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Comparison of Infinite Capacity $FM/FE_k/1$ Queuing Performance Using Fuzzy Queuing Model and Intuitionistic Fuzzy Queuing Model with Erlang Service Rates

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Abstract

In this work, we suggest an analytic technique with triangular fuzzy and triangular intuitionistic fuzzy numbers to compute the membership functions of considerable state-executing proportion in Erlang service models. The inter-entry rate, which is Poisson, and the admin (service) rate, which is Erlang, are both fuzzy-natured in this case, with FE_k designating the Erlang probabilistic deviation with k exponentially phase. The numeric antecedents are shown to validate the model's plausibility, $FM/FE_k/1$. A contextual inquiry is also carried out, comparing individual fuzzy figures. Intuitionistic fuzzy queuing models that are comprehensible are more categorical than fuzzy queuing models. Expanding the fuzzy queuing model to an intuitionistic fuzzy environment can boost the implementation of the queuing model. The purpose of this study is to assess the performance of a single server Erlang queuing model with infinite capacity using fuzzy queuing theory and intuitionistic fuzzy queuing theory. The fuzzy queuing theory model's performance evaluations are reported as a range of outcomes, but the intuitionistic fuzzy queuing theory model provides a myriad of values. In this context, the arrival and the service rate are both triangular and intuitionistic triangular fuzzy numbers. An assessment is made to find evaluation criteria using a design protocol in which fuzzy values are kept as they are and not made into crisp values, and two statistical problems are solved to understand the existence of the method.

Key Words: Queuing theory; Triangular fuzzy numbers; Triangular intuitionistic fuzzy numbers; Erlang service; Performance measures; Infinite capacity.

1. Introduction

Collaborating with fuzzy queuing theory necessitates first recognizing the real-world system of relevance, investigating it, and devising a mathematical model to represent it. The findings are then evaluated by examining this mathematical model, which purportedly pertains to the original system as well. The notions of compression and extrapolation are inherent in the process of developing a mathematical model; the analyzer must discard numerous elements that he or she feels are extraneous to the primary areas of focus. Approximations must be made in most circumstances when processing raw and often partial data into mathematical values that will enable the model to be scrutinized. It's not typical for an analyst to make endless hypotheses about the functioning of an actual system, based primarily on intuition and experience instead of any credible evidence that the system operates in this manner. From these considerations, it is reasonable to conclude that, in most contexts, assessments of parameters derived by a fuzzy queuing analysis should only be regarded as approximate indications of their magnitude in the real world. As a result, using fuzzy queuing theory to poke holes in existing operating systems, the dimensions to improve these structures, and the estimated values that some of the system's controllable variables must assume to reach a satisfactory degree of performance are quite beneficial.

The exponential family of probability distributions provides a lot less modelling flexibility than the Erlang family. When actual data does not support the significant exponential application, Erlang can provide increased functionality by effectively replicating reality. The relationship between Erlang and the exponential distribution with the Markovian property is another reason why it's beneficial in queuing analysis.

In recent decades, there have been various attempts on the topic of fuzzy queuing. Several researchers (Buckley, 1990; Li and Lee, 1989; Negi and Lee, 1992; Prade, 1980) were among the first to explore fuzzy queuing systems. Kao et al. (1999) used a parameterized planning methodology with α-cut tactic to scrutinize the fuzzy queuing system. Buckley et al. (2001) showed some examples in which they used the probability theory in the fuzzy theory to optimize a number of servers by evaluating the timeframe between two concurrent entrances and the serving time. Addressing the fuzzy parametric planning technique, Chen (2005) developed a model that used the fuzzy membership function and took into account the fuzzy response time and fuzzy cost. Chen (2006) devised a nonlinear arithmetic mixed-integer scheduling with state variables based on the probability hypothesis for a bunch arrival model with varying quantities in each group and fuzzy arrival and service times. Ritha et al. (2011) investigated the N policy for infinite lines with unclear arrival and service information. The uncertainty associated with the input parameters was estimated using fuzzy set theory, and the model was analyzed using the triangular membership function. Using two fuzzy queuing models of $M/M/1$ and $M/E_2/1$, Barak et al. (2012) investigated two models of organizing queuing systems and their impact on the costs of every system. They hypothesized that the rates of arrivals and the rate of service were ambiguous facts, as were the system expenses. They compared two realistic systems to investigate the various preconditions of the manufacturer's allotment in real-world queuing systems. Intuitionistic fuzzy queuing models have been characterized by multiple researchers, namely K.Atanavssov (1999); that publication has been the first approach to providing a broader and comprehensive account of intuitionistic fuzzy set theory and it is more adapted in a range of domains. K.Atanassov (1989) incorporated the intuitionistic fuzzy set, and he also departed from the temporal intuitionistic fuzzy sets. F. Ferdowsi (2019) proposed an intuitionistic fuzzy measure to handle the uncertainty where he used credibility measure to convert fuzzy to crisp model.

Erlang fuzzy queuing models have been propounded by numerous researchers, namely A. Mohammed Shapique (2016) used the DSW algorithm to define the membership functions of the Erlang queuing model. K. Usha Prameela et al. (2019) utilized α - cut method and DSW algorithmic rule with different types of fuzzy numbers to find the execution proportions in $FM/FE_k/1$ queuing model. V. Ashok Kumar (2011) created the nonlinear computing framework to extract the membership functions of the Erlang service queuing model. S. Hanumantha Rao et al. (2016) studied the Erlang queuing model with 2 phases and operated N-policy using generating functions to solve steady-state equations. S. Narayanamoorthy et al. (2020) used L-R arithmetic to solve Erlang fuzzy queuing model. In the intuitionistic Erlang queuing model, S. Narayanamoorthy et al. (2020) used Atanassov's extension principle and (α,β)- cut approach. G. A. Zverkina (2021) used the convergence rate's upper bound value and those values based on the state of the system. Mohamed Bisher Zeina (2020) used the neutrosophic statistical interval method to compute the performance measures. Konstantin Kogan et al. (2023) used an Erlang queuing model to analyze software license demand and calculate the likelihood that an access request will be denied.

A heuristic approach has been devised to solve Erlang queuing models using both fuzzy and intuitionistic fuzzy numbers without turning them into classical ones and competently retrieving the result. As a result, this paper introduces a new method for handling Erlang queuing problems that employ fuzzy and intuitionistic fuzzy numbers. That is, a straightforward approach to solving Erlang queuing issues is recommended. Performance metrics are given as values in queuing theory. Fuzzy queuing theory has a range of data, and intuitionistic fuzzy queuing theory provides a broad range of data. The performance measurements of the Erlang fuzzy queuing model are within the computed benchmarks of the Erlang intuitionistic fuzzy queuing model, according to the results of the investigation.

This paper examined the intuitionistic Erlang queuing model and its applications. As far as the authors are aware, no prior works have discussed the contrast between fuzzy and intuitionistic fuzzy in the context of queuing theory. The following are the study's core elements:

1. The novel Erlang queuing model under intuitionistic fuzzy sets is introduced.
2. It is recommended to formulate $FM/FE_k/1$ queue with an Erlang service rate.

3. A mathematical formulation is also addressed to exemplify the usability of the suggested queuing model.
4. Graphical representations are provided to assist the decision-maker in comprehending the solution.

The design of the article is summarised as follows: Component 2 provides the necessary definitions. Component 3 is where you'll find the system model. Manuals and preconceptions are displayed in component 4. A few basic theorems are discussed in component 5. The layout method for dealing with the current model is shown in component 6. The planned lining model is provided in component 7. Component 8 provides and solves a mathematical illustration to showcase the practicality of the suggested method. The outcomes and discourses are addressed in component 9. Component 10 is the penultimate component of the system.

