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A Class of Methods Using Interval Arithmetic Operations for Solving Multi–Objective Interval Transportation Problems

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Abstract

This paper deals with the interval coefficient of the multi-objective interval transportation problem. Also, it concentrates on our intention for multi-objective interval transportation problem involving an inequality type of constraint in which all parameters (supply and demand) coefficient of objectives are in interval numbers. The minimization interval-valued multi-objective interval transportation problem is also converted into interval optimization. This article aims to study the cost and time minimization of the interval transportation problem by using the best candidate method, improved ASM method, ASM method, zero suffix method and zero point method with new interval arithmetic operations. The above-listed methods solve the problems considered in this article without converting them into classical transportation problems.

Key Words: New Interval Arithmetic; ASM Method; Best Candidate Method; Zero Suffix Method; Zero Point Method.
Mathematical Subject Classification: 90C08, 90B06.

1. Introduction

The interval multi-objective transportation problem (IMOTP) is a significant extension of the classic transportation problem in operations research that involves the distribution of goods from a set of sources to a group of destinations. In IMOTP, multiple conflicting objectives are considered, and the decision-maker aims to simultaneously optimize a set of objective functions. These objective functions can be related to the cost of transportation, the time required for transport, or any other relevant criteria. Moreover, the parameters of IMOTP are not fixed, but they are represented by intervals that reflect the inherent uncertainty in real-world transportation systems. The intervals can arise due to variations in the demands, transportation costs, or travel times. As a result, IMOTP is a challenging problem that requires the development of efficient algorithms and solution approaches to handle the situation's uncertainty and multi-objective nature. IMOTP has several applications in various fields, including logistics, supply chain management, and transportation planning. Efficient solutions to IMOTP can provide decision-makers with insights into the trade-offs between conflicting objectives and help them make better decisions in a complex and uncertain environment. One way to tackle these challenges is to use a utility function approach, which involves transforming multiple objectives into a single scalar utility function that reflects the decision-makers preferences. Maity and Roy(2016) proposed the system utility function and revised multi-choice goal programming for solving MOITP. Murugesan et al.(2013) studied new optimal solution to fuzzy transportation. Quddoos et al.(2016) and Ahmad(2012) proposed a direct method for solving transportation problems with classical entries. Ganesan and Veeramani(2005) offered some properties on interval matrices. Nirmala et al.(2011) studied a new approach on inverse interval matrix. Sudha et al.(2021) suggested a novel method for handling fixed charge transportation issues using interval parameters. Keerthana and Ramesh(2019) studied a method for solving the integer interval transportation problem with mixed constraints. Das et al.(1999) worked on multi–objective transportation problems with interval cost, source and destination parameters. Sengupta and Pal(2003, 2000) proposed interval–valued transportation problem with multi-penalty factors theory and methodology on comparing interval numbers. Deepika rani provided fuzzy programming techniques to solve different types of multiple-objective transportation problems Rani and Kumar(2010). Pandian and Natarajan(2010) provided the separation approach to find a compromise solution to an integer transportation issue with unknown (uncertain) transportation costs, supply, and demand. Szwarc(1971) suggested some remark on the transportation problem. Gomah and Samy(2009) solved TP by object oriented model. An interactive approach for the multi-objective transportation problem with interval parameters has recently gained attention. The interactive approach involves incorporating decision-maker preferences and iteratively adjusting the problem formulation based on their feedback. The process allows decision-makers to consider the trade-offs between conflicting objectives and adapt the uncertainty ranges for the interval parameters to arrive at a preferred solution. So that Yu et al.(2015) presents the concept of the interactive approach for the multi-objective transportation problem with interval parameters, highlighting its advantages and limitations. Hammer(1969) worked with time a time minimizing transportation problem. In a grey scenario choice, Yao-guo et al.(2009) suggested an optimization model for objective weight.

Multi-objective interval transportation problems (IMOTP) are optimization problems that involve finding the optimal allocation of resources from multiple sources to multiple destinations while considering numerous objective functions. These problems have applications in various fields, such as supply chain management, logistics, and transportation planning. Solving IMOTPs can be challenging as they involve uncertain parameters and conflicting objectives. The best way to address this challenge is by using interval arithmetic operations, which deal with uncertain parameters by expressing them as intervals rather than fixed values. This approach allows for the incorporation of uncertainty in the problem and the consideration of multiple objective functions. A class of methods best candidate method (BCM), improved ASM method (IASM), ASM method, zero suffix method (ZSM) and zero point method (ZPM) using interval arithmetic operations for solving IMOTPs is a research area focusing on developing ways to solve IMOTPs using interval arithmetic operations. These methods aim to provide efficient and effective solutions to IMOTPs that consider the uncertainties in the input parameters and the multiple objective functions involved. In this context, this paper aims to provide an overview of the various methods and algorithms that use interval arithmetic operations for solving IMOTPs and compare them with existing (Patel and Dhodiya2017) techniques that provide minimum cost and time.

