \frac{d_2 n_2 e^{-\lambda\left(n_2 - n_3\right)}}{\lambda} - \frac{d_2 n_3}{\lambda} + \frac{d_2 e^{-\lambda\left(n_2 - n_3\right)}}{\lambda^2} - \frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^{\,2} - n_1 n_2}{2}\right) + c_2\left(\frac{4n_2^{\,3} - 3n_1 n_2^{\,2} - n_1^{\,3}}{12}\right) + c_3\left(\frac{3n_2^{\,4} - 2n_1 n_2^{\,3} - n_1^{\,4}}{12}\right)\end{array}\right]\end{array}\right\}\end{array}\right)\right]$$

$$= \left[\beta d_2\left(\begin{array}{l} -c_b\left\{\frac{d_2\left(n_2 - n_3\left[1-\beta\right]\right)}{\left(n_2 - n_1\right)}\left(n_2 - n_1\right) + d_2\left(n_3\left[1-\beta\right] - n_2\right)\right\}^2 + c_b\left(\beta d_2\right)^2 n_3^{\,2} + 2A\left(\beta d_2\right) \\ +2a\left(1-r\right)^{-1}\left\{\begin{array}{l}\left[\frac{d_2}{\lambda}\left(e^{-\lambda\left(n_2 - n_3\right)} - 1\right) - d_2\left(n_3 - n_2\right)\right] \\ +i\left[\begin{array}{l} -\frac{d_2 n_1 e^{-\lambda\left(n_2 - n_3\right)}}{2\lambda} + \frac{d_2 n_1}{2\lambda} + \frac{d_2 n_2 e^{-\lambda\left(n_2 - n_3\right)}}{\lambda} - \frac{d_2 n_3}{\lambda} + \frac{d_2 e^{-\lambda\left(n_2 - n_3\right)}}{\lambda^2} - \frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^{\,2} - n_1 n_2}{2}\right) + c_2\left(\frac{4n_2^{\,3} - 3n_1 n_2^{\,2} - n_1^{\,3}}{12}\right) + c_3\left(\frac{3n_2^{\,4} - 2n_1 n_2^{\,3} - n_1^{\,4}}{12}\right)\end{array}\right]\end{array}\right\}\end{array}\right)\right]$$

$$= \left[\beta d_2 \left(\begin{array}{l} c_b\left(\beta d_2\right)^2 n_3^{\ 2} + 2A\left(\beta d_2\right) \\ +2a\left(1-r\right)^{-1} \left\{ \begin{array}{l} \left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)} - 1\right) - d_2\left(n_3 - n_2\right) \right] \\ +i \left[\begin{array}{l} -\frac{d_2 n_1 e^{-\lambda(n_2-n_3)}}{2\lambda} + \frac{d_2 n_1}{2\lambda} + \frac{d_2 n_2 e^{-\lambda(n_2-n_3)}}{\lambda} - \frac{d_2 n_3}{\lambda} + \frac{d_2 e^{-\lambda(n_2-n_3)}}{\lambda^2} - \frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^{\ 2} - n_1 n_2}{2}\right) + c_2\left(\frac{4n_2^{\ 3} - 3n_1 n_2^{\ 2} - n_1^{\ 3}}{12}\right) + c_3\left(\frac{3n_2^{\ 4} - 2n_1 n_2^{\ 3} - n_1^{\ 4}}{12}\right) \end{array} \right] \end{array} \right\} \end{array} \right) \right] > 0$$

W

Theorem 1: *If the conditions in Lemma 1 are true, then the solution of Equation (30) is*

$$n^* = \sqrt{\frac{R}{P}}$$

Proof. The solution of Equation (30) is as follows

$$-Pn^4 - Qn^3 + Rn^2 + Sn + T = \left(-Pn^2 + R\right)n^2 = 0 \qquad \left[\text{From Lemma 1}\right]$$

$$\Rightarrow \qquad \text{either } n^* = 0, \ -\sqrt{\frac{R}{P}} \ \text{ or } \ \sqrt{\frac{R}{P}}$$

But

$$-\sqrt{\frac{R}{P}} < 0 \qquad \left[\text{since } P, R > 0, \text{ from Lemma 1}\right]$$

Since the length of the cycle is positive, then

$$n^* = \sqrt{\frac{R}{P}} \tag{31}$$

w

Theorem 2: *With the hypotheses of Lemma 1, the total cost function* $Z\left(n\right)$ *is a convex function of* n.

