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# Spatial-temporal factors affecting monthly rainfall in some Central Asian countries assuming a Weibull regression model

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**Abstract**

Climate change has been observed worldwide in the last years. Among the different effects of climate change, rain precipitation is one of the effects that most challenge the population of all countries in the world. The main goal of this study is to introduce a data analysis of monthly rainfall data related to five countries in Central Asia (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan) for a long period of time to discover the behavior of rain precipitation in these countries for each month of the year in the last decades and possible link with climate change. Since climate data are positive real values, Weibull regression models were fitted for the rain precipitation data (precipitation sums by climate station and year) under a classical inference approach in presence of some spatial factors as latitude and longitude of the climate stations in each country, temporal factors (linear year effects), altitude of the climate station and categorical factors (countries).The obtained results show that some factors have different effects in the monthly rainfall of the assumed countries during the follow-up assumed period, possibly linked to the climate change observed in the last decades worldwide.

**Key Words:** rainfall data; Weibull regression models; spatial-temporal factors; maximum likelihood estimators.

## 1. Introduction

In the last few decades, the world has seen drastic changes in the climate affecting temperature, level of the oceans and amount of annual rainfall among many other effects of climate change (https://www.ncdc.noaa.gov/monitoring-references/faq/indicators.php). Other effects of climate change could be observed in food production, the rise of sea levels, catastrophic flooding in many parts of the world and modifications on greenhouse gases which are essential to all living forms in the planet (https://www.un.org/en/sections/issues-depth/climate-change/). For example, temperature and precipitation changes over time have not been uniform across the planet or even in different regions of the same country as observed by many authors. As a special case, the average rainfall in the United States of America has increased since 1900, but some areas of the country had increases greater than the national rainfall average and some areas had lower than average (IPCC 2007, 2013). Thus, it is very important to study the changes occurring in the global temperature and rain precipitation in different regions of the world. This becomes even more important due to the different behaviors of the changes in different parts of the world since some regions may be more affected than others.

Many papers have been published related to climate change events, such as, change in precipitation volumes, temperature values, sea levels, among many others, and their implications. As special cases we can mention a study on the impact of climate change on water resources and flooding was published by Arnell and Lloyd-Hughes (2014); in other work, Costello et al. (2009) studied the relation between climate change and health effects; Lineman et al. (2015) introduced a study relating global warming and climate change; a study introduced by Kabir et al. (2016) analyzed the impact of climate change on the coastal areas of Bangladesh; sea level changes in relation to global warming was studied by Levermann et al. (2013); a paper introduced by Serdeczny et al. (2016) considered the impact of climate change on the sub-Saharan Africa; Turner et al. (2020) analyzed the threat posed by climate change on ecosystems; Tebaldi and Sansó (2009) introduced a study considering the joint change in temperature and precipitation from multiple climate models using a Bayesian point of view.

Olumuyiwa and Masengo (2017) introduced a review of case studies of climate change effects on the agricultural environment with emphasis on rainfall, as an important factor. Praveen et al. (2020) considered analyzes and forecasts for the long-term spatio-temporal changes in rainfall using data from 1901 to 2015 across India at meteorological divisional level using non-parametrical and machine learning approaches detecting an increasing rainfall trend in the period 1901–1950, while a significant decline rainfall trend was detected after 1951. Malmgren et al. (2020) analyzed precipitation trends in Sri Lanka since the 1870s and relationships to El Niño–Southern Oscillation using data from 15 climate stations in Sri Lanka discovering some significant temporal changes in precipitation at some climate stations. Lima et al. (2010) investigated the trends in annual and monthly precipitation in mainland Portugal considering long time series using data obtained from ten measuring climate stations scattered over mainland Portugal identifying a sequence of alternating decreasing and increasing trends in annual and monthly precipitation. Benestad (2013) related temporal variability in precipitation statistics with the global mean temperature using a multiple regression analysis. Donat et al. (2013), studied some relationships between climate extremes in the Arab region and certain prominent modes of variability, in particular El Niño-Southern Oscillation (ENSO) and North Atlantic Oscillation (NAO); they concluded that the relationships of the climate extremes with NAO are stronger, in general, than those with ENSO.