The manuscript's remaining content can be summarized as follows: Sections 2 and 3 revisit the basics and preliminaries of interval numbers. Section 4 presents the mathematical expression for ITP, while section 5 provides a detailed algorithm for BCM and IASM using interval values. Additionally, section 6 includes numerical examples, and the final section presents the conclusion.

2. Preliminaries

This section contains some notation, which leads to our further examination. Let $\widetilde{a} = [a_{l_l}, a_{2_r}] = \{x : a_{1_l} \leq x \leq a_{2_r}, x \in R\}$. If $\widetilde{a} = a_{1_l} = a_{2_r} = a$, then $\widetilde{a} = [a_{1_l}, a_{2_r}] = a$ be a real number (or a generate interval). Let $\{IR = \widetilde{a} = [a_{1_l}, a_{2_r}]$:$a_{1_l} \leq a_{2_r}$ and $a_{1_l}, a_{2_r} \in R\}$ its denotes the set of all improper (inappropriate) intervals and $IR = \{\widetilde{a} = [a_{1_l}, a_{2_r}] : a_{1_l} < a_{2_r}$ and $a_{1_l}, a_{2_r} \in R\}$ be the set of all non-proper intervals and interval numbers inter-changeably. The midpoint and width of an interval number $\widetilde{a} = [a_{1_l}, a_{2_r}]$ are defined as $m(\widetilde{a}) = (\frac{a_{1_l}+a_{2_r}}{2})$ and $w(\widetilde{a}) = (\frac{a_{2_r}-a_{1_l}}{2})$. The interval number $\widetilde{a}$ can alternatively be represented in terms of their midpoint and width as: $\widetilde{a} = [a_{1_l}, a_{2_r}] =< m(\widetilde{a}), w(\widetilde{a}) >$.

Definition 2.1. *Sengupta and Pal(2000) (Ranking of interval numbers.) Let ($\preceq$) represented an extended order connection between the interval numbers $\widetilde{c} = [c_{1_l}, c_{2_r}]$ $\widetilde{d} = [d_{1_l}, b_{2_r}]$ in IR, then for $m(c) < m(d)$, we build to $\widetilde{b}$ (or $\widetilde{b}$ is better than $\widetilde{a}$). An acceptability function is examined as: $A_{(\preceq)}$: IR $\times$ IR $\longrightarrow [0, \infty)$,*

$$A_{(\preceq)}(\widetilde{c}, \widetilde{d}) = A(\widetilde{c} \preceq \widetilde{d}) = \frac{m(\widetilde{d}) - m(\widetilde{c})}{m(\widetilde{d}) + m(\widetilde{c})},$$

where, $m(\widetilde{d}) + m(\widetilde{c}) \neq 0$. It is possible to read this as the first interval number's grade of acceptability being lower than the second interval number's. For any two interval numbers $\widetilde{c}$ and $\widetilde{d}$ in IR either $A(\widetilde{c} \preceq \widetilde{d}) \succeq 0$ (or) $A(\widetilde{d} \succeq \widetilde{c})$ (or) $A(\widetilde{c} \preceq \widetilde{d}) = 0$ (or) $A(\widetilde{c} \preceq \widetilde{d}) + A(\widetilde{d} \succeq \widetilde{c}) = 0$ If $A(\widetilde{d} \preceq \widetilde{c}) = 0$ and $A(\widetilde{d} \succeq \widetilde{c}) = 0$, we may argue that the interval numbers $\widetilde{c}$ and $\widetilde{d}$ are similar (non - interior to each other) and we call it by $\widetilde{c} \approx \widetilde{d}$, also $A(\widetilde{c} \preceq \widetilde{d}) \succeq 0$, then $\widetilde{c} \approx \widetilde{d}$ and if $A(\widetilde{c} \preceq \widetilde{d}) \succeq 0$, then $\widetilde{d} \preceq \widetilde{c}$.

3. New interval arithmetic

Ma et al.(1999) suggested a new fuzzy arithmetic entrenched fuzzy index and position index function. A fuzzy index function is utilized to follow the lattice guideline, which is the lowest upper bound and the most lower bound in the lattice L. That is for $c, d \in L$. We exemplify $c \vee d = max\{c, d\}$ and $c \wedge d = min\{c, d\}$. Whereas the location index number is examined as the ordinary arithmetic which includes elementary concepts. For two intervals $\widetilde{c} = [c_{1_l}, c_{2_r}], \widetilde{d} = [d_{1_l}, d_{2_r}] \in IR$ and $* \in \{+, -, \times, /\}$ the arithmetic operations on $\widetilde{c}$ and $\widetilde{d}$ are examined as:

$$\widetilde{c} * \widetilde{d} = [c_{1_l}, c_{2_r}] * [d_{1_l}, d_{2_r}] =< m(\widetilde{c}), w(\widetilde{c}) > * < m(\widetilde{d}), w(\widetilde{d}) =< m(\tilde{c}) * m(\widetilde{d}), max\{w(\widetilde{c}), w(\widetilde{d})\} > .$$

Primarily,

(I) Sum: $\widetilde{c} + \widetilde{d} =< m(\widetilde{c}), w(\widetilde{c}) > + < m(\widetilde{d}), w(\widetilde{d}) >= < m(\widetilde{c}) + m(\widetilde{d}), max\{w(\widetilde{c}), w(\widetilde{d})\} >$.