Proof: To check for the sufficient condition of optimality of $Z\left(n\right)$, we look for $\frac{d^2 Z\left(n\right)}{dn^2}$, that is

$$\frac{d^2 Z\left(n\right)}{dn^2} = \frac{2A}{n^3} + \frac{c_b \beta d_2 n_3^{\ 2}}{n^3} + \frac{2a\left(\beta d_2\right)^2\left(1-r\right)^{-1}}{\left(d_1\left(n_2 - n_1\right) + d_2\left(n_3 - n_2\right) + \beta d_2\left(n - n_3\right)\right)^3} \left\{ \begin{array}{l} \left[\frac{d_2}{\lambda}\left(e^{-\lambda(n_2-n_3)} - 1\right) - d_2\left(n_3 - n_2\right) \right] \\ +i \left[\begin{array}{l} -\frac{d_2 n_1 e^{-\lambda(n_2-n_3)}}{2\lambda} + \frac{d_2 n_1}{2\lambda} + \frac{d_2 n_2 e^{-\lambda(n_2-n_3)}}{\lambda} - \frac{d_2 n_3}{\lambda} + \frac{d_2 e^{-\lambda(n_2-n_3)}}{\lambda^2} - \frac{d_2}{\lambda^2} \\ +c_1\left(\frac{n_2^{\ 2} - n_1 n_2}{2}\right) + c_2\left(\frac{4n_2^{\ 3} - 3n_1 n_2^{\ 2} - n_1^{\ 3}}{12}\right) + c_3\left(\frac{3n_2^{\ 4} - 2n_1 n_2^{\ 3} - n_1^{\ 4}}{12}\right) \end{array} \right] \end{array} \right\} \tag{32}$$

Applying Lemma 1 into equation (32), the equation becomes

$$= \frac{2A}{n^3} + \frac{c_b \beta d_2 n_3^{\ 2}}{n^3} + \frac{2a\left(1-r\right)^{-1}}{\left(\beta d_2\right)n^3} \left[\begin{array}{l} -\frac{d_2 n_1 e^{-\lambda(n_2-n_3)}}{2\lambda} + \frac{d_2 n_1}{2\lambda} + \frac{d_2 n_2 e^{-\lambda(n_2-n_3)}}{\lambda} - \frac{d_2 n_3}{\lambda} + \frac{d_2 e^{-\lambda(n_2-n_3)}}{\lambda^2} - \frac{d_2}{\lambda^2} \\ +\left[n_2 - n_1\right]\left\{ c_1\left(\frac{n_2}{2}\right) + c_2\left(\frac{4n_2^{\ 2} + n_1 n_2 + n_1^{\ 2}}{12}\right) + c_3\left(\frac{3n_2^{\ 3} + n_1 n_2^{\ 2} + n_1^{\ 2} n_2 + n_1^{\ 3}}{12}\right) \right\} \end{array} \right]$$

$$> \frac{2A}{n^3} + \frac{c_b \beta d_2 n_3^{\ 2}}{n^3} \geq 0$$

$$\therefore \qquad \frac{d^2 Z(n)}{dn^2} > 0$$

Therefore, we conclude that $Z(n)$ is a minimization function of n.

3.10 EPQ of the Model

The EPQ of the corresponding best cycle length n^* will be computed from:

$$EPQ^* = \text{total demand before deterioration starts} + \text{total demand after deterioration starts} + \text{total number of deteriorated items} + \textit{Total demand during shortage period}$$

$$= c_1(n_2 - n_1) + \frac{c_2(n_2^2 - n_1^2)}{2} + \frac{c_3(n_2^3 - n_1^3)}{3} + d_2(n_3 - n_2) + \frac{d_2}{\lambda}\left(e^{-\lambda(n_2 - n_3)} - 1\right) - d_2(n_3 - n_2) + \frac{\beta d_2}{2}(n - n_3)^2$$

$$\Rightarrow \qquad EPQ^* = c_1(n_2 - n_1) + \frac{c_2(n_2^2 - n_1^2)}{2} + \frac{c_3(n_2^3 - n_1^3)}{3} + \frac{d_2}{\lambda}\left(e^{-\lambda(n_2 - n_3)} - 1\right) + \frac{\beta d_2}{2}(n - n_3)^2 \qquad (33)$$

4.0 Numerical Example

The parameters used in this example are improved from Dari and Sani (2020) by adding the value for the parameter β which was not reflected in their Model.

d_2	2100 units per unit time
A	₦3300 per production run
i	0.1 per unit per unit time
λ	0.2 in a unit time
β	0.9 in a unit time
n_1	0.05753 year
n_2	0.08767 year
n_3	0.13699 year
a	200
r	0.7
c_1	8400
c_2	4
c_3	3

Replacing these values into Equations (27), (31), (26) and (33), we get the best variable price per year, the best cycle length, the cost of backorder and the EPQ value per year respectively as

Z = **₦17907.54** per year**,** ***n**=0.23288** year, C_B = ₦868.9 per year **and** EPQ**= 365.93 units**

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5.0 Sensitivity Analysis

We examine the impact of changes in certain parameters of the system in example 1, and we obtain the following results.