Longobardi and Villani (2010) considered analysis of rainfall time series for the period 1918–1999 and a wide area, detecting potential trends and assessing their significance considering data from 211 gauged stations, mainly located in southern Italy. Caloiero et al. (2011), considered a statistical analysis of annual and seasonal precipitation performed over 109 cumulated rainfall series with more than 50 years of data observed in a region of Southern Italy (Calabria) showing a decreasing trend for annual and winter–autumn rainfall and an increasing trend for summer precipitation. Clarke et al. (2011) studied short-duration rainfall data related to design of stormwater infrastructure considering data of 13 Canadian climate stations. Miao et al. (2012) presented a precipitation trend and periodic analysis at the seasonal scale on a 286–year data series (1724–2009) for Beijing, China based on different statistical models. Ambun et al. (2013) introduced a statistical analysis, based on linear regression models, of monthly, annual and seasonal trends of rainfall in Kuching, Malaysia from 1968 to 2010.

Arnbjerg-Nielsen et al. (2013) introduced a review of current methods for assessing future changes in urban rainfall extremes and their effects on urban drainage systems, due to anthropogenic-induced climate change. Kwarteng et al. (2009) considered a statistical analysis of the characteristics of rainfall in Oman using data recorded between 1977 and 2003 where the data was divided into six geomorphic compartments to represent the various topographic regions in Oman. Gunawardhana and Al-Rawas (2014) analyzed daily precipitation and temperature records in Muscat, Oman, mainly focusing on extremes. Modarres and Rodrigues da Silva (2007) considered time-series of annual rainfall, number of rainy-days per year and monthly rainfall of 20 stations, to analyze climate variability in semi-arid regions of Iran. Dinpashoh et al. (2004) selected variables to be used to regionalize Iran's precipitation climate using factor analysis and clustering techniques considering data from 77 climate stations in Iran from 1956 to 1998.

Raziei et al. (2014) investigated spatial patterns of monthly, seasonal, and annual precipitation over Iran and the corresponding long-term trends for the period 1951–2009 using the Global Precipitation Climatology Centre gridded dataset. Almazroui et al. (2012) studied the Arabian Peninsula's seasonal climate using observational and gridded data from surface that, irrespective of season, rainfall insignificantly increased in the first period (1979–1993), and then significantly decreased in the second period (1994–2009). Hasanean and Almazroui (2015) presented a review of Saudi Arabia (SA) climate, indicating that a great inter-annual change in the rainfall over the SA was observed for the period (1978–2009). Al-Mamoon and Rahman (2012), examined the trends of daily extreme rainfall events from 30 rain gauges located in Qatar using rainfall data covering from 1962 to 2011. Jones et al. (2015) studied the temporal variation of precipitation in the Upper Tennessee River basin using datasets from the Tennessee Valley Authority (TVA) rain gauge network consisting of 56 rain gauges (1990–2010), and the National Weather Service (NWS) analyzing mean areal precipitation values for 78 subbasins (1950–2009).

Kang and Yusof (2012) considered different homogeneity tests to detect the inhomogeneity of the daily rainfall data with at most 10% missing values for rainfall series of three climate stations (Damansara, Johor and Kelantan) of Malaysia using annual mean, annual maximum, and annual median. Marengo et al. (2020) studied trends in extreme rainfall events in the Metropolitan Area of São Paulo (MASP), Brazil in relation to hydrometeorological hazards that trigger natural disasters, such as flash floods, landslides, and droughts, that affect the population and local economies. Cong and Brady (2012) considered five families of copula models to model the interdependence between rainfall and temperature considering historical climate data of a leading agricultural province (Scania) in Sweden that is affected by a maritime climate. Mekis et al. (2018) introduced an overview of the present status and procedures related to surface precipitation observations at Environment and Climate Change Canada (ECCC). Devine and Mekis (2008) used daily historical rain-gauge data from several Canadian sources and field experiments to compare to the World Meteorological Organization (WMO) pit gauge rainfall measurements to determine the accuracies for different operational rain gauges.

Altın et al. (2012) presented a study using different statistical methodologies, showing the change in precipitation and temperature of the Central Anatolia region where a semi-arid climate prevails. Twumasi et al. (2020) examined the long-term climate variations in Central African Countries (Gabon, Cameroon, Republic of Congo, Central Africa Republic, Chad, and Democratic Republic of Congo) for the period 1901 to 2015, investigating the possible influence of increases in greenhouse gas concentrations using data collected from the World Bank Group Climate Change Knowledge Portal. Moreover, assuming non-transformed climate data, that is, the data in the original scale, some authors explore the use of standard existing lifetime probability distributions as the exponential, the Weibull, gamma, generalized gamma or log-normal distributions in the analysis of the data given the usual presence of asymmetry especially considering rain precipitation data (Singh 1987; Christopher et al. 2010; Alonge and Afullo 2012).