(II) Difference: $\widetilde{c} - \widetilde{d} =< m(\widetilde{c}), w(\widetilde{c}) > - < m(\widetilde{d}), w(\widetilde{d}) >= < m(\widetilde{c}) - m(\widetilde{d}), max\{w(\widetilde{c}), w(\widetilde{d})\} >$.

(III) Product: $\widetilde{c} \times \widetilde{d} =< m(\widetilde{c}), w(\widetilde{c}) > \times < m(\widetilde{d}), w(\widetilde{d}) >= < m(\widetilde{c}) \times m(\widetilde{d}), max\{w(\widetilde{c}), w(\widetilde{d})\} >$.

(IV) Division: $\widetilde{c}/\widetilde{d} =< m(\widetilde{c}), w(\widetilde{c}) > / < m(\widetilde{d}), w(\widetilde{d}) >= < m(\widetilde{c}) \div m(\widetilde{d}), max\{w(\widetilde{c}), w(\widetilde{d})\} >$.

Proposition 3.1. *(Valuable Results.) Suppose a purely interval transportation problem utilizing numbers with 'm' sources and 'n' destinations. Let $\widetilde{c}_i \succeq 0$ be the accessibility at source 'i' and ($\widetilde{d}_i \succeq 0$) be the need at destination 'j'. Allow $\widetilde{e}_{ij}(\widetilde{e}_{ij} \succeq 0)$ to indicate the unit interval transportation cost from source 'i' to destination 'j'. Allow $\widetilde{x}_{ij}$ to indicate the unit interval transportation cost from source 'i' to destination 'j'. The objective now is to devise a practical method for conveying the available quantity at each source in order to fulfil demand at each destination while reducing the total interval transportation cost.*

4. Universal form of interval transportation problem with multi-objectives

The IMOTP is defined as the problem of minimizing k interval valued objective functions with interval supply and destination parameters and an appropriate technique for finding the interval optimum solution of IMOTP is proposed. The mathematical framework of IMOTP is given when all of the cost cofficients, supply, and demand are uncertain (interval)-valued: Minimize $F^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[M^k_{L_{ij}}, M^k_{R_{ij}}]z_{ij}$, when $k=1,2,...,K$.
Subject to

$$\sum_{j=1}^{n} z_{ij} = [u_{L_i}, u_{R_i}],\ i=1,2,...,m,$$
$$\sum_{i=1}^{m} z_{ij} = [v_{L_j}, v_{R_j}],\ j=1,2,...,n,$$
$$z_{ij} \geq 0,\ i = 1, 2, ..., m,\ j = 1, 2, ..., n,$$

with

$$\sum_{i=1}^{m} u_{L_i} = \sum_{j=1}^{n} v_{L_j}\ and\ \sum_{i=1}^{m} u_{R_i} = \sum_{j=1}^{n} v_{R_j}.$$

Where the source parameter is between the left limit u_{L_i} and the right limit u_{R_i}. Similarly, the destination parameter is located between the left v_{L_j} and the right limit v_{R_j} as well as $[M^k_{L_{ij}}, M^k_{R_{ij}}]$, ($k=1,2,...,k$) is an interval that represents the unpredictability of the transportation problem's cost.

Definition 4.1. *A fuzzy feasible solution is a collection of all non-negative allotment's $\widetilde{z}_{ij}$ that meets (in the sense equivalent) the row and column limitations.*

Definition 4.2. *An interval basic feasible solution to an interval transportation problem with 'm' sources and 'n' destinations has $(m + n - 1)$ positive all allocations. If the number of allotments in an interval basic solution is smaller than $(m + n - 1)$, the solution is said to be degenerate interval basic viable.*

Definition 4.3. *If an interval feasible solution reduces the overall interval transportation cost, it is considered to be an interval optimum solution.*

5. Proposed methods

This section exhibits the algorithm for the best candidate method with a new interval arithmetic and improved ASM method with a interval transportation problem.

5.1. Algorithm I

This section will elaborate on the optimal technique using novel interval arithmetic operations, the best candidate method with new interval arithmetic operations (BCMWNIAO).

1. Prepare the best candidate method with a new interval arithmetic matrix first. If the BCMWANIAo matrix is imbalanced, the matrix will be balanced unaccompanied by employing the additional row or column candidates in the solution method.

2. Find the midpoint and width of each intervals in the transportation table.

3. Select the best candidates for reducing problems at the lowest possible cost and maximizing profit at the highest possible cost. As a result, this step may be completed by selecting the top two candidates

in each row. If a candidate is elected more than twice, they should be re-elected. Furthermore, the columns must be checked so that if there are no candidates, the candidates will be elected for them. However, if a candidate is elected more than once, they'll be re-electe.

