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Assessing the Effect of Non-response in Stratified Random Sampling using Enhanced Ratio Type Estimators under Double Sampling Strategy

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Abstract

In this paper, separate and combined ratio type estimators have been proposed in presence of non-response for estimating the population mean under stratified random sampling when the non-response occurs both on study and the auxiliary variables and the population mean of the auxiliary variable is unknown. The expressions for the biases and mean square errors (MSEs) of the proposed estimators have been derived to the first order of approximation. The proposed estimators have been compared with the other existing estimators using MSE criterion, and the condition under which the proposed estimators perform better than existing estimators have been obtained. In addition to the theoretical research, an empirical study was conducted.

Key Words: Non-response; Double sampling; Separate and Combined estimators; Bias; Mean square error; Relative efficiency

Mathematical Subject Classification: 62D05

1. Introduction:

In a sample survey, it is intended that information will be collected from all of the sample's selected units; however this is usually not practicable due to non-response. Some units may not answer, or may not be reached at all throughout the survey period. Non-response increases the sampling variance of estimates since the effective sample size is reduced from the original needed size, and it also causes estimation bias when the non-respondents differ from respondents in the characteristics observed. Hansen and Hurwitz (1946) were the first to address the problem of non-response in mail surveys, introducing a strategy of sub-sampling non-respondents to estimate the population mean in the context of non-response. In the context of stratified random sampling in the presence of non-response, several authors have made major contributions. Under non-response, Khare (1987) offered various estimating strategies for determining the sample design in each stratum of a stratified population. Chaudhary et al. (2011) proposed various new stratified random sampling allocation schemes based on response or/and non-response rates, and compared them to proportional and Neyman allocation schemes. When the non-response is detected only on the study variable, Chaudhary et al. (2009) introduced a family of estimators for calculating the mean of a stratified population using information from an auxiliary variable. It should be remembered that when the parametric value(s) of an auxiliary variable are readily available, one can easily use that information to estimate the parameter of the study variable; otherwise, the twofold (or two-phase) sampling strategy should be used. In the case of non-response, Chaudhary and Kumar (2015) and Chaudhary and Saurabh (2016) developed different families of estimators of population mean in stratified random sampling using a double sampling strategy. Chaudhary and Saurabh (2017) proposed a combined-type family of estimators for estimating population mean in stratified random sampling under non-response, based on the idea of a two-phase sampling strategy. Chaudhary and Kumar (2017) estimating the population mean in stratified random sampling using double sampling scheme under non-response.

In this present study we have proposed some separate and combined ratio type estimators for estimating the mean using a double sampling technique in case of non-response on both the study and auxiliary variables and the population mean of the auxiliary variable is unknown.

Let N_h be the size of the h^{th} (h=1, 2, …, L) stratum such that $\sum_{h=1}^{L} N_h = N$. Let y_{hi} and x_{hi} be the values of the study variable (y) and the auxiliary variables (x) on the i^{th} unit in the h^{th} stratum, respectively. Let $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ be the sample means that correspond to the population means $\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$ and $\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$ respectively in the h^{th} stratum. When $\bar{X}_h$ is unknown, a preliminary big sample of size $n'_h (n'_h < N_h)$ is required from h^{th} stratum. At the first phase, it is observed that out of n'_h units n'_{h1} units supply and n'_{h2} units do not supply the information on auxiliary variable in the h^{th} stratum. Now we select a sub-sample of r'_{h2} units from n'_{h2} units by simple random sampling WOR scheme $r'_{h2} = \frac{n'_{h2}}{k'_h} (k'_h > 1)$ where k'_h is the inverse sampling rate at the first phase and collect the information from all the n'_{h2} units. In the second phase, a subsample of size $n_h (< n'_h)$ is selected using the simple random sampling without replacement (*SRSWOR*) scheme, such that $\sum_{h=1}^{L} n_h = n$ and data is collected on y_h and x_h. Let $\bar{y}_{st} = \sum_{i=1}^{n_h} W_h \bar{y}_h$ and $\bar{x}_{st} = \sum_{i=1}^{n_h} W_h \bar{x}_h$ be the sample means obtained from the second phase of sampling and $\bar{x}'_{st} = \sum_{h=1}^{L} W_h \bar{x}'_h$ be the sample means obtained from the first phase of sampling, where $\bar{y}_h$ and $\bar{x}_h$ are the sample means of y and x in the h^{th} stratum, respectively, and $W_h = \frac{N_h}{N}$ are the stratum weights that are known. It is presumed that in the first phase, a sample of size $n'_h (n'_h < N_h)$ units are picked from the hth stratum using SRSWOR and the auxiliary variables are observed. In the second phase, a subsample of size $n_h (n_h < n'_h)$ units are chosen, and observations are performed on both the study and the auxiliary variablesfor the second phase sample of size n_h, it is assumed that n_{h1} units result in responses and n_{h2} units result in non-responses. Let N_{h1} and N_{h2}be the number of population units in the response and non-response categories, respectively. Using the Hansen and Hurwitz (1946) technique, a random subsample of size $r_{h2} (r_{h2} < n_{h2})$ units are chosen, and a response is acquired via interview by assuming$r_{h2} = \frac{n_{h2}}{k_h} (k_h > 1)$.