In this work, we study the behavior of annual rainfall precipitation (precipitation sums by climate station and year) in each month of the year (January to December) in a region of the world where climate change has had a great impact: Central Asia. In this way, we consider monthly rain precipitation data and possible dependence of rain precipitation with some temporal factors, or some spatial factors related to location of each observational climate station where the rainfall index was reported, as longitude (denoted as long) and latitude (denoted as lat) and other factors as altitude (denoted as alt) in different countries in Central Asia considering a data set introduced by Williams and Konovalov (2008).

The climate data considered in this study refers do rain precipitation data in five countries located in Central Asia: Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan (map of region in Figure A.1). The data set reported by Williams and Konovalov (2008) provides temperature and precipitation data from 298 surface meteorological stations in the Northern Tien Shan and Pamir Mountain Ranges of Central Asia, specifically from stations in Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan. The period of record covered by each station is variable, however, most stations have almost 100 years of observations with the earliest record from 1879 and the latest from 2003. The data set was compiled from meteorological measurements conducted by the National Hydrometeorological Services (NHMS) of the Central Asian countries. Rain precipitation data reported in each climate station are given by monthly sums. Besides rain precipitation data, also there are other information of each climate station, as population of the region where it is located the climate station, vegetation of the region and topography. The original data set consists of monthly rainfall (monthly sums) reported in different observational sites for a long period. Since the data set have many missing observations (many months with no data, especially in the first years of the follow-up periods), in this study we considered only complete data, that is, rainfall of years when there are the data for the twelve months of each year.

Figure A.2-A.6 (Appendix A) shows the histograms of the monthly rain precipitations (total rain precipitation for each month) for the five countries (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan) for a long period stating in the year 1879 and ending in the year 1999. From Figure A.2-A.6, we observe asymmetrical behavior for the rain precipitation data (monthly sums) for the twelve months considering the five countries, which requires the use of an asymmetric probability distribution in the data analysis assuming the data in the original scale. In practice, sometimes it is possible to transform the rain precipitation data, for example using a Box-Cox (1964) transformation, and the use of standard statistical techniques that assume a normal distribution such as linear regression models with normal errors or ANOVA (analysis of variance) models. Despite this possibility, the use of variable transformations can present difficulties for interpretations by the climate researchers usually interested in the results obtained using statistical models fitted for the data in the original scale. Different asymmetrical distributions for positive values could be used in the data analysis of the rain precipitation data set. In this study, we use a special parametric probability distribution commonly assumed in the statistical analysis of lifetime data given the great flexibility of fit: the Weibull distribution.

In this way, we assume a Weibull distribution in presence of some spatial-temporal covariates as latitude (lat), longitude (long), years and altitude (alt) of the observational rain stations considering the rain precipitation data separately for each month of the year (12 data sets). As rain precipitation varies seasonally in different months of the year associated with the year seasons, the twelve Weibull regression models could be important to verify which months of the year in different locations, regions, and countries the effect of climate change could be more meaningful. The results of the statistical analysis can be of great interest to these countries, especially in rain forecasts and agricultural planning. After a careful preliminary analysis of the database for missed observations, a total of N =13,888 reported month rain precipitation measures were considered in this study after deleting missing observations in the rainfall report introduced by Williams and Konovalov (2008).

Following SMART (Specific, Measurable, Achievable, Relevant, and Time bounded) structure, the main objectives of this study are:

- **Specific:** As a first goal of the study, we want to fit Weibull regression models for the rain precipitations (monthly sums) considering data in the original scale separately for each month of the year (twelve data sets from January to December in the period from 1879 to 1999) in each one of the five considered countries (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan) of Central Asia in presence of spatial factors (longitude, latitude), temporal factor (years), altitude of the observational location and also the categorical variables related to countries (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan). As a second goal of the study, we want to fit a Weibull distribution not considering the presence of covariates for the rain precipitation averages (averages of the precipitation sums over all stations for a specified month and region) to estimate the rain precipitation means in each month of the year and in each region. As a third goal of the study, we want to fit a Weibull distribution not considering the presence of covariates for the rain precipitation averages (averages of the rain precipitation sums over all stations for a specified geographical area) to estimate the rain precipitation means in each geographical area (agro, desert, forest, semi-desert, and urban area).
- **Measurable:** Use of a data set compiled from meteorological measurements conducted by the National Hydrometeorological Services (NHMS) of the Central Asian countries. Besides the rain precipitation data, also have other information of each climate station, as population of the region where it is located the climate station, vegetation of the region and topography. The data set provides temperature and precipitation data from 298 meteorological stations in the Northern Tien Shan and Pamir Mountain Ranges of Central Asia, specifically from stations in Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, and Uzbekistan. The period of record covered by each station is variable, however, most stations have almost 100 years of observations with the earliest record from 1879 and the latest from 2003 (N =13,888 is the sample size used in the study).
- **Achievable:** From the fitted Weibull distributions, we want discover the significant effects of the assumed covariates in each month rain precipitation measure (monthly sums); we also want to get accurate estimates for the means, of the rain precipitations for the twelve months of the year in each one of the five countries (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan) and accurate estimates for the means of the rain precipitations in each geographical area (agro, desert, forest, semi-desert and urban area) assuming a Weibull distribution.
- **Relevant:** To discover from the data analysis the most relevant factors affecting the variability of the rain precipitation measures (monthly sums) considering the follow-up period 1879-1999. A great interest here is to verify if the effect of different factors is similar or different in each month of the year in the follow-up period of 120 years. Other important point: to verify the behavior of the rain precipitation means in each month of the year in the five different regions and different geographical areas.
- **Time bounded:** From the statistical data analysis assuming a Weibull distribution in presence or not of covariates, we want to discover the significant factors affecting the rain precipitation (monthly sums) in each month of the year in different countries, to get accurate means for rain precipitation in each month in each country and to get accurate rain precipitation means for different geographical areas (agro, desert, forest, semi-desert, and urban area). These results could be of interest to authorities to manage the use of water by the population and plan agricultural plantations in the region considered in the study (Central Asia).

## 2. Methodology

One of the most popular distributions used to analyze positive observations, in particular considering lifetime data, is given by the Weibull distribution (Weibull, 1951). Among the great advantages of the Weibull distribution, we can highlight its versatility and facility of use. The distribution provides a good fit for a wide range / variety of data sets (Lawless, 1982; Nelson, 2004). In this study, we have the presence of some covariates that affect the responses (rain average). In this sense, we assume a Weibull parametric regression model, affecting one parameter of the Weibull distribution. In this way, considering T as a random variable denoting the response of interest (rain precipitation), we assume a first-order Weibull regression model in the data analysis. That is, we assume a Weibull distribution for T with a probability density function (pdf) given by,

$$f(t) = \frac{\alpha}{\lambda^{\alpha}} t^{\alpha-1} \exp\left\{-\left(\frac{t}{\lambda}\right)^{\alpha}\right\} \tag{1}$$

where $t > 0$ denotes the rain precipitation sum reported in each month in every observational station. The parameters $\lambda$ and $\alpha$ denote, respectively, the scale and shape parameters of the distribution. Different values of $\alpha$ lead to different forms for the distribution, which makes it very flexible in the data analysis. Note that if $\alpha = 1$, we have the exponential distribution, that is, the exponential distribution is a special case of the Weibull distribution.

The Weibull distribution possibly is the most used parametric distribution in medical studies (survival data), reliability studies in industrial and engineering applications among many others. Thus, assuming a first-order Weibull regression model for the rain precipitations, let us assume the regression model defined by,

$$\log \lambda_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \beta_5 x_{5i} + \beta_6 x_{6i} + \beta_7 x_{7i} + \beta_8 x_{8i} \tag{2}$$

where $i = 1,2,\ldots,n$ (sample size); $x_{1i}$ denotes longitude (long); $x_{2i}$ denotes latitude (lat); $x_{3i}$ denotes altitude (alt); $x_{4i}$ denotes years; $x_{5i}$ is a dummy variable ($x_{5i} = 1$ for Kyrgyzstan and $x_{5i} = 0$ for other countries); $x_{6i}$ is a dummy variable ($x_{6i} = 1$ for Tadjikistan and $x_{6i} = 0$ for other countries); $x_{7i}$ is a dummy variable ($x_{7i} = 1$ for Turkmenistan and $x_{7i} = 0$ for other countries) and $x_{8i}$ is a dummy variable ($x_{8i} = 1$ for Uzbekistan and $x_{8i} = 0$ for other countries) where Kazakhstan is considered as a reference. Note that the regression model given by (2) defines a regression model in the scale parameter (Lawless, 1982) assuming the same shape parameter. In this case, the expected value for the rain precipitation is given by,

$$E\,(T_i) = \Gamma\left[1 + \frac{1}{\alpha}\right]\exp\{\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \beta_5 x_{5i} + \beta_6 x_{6i} + \beta_7 x_{7i} + \beta_8 x_{8i}\} \tag{3}$$

In the estimation of the parameters of the Weibull regression model defined by (1) and (2), we consider the standard maximum likelihood methodology (Lawless, 1982) using existing iterative numerical techniques and usual normal asymptotical approximations to get hypothesis tests and confidence intervals for the parameters of the model.