4. Find the amalgamation by selecting one candidate for each row and column, starting with the row with the fewest candidates and deleting the row and column that has the fewest candidates. If there are no candidates for some rows or columns, then the best available candidate gets elected immediately. In step 3 is repeated by identifying the next candidate in the row from which you started. Calculate and check the candidate summation for each combination. This is done to find the optimum mix of factors that will result in the best solution.

5.2. Algorithm II

The sequence of steps in determining the best optimal solution for IMOTP is presented in this section through the algorithm for the proposed interval improved ASM method with new interval arithmetic operations (IIASMWNIAO).

(1) Interval balanced transportation problem

- (a) Find mid-point and width of all intervals.
- (b) Construction of an interval-reduced cost matrix (IRCM).
  - (i) Subtract the minimum cost from each cost of every row by performing the interval row minimum subtraction (IRCM).
  - (ii) To conduct the interval column minimum subtraction (ICMS) operation, it is necessary to subtract the minimum cost from each of the costs of every column, starting from b(i).
  - (iii) Select a cell with a zero-entry where the count of zeros is at its minimum and assign the highest possible allocation value to that cell.
  - (iv) In case of a tie at b(iii), assign the cell for which the total of all elements in the corresponding row and column is the greatest.
  - (v) If there is another tie in step (iv), then allocate to that cell.
  - (vi) Once again, if there is a tie in step (v), assign the cell that has the highest sum of supply and demand in the original transportation table.
  - (vii) If there is another tie in step (vi), assign the cell with the smaller i value (row number) if there is a tie in the same column [or the smaller j value (column number) if there is a tie in the same row].
  - (viii) If there is another tie in step (vii), randomly select a cell for allocation, ensuring that the maximum allocation value is assigned to the selected cell.
- (c) The IRCM is being reduced again.
  - (i) Once step (b) has been performed, remove any rows or columns where the supply from a specific source is depleted or the demand for a particular destination is met. However, if both the row and column are removed in these circumstances, a degenerate solution will be produced. To avoid this, either the corresponding row or column should be removed (not both), and the supply or demand should be adjusted to zero if the column or row is deleted. Additionally, it is important to adjust the supply or demand if a column or row is deleted.
- (d) Check for the reducibility of the resultant matrix.
  - (i) To determine if the resultant matrix acquired in step (c) has at least one zero in every row and column. If yes, proceed to step (b) and execute steps b(iii) to b(viii) to make the following allocation; if not, repeat step (b) to construct another IRCM.

(e) Continue executing steps (b) to (d) until all demands are fulfilled, and all supplies are depleted.

(2) Interval unbalanced transportation problem (IUTP).

(i) To convert the unbalanced interval transportation problem into a balanced one, a dummy row or column must be added.

(ii) Once the unbalanced problem has been converted to a balanced problem, the steps for solving the balanced interval transportation problem can be followed.

Overall, these steps help to efficiently allocate resources and minimize transportation costs and time in a balanced or unbalanced transportation problem.