Following is the Hansen and Hurwitz (1946) estimator,

Thus, the estimate of $\bar{X}_h$at the first phase is given by

$$\bar{x}_h^{*\prime} = \frac{n'_{h1}\bar{x}'_{h1} + n'_{h2}\bar{x}'_{rh2}}{n'_h}$$

Where $\bar{x}'_{h1}$ and $\bar{x}'_{rh2}$ are respectively the means based on n'_{h1} responding units and r'_{h2}non-responding units in the h^{th} stratum. Hence the estimator $\bar{X}$ at the first phase is given by

$$\bar{x}_{st}^{*\prime} = \sum_{h=1}^{L} W_h \bar{x}_h^{*\prime}$$

The variance of the estimator $\bar{x}_{st}^{*\prime}$ is given as

$$Var(\bar{x}_{st}^{*\prime}) = \sum_{h=1}^{L} W_h^2 \left\{ \left(\frac{1}{n'_h} - \frac{1}{N_h} \right) S_{hx}^2 + \frac{(k'_h - 1)}{n'_h} W_{h2} S_{hx(2)}^2 \right\}$$

Where S_{hx}^2 and $S_{hx(2)}^2$ are the population mean squares of entire group and non-response group respectively in the hth stratum for the auxiliary variable. W_{h2} is the non-response rate in the h^{th} stratum

The Hansen and Hurwitz (1946) type estimators of $\bar{Y}$ and $\bar{X}$ at the second phase are respectively given by

$\bar{y}_{st}^* = \sum_{h=1}^{L} W_h \bar{y}_h^*$ and $\bar{x}_{st}^* = \sum_{h=1}^{L} W_h \bar{x}_h^*$ respectively be the stratified sample means of y and x in the h^{th} stratum under non-response,

where $\bar{y}_h^* = \frac{n_{h1}\bar{y}_{nh1}+n_{h2}\bar{y}_{rh2}}{n_h}$, $\bar{x}_h^* = \frac{n_{h1}\bar{x}_{nh1}+n_{h2}\bar{x}_{rh2}}{n_h}$, and $(\bar{y}_{nh1}, \bar{x}_{nh1})$ and $(\bar{y}_{rh2}, \bar{x}_{rh2})$ respectively be the sample means based on n_{h1} and r_{h2} units.
The variance of $\bar{y}_{st}^*$ and $\bar{x}_{st}^*$ are respectively given by

$$Var(\bar{y}_{st}^*) = \bar{y}_1^* = \sum_{h=1}^{L} W_h^2 \left\{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{hy}^2 + \frac{(k_h-1)}{n_h} W_{h2} S_{hy(2)}^2\right\} \quad (1)$$

$$Var(\bar{x}_{st}^*) = \sum_{h=1}^{L} W_h^2 \left\{\left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{hx}^2 + \frac{(k_h-1)}{n_h} W_{h2} S_{hx(2)}^2\right\}$$

Where S_{hy}^2 and $S_{hy(2)}^2$ are the population mean squares of entire group and non-response group respectively in the h[th] stratum for the study variable.

To obtain the bias and MSE of the proposed estimators let us assume,

$\xi_{0h}^* = \frac{\bar{y}_h^* - \bar{Y}_h}{\bar{Y}_h}$: $\xi_{1h}^* = \frac{\bar{x}_h^* - \bar{X}_h}{\bar{X}_h}$: $\xi_{1h}^{*\prime} = \frac{\bar{x}_h^{*\prime} - \bar{X}_h}{\bar{X}_h}$

$\xi_{0st}^* = \frac{\bar{y}_{st}^* - \bar{Y}}{\bar{Y}}$: $\xi_{1st}^* = \frac{\bar{x}_{st}^* - \bar{X}}{\bar{X}}$: $\xi_{1st}^{*\prime} = \frac{\bar{x}_{st}^{*\prime} - \bar{X}}{\bar{X}}$

Expectation of relative error terms for Separate estimators.

$E(\xi_{0h}^*) = E(\xi_{1h}^*) = E(\xi_{1h}^{*\prime}) = 0$ and

$$E(\xi_{0h}^{*2}) = \frac{1}{\bar{Y}_h^2}\left[\theta_h S_{hy}^2 + \frac{W_{2h}(k_h-1)}{n_h} S_{hy(2)}^2\right] = A_h; \qquad E(\xi_{1h}^{*2}) = \frac{1}{\bar{X}_h^2}\left[\theta_h S_{hx}^2 + \frac{W_{2h}(k_h-1)}{n_h} S_{hx(2)}^2\right] = B_h$$

$$E(\xi_{0h}^* \xi_{1h}^*) = \frac{1}{\bar{Y}_h \bar{X}_h}\left[\begin{array}{c}\theta_h S_{hxy} \\ + \frac{w_{2h}(k_h-1)}{n_h} S_{hxy(2)}\end{array}\right] = C_h; \qquad E(\xi_{1h}^{*\prime 2}) = E(\xi_{1h}^* \xi_{1h}^{*\prime}) = \frac{1}{\bar{X}_h^2}\left[\begin{array}{c}\theta_h' S_{hx}^2 \\ + \frac{W_{2h}(K_h'-1)}{n_h'} S_{hx(2)}^2\end{array}\right] = G_h$$

$$E(\xi_{0h}^* \xi_{1h}^{*\prime}) = \frac{1}{\bar{Y}_h \bar{X}_h}\left[\begin{array}{c}\theta_h' S_{hxy} \\ + \frac{W_{2h}(K_h'-1)}{n_h'} S_{hxy(2)}\end{array}\right] = H_h$$

Expectation of relative error terms for combined estimators.