Table 1: ***sensitivity analysis to see the effect of changes in some of the parameters***

Parameters	% change in parameter	% change in Z	% change in n^*	% change in C_B	% change in EPQ
A	+50	36.9649616	14.11764737	80.32653063	1.907449432
	+25	19.12906405	7.058823469	37.22448982	0.88392865
	0	**0**	**0**	**0**	**0**
	-25	-20.75991819	-9.411764625	-40.48979592	-0.961458579
	-50	-43.65084226	-18.82352925	-70.53061224	-1.674832315
λ	+50	0.003302465	0	0	0.070369957
	+25	0.001649864	0	0	0.035143986
	0	**0**	**0**	**0**	**0**
	-25	-0.001647184	0	0	-0.03508933
	-50	-0.003291631	0	0	-0.070124004
r	+50	0.22305	0	0	0
	+25	0.044421	0	0	0
	0	**0**	**0**	**0**	**0**
	-25	-0.01169	0	0	0
	-50	-0.01709	0	0	0
d_2	+50	7.788385	-11.7647	-23.4694	13.66344
	+25	4.413517	-7.05882	-14.1837	6.77358
	0	**0**	**0**	**0**	**0**
	-25	-6.0849	9.411765	13.20408	-6.79684
	-50	-15.1739	28.23529	42.08163	-13.2215
i	+50	0.012603	0	0	0
	+25	0.006302	0	0	0
	0	**0**	**0**	**0**	**0**
	-25	-0.0063	0	0	0
	-50	-0.0126	0	0	0
n_1	+50	-0.01495	0	0	-66.0392
	+25	-0.00723	0	0	-33.0194
	0	**0**	**0**	**0**	**0**
	-25	0.006516	0	0	33.0192
	-50	0.012162	0	0	66.03817
n_2	+50	0.034018	0	0	75.33797
	+25	0.012408	0	0	37.64096
	0	**0**	**0**	**0**	**0**
	-25	-0.00487	0	0	-37.5847
	-50	-0.18471	0	0	-74.6883
n_3	+50	-23.6602	18.82353	-44.8163	38.90614
	+25	-13.1657	9.411765	-24.0612	19.34537
	0	**0**	**0**	**0**	**0**
	-25	16.68105	-7.05882	40.59184	-18.8169
	-50	37.78834	-14.1176	88.08163	-37.3349
c_b	+50	7.793762	-11.7647	-23.4694	-1.16306
	+25	4.416491	-7.05882	-14.1837	-0.74436
	0	**0**	**0**	**0**	**0**
	-25	-6.08878	9.411765	13.20408	1.209598
	-50	-15.1832	28.23529	42.08163	4.373158

From Table 1, these can be observed:

i. As A rises, Z, n^*, C_B and EPQ all rise. Naturally, any increment in the set-up cost will result to an increase to the total variable cost. Therefore, the model will try to increase the best cycle length and the EPQ so as to minimize the set-up cost.

ii. As λ rises, Z rises, EPQ declines whereas n^* and C_B remain constant. This is usually expected because if the rate of deterioration high, the model seeks to produce little items to avoid the spoilage. Thus, many items are going to be produced (EPQ) and so Z is increased.

iii. As r rises, Z rises whereas EPQ, n^* and C_B remain constant. Naturally, the higher the reliability of an item, the higher the cost of such item, thus the total variable cost will be high. Therefore, to reduce the cost, the model seeks to maintain the optimal cycle length, the optimal total backorder cost and the optimal EPQ so as to increase the revenue.

iv. As d_2 rises, Z and EPQ rise whereas n^* and C_B decline. This is obvious case because if the rate of demand of an item is high, the producer tends to produce more (EPQ) and Z will ultimately increase. Therefore, the model pursues to maintain a smaller cycle size and the backorder cost will eventually reduce.

v. As i rises, Z rises while EPQ, n^* and C_B remain constant. That must be the case since if i is high, it will amount to an increase in the total variable cost. Consequently, to lessen the cost, the model pursues a constant cycle size, total backorder cost and EPQ.

vi. As n_1 rises, Z and EPQ decline while n^* and C_B stay constant (may be due to the value of the range we considered in our study). This is so probably because if the total variable cost per unit time is small, the producer will spend more time in producing his goods (few items with high reliability). Therefore, EPQ will also be smaller while the total cost of backorder will be a constant.

vii. As n_2 rises, Z and EPQ rise while n^* and C_B stay constant. This is so probably because if the total variable cost per unit time is high, the producer will produce more goods (with high reliability). Therefore, the model seeks to maintain the optimal cycle length and the best total backorder cost so as to increase the revenue.

viii. As n_3 rises, Z and C_B decline while L^* and EPQ rise. This is probably since as the period of backorder n is high, more items are going to be produced so as to reduce the total variable cost. Hence, the total variable cost and the backorder cost will reduce.

ix. As c_b rises, Z rises, n^*, C_B and EPQ decline. This is obvious because if the unit cost of a backorder is increases, the Z will eventually increases. Therefore, the model pursues a smaller cycle length, lower total backorder cost and fewer items are going to be produce so as to reduce the total variable cost.

6.0 Conclusions

In this paper, an EPQ model for items with non-instantaneous deterioration, reliability consideration and shortages is developed. A numerical example and a careful study were conducted to see the impact of the changes on some of the system parameters. From the results of the study, it has been shown that:

I. The decision variables C_B, Z and n^* are more affected by A, d_2, n_3 and c_b while they are less affected by r, n_1, n_2, i and λ.

II. On the other hand, the decision variable EPQ is more sensitive to the changes in n_1, n_2, n_3 and d_2 while they are less sensitive to changes in r, A, i and λ.

III. EPQ, C_B, Z and n^* all rise with rises in A, but Z rises and n^* declines with rise in both d_2 and c_b.

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