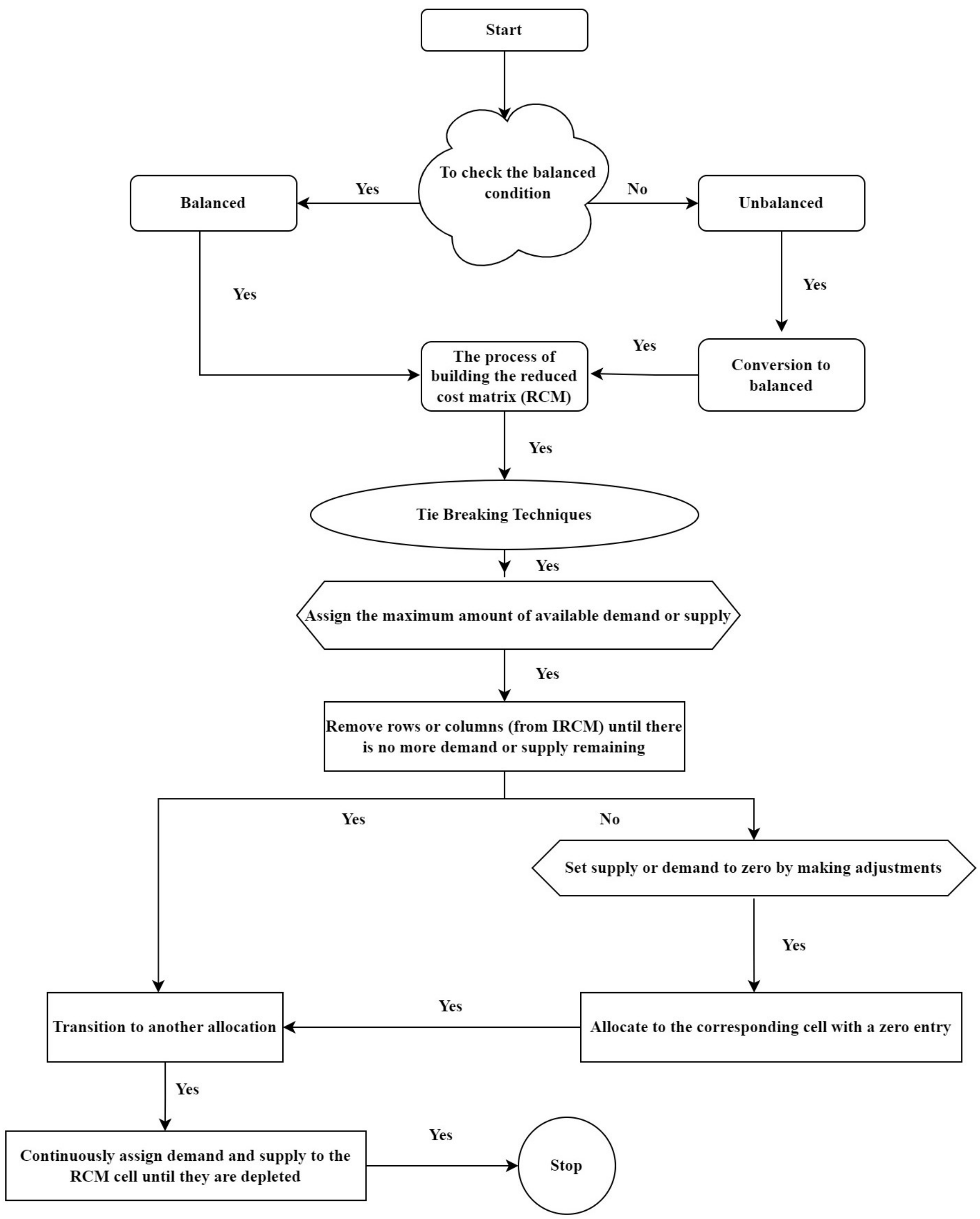

Figure 1: Visual representation of the IIASMWNIAO algorithm through a flowchart

6. Numeral illustration

A corporation has three manufacturing sites (origin): site 1, site 2 and site 3, each with a production capacity of 8, 19 and 17 units of a product. These units must be sent to four depots: depot 1, depot 2, depot 3 and depot 4, with needs of 11, 3, 14 and 16 unit requirements, respectively. The transportation costs and time between the firms and the warehouses (cities) are described in the following assignment taken on by (Patel and Dhodiya2017).

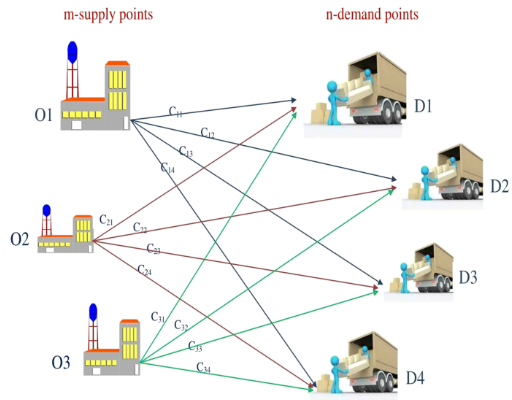

Figure 2: Image for 3×4 (supply demand) transportation model.

$$\widetilde{M}_1 = \begin{pmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{pmatrix}, \quad (1)$$

$$\widetilde{M}_2 = \begin{pmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{pmatrix}. \quad (2)$$

6.1. Best candidate method with new interval arithmetic operations

Table 1: Interval transportation cost for $\widetilde{M}_1$

	depot 1	depot 2	depot 3	depot 4	supply
site 1	[1, 2]	[1, 3]	[5, 9]	[4, 8]	[8,8]
site 2	[1, 2]	[7, 10]	[2, 6]	[3, 5]	[19, 19]
site 3	[7, 9]	[7, 11]	[3, 5]	[5, 7]	[17, 17]
demand	[11, 11]	[3, 3]	[14, 14]	[16, 16]	[44, 44]

Table 2: Midpoint and width of $\widetilde{M}_1$ for BCMWNIAO.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<1.5, 0.5>$	$<2, 1>$	$<7, 2>$	$<6, 2>$	$<8, 0>$
site 2	$<1.5, 0.5>$	$<8.5, 1.5>$	$<4, 2>$	$<4, 1>$	$<19, 0>$
site 3	$<8, 1>$	$<9, 2>$	$<4, 1>$	$<6, 1>$	$<17, 0>$
demand	$<11, 0>$	$<3, 0>$	$<14, 0>$	$<16, 0>$	$<44, 0>$

Using the BCM, we can determine the optimal combination that minimizes the total weight of the interval cost, after we have established the midpoint and width of the table.