## 3. Results

In this section, we present the results of the data analysis of the rain precipitation data related to the five countries in Central Asia (211- Kazakhstan, 213 - Kyrgyzstan, 227 -Tadjikistan, 229 - Turkmenistan and 231 - Uzbekistan).

### 3.1. Month Rain Precipitation

Table B.1 in Appendix shows the maximum likelihood estimators (MLE), the standard-errors (SE) and p-values associated to hypotheses tests for the regression parameters $\beta_0$ , $\beta_1$ , $\beta_2$ , $\beta_3$ , $\beta_4$ , $\beta_5$ , $\beta_6$ , $\beta_7$ and $\beta_8$ ($H_0$: $\beta_j = 0$ versus $H_1$: $\beta_j \neq 0$, $j = 0,1, 2,\ldots, 8$) and for the shape parameter $\alpha$ of the regression Weibull model defined by (1) and (2) assuming the rain precipitation sums for each one of the 12 months during the follow-up period of many years (use of the Minitab software). The needed assumptions for the Weibull regression model defined by (2) were verified from standard residual Cox-Snell plots usually available in the Minitab software to check the fit of the Weibull regression model for the data.

From the obtained results of Table B.1, we can conclude that, for all months, we observe a significant effect of years (p-value < 0.05) where in these months, the maximum likelihood estimators have positive signs indicating that the month rain precipitations are increasing over the years, except for months June and August where the statistical data analysis do not show difference of rain precipitations over the years (p-value > 0.05). In addition, for all 12 months, we observe significant effects of the covariates longitude (long), latitude (lat) and altitude (alt) (p-value < 0.05) in the response month rain precipitation. Longitude (long) in all months have MLE for the corresponding regression parameter with positive sign; latitude (lat) has negative signs for the MLE of the regression parameters considering the months January, February, March, April, May, November and December and positive signs for the MLE of the regression parameters considering the months June, July, August, September, and October. Altitude (alt) have positive signs for the MLE of the regression parameters considering all months (rain precipitation higher for greater altitudes). That is,

- **Countries in January:** Kyrgyzstan, Tadjikistan and Turkmenistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these three countries have month rain precipitations smaller than the rain precipitation in Kazakhstan (reference). Uzbekistan (p-value < 0.05) has positive MLE for the corresponding regression parameter, an indication that the month rain precipitation is larger than for Kazakhstan (reference).

- **Countries in February:** Kyrgyzstan, Tadjikistan and Turkmenistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these three countries have month rain precipitations smaller than the month rain precipitation in Kazakhstan (reference). The month rain precipitation in Uzbekistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).
- **Countries in March:** the same conclusions as observed in February are observed for the month March.
- **Countries in April:** Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these four countries have month rain precipitations smaller than the month rain precipitation in Kazakhstan (reference).
- **Countries in May:** the same conclusions as observed in April are observed for the month May.
- **Countries in June:** Turkmenistan and Uzbekistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these two countries have month rain precipitations smaller than the rain precipitation in Kazakhstan (reference). The month rain precipitation in Kyrgyzstan (p-value < 0.05) is greater than the month rain precipitation Kazakhstan (reference) since the corresponding MLE is positive. The month rain precipitation in Tadjikistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).
- **Countries in July:** Kyrgyzstan, Tadjikistan and Turkmenistan (p-value < 0.05) have positive MLE for their respective regression parameters, indicating that these three countries have month rain precipitations larger than the month rain precipitation in Kazakhstan (reference). Uzbekistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).
- **Countries in August:** Kyrgyzstan and Turkmenistan (p-value < 0.05) have positive MLE for their respective regression parameters, indicating that these two countries have month rain precipitations greater than the month rain precipitation in Kazakhstan (reference). The month rain precipitation in Uzbekistan (p-value < 0.05) is smaller than the month rain precipitation Kazakhstan (reference) since the corresponding MLE is negative. The month rain precipitation in Tadjikistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).
- **Countries in September:** Tadjikistan and Uzbekistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these two countries have month rain precipitations smaller than the month rain precipitation in Kazakhstan (reference). The month rain precipitation in Kyrgyzstan (p-value < 0.05) is greater than the month rain precipitation in Kazakhstan (reference) since the corresponding MLE is positive. The month rain precipitation in Turkmenistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).
- **Countries in October:** Turkmenistan (p-value < 0.05) has negative MLE for its respective regression parameter, indicating that this country has month rain precipitation smaller than the month rain precipitation in Kazakhstan (reference); the other three countries (Kyrgyzstan, Tadjikistan and Uzbekistan ) have p-value > 0.05 which does not indicate difference with the month rain precipitation in Kazakhstan (reference).
- **Countries in November:** Kyrgyzstan, Tadjikistan and Turkmenistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these three countries have month rain precipitation smaller than the month rain precipitation in Kazakhstan (reference). Uzbekistan have p-value > 0.05, which does not indicate difference with the month rain precipitation in Kazakhstan (reference).
- **Countries in December:** Kyrgyzstan and Turkmenistan (p-value < 0.05) have negative MLE for their respective regression parameters, indicating that these two countries have month rain precipitation smaller than the month rain precipitation in Kazakhstan (reference). The month rain precipitation in Uzbekistan (p-value < 0.05) is greater than the month rain precipitation Kazakhstan (reference) since the corresponding MLE is positive. The month rain precipitation in Tadjikistan (p-value > 0.05) does not show statistically difference of the month rain precipitation in Kazakhstan (reference).