$E(\xi_{0st}^*) = E(\xi_{1st}^*) = E(\xi_{1st}^{*\prime}) = 0$and

$$E(\xi_{0st}^{*2}) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L} W_h^2 \left[\begin{array}{c}\theta_h S_{hy}^2 \\ + \frac{W_{2h}(k_h-1)}{n_h} S_{hy(2)}^2\end{array}\right] = A; \qquad E(\xi_{1st}^{*2}) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 \left[\begin{array}{c}\theta_h S_{hx}^2 \\ + \frac{W_{2h}(k_h-1)}{n_h} S_{hx(2)}^2\end{array}\right] = B$$

$$E(\xi_{0st}^* \xi_{1st}^*) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2 \left[\begin{array}{c}\theta_h S_{hxy} \\ + \frac{w_{2h}(k_h-1)}{n_h} S_{hxy(2)}\end{array}\right] = C;$$

$$E(\xi_{1st}^{*\prime 2}) = E(\xi_{1st}^* \xi_{1st}^{*\prime}) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L} W_h^2 \left[\theta_h' S_{hx}^2 + \frac{W_{2h}(K_h'-1)}{n_h'} S_{hx(2)}^2\right] = G$$

$$E(\xi_{0st}^* \xi_{1st}^{*\prime}) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L} W_h^2 \left[\theta_h' S_{hxy} + \frac{W_{2h}(K_h'-1)}{n_h'} S_{hxy(2)}\right] = H$$

Where

$$\theta_h = \frac{N_h - n_h}{N_h n_h}, \quad \text{and} \theta'_h = \frac{N_h - n'_h}{N_h n'_h}$$

2. Estimators in Literature:

Let's discuss a few of the mean estimators that are already used in simple random sampling schemes under non-response before introducing our proposed estimator.

Chaudhary and Kumar (2018) proposed estimator for estimating the population mean in stratified sampling with one auxiliary variable under non-response using two phase sampling scheme.

1. When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the separate ratio estimator is given as,

$$\bar{y}_2^* = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h^*} \bar{x}_h'^* \tag{2}$$

The MSE of $\bar{y}_2^*$ is given as

$$MSE(\bar{y}_2^*) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \{A_h + B_h - G_h - 2(C_h - H_h)\} \tag{3}$$

2. When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the combined ratio estimator is given as,

$$\bar{y}_3^* = \frac{\bar{y}_{st}^*}{\bar{x}_{st}^*} \bar{x}_{st}'^* \tag{4}$$

The MSE of $\bar{y}_3^*$ is given as

$$MSE(\bar{y}_3^*) = \bar{Y}^2 \{A + B - G - 2(C - H)\} \tag{5}$$

3. When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the separate product estimator is given as.

$$\bar{y}_4^* = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h'^*} \bar{x}_h^* \tag{6}$$

The MSE of $\bar{y}_4^*$ is given as

$$MSE(\bar{y}_4^*) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \{A_h + B_h - G_h + 2(C_h - H_h)\} \tag{7}$$

4.When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the combined product estimator is given as,

$$\bar{y}_5^* = \frac{\bar{y}_{st}^*}{\bar{x}_{st}'^*} \bar{x}_{st}^* \tag{8}$$

The MSE of $\bar{y}_5^*$ is given as

$$MSE(\bar{y}_5^*) = \bar{Y}^2\{A + B - G + 2(C - H)\} \tag{9}$$

5. When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the separate regression estimator is given as,

$$\bar{y}_6^* = \sum_{h=1}^{L} W_h \left[\bar{y}_h^* + b_{hyx}^*(\bar{x}_h'^* - \bar{x}_h^*)\right] \tag{10}$$

The MSE of$\bar{y}_6^*$ is given as

$$MSE(\bar{y}_6^*) = \sum_{h=1}^{L} W_h^2 \left\{\bar{Y}_h^2 A_h + \beta_{hyx}^2 \bar{X}_h^2 (B_h - G_h) + 2\beta_{hyx}\bar{Y}_h\bar{X}_h(H_h - C_h)\right\} \tag{11}$$

6. When there is a non-response on both the study variable and the auxiliary variable and the population mean of the auxiliary variable is unknown, the combined regression estimator is given as,

$$\bar{y}_7^* = \bar{y}_{st}^* + b_{yx}^*(\bar{x}_{st}'^* - \bar{x}_{st}^*) \tag{12}$$

The MSE of$\bar{y}_7^*$ is given as

$$MSE(\bar{y}_7^*) = \bar{Y}^2 A + \beta_{yx}^2 \bar{X}^2 (B - G) + 2\beta_{yx}\bar{Y}\bar{X}(H - C) \tag{13}$$

3. PROPOSED SEPARATE AND COMBINED RATIO ESTIMATORS

Motivated from classical ratio estimator and Srivenkataramana (1980) transformation, we have to develop a separate ratio estimator $\left(\bar{y}_{zrsp}^*\right)$ in Theorem 3.1 and combined ratio estimator $\left(\bar{y}_{zrcp}^*\right)$ in Theorem 3.2 in presence of non-response under two-phase sampling scheme when there is non-response on both the study variable Y and the auxiliary variable X, and the auxiliary variable's population mean is unknown.

Theorem: 3.1

$$\bar{y}_{zrsp}^* = \sum_{h=1}^{L} W_h \left\{\bar{y}_h^* \frac{(n_h' - n_h)\bar{x}_h^{*\prime}}{n_h'\bar{x}_h^{*\prime} - n_h\bar{x}_h^*} + \sigma^* \left(\bar{x}_h^{*\prime} - \frac{(n_h'\bar{x}_h^{*\prime} - n_h\bar{x}_h^*)}{n_h' - n_h}\right)\right\} \tag{14}$$

The Bias, MSE and minimum MSE of the proposed estimator $\bar{y}_{zrsp}^*$ are given as

$$Bias\left(\bar{y}_{zrsp}^*\right) = \sum_{h=1}^{L} W_h \bar{Y}_h \left\{\begin{matrix}(t_{1h}^2 - t_{1h})G_h + (t_{2h} - 2t_{1h}t_{2h})G_h + \\ t_{2h}^2 B_h + (1 - t_{1h})H_h + t_{2h}C_h\end{matrix}\right\} \tag{15}$$

$$MSE\left(\bar{y}_{zrsp}^*\right) = \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 Z_{7h} + \sigma^* \bar{X}_h^2 Z_{8h} + 2\sigma^* \bar{X}_h \bar{Y}_h Z_{9h}\right] \tag{16}$$

$$MSE\left(\bar{y}_{zrsp}^*\right)_{min} = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[Z_{7h} - \frac{Z_{9h}^2}{Z_{8h}}\right] \tag{17}$$