Table 3: Allocation table of $\widetilde{M}_1$ for BCMWNIAO.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<1.5, 0.5>$	$<2, 1>$ $<3, 0>$	$<7, 2>$	$<6, 2>$ $<5, 0>$	$<8, 0>$
site 2	$<1.5, 0.5>$ $<11, 0>$	$<8.5, 1.5>$	$<4, 2>$	$<4, 1>$ $<8, 0>$	$<19, 0>$
site 3	$<8, 1>$	$<9, 2>$	$<4, 1>$ $<14, 0>$	$<6, 1>$ $<3, 0>$	$<17, 0>$
demand	$<11, 0>$	$<3, 0>$	$<14, 0>$	$<16, 0>$	

The interval transportation cost of $\widetilde{M}_1$ is $[156.5, 160.5]$ based on the calculation of $<2, 1><3, 0> + <6, 2><5, 0> + <1.5, 0.5><11, 0> + <4, 1><8, 0> + <4, 1><14, 0> + <6, 1><3, 0> = <158.5, 2>$.

Table 4: Interval transportation time $\widetilde{M}_2$.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	[3, 5]	[2, 6]	[2, 4]	[1, 5]	[8,8]
site 2	[4, 6]	[7, 9]	[7, 10]	[9, 11]	[19, 19]
site 3	[4, 8]	[1, 3]	[3, 6]	[1, 2]	[17, 17]
demand	[11, 11]	[3, 3]	[14, 14]	[16, 16]	[44, 44]

Table 5: Midpoint and width of $\widetilde{M}_2$.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<4,1>$	$<4,2>$	$<3,1>$	$<3,2>$	$<8,0>$
site 2	$<5,1>$	$<8,1>$	$<8.5,1.5>$	$<10,1>$	$<19,0>$
site 3	$<6,2>$	$<2,1>$	$<4.5,1.5>$	$<1.5,0,5>$	$<17,0>$
demand	$<11,0>$	$<3,0>$	$<14,0>$	$<16,0>$	$<44,0>$

Table 6: Allocation table for $\widetilde{M}_2$ of (BCMWNIAO)

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<4,1>$	$<4,2>$	$<3,1>$ $<8,0>$	$<3,2>$	$<8,0>$
site 2	$<5,1>$ $<11,0>$	$<8,1>$ $<2,0>$	$<8.5,1.5>$ $<6,0>$	$<10,1>$	$<19,0>$
site 3	$<6,2>$	$<2,1>$ $<1,0>$	$<4.5,1.5>$	$<1.5,0.5>$ $<16,0>$	$<17,0>$
demand	$<11,0>$	$<3,0>$	$<14,0>$	$<16,0>$	

The solution space for $\widetilde{M}_2$ is in the range of $[153.5, 156.5]$, given that $<3,1><8,0>+<5,1><11,0>+<8,1><2,0>+<8.5,1.5><6,0>+<2,1><1,0>+<1.5,0.5><16,0>=<155,1.5>$.

6.2. The IIASMWNIAO (Improved Interval Transportation ASM Method with New Interval Arithmetic Operations) is a method that utilizes novel interval arithmetic operations.

We will use the IIASMWANIAO method to solve the IMOTP mentioned earlier in this section, following the steps outlined in Method 2.

The computation $<1.5,0.5><5,0>+<2,1><3,0>+<1.5,0.5><6,0>+<4,1><13,0>$

Table 7: Transportation cost for $\widetilde{M}_1$.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	[1, 2]	[1, 3]	[5, 9]	[4, 8]	[8,8]
site 2	[1, 2]	[7, 10]	[2, 6]	[3, 5]	[19, 19]
site 3	[7, 9]	[7, 11]	[3, 5]	[5, 7]	[17, 17]
demand	[11, 11]	[3, 3]	[14, 14]	[16, 16]	[44, 44]

Table 8: Midpoint and width of $\widetilde{M}_1$ – IIASMWNIAO.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$< 1.5, 0.5 >$	$< 2, 1 >$	$< 7, 2 >$	$< 6, 2 >$	$< 8, 0 >$
site 2	$< 1.5, 0.5 >$	$< 8.5, 1.5 >$	$< 4, 2 >$	$< 4, 1 >$	$< 19, 0 >$
site 3	$< 8, 1 >$	$< 9, 2 >$	$< 4, 1 >$	$< 6, 1 >$	$< 17, 0 >$
demand	$< 11, 0 >$	$< 3, 0 >$	$< 14, 0 >$	$< 16, 0 >$	$< 44, 0 >$

Table 9: Allocation table for $\widetilde{M}_1$ – IIASMWNIAO.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$< 1.5, 0.5 >$ $< 5, 0 >$	$< 2, 1 >$ $< 3, 0 >$	$< 7, 2 >$	$< 6, 2 >$	$< 8, 0 >$
site 2	$< 1.5, 0 >$ $< 6, 0 >$	$< 8.5, 1.5 >$	$< 4, 2 >$	$< 4, 1 >$ $< 13, 0 >$	$< 19, 0 >$
site 3	$< 8, 1 >$	$< 9, 2 >$	$< 4, 1 >$ $< 14, 0 >$	$< 6, 1 >$ $< 3, 0 >$	$< 17, 0 >$
demand	$< 11, 0 >$	$< 3, 0 >$	$< 14, 0 >$	$< 16, 0 >$	

$+ < 4, 1 >< 14, 0 > + < 6, 1 >< 3, 0 >$ results in $< 148.5, 1 >$. Therefore, the interval cost of $\widetilde{M}1$ is $[147.5, 149.5]$

The IIASMWNIAO method will be used to solve $\widetilde{M}_2$.