2. Preliminaries

The motive of this division is to give some basic definitions, annotations, and outcomes that are used in our subsequent calculations.

Definition 2.1: (Ming Ma et al. 1999) A fuzzy set $\tilde{A}$ is defined on R, the set of real numbers is called a **fuzzy number** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$, such that $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$
(b) $\tilde{A}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$
(c) $\tilde{A}$ is piecewise continuous.

Definition 2.2: (Ming Ma et al. 1999) A fuzzy number $\tilde{A}$ is defined on R, the set of real numbers is said to be a **triangular fuzzy number (TFN)** if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which satisfy the following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-\tilde{\alpha}_1}{\tilde{\alpha}_2-\tilde{\alpha}_1} \text{ for } \tilde{\alpha}_1 \leq x \leq \tilde{\alpha}_2 \\ 1 \text{ for } x = \tilde{\alpha}_2 \\ \frac{\tilde{\alpha}_3-x}{\tilde{\alpha}_3-\tilde{\alpha}_2} \text{ for } \tilde{\alpha}_2 \leq x \leq \tilde{\alpha}_3 \\ 0 \text{ otherwise} \end{cases}$$

Definition 2.3: (Ming Ma et al. 1999) Let the two triangular fuzzy numbers be $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2)$ and then the **arithmetic operations on TFN** be given as follows:

(A)Addition

$$\tilde{P} + \tilde{Q} \approx (\tilde{m}_1 + \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \quad (1)$$

(B)Subtraction

$$\tilde{P} - \tilde{Q} \approx (\tilde{m}_1 - \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \quad (2)$$

(C) Multiplication

$$\tilde{P}.\tilde{Q} \approx (\tilde{m}_1.\tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}) \quad (3)$$

(D) Division

$$\frac{\tilde{P}}{\tilde{Q}} \approx \left(\frac{\tilde{m}_1}{\tilde{m}_2}, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}\right) \quad (4)$$

(E) Scalar Multiplication

$$k\tilde{P} \approx \begin{cases} (ka_2, \alpha_1, \beta_1), k \geq 0 \\ (-ka_2, \alpha_1, \beta_1), k < 0 \end{cases} \quad (5)$$

Definition 2.4: For every triangular fuzzy number $\tilde{P} \approx (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3) \in F(R)$ **ranking function** $\mathfrak{R}: F(R) \to R$ is defined by graded mean as

$$\Re(\tilde{P}) = \frac{(\tilde{a}_1 + 4\tilde{a}_2 + \tilde{a}_3)}{6}$$

For any two TFN $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$ and $\tilde{Q} \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ We have the following comparisons,

$$(a)\tilde{P} \succ \tilde{Q} \Leftrightarrow \Re(\tilde{P}) > \Re(\tilde{Q})$$

$$(b)\tilde{P} \prec \tilde{Q} \Leftrightarrow \Re(\tilde{P}) < \Re(\tilde{Q})$$

$$(c)\tilde{P} \approx \tilde{Q} \Leftrightarrow \Re(\tilde{P}) = \Re(\tilde{Q})$$

$$(d)\tilde{P} - \tilde{Q} \approx 0 \Leftrightarrow \Re(\tilde{P}) - \Re(\tilde{Q}) = 0$$

A triangular fuzzy number $\tilde{P} \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) \in F(R)$ is known to be **positive** if $\Re(\tilde{P}) > 0$ and defined by $\tilde{P} \succ 0$.

Definition 2.5:(Shaw et al. 2012) Let a non–empty set be X . An **Intuitionistic fuzzy set (IFS)** $\tilde{A}'$ is defined as $\tilde{A}' = \{(x, \mu_{\tilde{A}'}(x), \gamma_{\tilde{A}'}(x)/x \in X)\}$, where $\mu_{\tilde{A}'}: X \to [0,1]$ and $\gamma_{\tilde{A}'}: X \to [0,1]$ denotes the degree of membership and degree of non–membership functions respectively where $x \in X$, for every $x \in X, 0 \leq \mu_{\tilde{A}'}(x) + \gamma_{\tilde{A}'}(x) \leq 1$

Definition 2.6:(Shaw et al. 2012) An intuitionistic fuzzy set $\tilde{A}'$ described on R, the real numbers are said to be an **Intuitionistic fuzzy number (IFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and its non – membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ should be agreeable to the following conditions:

i) $\tilde{A}'$ is normal, which means that there exists an $x \in R$, such that $\mu_{\tilde{A}'}(x) = 1, \gamma_{\tilde{A}'}(x) = 0$

ii) $\tilde{A}'$ is convex for the membership functions $\mu_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\mu_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\tilde{A}'}(x_1), \mu_{\tilde{A}'}(x_2)\}$

iii) $\tilde{A}'$ is concave for the non – membership function $\gamma_{\tilde{A}'}$, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$ such that $\gamma_{\tilde{A}'}(\lambda x_1 + (1-\lambda)x_2) \leq max\{\gamma_{\tilde{A}'}(x_1), \gamma_{\tilde{A}'}(x_2)\}$

Definition 2.7:(Shaw et al. 2012) A fuzzy number $\tilde{A}'$ on R is said to be a **triangular intuitionistic fuzzy number (TIFN)** if its membership function $\mu_{\tilde{A}'}: R \to [0,1]$ and non – membership function $\gamma_{\tilde{A}'}: R \to [0,1]$ have the following conditions:

$$\mu_{\tilde{A}'}(x) = \begin{cases} \frac{x-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1} \text{ for } \tilde{a}_1 \leq x \leq \tilde{a}_2 \\ 1 \text{ for } x = \tilde{a}_2 \\ \frac{\tilde{a}_3-x}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}_3 \\ 0 \text{ otherwise} \end{cases}$$

and

$$\gamma_{\tilde{A}'}(x) = \begin{cases} 1 \text{ for } x < \tilde{a}'_1, x > \tilde{a}'_3 \\ \frac{\tilde{a}_2-x}{\tilde{a}_2-\tilde{a}'_1} \text{ for } \tilde{a}'_1 \leq x \leq \tilde{a}_2 \\ 0 \text{ for } x = \tilde{a}_2 \\ \frac{x-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2} \text{ for } \tilde{a}_2 \leq x \leq \tilde{a}'_3 \end{cases}$$

and is given by $\tilde{A}' = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ where $\tilde{a}'_1 \leq \tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}'_3$

Cases: Let $\tilde{A}' = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ be a TIFN then the following cases arise.

Case:1 If $\tilde{a}'_1 = \tilde{a}_1, \tilde{a}'_3 = \tilde{a}_3$ then $\tilde{A}'$ represent a TFN.

Case:2 If $\tilde{a}'_1 = \tilde{a}_1 = \tilde{a}_2 = \tilde{a}'_3 = \tilde{a}_3 = \tilde{m}$ then $\tilde{A}'$ represent a real number $\tilde{m}$. The parametric form of TIFN $\tilde{A}'$ is represented as $\tilde{A}' = (\alpha, m, \beta; \alpha', m, \beta')$ where $\tilde{\alpha}, \tilde{\alpha}' \& \tilde{\beta}, \tilde{\beta}'$represents the left spread and right spread of membership functions and non – membership functions respectively.

Definition 2.8: The extension of fuzzy arithmetic operations of Ming Ma et al. (1999) to the set of TIFN based upon both location indices and functions of fuzziness indices. The location indices number is taken in the regular arithmetic while the functions of fuzziness indices are assumed to follow the lattice rule, which is the least upper bound in the lattice $\tilde{I}'$. For any two arbitrary TIFN $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ and $* \in \{+, -, \times, \div\}$, then the **arithmetic operations on TIFN** are defined by $\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$.