Proof:

The proposed estimator $\bar{y}_{zrsp}^*$ can be written as

$$\bar{y}^{*}_{zrsp} = \sum_{h=1}^{L} W_h J_h \tag{18}$$

Where:

$$J_h = \bar{y}^{*}_h \frac{(n'_h - n_h)\bar{x}^{*\prime}_h}{n'_h \bar{x}^{*\prime}_h - n_h \bar{x}^{*}_h} + \sigma^* \left(\bar{x}^{*\prime}_h - \frac{(n'_h \bar{x}^{*\prime}_h - n_h \bar{x}^{*}_h)}{n'_h - n_h} \right) \tag{19}$$

For easy simplification we let

$$\bar{x}^{R^*}_h = \frac{n'_h \bar{x}^{*\prime}_h - n_h \bar{x}^{*}_h}{(n'_h - n_h)}$$

Therefore equation (19) can be written as

$$J_h = \bar{y}^{*}_h \frac{\bar{x}^{*\prime}_h}{\bar{x}^{R^*}_h} + \sigma^* \left(\bar{x}^{*\prime}_h - \bar{x}^{R^*}_h \right) \tag{20}$$

First, we solve $\bar{x}^{R^*}_h$ using error terms we get

$$\bar{x}^{R^*}_h = \frac{n'_h \bar{x}^{*\prime}_h - n_h \bar{x}^{*}_h}{(n'_h - n_h)}$$

$$\bar{x}^{R^*}_h = \frac{n'_h (1 + \xi^{*\prime}_{1h})\bar{X}_h - n_h (1 + \xi^{*}_{1h})\bar{X}_h}{(n'_h - n_h)}$$

$$\bar{x}^{R^*}_h = \bar{X}_h \left\{ 1 + \frac{n'_h}{(n'_h - n_h)} \xi^{*\prime}_{1h} - \frac{n_h}{(n'_h - n_h)} \xi^{*}_{1h} \right\}$$

$$\bar{x}^{R^*}_h = \bar{X}_h \{ 1 + t_{1h} \xi^{*\prime}_{1h} - t_{2h} \xi^{*}_{1h} \}$$

Where:

$t_{1h} = \frac{n'_h}{(n'_h - n_h)}$ and $t_{2h} = \frac{n_h}{(n'_h - n_h)}$

Therefore, the equation (20) becomes

$$J_h = \bar{Y}_h (1 + \xi^{*}_{0h})(1 + \xi^{*\prime}_{1h})(1 + t_{1h}\xi^{*\prime}_{1h} - t_{2h}\xi^{*}_{1h})^{-1} + \sigma^* [(1 + \xi^{*\prime}_{1h})\bar{X}_h - \bar{X}_h (1 + t_{1h}\xi^{*\prime}_{1h} - t_{2h}\xi^{*}_{1h})]$$

$$J_h - \bar{Y}_h = \left\{ \begin{array}{c} \bar{Y}_h \left\{ \begin{array}{c} (1 - t_{1h})\xi^{*\prime}_{1h} + (t^2_{1h} - t_{1h})\xi^{*\prime 2}_{1h} \\ +(t_{2h} - 2t_{1h}t_{2h})\xi^{*\prime}_{1h}\xi^{*}_{1h} + t_{2h}\xi^{*}_{1h} + t^2_{2h}\xi^{*2}_{1h} \\ +\xi^{*}_{0h} + (1 - t_{1h})\xi^{*}_{0h}\xi^{*\prime}_{1h} + t_{2h}\xi^{*}_{0h}\xi^{*}_{1h} \end{array} \right\} \\ +\sigma^* \bar{X}_h \{ (1 - t_{1h})\xi^{*\prime}_{1h} + t_{2h}\xi^{*}_{1h} \} \end{array} \right\} \tag{21}$$

Taking Expectation on equation (21) both sides we get the bias of J_h

$$Bias(J_h) = \bar{Y}_h \left\{ \begin{array}{c} (t^2_{1h} - t_{1h})G_h + (t_{2h} - 2t_{1h}t_{2h})G_h \\ +t^2_{2h}B_h + (1 - t_{1h})H_h + t_{2h}C_h \end{array} \right\} \tag{22}$$

The Bias of the proposed estimator $\bar{y}^{*}_{zrsp}$ is given by using equation (18) we get

$$Bias(\bar{y}^{*}_{zrsp}) = \sum_{h=1}^{L} W_h \, Bias(J_h) \tag{23}$$

Substituting equation (22) in equation (23) we get the bias of proposed estimator $\bar{y}^{*}_{zrsp}$

$$Bias(\bar{y}^{*}_{zrsp}) = \sum_{h=1}^{L} W_h \bar{Y}_h \begin{Bmatrix} (t_{1h}^2 - t_{1h})G_h + (t_{2h} - 2t_{1h}t_{2h})G_h + \\ t_{2h}^2 B_h + (1 - t_{1h})H_h + t_{2h}C_h \end{Bmatrix} \tag{24}$$

Squaring equation (21) and then taking expectation we get the MSE of J_h up to first order of approximation

$$MSE(J_h) = \begin{Bmatrix} \bar{Y}_h^2 [A_h + t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h + 2t_{2h}C_h + 2(1 - t_{1h})H_h] \\ +\sigma^{*2} \bar{X}_h^2 [t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h] \\ +2\sigma^* \bar{X}_h \bar{Y}_h [t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h + t_{2h}C_h + (1 - t_{1h})H_h] \end{Bmatrix} \tag{25}$$