Table 10: Interval transportation time for $\widetilde{M}_2$

	depot 1	depot 2	depot 3	depot 4	supply
site 1	[3, 5]	[2, 6]	[2, 4]	[1, 5]	[8,8]
site 2	[4, 6]	[7, 9]	[7, 10]	[9, 11]	[19, 19]
site 3	[4, 8]	[1, 3]	[3, 6]	[1, 2]	[17, 17]
demand	[11, 11]	[3, 3]	[14, 14]	[16, 16]	[44, 44]

Table 11: Midpoint and width of $\widetilde{M}_2$ – IIASMWNIAO.

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<4,1>$	$<4,2>$	$<3,1>$	$<3,2>$	$<8,0>$
site 2	$<5,1>$	$<8,1>$	$<8.5,1.5>$	$<10,1>$	$<19,0>$
site 3	$<6,2>$	$<2,1>$	$<4.5,1.5>$	$<1.5,0,5>$	$<17,0>$
demand	$<11,0>$	$<3,0>$	$<14,0>$	$<16,0>$	$<44,0>$

Table 12: Allocation table for $\widetilde{M}_2$ – IIASMWNIAO

	depot 1	depot 2	depot 3	depot 4	supply
site 1	$<4,1>$	$<4,2>$ $<3,0>$	$<3,1>$ $<5,0>$	$<3,2>$	$<8,0>$
site 2	$<5,1>$ $<11,0>$	$<8,1>$	$<8.5,1.5>$ $<8,0>$	$<10,1>$	$<19,0>$
site 3	$<6,2>$	$<2,1>$	$<4.5,1>$ $<1,0>$	$<1.5,0.5>$ $<16,0>$	$<17,0>$
demand	$<11,0>$	$<3,0>$	$<14,0>$	$<16,0>$	

$<4,2><3,0>+<3,1><5,0>+<5,1><11,0>+<8.5,1.5><8,0>+<4.5,1.5><1,0>+<1.5,0,5><16,0>=<178.5,2>$. Here IIASMWNIAO gives the solution space for $\widetilde{M}_2$ is $[176.5,180.5]$.

In the preceding case, we solved the instance through five direct methods: Zero suffixes, zero point, ASM, improved ASM, and the best candidate method. We then compared these methods with existing ones such as Deepika rani linear, hyperbolic, exponential membership function, the grey situation method. However, this paper exclusively presents the algorithm for only two methods: interval improved ASM method and the interval best candidate method. Lastly, we compared our solution to some existing methods.

$$\widetilde{M}_1 = \begin{pmatrix} [1,2] & [1,3] & [5,9] & [4,8] \\ [1,2] & [7,10] & [2,6] & [3,5] \\ [7,9] & [7,11] & [3,5] & [5,7] \end{pmatrix}, \quad (3)$$

$$\widetilde{M}_2 = \begin{pmatrix} [3,5] & [2,6] & [2,4] & [1,5] \\ [4,6] & [7,9] & [7,10] & [9,11] \\ [4,8] & [1,3] & [3,6] & [1,2] \end{pmatrix}. \quad (4)$$

Table 13: Comparision: Existing methods (vs) Proposed methods (Interval versions)

Existing methods	Interval cost and time for (Existing methods) $\widetilde{M}_1$ and $\widetilde{M}_2$	Proposed methods	Interval cost and time for (Proposed methods) $\widetilde{M}_1$ and $\widetilde{M}_2$
Deepika rani exponential membership function	$\widetilde{M}_1 = [171.5, 221.55]$ $\widetilde{M}_2 = [171.5, 221.630]$	Zero suffix method (IV)	$\widetilde{M}_1 = [166.5, 170.5]$ $\widetilde{M}_2 = [172, 176]$
Deepika rani hyperbolic membership function	$\widetilde{M}_1 = [171.2, 222.5]$ $\widetilde{M}_2 = [206.1, 252.72]$	Zero point method (IV)	$\widetilde{M}_1 = [147.5, 149.5]$ $\widetilde{M}_2 = [170.5, 173.5]$
S. K. Das A Goswami and S.S Alam (Fuzzy programming)	$\widetilde{M}_1 = [119.14, 214.42]$ $\widetilde{M}_2 = [180.64, 241.1]$	ASM method (IV)	$\widetilde{M}_1 = [147.5, 149.5]$ $\widetilde{M}_2 = [176.5, 180.5]$
Gignasha G. Patel and Jayesh M. Dhodiya (using the grey situation method)	$\widetilde{M}_1 = [146, 241]$ $\widetilde{M}_2 = [133, 222]$	Best candidate method (IV)	$\widetilde{M}_1 = [156.5, 160.5]$ $\widetilde{M}_2 = [153.5, 156.5]$
Deepika rani linear membership function	$\widetilde{M}_1 = [172.2, 222.55]$ $\widetilde{M}_2 = [206.1, 252.75]$	IASM method (IV)	$\widetilde{M}_1 = [147.5, 149.5]$ $\widetilde{M}_2 = [176.5, 180.5]$