In particular, for any two TIFNs $\tilde{P}' \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$ and $\tilde{Q}' \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ the arithmetic operations are defined as

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1') * (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}_1, \tilde{\alpha}_2\}, max\{\tilde{\beta}_1, \tilde{\beta}_2\}; \tilde{m}_1 * \tilde{m}_2, max\{\tilde{\alpha}_1', \tilde{\alpha}_2'\}, max\{\tilde{\beta}_1', \tilde{\beta}_2'\})$$

$$\tilde{P}' * \tilde{Q}' = (\tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1 \vee \tilde{\alpha}_2, \tilde{\beta}_1 \vee \tilde{\beta}_2; \tilde{m}_1 * \tilde{m}_2, \tilde{\alpha}_1' \vee \tilde{\alpha}_2', \tilde{\beta}_1' \vee \tilde{\beta}_2')$$

In particular, for any two TIFN $\tilde{P}' \approx (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3; \tilde{a}_1', \tilde{a}_2', \tilde{a}_3') \approx (\tilde{m}_1, \tilde{\alpha}_1, \tilde{\beta}_1; \tilde{m}_1, \tilde{\alpha}_1', \tilde{\beta}_1')$, $\tilde{Q}' \approx (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3; \tilde{b}_1', \tilde{b}_2', \tilde{b}_3') \approx (\tilde{m}_2, \tilde{\alpha}_2, \tilde{\beta}_2; \tilde{m}_2, \tilde{\alpha}_2', \tilde{\beta}_2')$ we define:

Addition

$$\tilde{P}' + \tilde{Q}' = (m_1 + m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 + m_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \quad (6)$$

Subtraction

$$\tilde{P}' - \tilde{Q}' = (m_1 - m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 - m_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \quad (7)$$

Multiplication

$$\tilde{P}' \times \tilde{Q}' = (m_1 \times m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 \times m_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \quad (8)$$

Division

$$\tilde{P}' \div \tilde{Q}' = (m_1 \div m_2, max\{\alpha_1, \alpha_2\}, max\{\beta_1, \beta_2\}; m_1 \div m_2, max\{\alpha_1', \alpha_2'\}, max\{\beta_1', \beta_2'\}) \quad (9)$$

Scalar Multiplication

$$k\tilde{P}' = \begin{cases} (ka_2, \alpha_1, \beta_1; ka_2, \alpha_1', \beta_1'), & \text{for } k \geq 0 \\ (-ka_2, \alpha_1, \beta_1; -ka_2, \alpha_1', \beta_1'), & \text{for } k < 0 \end{cases} \quad (10)$$

Definition 2.9: Consider an arbitrary TIFN $\tilde{P}' = (a_1, a_2, a_3; a_1', a_2, a_3') = (m, \alpha, \beta; m, \alpha', \beta')$ and the magnitude of TIFN $\tilde{P}'$ is given by

$$mag(\tilde{P}') = \frac{1}{2}\int_0^1 (\beta + \beta' + 6m - \alpha - \alpha') f(r) dr$$

In real-life scenarios, decision-makers select the value of $\tilde{f}(\tilde{r}')$ based on their circumstances. Here for our ease, we choose $\tilde{f}(\tilde{r}') = \tilde{r}'^2$

$$mag(\tilde{P}') = \left(\frac{\beta + \beta' + 6m - \alpha - \alpha'}{6}\right)$$

For any two TIFN $\tilde{P}' \approx (m_1, \alpha_1, \beta_1; m_1, \alpha_1', \beta_1')$, $\tilde{Q}' \approx (m_2, \alpha_2, \beta_2; m_2, \alpha_2', \beta_2')$ in $F(R)$, we define

$$(a)\tilde{P}' \geq \tilde{Q}' \Leftrightarrow mag(\tilde{P}') \geq mag(\tilde{Q}')$$
$$(b)\tilde{P}' \leq \tilde{Q}' \Leftrightarrow mag(\tilde{P}') \leq mag(\tilde{Q}')$$
$$(c)\tilde{P}' \approx \tilde{Q}' \Leftrightarrow mag(\tilde{P}') = mag(\tilde{Q}')$$

3. Model description

In this $FM/FE_k/1$ approach, the user arrives at a solo server facility with a Poisson arrival rate $\tilde{\lambda}$ & $\tilde{\lambda}'$ and an Erlang service rate $\tilde{\mu}$ & $\tilde{\mu}'$. The unit is divided into K stages here. The introduction of a brand adds k phases of service, while the departure of one unit reduces the number of phases of service. The services criteria are presumed to be first in, first out, and the system capacity is unbounded. The Erlang k type distribution consists of independent and identically distributed and comparable exponential phases, each with a mean of $\frac{1}{k\tilde{\mu}}$. Figure 1 depicts this situation.

Both TFN and TIFN are used to calculate the arrival and service rates. The main purpose is to determine performance measurements using both fuzzy and intuitionistic fuzzy numbers, and models are compared based on the mean number of customers in the queue and system, while also their transit (awaiting) time in the queue and system. The service rates are assumed to be Erlang's nature in this scenario. The disputes are resolved by keeping the fuzziness values until the end, i.e., without altering them to crisp. As a result, it is more applicable in real-life circumstances.

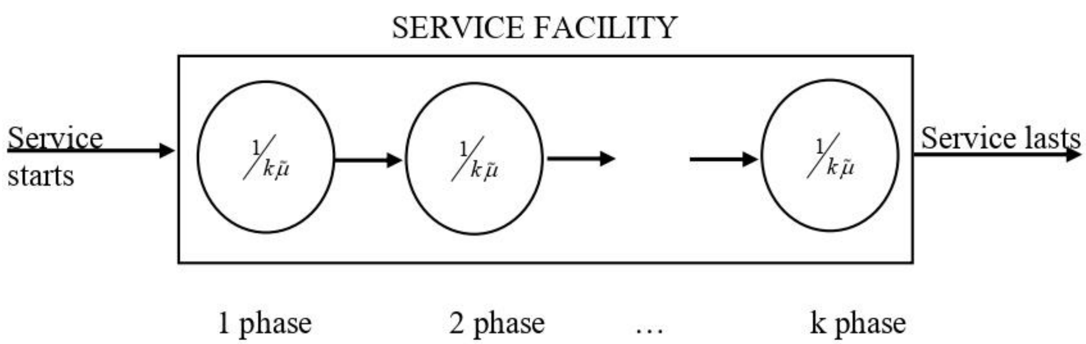


Figure 1: Erlang service time

4. Speculations and Syntaxes

4.1. Speculations

i) Consider the endless capacity $FM/FE_k/1$ fuzzy queuing model under FCFS discipline with Erlang service rate.

ii) The system is served by a single server.

iii) The time between arrivals follows a Poisson distribution.

iv) Service (Erlang) time is distributed exponentially.

v) The arrival and service rate are taken as TFN and TIFN.

4.2. Syntaxes

Here we are using the following notations:

$\tilde{\lambda}, \tilde{\lambda}'$ → The mean no. of consumers who arrive in a predetermined period of time.

$\tilde{\mu}, \tilde{\mu}'$ → The mean no. of consumers being serviced per unit of time.

$\tilde{\rho}$ →Traffic intensity.

$\tilde{N}_q, \tilde{N}_q'$ → The mean no. of consumers in the line.

$\tilde{N}_s, \tilde{N}_s'$ →The mean no. of consumers in the system.

$\tilde{T}_q, \tilde{T}_q^{'}$ → The mean sojourn time of the consumers in the queue.

$\tilde{T}_s, \tilde{T}_s^{'}$ → The mean sojourn time of the consumers in the system.