For simplicity we write the MSE of J_h as

$$MSE(J_h) = [\bar{Y}_h^2 Z_{7h} + \sigma^{*2} \bar{X}_h^2 Z_{8h} + 2\sigma^* \bar{X}_h \bar{Y}_h Z_{9h}] \tag{26}$$

Where:

$Z_{7h} = A_h + t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h + 2t_{2h}C_h + 2(1 - t_{1h})H_h$

$Z_{8h} = t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h$

$Z_{9h} = t_{2h}^2 B_h + (1 - t_{1h})^2 G_h + 2(1 - t_{1h})t_{2h}G_h + t_{2h}C_h + (1 - t_{1h})H_h$

The MSE of the proposed estimator $\bar{y}^{*}_{Zrsp}$ is given by using equation (18) we get

$$MSE(\bar{y}^{*}_{zrsp}) = \sum_{h=1}^{L} W_h^2 \, MSE(J_h) \tag{27}$$

Substituting equation (26) in equation (27) we get the bias of proposed estimator $\bar{y}^{*}_{zrsp}$

$$MSE(\bar{y}^{*}_{zrsp}) = \sum_{h=1}^{L} W_h^2 [\bar{Y}_h^2 Z_{7h} + \sigma^{*2} \bar{X}_h^2 Z_{8h} + 2\sigma^* \bar{X}_h \bar{Y}_h Z_{9h}] \tag{28}$$

For obtaining the optimum values of σ^* , differentiating equation (28)) w.r.t σ^* and equating to zero we have

$$\frac{\partial MSE(\bar{y}^{*}_{zrsp})}{\partial \sigma^*} = 0$$

$$\sigma^*_{opt} = -\frac{\bar{Y}_h Z_{9h}}{\bar{X}_h Z_{8h}} \tag{29}$$

Using the value of σ^*_{opt} in equation (28), we get the minimal MSE of the proposed estimators

$$MSE(\bar{y}^{*}_{zrsp})_{min} = \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 Z_{7h} + \left(-\frac{\bar{Y}_h Z_{9h}}{\bar{X}_h Z_{8h}}\right)^2 \bar{X}_h^2 Z_{2h} + 2\left(-\frac{\bar{Y}_h Z_{9h}}{\bar{X}_h Z_{8h}}\right) \bar{X}_h \bar{Y}_h Z_{9h} \right]$$

$$MSE(\bar{y}^{*}_{zrsp})_{min} = \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h^2 Z_{7h} + \bar{Y}_h^2 \frac{Z_{9h}^2}{Z_{8h}} - 2\bar{Y}_h^2 \frac{Z_{9h}^2}{Z_{8h}} \right]$$

$$MSE(\bar{y}^{*}_{zrsp})_{min} = \sum_{h=1}^{L} W_h^2 \, \bar{Y}_h^2 \left[Z_{7h} - \frac{Z_{9h}^2}{Z_{8h}} \right] \tag{30}$$

Theorem 3.2

$$\bar{y}_{zrcp}^{*} = \bar{y}_{st}^{*}\frac{(n'-n)\bar{x}_{st}^{*\prime}}{n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*}} + \gamma^{*}\left(\bar{x}_{st}^{*\prime} - \frac{(n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*})}{n'-n}\right) \tag{31}$$

The bias and MSE of the proposed estimator $\bar{y}_{zrcp}^{*}$ are given as

$$Bias\left(\bar{y}_{zrcp}^{*}\right) = \bar{Y}\{(t_1^2-t_1)G + (t_2-2t_1t_2)G + t_2^2B + (1-t_1)H + t_2C\} \tag{32}$$

$$MSE\left(\bar{y}_{zrcp}^{*}\right) = \bar{Y}^2Z_7 + \gamma^{*2}\bar{X}^2Z_8 + 2\gamma^{*}\bar{X}\bar{Y}Z_9 \tag{33}$$

$$MSE\left(\bar{y}_{zrcp}^{*}\right)_{min} = \bar{Y}^2\left[Z_7 - \frac{Z_9^2}{Z_8}\right] \tag{34}$$

Proof:

$$\bar{y}_{zrcp}^{*} = \bar{y}_{st}^{*}\frac{(n'-n)\bar{x}_{st}^{*\prime}}{n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*}} + \gamma^{*}\left(\bar{x}_{st}^{*\prime} - \frac{(n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*})}{n'-n}\right) \tag{35}$$

For easy simplification we let

$$\bar{x}_{st}^{S^*} = \frac{n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*}}{(n'-n)}$$

Therefore equation (35) can be written as

$$\bar{y}_{zrcp}^{*} = \bar{y}_{st}^{*}\frac{\bar{x}_{st}^{*\prime}}{\bar{x}_{st}^{S^*}} + \gamma^{*}\left(\bar{x}_{st}^{*\prime} - \bar{x}_{st}^{S^*}\right) \tag{36}$$

First, we solve $\bar{x}_{st}^{S^*}$ in error terms we get

$$\bar{x}_{st}^{S^*} = \frac{n'\bar{x}_{st}^{*\prime}-n\bar{x}_{st}^{*}}{(n'-n)}$$

$$\bar{x}_{st}^{S^*} = \frac{n'(1+\xi_{1st}^{*\prime})\bar{X} - n(1+\xi_{1st}^{*})\bar{X}}{(n'-n)}$$

$$\bar{x}_{st}^{S^*} = \bar{X}\left\{1 + \frac{n'}{(n'-n)}\xi_{1st}^{*\prime} - \frac{n}{(n'-n)}\xi_{1st}^{*}\right\}$$

$$\bar{x}_{st}^{S^*} = \bar{X}\{1 + t_{1h}\xi_{1st}^{*\prime} - t_{2h}\xi_{1st}^{*}\}$$

Where:

$t_1 = \frac{n'}{(n'-n)}$ and $t_2 = \frac{n}{(n'-n)}$

Therefore, the estimator $\bar{y}_{zrcp}^{*}$ in equation (31) using error terms is given as

$$\bar{y}_{zrcp}^{*} = \begin{Bmatrix} \bar{Y}(1+\xi_{0st}^{*})\left(1+\xi_{1st}^{*\prime}\right)\left(1+t_1\xi_{1st}^{*\prime} - t_2\xi_{1st}^{*}\right)^{-1} \\ +\gamma^{*}[(1+\xi_{1st}^{*\prime})\bar{X} - \bar{X}(1+t_1\xi_{1st}^{*\prime} - t_2\xi_{1st}^{*})] \end{Bmatrix}$$

$$\bar{y}_{zrcp}^{*}-\bar{Y}=\bar{Y}\begin{Bmatrix}(1-t_1)\xi_{1st}^{*'}+(t_1^2-t_1)\xi_{1st}^{*'2}+(t_2-2t_1t_2)\xi_{1st}^{*'}\xi_{1st}^{*}\\+t_2\xi_{1st}^{*}+t_2^2\xi_{1st}^{*2}+\xi_{0st}^{*}+(1-t_1)\xi_{0st}^{*}\xi_{1st}^{*'}+t_2\xi_{0st}^{*}\xi_{1st}^{*}\\+\mu^{*}\bar{X}\{(1-t_1)\xi_{1st}^{*'}+t_2\xi_{1st}^{*}\}\end{Bmatrix} \tag{37}$$

Taking expectation on equation (37) on both sides we get the bias of the proposed estimator $\bar{y}_{zrcp}^{*}$ up to first order of approximation and is given as

$$Bias(\bar{y}_{zrcp}^{*})=\bar{Y}\begin{Bmatrix}(t_1^2-t_1)G+(t_2-2t_1t_2)G\\+t_2^2B+(1-t_1)H+t_2C\end{Bmatrix} \tag{38}$$

Squaring equation (37) and then taking expectation we get the MSE of $\bar{y}_{zrcp}^{*}$

$$MSE(\bar{y}_{zrcp}^{*})=\begin{Bmatrix}\bar{Y}^2[A+t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G+2t_2C+2(1-t_1)H]\\+\gamma^{*2}\bar{X}^2[t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G]\\+2\gamma^{*}\bar{X}\bar{Y}[t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G+t_2C+(1-t_1)H]\end{Bmatrix} \tag{39}$$

$$MSE(\bar{y}_{zrcp}^{*})=\bar{Y}^2Z_7+\gamma^{*2}\bar{X}^2Z_8+2\gamma^{*}\bar{X}\bar{Y}Z_9 \tag{40}$$

Where:

$Z_7=A+t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G+2t_2C+2(1-t_1)H$

$Z_8=t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G$

$Z_9=t_2^2B+(1-t_1)^2G+2(1-t_1)t_2G+t_2C+(1-t_1)H$

For obtaining the optimum values of γ^{*} , differentiating equation (40) w.r.t γ^{*} and equating to zero we have

$$\frac{\partial MSE(\bar{y}_{zrcp}^{*})}{\partial\gamma^{*}}=0$$

$$\gamma_{opt}^{*}=-\frac{\bar{Y}Z_9}{\bar{X}Z_8} \tag{41}$$

Using the value of γ_{opt}^{*} in equation (40), we get the minimal MSE of the proposed estimators

$$MSE(\bar{y}_{zrcp}^{\prime*})_{min}=\bar{Y}^2Z_7+\left(-\frac{\bar{Y}Z_9}{\bar{X}Z_8}\right)^2\bar{X}^2Z_8+2\left(-\frac{\bar{Y}Z_9}{\bar{X}Z_8}\right)\bar{X}\bar{Y}Z_9$$

$$MSE(\bar{y}_{zrcp}^{\prime*})_{min}=\bar{Y}^2Z_7+\bar{Y}^2\frac{Z_9^2}{Z_8}-2\bar{Y}^2\frac{Z_9^2}{Z_8}$$

$$MSE(\bar{y}_{zrcp}^{\prime*})_{min}=\bar{Y}^2\left[Z_7-\frac{Z_9^2}{Z_8}\right] \tag{42}$$

4. Efficiency Comparison: In this section we compare the efficiency of separate and combined ratio estimator with the existing estimators when there is a non-response on both the study and the auxiliary variable and the population mean for the auxiliary variable is unknown.

4.1. For Separate Ratio Estimator

1.y_{zrsp}^{*} Perform better than $\bar{y}_1^{*}$ if:

$$MSE(y_{zrsp}^{*})<MSE(\bar{y}_1^{*})$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right]-\bar{Y}^2A<0$$

2.y_{zrsp}^* Perform better than $\bar{y}_2^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_2^*)$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[\left\{Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right\}-\{A_h+B_h-G_h-2(C_h-H_h)\}\right]<0$$

3.y_{zrsp}^* Perform better than $\bar{y}_3^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_3^*)$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right]-\bar{Y}^2\{A+B-G-2(C-H)\}<0$$

4.y_{zrsp}^* Perform better than $\bar{y}_4^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_4^*)$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[\left\{Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right\}+\{A_h+B_h-G_h-2(C_h-H_h)\}\right]<0$$

5.y_{zrsp}^* Perform better than $\bar{y}_5^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_5^*)$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[Z_{1h}-\frac{Z_{3h}^2}{Z_{2h}}\right]-\bar{Y}^2\{A+B-G+2(C-H)\}<0$$