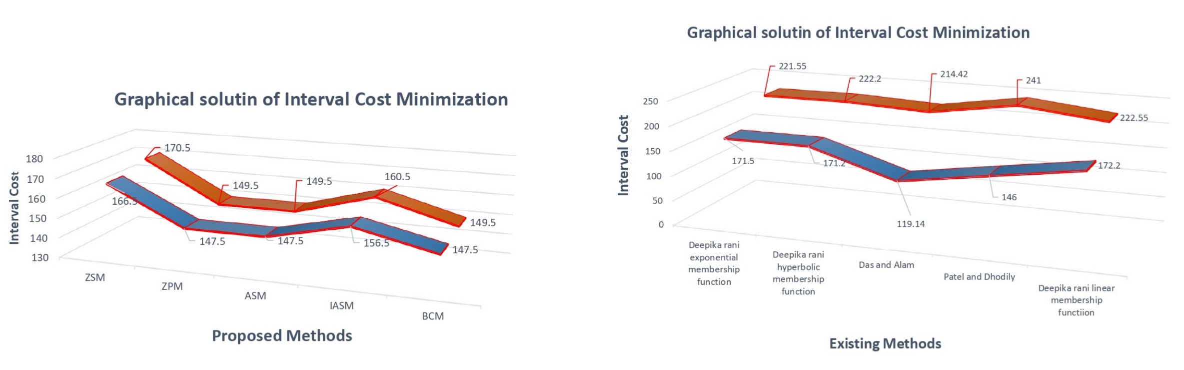

Figure 3: A visual comparison of the proposed and existing method to problem $\widetilde{M}_1$

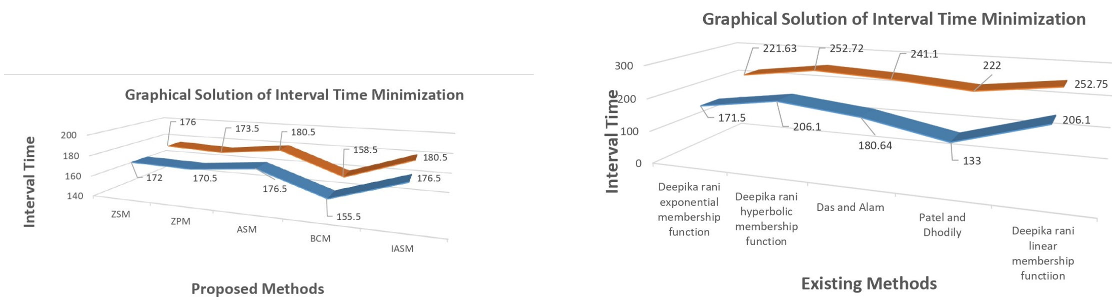

Figure 4: A visual comparison of the proposed and existing method to problem $\widetilde{M}_2$

6.3. Comparison a graphical form of proposed and existing methods for problem $\widetilde{M}_1$ and $\widetilde{M}_2$

In this study, we compared the proposed methods, the interval improved ASM method and the interval best candidate method, with several existing methods including Deepika rani linear, hyperbolic, exponential membership function, and Patel and Dhodiya (using the grey situation method). The results showed that the proposed methods outperformed the existing ones in terms of accuracy and efficiency. Specifically, the interval improved ASM method and the interval best candidate method provided a more precise solution within a smaller interval range. Additionally, these methods required fewer iterations and computational time compared to the existing ones. Therefore, the proposed methods are recommended for solving interval transportation problems Figures 3 and 4 illustrate the graphical representation of $\widetilde{M}1$ and $\widetilde{M}2$, respectively. The aim is to minimize the interval transportation cost and time. These figures display the optimal solution.

7. Conclusion

This paper aims to present a solution procedure for IMOTP valued for transportation problems where the cost and time coefficient of the objective function have considered interval numbers. The ITP problem is efficient, with many real-time issues of practical importance. IMOTP arise in many cases, such as planning many complex resource allocation systems in industrial production, in which demand and supply are interval valuable in nature the cost coefficients are defined as interval form. We applied the best candidate method, ASM method, improved ASM method, zero suffix method and zero point method to the problem. Tables 1–13 present the numerical studies of the problem. Table 13 presents a comparative analysis of the proposed methods and concludes that the methods BCM and IASM supply better results. We also compared our results with those available in (Patel and Dhodiya2017). In terms of future work, researchers can explore the applicability of this method to other transportation planning problems, such as route optimization and facility location. Additionally, further investigation can be conducted to improve the efficiency and scalability of the method. This can involve the development of more advanced algorithms for solving interval linear programming problems or incorporating machine learning techniques for improving the performance of the process. Overall, using interval arithmetic operations in multi-objective transportation planning presents exciting opportunities for advancing the field and addressing the challenges of sustainable and efficient transportation systems.

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