$\tilde{P}, \tilde{P}^{'}$ → Interarrival rate.

$\tilde{Q}, \tilde{Q}^{'}$ → Service rate.

X → Arrival time schedule.

Y →Service time schedule.

FM →Fuzzified exponential diffusion.

FE_k →Fuzzified Erlang diffusion.

5. NON-POISSON QUEUING MODEL/ IMBEDDED MARKOV CHAIN TECHNIQUE

In this queuing System of Poisson arrivals, Erlang service time with k phases and a single server.

Let, $\tilde{P}_n'(t)$ = Probability that there are n phases in the system (waiting and in-service) at time $\tilde{t}'$.

n = total number of phases in the system (waiting and in-service) and

k = number of phases in one entity

Therefore, $\tilde{\lambda}_n' = \tilde{\lambda}'$ phases arrive per unit time

$\tilde{\mu}_n' = k\tilde{\mu}'$ phases served per unit time

Theorem 5.1. In the $(FM/FE_k/1)$: $(\infty/FIFO)$ queuing model, at the initial state, when there are no units in the state $n = 0$ at the time $(\tilde{t}' + \Delta\tilde{t}')$, prove that the rate is $\tilde{\lambda}'\tilde{P}_0' = k\tilde{\mu}'\tilde{P}_1'$.
Proof. For $n = 0$,
The possibility that somehow there won't be any entities in the system at $(\tilde{t}' + \Delta\tilde{t}')$ will be equal to the sum of the next two separate probabilities:

i) $\tilde{P}_0'(\tilde{t}')\left(1 - \tilde{\lambda}'\Delta\tilde{t}'\right)$ is the statistical likelihood that there would be no entity in the system at $\tilde{t}'$ and no arrival at $\Delta\tilde{t}'$

ii) $\tilde{P}_1'(\tilde{t}').k\tilde{\mu}'\Delta\tilde{t}'.\left(1 - \tilde{\lambda}'\Delta\tilde{t}'\right) \cong \tilde{P}_1'(\tilde{t}')k\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}')$ is the statistical likelihood that one unit is present in the system at $\tilde{t}'$, one unit is serviced at $\Delta\tilde{t}'$, and no arrival at $\Delta\tilde{t}'$.

As a result, the probability of $n = 0$ is

$$\tilde{P}_0'(\tilde{t}' + \Delta\tilde{t}') = \tilde{P}_0'(\tilde{t}')\left(1 - \tilde{\lambda}'\Delta\tilde{t}'\right) + \tilde{P}_1'(\tilde{t}')k\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}') \tag{11}$$

$$\tilde{P}_0'(\tilde{t}' + \Delta\tilde{t}') - \tilde{P}_0'(\tilde{t}') = -\tilde{\lambda}'\Delta\tilde{t}'\tilde{P}_0'(\tilde{t}') + \tilde{P}_1'(\tilde{t}')k\tilde{\mu}'\Delta\tilde{t}' + o(\Delta\tilde{t}')$$

Now dividing the aforementioned equation by $\Delta\tilde{t}'$ and assuming the limit as $\Delta\tilde{t}' \to 0$, hence it transforms into:

$$\Delta\tilde{t}' \to 0\left[\frac{\tilde{P}_0'(\tilde{t}' + \Delta\tilde{t}') - \tilde{P}_0'(\tilde{t}')}{\Delta\tilde{t}'}\right] = -\tilde{\lambda}'\tilde{P}_0'(\tilde{t}') + k\tilde{\mu}'\tilde{P}_1'(\tilde{t}'); n = 0$$

$$\tilde{P}_0'(\tilde{t}') = -\tilde{\lambda}'\tilde{P}_0'(\tilde{t}') + k\tilde{\mu}'\tilde{P}_1'(\tilde{t}') \text{ for } n = 0 \tag{12}$$

In the scenario of a steady state where $\tilde{t}' \to \infty$, $\tilde{P}_n'(\tilde{t}') \to \tilde{P}_n'$ (independent of $\tilde{t}'$) and consequently $\tilde{P}_n'(\tilde{t}') \to 0$. Eventually, the steady-state difference equations of the system are provided by

$0 = -\tilde{\lambda}'\tilde{P}_0' + k\tilde{\mu}'\tilde{P}_1'$ (From 12)

Furthermore, $\tilde{\lambda}'\tilde{P}_0' = k\tilde{\mu}'\tilde{P}_1'; n = 0$

$$\tilde{P}_1' = \left(\frac{\tilde{\lambda}'}{k\tilde{\mu}'}\right)\tilde{P}_0'$$

$$\tilde{P}_1' = \tilde{\rho}'\tilde{P}_0' \text{ for } n = 0 \tag{13}$$

Theorem 5.2. In the $(FM/FE_k/1):(\infty/FIFO)$ queuing model, at the state $n \geq 1$, prove that the rate is $(\tilde{\lambda}' + k\tilde{\mu}')\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-k}' + k\tilde{\mu}'\tilde{P}_{n+1}$.
Proof. For $n \geq 1$,
When independent conditions are incorporated together, the probability will be shown as:

i) At any time $\tilde{t}'$, there are n entities in the system are $\tilde{P}_n'(\tilde{t}')$. However, at $\Delta\tilde{t}'$, there are no arrivals and no services respectively, $(1 - \tilde{\lambda}'\Delta\tilde{t}')$ and $(1 - k\tilde{\mu}'\Delta\tilde{t}')$. As a consequence, the probability is provided as

$$\Rightarrow \tilde{P}_n'(\tilde{t}')(1 - \tilde{\lambda}'\Delta\tilde{t}')(1 - k\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow \tilde{P}_n'(\tilde{t}')\left(1 - \Delta\tilde{t}'(\tilde{\lambda}' + k\tilde{\mu}')\right) + o_1(\Delta\tilde{t}') \tag{14}$$

ii) At any time $\tilde{t}'$, there are $(n-k)$ entities in the system are $\tilde{P}_{n-k}'(\tilde{t}')$; there is one entrance at $\Delta\tilde{t}'$ is $\tilde{\lambda}'\Delta\tilde{t}'$ and no assistance at $\Delta\tilde{t}'$is $(1 - k\tilde{\mu}'\Delta\tilde{t}')$. As a result, the probability is given as

$$\Rightarrow \tilde{P}_{n-k}'(\tilde{t}')\tilde{\lambda}'\Delta\tilde{t}'(1 - k\tilde{\mu}'\Delta\tilde{t}')$$

$$\Rightarrow \tilde{\lambda}'\Delta\tilde{t}'\tilde{P}_{n-k}'(\tilde{t}') + o_2(\Delta\tilde{t}') \tag{15}$$

iii) At any time $\tilde{t}'$, there are $(n+1)$ units in the system are $\tilde{P}_{n+1}'(\tilde{t}')$; there is no inflow at $\Delta\tilde{t}'$ is $(1 - \tilde{\lambda}'\Delta\tilde{t}')$ and one service at $\Delta\tilde{t}'$ is $k\tilde{\mu}'\Delta\tilde{t}'$. As an outcome, the probability is given as

$$\Rightarrow \tilde{P}_{n+1}'(\tilde{t}')(1 - \tilde{\lambda}'\Delta\tilde{t}')k\tilde{\mu}'\Delta\tilde{t}'$$

$$\Rightarrow k\tilde{\mu}'\Delta\tilde{t}'\tilde{P}_{n+1}'(\tilde{t}') + o_3(\Delta\tilde{t}') \tag{16}$$