6.y_{zrsp}^* Perform better than $\bar{y}_6^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_6^*)$$

$$\sum_{h=1}^{L} W_h^2\left\{\bar{Y}_h^2\left[Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right]-\left\{\bar{Y}_h^2A_h+\beta_{hyx}^2\bar{X}_h^2(B_h-G_h)+2\beta_{hyx}\bar{Y}_h\bar{X}_h(H_h-C_h)\right\}\right\}$$

7.y_{zrsp}^* Perform better than $\bar{y}_7^*$ if:

$$MSE\left(y_{zrsp}^*\right)<MSE(\bar{y}_1^*)$$

$$\sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\left[Z_{7h}-\frac{Z_{9h}^2}{Z_{8h}}\right]-\left\{\bar{Y}^2A+\beta_{yx}^2\bar{X}^2(B-G)+2\beta_{yx}\bar{Y}\bar{X}(H-C)\right\}<0$$

4.2 For Combined Ratio Estimator

1.y^*_{zrcp} Perform better than $\bar{y}^*_1$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_1)$$

$$Z_7 - \frac{Z_9^2}{Z_8} - A < 0$$

2.y^*_{zrcp} Perform better than $\bar{y}^*_2$ if

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_2)$$

$$\bar{Y}^2\left[Z_7 - \frac{Z_9^2}{Z_8}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\{A_h + B_h - G_h - 2(C_h - H_h)\}$$

3.y^*_{zrcp} Perform better than $\bar{y}^*_3$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_3)$$

$$\left[Z_7 - \frac{Z_9^2}{Z_8} - \left(A + B - G - 2(C - H)\right)\right] < 0$$

4.y^*_{zrcp} Perform better than $\bar{y}^*_4$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_4)$$

$$\bar{Y}^2\left[Z_7 - \frac{Z_9^2}{Z_8}\right] - \sum_{h=1}^{L} W_h^2\,\bar{Y}_h^2\{A_h + B_h - G_h + 2(C_h - H_h)\} < 0$$

5.y^*_{zrcp} Perform better than $\bar{y}^*_5$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_5)$$

$$\left[Z_7 - \frac{Z_9^2}{Z_8} - \{A + B - G + 2(C - H)\}\right] < 0$$

6.y^*_{zrcp} Perform better than $\bar{y}^*_6$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_6)$$

$$\bar{Y}^2\left[Z_7 - \frac{Z_9^2}{Z_8}\right] - \sum_{h=1}^{L} W_h^2\left\{\bar{Y}_h^2 A_h + \beta_{hyx}^2\bar{X}_h^2(B_h - G_h) + 2\beta_{hyx}\bar{Y}_h\bar{X}_h(H_h - C_h)\right\} < 0$$

7.y^*_{zrcp} Perform better than $\bar{y}^*_7$ if:

$$MSE\left(y^*_{zrcp}\right) < MSE(\bar{y}^*_7)$$

$$\bar{Y}^2\left[Z_7 - \frac{Z_9^2}{Z_8}\right] - \left(\bar{Y}^2 A + \beta_{yx}^2\bar{X}^2(B - G) + 2\beta_{yx}\bar{Y}\bar{X}(H - C)\right) < 0$$

5. Numerical Illustration: We assessed the performance of proposed estimators in terms of the MSE in comparison to other competing estimators. We choose actual dataset for this purpose:

.

Population: (Source: Koyuncu and Kadilar (2009)). We consider No. of teachers as study variable *(Y)* and No. of students in primary and secondary schools as auxiliary variable *(X)*, for 923 districts at six 6 regions (1: Marmara, 2: Agean, 3: Mediterranean, 4: Central Anatolia, 5: Black Sea, and 6: East and Southeast Anatolia) in Turkey in 2007.

Table 1: The descriptive statistics for Population are

h	1	2	3	4	5	6
N_h	127	117	103	170	205	201
n_{1h}	70	50	75	95	70	90
n_{2h}	31	21	29	38	22	39
S_{yh}	883.84	644.92	1033.40	810.58	403.65	771.72
S_{xh}	30486.7	15180.77	27549.78	18218.93	8497.77	23094.14
$\bar{Y}_h$	703.74	413	573.17	424.66	267.03	393.84
$\bar{X}_h$	20804.59	9211.79	14309.3	9478.85	5569.95	12997.59
ρ_{yxh}	0.936	0.996	0.994	0.983	0.989	0.965
$\mathbf{W_{2h} = 10\%\ Non-response}$						
$S_{hy(2)}$	510.57	386.77	1872.88	1603.3	264.19	497.84
$S_{hx(2)}$	9446.93	9198.29	52429.99	34794.9	4972.56	12485.1
$\rho_{hxy(2)}$	0.9961	0.9975	0.9998	0.9741	0.995	0.9284
$\mathbf{W_{2h} = 20\%\ Non-response}$						
$S_{hy(2)}$	396.77	406.15	1654.40	1333.35	335.83	903.91
$S_{hx(2)}$	7439.16	8880.46	45784.78	29219.3	6540.43	28411.44
$\rho_{hxy(2)}$	0.9954	0.9931	0.9761	0.9761	0.9966	0.9869