By adding (14), (15) & (16), we obtain

$$\tilde{P}_n'(\tilde{t}' + \Delta\tilde{t}') = \tilde{P}_n'(\tilde{t}')\left[1 - (\tilde{\lambda}' + k\tilde{\mu}')\Delta\tilde{t}'\right] + \tilde{\lambda}'\Delta\tilde{t}'\tilde{P}_{n-k}'(\tilde{t}') + k\tilde{\mu}'\Delta\tilde{t}'\tilde{P}_{n+1}'(\tilde{t}') + o(\Delta\tilde{t}') \tag{17}$$

The above equation now becomes $\tilde{P}_n'(\tilde{t}')$ by dividing it by $\Delta\tilde{t}'$ and presuming that $\Delta\tilde{t}' \to 0$ is the limit; hence it transforms into:

$$\tilde{P}_n'(\tilde{t}') = -(\tilde{\lambda}' + k\tilde{\mu}')\tilde{P}_n'(\tilde{t}') + \tilde{\lambda}'\tilde{P}_{n-k}'(\tilde{t}') + k\tilde{\mu}'\tilde{P}_{n+1}'(\tilde{t}'); n \geq 1 \tag{18}$$

In the circumstance of a steady state where $\tilde{t}' \to \infty$, $\tilde{P}_n'(\tilde{t}') \to \tilde{P}_n'$ (independent of $\tilde{t}'$) and hence $\tilde{P}_n'(\tilde{t}') \to 0$. So, the system of steady-state difference equations is given by

$0 = -(\tilde{\lambda}' + k\tilde{\mu}')\tilde{P}_n' + \tilde{\lambda}'\tilde{P}_{n-k}' + k\tilde{\mu}'\tilde{P}_{n+1}'; n \geq 1$ (From 18)

Hence, $(\tilde{\lambda}' + k\tilde{\mu}')\tilde{P}_n' = \tilde{\lambda}'\tilde{P}_{n-k}' + k\tilde{\mu}'\tilde{P}_{n+1}'; n \geq 1$ (19)

Divide by $k\tilde{\mu}'$ throughout the above equation and also using $\tilde{\rho}' = \left(\frac{\tilde{\lambda}'}{k\tilde{\mu}'}\right)$, we obtain

$$(1 + \tilde{\rho}')\tilde{P}_n' = \tilde{\rho}'\tilde{P}_{n-k}' + \tilde{P}_{n+1}'; n \geq 1 \tag{20}$$

Theorem 5.3. In the $(FM/FE_k/1):(\infty/FIFO)$ queuing model, solve the system of equations and prove the steady state equation of the model.
Proof. By the definition of generating function,

$$G.F \equiv P'(Z) = \sum_{n=0}^{\infty} \tilde{P}'_n Z^n \tag{21}$$

Multiply equation (21) by equation (20), and we obtain

$$(1+\tilde{\rho}')\sum_{n=1}^{\infty} \tilde{P}'_n Z^n = \tilde{\rho}' \sum_{n=1}^{\infty} \tilde{P}'_{n-k} Z^n + \sum_{n=1}^{\infty} \tilde{P}'_{n+1} Z^n \tag{22}$$

Using equation (13), we have $\tilde{P}'_1 = \tilde{\rho}'\tilde{P}'_0$

Adding both equations (13) and (22), we obtain

$$\tilde{\rho}'\tilde{P}'_0 + (1+\tilde{\rho}')\sum_{n=1}^{\infty} \tilde{P}'_n Z^n = \tilde{P}'_1 + \tilde{\rho}' \sum_{n=1}^{\infty} \tilde{P}'_{n-k} Z^n + \sum_{n=1}^{\infty} \tilde{P}'_{n+1} Z^n \tag{23}$$

By adding and subtracting $\tilde{P}'_0$ on L.H.S. of (23), we can write it as

$$\tilde{\rho}'\tilde{P}'_0 + (1+\tilde{\rho}')\sum_{n=1}^{\infty} \tilde{P}'_n Z^n + \tilde{P}'_0 - \tilde{P}'_0 = \tilde{P}'_1 + \tilde{\rho}' \sum_{n=1}^{\infty} \tilde{P}'_{n-k} Z^n + \sum_{n=1}^{\infty} \tilde{P}'_{n+1} Z^n$$

$$(1+\tilde{\rho}')[\tilde{P}'_0 + \sum_{n=1}^{\infty} \tilde{P}'_n Z^n] - \tilde{P}'_0 = \tilde{\rho}' \sum_{n=1}^{\infty} \tilde{P}'_{n-k} Z^n + \sum_{n=0}^{\infty} \tilde{P}'_{n+1} Z^n$$

Multiply and divide by Z on R.H.S. of the 2nd term alone, and we obtain

$$(1+\tilde{\rho}')\sum_{n=0}^{\infty} \tilde{P}'_n Z^n - \tilde{P}'_0 = \left(\frac{1}{Z}\right)\sum_{n=0}^{\infty} \tilde{P}'_{n+1} Z^{n+1} + \tilde{\rho}' \sum_{n=0}^{\infty} \tilde{P}'_{n-k} Z^n \tag{24}$$

Substitute $(n+1) = i; (n-k) = m \Rightarrow n = m+k$ put the value of m in the R.H.S. of the equation (24) and adjust limits,

$$(1+\tilde{\rho}')\sum_{n=0}^{\infty} \tilde{P}'_n Z^n - \tilde{P}'_0 = \left(\frac{1}{Z}\right)\sum_{i=1}^{\infty} \tilde{P}'_i Z^i + \tilde{\rho}' \sum_{m=0}^{\infty} \tilde{P}'_m Z^{m+k} \tag{25}$$

By adding and subtracting $\tilde{P}'_0$ on the first term R.H.S. of (25), we can write it as

$$(1+\tilde{\rho}')\sum_{n=0}^{\infty} \tilde{P}'_n Z^n - \tilde{P}'_0 = \left(\frac{1}{Z}\right)\sum_{i=1}^{\infty} \tilde{P}'_i Z^i + \tilde{P}'_0 - \tilde{P}'_0 + \tilde{\rho}' \sum_{m=0}^{\infty} \tilde{P}'_m Z^{m+k}$$

$$(1+\tilde{\rho}')\sum_{n=0}^{\infty} \tilde{P}'_n Z^n - \tilde{P}'_0 = \left(\frac{1}{Z}\right)\sum_{i=0}^{\infty} \tilde{P}'_i Z^i - \tilde{P}'_0 + \tilde{\rho}' Z^k \sum_{m=0}^{\infty} \tilde{P}'_m Z^m$$

$$(1+\tilde{\rho}')[P'(Z)] - \tilde{P}'_0 = \left(\frac{1}{Z}\right)[P'(Z) - \tilde{P}'_0] + \tilde{\rho}' Z^k P'(Z) \text{ (As } \sum_{n=0}^{\infty} \tilde{P}'_n Z^n = P'(Z))$$

$$(1+\tilde{\rho}')[P'(Z)] - \left(\frac{1}{Z}\right)P'(Z) - \tilde{\rho}' Z^k P'(Z) = -\frac{1}{Z}\tilde{P}'_0 + \tilde{P}'_0$$

$$\left(\frac{1}{Z}\right)P'(Z) + \tilde{\rho}' Z^k P'(Z) - (1+\tilde{\rho}')[P'(Z)] = \frac{1}{Z}\tilde{P}'_0 - \tilde{P}'_0$$

$$P'(Z)\left[\frac{1}{Z} + \tilde{\rho}' Z^k - (1+\tilde{\rho}')\right] = \tilde{P}'_0\left(\frac{1}{z} - 1\right)$$

$$P'(Z) = \frac{\tilde{P}'_0(1-Z)}{1+\tilde{\rho}' Z^{k+1} - (1+\tilde{\rho}')Z} = \frac{\tilde{P}'_0(1-Z)}{(1-Z) - \tilde{\rho}' Z(1-Z^k)}$$

$$P'(Z) = \frac{\tilde{P}'_0}{1-\tilde{\rho}' Z\left(\frac{1-Z^k}{1-Z}\right)} = \tilde{P}'_0\left[1 - \tilde{\rho}' Z\left(\frac{1-Z^k}{1-Z}\right)\right]^{-1}$$

$P'(Z) = \tilde{P}_0' \left[1 + \tilde{\rho}' Z \left(\frac{1-Z^k}{1-Z}\right) + (\tilde{\rho}' Z)^2 \left(\frac{1-Z^k}{1-Z}\right)^2 + \cdots\right]$

$P'(Z) = \tilde{P}_0' \sum_{n=0}^{\infty} (\tilde{\rho}' Z)^n \left(\frac{1-Z^k}{1-Z}\right)^n$

As the sum of the series $(1 + Z + Z^2 + \cdots + Z^{k-1}) = \left(\frac{1-Z^k}{1-Z}\right)$. Hence,

$$P'(Z) = \tilde{P}_0' \sum_{n=0}^{\infty} (\tilde{\rho}' Z)^n [1 + Z + Z^2 + \cdots + Z^{k-1}]^n \quad (26)$$

To find $\tilde{P}_0'$, (put $z = 1$)

$P'(1) = \tilde{P}_0' \sum_{n=0}^{\infty} \tilde{\rho}'^n \, [1 + 1 + \cdots + k \; times]^n$

$P'(1) = \tilde{P}_0' \sum_{n=0}^{\infty} \tilde{\rho}'^n k^n$ (Put $\sum_{n=0}^{\infty} (\tilde{\rho}' k)^n = \frac{1}{1-\tilde{\rho}' k}$ sum of infinite G.P.)

$P'(1) = \frac{\tilde{P}_0'}{(1-\tilde{\rho}' k)}$

By the law of total probability, we have $P'(1) = \sum_{n=0}^{\infty} \tilde{P}_n' = 1$

$\frac{\tilde{P}_0'}{(1-\tilde{\rho}' k)} = 1$

$$\tilde{P}_0' = 1 - \tilde{\rho}'^k \quad (27)$$

$$\tilde{P}_n' = \tilde{\rho}_n (1 - \tilde{\rho}'^k) \quad (28)$$

6. $(\boldsymbol{FM/FE_k/1})$: $(\infty/\boldsymbol{FCFS})$ Queues

In this model, the customer arrives at one server facility with a Poisson arrival rate $\tilde{\lambda}, \tilde{\lambda}'$ and Erlang service rate $\tilde{\mu}, \tilde{\mu}'$. We assume a single-server Erlang fuzzy queuing system with infinite capacity. The customers are served in k phases. The inter-arrival rate $\tilde{P}$ and the service rate $\tilde{Q}$ are nearly comprehended and depicted by a fuzzy set,

$$\tilde{P} = \{p, \mu_{\tilde{P}}(p)/p \in X\}$$
$$\tilde{Q} = \{q, \mu_{\tilde{Q}}(q)/q \in Y\}$$

In this, X is the inter-entrance period configuration and Y is the service time configuration. $\mu_{\tilde{P}}(p)$ is the inter-entrance time's membership function and $\mu_{\tilde{Q}}(q)$ is the enlistment capacity of the service time. In addition to that, consider a single server intuitionistic Erlang fuzzy queuing system with infinite capacity. The inter-arrival rate $\tilde{P}'$ and the service rate $\tilde{Q}'$ are nearly comprehended and depicted by an intuitionistic fuzzy set,

$$\tilde{P}' = \{p, \mu_{\tilde{P}'}(p), \gamma_{\tilde{P}'}(p)/p \in X\}$$
$$\tilde{Q}' = \{q, \mu_{\tilde{Q}'}(q), \gamma_{\tilde{Q}'}(q)/q \in Y\}$$

In this, X is the inter-entrance duration customization and Y is the service time customization. $\mu_{\tilde{P}'}(p) \& \gamma_{\tilde{P}'}(p)$ is the membership and non-membership functions respectively of the inter-arrival time. $\mu_{\tilde{Q}'}(q) \& \gamma_{\tilde{Q}'}(q)$ is the membership and non-membership functions, respectively of the service time.

7. Single server Erlang fuzzy queuing model with infinite capacity

Let $\tilde{\lambda}$ and $\tilde{\lambda}'$ be the fuzzy and intuitionistic fuzzy arrival rates respectively. Let $\tilde{\mu}$ and $\tilde{\mu}'$ be the fuzzy and intuitionistic fuzzy service rates, respectively. At the steady-state, the FCFS discipline is upheld, and the capacity is unlimited.

The following are the fabrication characteristics of the above model:

i) The mean no. of customers in the queue,

$$\tilde{N}_q = \frac{k+1}{2k}\frac{\tilde{\lambda}^2}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})} \quad (29)$$

ii) The mean no. of customers in the system,

$$\tilde{N}_s = \frac{k+1}{2k}\frac{\tilde{\lambda}^2}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})} + \frac{\tilde{\lambda}}{\tilde{\mu}} \quad (30)$$

iii) The mean waiting time of the customers in the queue,

$$\tilde{T}_q = \frac{k+1}{2k}\frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})} \quad (31)$$

iv) The mean waiting time of the customers in the system,

$$\tilde{T}_s = \tilde{T}_q + \frac{1}{\tilde{\mu}} \quad (32)$$

8. Mathematical description

Consider the characteristics presented by a set of k traffic signals in $(FM/FE_k/1):(\infty/FCFS)$ fuzzy queuing system. To investigate the system as a queuing problem, we will use the terms to signify the customer's (vehicles) arrival rate per minute and denote the service rate per minute (i.e., for a shorter amount of time, the traffic signals will look green, permitting vehicles to start moving in under a minute.). After that, assume an endless source population, a limitless queue length, and an FCFS queue discipline. In the examination of the queuing problem associated with k traffic signals, each of the output parameters $\tilde{N}_q$ & $\tilde{N}_q'$, $\tilde{T}_q$ & $\tilde{T}_q'$, $\tilde{N}_s$ & $\tilde{N}_s'$ and $\tilde{T}_s$ & $\tilde{T}_s'$ is skeptical. Let $k = 2$ (i.e., a queuing system with two traffic signals), and the customer (vehicle) arrival rate per minute is $\tilde{\lambda} = 3$ and $\tilde{\mu} = 13$ (i.e., thirteen vehicles are permitted to move through the system in one minute) for the sake of convenience.

8.1 Single server fuzzy Erlang queuing model with infinite capacity

Consider an automated process in which the service is divided into two phases, with an arrival rate $\tilde{\lambda} = (2,3,4)$ and service rate $\tilde{\mu} = (12,13,14)$ with $k = 2$. Determine the TFN in the form of $(\tilde{m}, \tilde{\alpha}, \tilde{\beta})$ as $\tilde{\lambda} = (3,1,1)$ and $\tilde{\mu} = (13,1,1)$. To determine the values of a no. of customers and their sojourn time in the queue as well as a system respectively using suitable formulas among (29), (30), (31), & (32). It is necessary to use the appropriate arithmetic operations described in (1), (2), (3), (4), and (5) for add, sub, multiply, divide, and scalar multiply, respectively. For instance, the value of $\tilde{N}_q$ is calculated and given in Appendix A.

The metrics of performance are calculated and tabulated in Table 1.