Table 2: *MSE* of proposed and existing estimators

$\mathbf{W_{2h} = 10\%\ Non-response}$					
Estimators	$\mathbf{K_h = K'_h = 2}$	$\mathbf{K_h = K'_h = 2.5}$	$\mathbf{K_h = K'_h = 3}$	$\mathbf{K_h = K'_h = 3.5}$	$\mathbf{K_h = K'_h = 4}$
$\bar{y}_1^*$	2769.69	2996.32	3222.95	3449.58	3676.20
$\bar{y}_2^*$	866.07	962.42	1058.77	1155.12	1251.47
$\bar{y}_3^*$	917.13	1014.39	1111.65	1208.92	1306.18
$\bar{y}_4^*$	8900.42	9533.26	10166.10	10798.94	11431.79
$\bar{y}_5^*$	8794.00	9380.77	9967.54	10554.31	11141.09
$\bar{y}_6^*$	847.65	943.81	1039.97	1136.12	1232.28
$\bar{y}_7^*$	847.65	943.81	1039.97	1136.12	1232.28
y_{zrsp}^*	847.54	943.59	1039.62	1135.62	1231.59
y_{zrcp}^*	847.54	943.59	1039.62	1135.62	1231.59
$\mathbf{W_{2h} = 20\%\ Non-response}$					
$\bar{y}_1^*$	3162.68	3585.80	4008.91	4432.03	4855.15
$\bar{y}_2^*$	1035.82	1217.05	1398.27	1579.50	1760.73
$\bar{y}_3^*$	1094.02	1279.73	1465.43	1651.14	1836.85
$\bar{y}_4^*$	9987.72	11164.21	12340.70	13517.19	14693.69

$\bar{y}_5^*$	9922.50	11073.51	12224.53	13375.55	14526.57
$\bar{y}_6^*$	1017.00	1197.84	1378.67	1559.51	1740.34
$\bar{y}_7^*$	1017.00	1197.84	1378.67	1559.51	1740.34
y_{zrsp}^*	1016.74	1197.33	1377.86	1558.35	1738.80
y_{zrcp}^*	1016.74	1197.33	1377.86	1558.35	1738.80

Table 3: Percent relative efficiency of proposed and existing estimators with respect to Hansen and Hurwitz estimator

$\mathbf{W_{2h} = 10\%\ Non-response}$					
Estimators	$\mathbf{K_h = K_h' = 2}$	$\mathbf{K_h = K_h' = 2.5}$	$\mathbf{K_h = K_h' = 3}$	$\mathbf{K_h = K_h' = 3.5}$	$\mathbf{K_h = K_h' = 4}$
$\bar{y}_1^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_2^*$	319.80	311.33	304.41	298.63	293.75
$\bar{y}_3^*$	301.99	295.38	289.92	285.35	281.45
$\bar{y}_4^*$	31.12	31.43	31.70	31.94	32.16
$\bar{y}_5^*$	31.50	31.94	32.33	32.68	33.00
$\bar{y}_6^*$	326.75	317.47	309.91	303.63	298.32
$\bar{y}_7^*$	326.75	317.47	309.91	303.63	298.32
y_{zrsp}^*	326.79	317.54	310.01	303.76	298.49
y_{zrcp}^*	326.79	317.54	310.01	303.76	298.49
$\mathbf{W_{2h} = 20\%\ Non-response}$					
$\bar{y}_1^*$	100.00	100.00	100.00	100.00	100.00
$\bar{y}_2^*$	305.33	294.63	286.70	280.60	275.75
$\bar{y}_3^*$	289.09	280.20	273.56	268.42	264.32
$\bar{y}_4^*$	31.67	32.12	32.49	32.79	33.04
$\bar{y}_5^*$	31.87	32.38	32.79	33.14	33.42
$\bar{y}_6^*$	310.98	299.36	290.78	284.19	278.98
$\bar{y}_7^*$	310.98	299.36	290.78	284.19	278.98
y_{zrsp}^*	311.06	299.48	290.95	284.41	279.22
y_{zrcp}^*	311.06	299.48	290.95	284.41	279.22

- Table 2 shows the *MSE* of the existing and proposed estimators. The MSE of the proposed separate and combined ratio estimators are same. The *MSEs* are calculated at different values of K_h and non-response rates (10% and 20% no-response rates).
- Table 3 shows that the proposed estimators i.e., y_{zrsp}^* and y_{zrcp}^* are more efficient than existing estimators. The proposed estimators y_{zrsp}^* and y_{zrcp}^* are equally efficient.
- The percent relative efficiency of the proposed estimators y_{zrsp}^* and y_{zrcp}^* at 10% non-response rate and at K_h=2 with Hansen and Hurwitz estimator is 326.79. Also, the efficiency decreases with increasing the value of K_h.
- The percent relative efficiency of the proposed estimators y_{zrsp}^* and y_{zrcp}^* at 20% non-response rate and at K_h=2 with Hansen and Hurwitz estimator is 311.06. Also, the efficiency decreases with increasing the value of K_h.

6. Conclusion

- The problem of estimating the population mean in presence of non-response using auxiliary information when non-response occurs both on the study and auxiliary variable and the population mean of the auxiliary variable is unknown under stratified random sampling was addressed in this paper. In the presence of non-response, we proposed a separate $\left(\boldsymbol{y}_{\boldsymbol{zrsp}}^{*}\right)$ and combined ratio $\left(\boldsymbol{y}_{\boldsymbol{zrcp}}^{*}\right)$ estimator up to the first order of approximation. The properties of estimator were obtained such as bias, mean square error and minimum mean square error and the condition under which the proposed estimators are better than existing estimators have also been obtained. Numerical illustration is also carried out to support the theoretical findings and from where we see that both the estimators i.e., proposed separate ratio estimator$\left(\boldsymbol{y}_{\boldsymbol{zrsp}}^{*}\right)$ and combined ratio estimator$\left(\boldsymbol{y}_{\boldsymbol{zrcp}}^{*}\right)$ are equally efficient. The percent relative efficiency of the proposed estimators y_{zrsp}^{*} and y_{zrcp}^{*} at 10% non-response rate and at K_h=2 with Hansen and Hurwitz estimator is 326.79. Also, the efficiency decreases with increasing the value of K_h. The percent relative efficiency of the proposed estimators y_{zrsp}^{*} and y_{zrcp}^{*} at 20% non-response rate and at K_h=2 with Hansen and Hurwitz estimator is 311.06. Also, the efficiency decreases with increasing the value of K_h. We infer from the discussion that these estimators are reliable in practice, and we recommend them for practical use.

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