Table 1: Performance Measures using triangular fuzzy numbers

S. No	Parameters	Quantifiable Metrics Using TFN
1	$\tilde{N}_q$	$(-0.9481, 0.0519, 1.0519)$
2	$\tilde{N}_s$	$(-0.7174, 0.2826, 1.2826)$
3	$\tilde{T}_q$	$(-0.9827, 0.0173, 1.0173)$
4	$\tilde{T}_s$	$(-0.9058, 0.0942, 1.0942)$

8.2. Single server fuzzy Erlang queuing model with infinite capacity

Consider an automated process in which the service is divided into two phases, with an arrival rate $\tilde{\lambda}' = (2,3,4;1,3,5)$ and service rate $\tilde{\mu}' = (12,13,14;11,13,15)$ with $k = 2$. Determine the TIFN in the form of $(m, \alpha, \beta; m, \alpha', \beta')$ as $\tilde{\lambda}' = (3,1,1;3,2,2)$ and $\tilde{\mu}' = (13,1,1;13,2,2)$. To determine the values of a no. of customers and their sojourn time in the queue as well as a system respectively using suitable formulas among (29), (30), (31), & (32). It is necessary to use the appropriate arithmetic operations described in (6), (7), (8), (9), and (10) for add, sub, multiply, divide, and scalar multiply, respectively. For instance, the value of $\tilde{N}_q'$ is calculated and given in Appendix B.

The metrics of performance are calculated and tabulated in Table 2.

Table 2: Performance Measures using triangular intuitionistic fuzzy numbers

S. No	Parameters	Quantifiable Metrics Using TIFN
1	$\tilde{N}_q'$	$(-0.9481,0.0519,1.0519;-1.9481,0.0519,2.0519)$
2	$\tilde{N}_s'$	$(-0.7174,0.2826,1.2826;-1.7174,0.2826,2.2826)$
3	$\tilde{T}_q'$	$(-0.9827,0.0173,1.0173;-1.9827,0.0173,2.0173)$
4	$\tilde{T}_s'$	$(-0.9058,0.0942,1.0942;-1.9058,0.0942,2.0942)$

The following figures depict the visualizations of Tables 1 and 2.

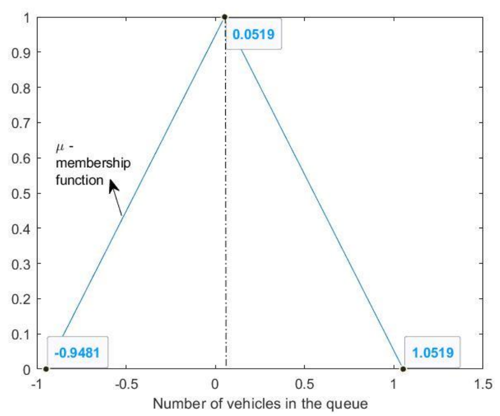


Figure 2: Number of vehicles in the queue $\tilde{N}_q$

Figure 3: Number of vehicles in the system $\tilde{N}_s$

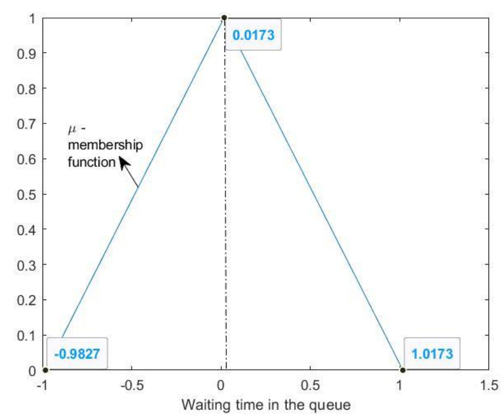
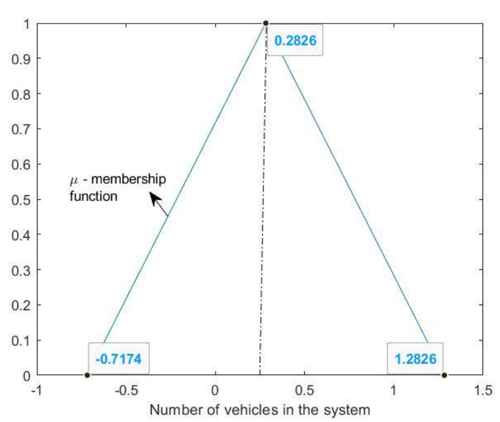


Figure 4: Waiting time of vehicles in the queue $\tilde{T}_q$



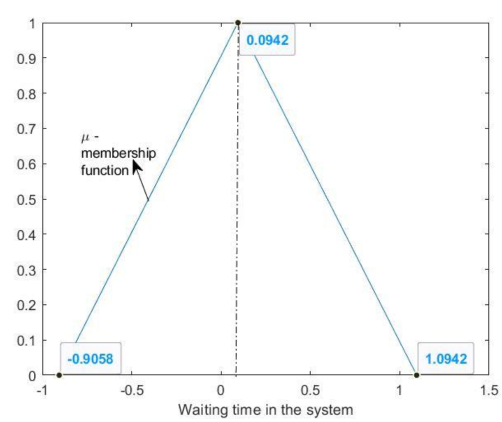


Figure 5: Waiting time of vehicles in the system $\tilde{T}_s$

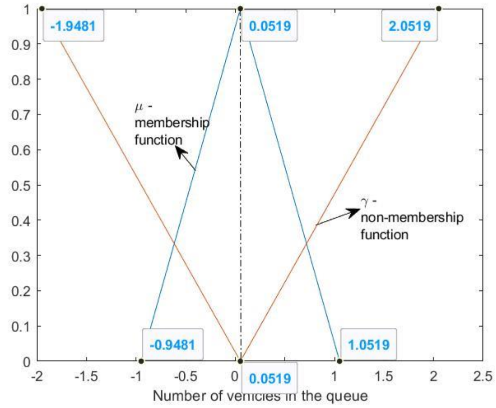


Figure 6: The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the number of vehicles in the queue $\widetilde{N}_q^{'}$

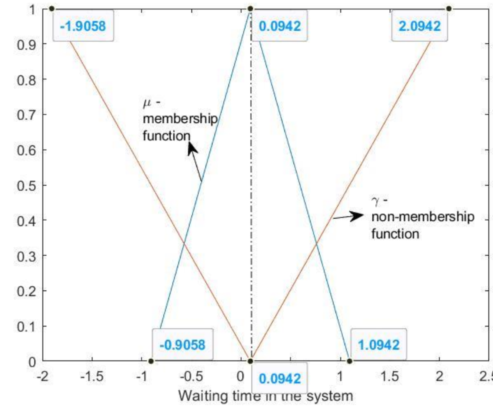


Figure 8: The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of waiting time of vehicles in the system $\tilde{T}_s^{'}$

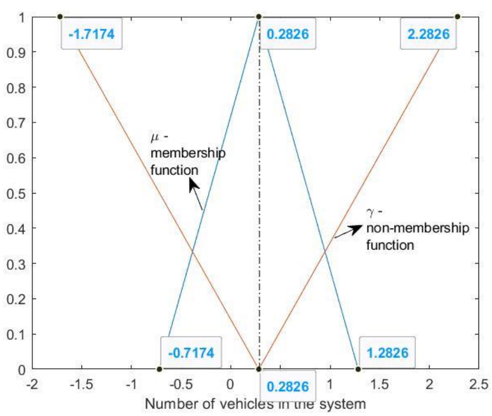


Figure 7: The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of the number of vehicles in the system $\widetilde{N}_s^{'}$

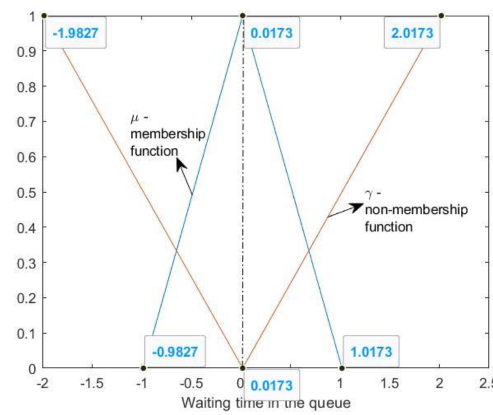


Figure 9: The membership($\tilde{\mu}$) and the non-membership($\tilde{\gamma}$) functions of waiting time of vehicles in the queue $\tilde{T}_q^{'}$

9. Results and Conclusions

The results are presented in Tables 1-2, which illustrate distinct assessments for a variety of membership functions (TFN and TIFN).

i) The mean value of $\widetilde{N}_q = 0.0519$ and the left spread and right spread values are -0.9481 and 1.0519 respectively signalling that the queue length of the vehicles is closely between -0.9481 and 1.0519. Its most typical value is 0.0519.

ii) The mean value of $\widetilde{N}_s = 0.2826$ and the left spread and right spread values are -0.7174 and 1.2826 respectively signalling that the system length of the vehicles is closely between -0.7174 and 1.2826. Its most typical value is 0.2826.

iii) The mean value of $\tilde{T}_q = 0.0173$ and the left spread and right spread values are -0.9827 and 1.0173 respectively signalling that the waiting time of the vehicles in the queue is closely between -0.9827 and 1.0173. Its most typical value is 0.0173(1 second).

iv) The mean value of $\tilde{T}_s = 0.0942$ and the left spread and right spread values are -0.9058 and 1.0942 respectively signalling that the waiting time of the vehicles in the system is closely between -0.9058 and 1.0942. Its most typical value is 0.0942(6 seconds).

v) The mean value of $\widetilde{N}_q^{'} = 0.0519$ and the left and right fuzziness of the membership$(\tilde{\mu})$ functions are -0.9481 and 1.0519 respectively and the left and right fuzziness of non-membership$(\tilde{\gamma})$ functions are -1.9481 and 2.0519 respectively. Its most typical value is 0.0519.

vi) The mean value of $\widetilde{N}_s^{'} = 0.2826$ and the left and right fuzziness of the membership$(\tilde{\mu})$ functions are -0.7174 and 1.2826 respectively and the left and right fuzziness of non-membership$(\tilde{\gamma})$ functions are -1.7174 and 2.2826 respectively. Its most typical value is 0.2826.

vii) The mean value of $\widetilde{T}_q^{'} = 0.0173$ and the left and right fuzziness of the membership$(\tilde{\mu})$ functions are -0.9827 and 1.0173 respectively and the left and right fuzziness of non-membership$(\tilde{\gamma})$ functions are -1.9827 and 2.0173 respectively. Its most typical value is 0.0173(1 second).

viii) The mean value of $\widetilde{T}_s^{'} = 0.0942$ and the left and right fuzziness of the membership$(\tilde{\mu})$ functions are -0.9058 and 1.0942 respectively and the left and right fuzziness of non-membership$(\tilde{\gamma})$ functions are -1.9058 and 2.0942 respectively. Its most typical value is 0.0942(6 seconds).

10. Conclusion

IFS appears to have some propensity for Decision Making Problems. Decision-makers may be unable to properly communicate their opinions on the situation in some cases due to a lack of meticulous, reliable, or precise information about the problem or because they are reluctant explicitly delineate the degree to which one alternative is preferable to others. The decision-maker may exhibit certain preferences for alternatives in some cases, but it's possible that they aren't completely convinced. Many academics have concentrated on IFS theory to address the demand for optimization issues involving ambivalence and inconsistency.

The $(FM/FE_k/1):(\infty/FCFS)$ queuing model is investigated in this work using the algebraic operations technique. Assume that the arrival and service times are ambiguous. The results of this procedure are also a little hazy. This technique's effectiveness is demonstrated numerically. It has been discovered that boosting the confluence of variables can enhance the success of the lining model. The proposed model will aid enterprises, distributors, and merchants in properly determining the queuing system's optimal performance indicators. There are several ways to expand the scope of the report. One of them is to interpret the arrival rate and service rate as a fluctuating random variable or fuzzy random variable. Furthermore, it is widely known in the fuzzy logic literature that analytical conclusions derived in an intuitionistic fuzzy environment are more beneficial for software developers and programmers than those acquired in a fuzzy model since they are more realistic and instructive.

The mean queue and system lengths, as well as the mean sojourn time of a single server fuzzy queue in an Erlang service and system with unlimited capacity, are interpreted in this study. The fuzzy and intuitionistic fuzzy queue under the Erlang service rate is more precisely stated, and the prediction model is used to provide scientific results. TFN and TIFN numerical descriptions are used to assess the soundness of the proposed queuing system. The intuitionistic fuzzy queuing model is substantially more productive and convenient in assessing and evaluating measures of queuing models because this intuitionistic fuzzy theory is more flexible and scalable. Fuzzy results deliver a range of solutions, whereas intuitionistic fuzzy results provide a broader range of solutions. Hence the decision-makers can choose the best among the worst in an uncertain environment. The analysis's research shows that the fuzzy queuing model's performance measurements fall within the ballpark of the intuitionistic fuzzy queuing model's estimated performance measures. As a result, intuitionistic fuzzy queuing is one of the better ways of computing assessment tasks since the information compiled from the application is easier to grasp and interpret, according to this study. In the future, the proposed queuing model may eventually be expanded to include multiple objectives in Erlang. The proposed model can be used to explore new facets of intuitionistic set extensions like neutrosophic sets.

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Appendix A

The mean no. of customers in the queue, $\widetilde{N}_q = \frac{k+1}{2k}\frac{\tilde{\lambda}^2}{\tilde{\mu}(\tilde{\mu}-\tilde{\lambda})}$ (From 29)

Since the value of $\tilde{\lambda} = (3,1,1)$, $\tilde{\mu} = (13,1,1)$ and $k = 2$, the value of $\widetilde{N}_q$ is calculated as follows:

$$\widetilde{N}_q = \frac{3}{4}\frac{(3,1,1)^2}{(13,1,1)(10,1,1)}$$

$$\widetilde{N}_q = (0.75)\frac{(9,1,1)}{(130,1,1)}$$

$$\widetilde{N}_q = (0.75)(0.0692,1,1)$$

$$\widetilde{N}_q = (0.0519,1,1)$$

$$\widetilde{N}_q = (-0.9481,0.0519,1.0519)$$

Similarly, calculate the remaining parameters for the fuzzy queuing model.

Appendix B

Since the value of $\tilde{\lambda}' = (3,1,1;3,2,2)$, $\tilde{\mu}' = (13,1,1;13,2,2)$ and $k = 2$, the value of $\widetilde{N}_q'$ is calculated as follows:

$$\widetilde{N}_q' = \frac{3}{4}\frac{(3,1,1;3,2,2)^2}{(13,1,1;13,2,2)(10,1,1;10,2,2)}$$

$$\widetilde{N}_q' = \frac{3}{4}\frac{(9,1,1;9,2,2)}{(130,1,1;130,2,2)}$$

$$\widetilde{N}_q' = \frac{(27,1,1;27,2,2)}{(520,1,1;520,2,2)}$$

$$\widetilde{N}_q' = (0.0519,1,1;0.0519,2,2)$$

$$\widetilde{N}_q' = (-0.9481,0.0519,1.0519;-1.9481,0.0519,2.0519)$$

Similarly, calculate the remaining parameters for the intuitionistic fuzzy queuing model.