### 3.2. Month Rain Precipitation Averages

Table B.2 at the end of the manuscript shows the maximum likelihood estimators (MLE) for the means (3) assuming the Weibull distribution (1) not considering the presence of the covariates for the rain precipitation averages (averages of the precipitation sums over all stations for a specified month and region) of each one of the 12 months during the follow-up period of 120 years (use of the Minitab software) considering the five countries in Central Asia (211-Kazakhstan, 213-Kyrgyzstan, 227-Tadjikistan, 229-Turkmenistan and 231-Uzbekistan). Table B.2, also shows the sample means and sample standard deviations of the data and 95% confidence intervals for the means obtained using normal asymptotical methods for MLE (use of the software Minitab). We observe accurate 95% confidence intervals for the means are obtained for the rain precipitation means in all cases. It is important to point out that large sample sizes are observed in all cases. The sample sizes for each country are respectively given by: 211-Kazakhstan (N = 2947), 213-Kyrgyzstan (N = 2655), 227-Tadjikistan (N = 1821), 229-Turkmenistan (N = 1055) and 231-Uzbekistan (N = 5410). Figure A.7 shows the observed sample means and the estimated Weibull means estimated by maximum likelihood method indicating an excellent fit of the Weibull distribution for the rain precipitation data in all countries.

### 3.3. Month Rain Precipitation Average in Different Geographical Areas

The observational locations in the five countries are in different geographical areas where the rain precipitation usually is very different depending on the local characteristics. Considering some special areas where it is expected great differences in the yearly rain precipitations (agro, desert, forest, semi-desert and urban area), the sample precipitations averages (standard-deviations) observed in the follow-up periods are given, respectively, by: 25.570 (7.164) for agro area (N=109); 10.620 (3.763) for desert area (N=90); 59.67 (19.65) for forest area (N=71); 15.937 (4.074) for semi-desert (N=104) area and 25.635 (4.240) for urban area (N=109). Figure A.8 shows the histograms of the yearly rain means considering the five areas. where it is observed that some regions (agro, forest and urban areas have more asymmetrical shapes for the histograms indicating that these regions the precipitation have a more not normal behavior in the observed follow-up periods. In this case, we also assume Weibull distributions in the data analysis. Finally, Table B.3 presents, respectively, the MLE for the shape, scale and mean (3) for the Weibull distribution for each region (agro, forest, desert, semi-desert and urban) obtained using the software Minitab. From the results of Table B.1, we observe that the obtained maximum likelihood estimates (MLE) for the precipitation means are very close to the sample precipitation averages in each area, an indication of the good fit of the Weibull distribution for the data.

Figure A.9 shows the fitted Weibull distributions for each considered area considering the MLE of the shape and scale parameters of the Weibull distribution in each area presented in Table 3, from where, it is possible to get all inferences of interest as the probabilities for the rain precipitation to be in specified intervals of interest. From Figure A.9, it is possible to observe that the rain precipitation means are very concentrated (small variability) for the desert and semi-desert areas, while for the forest and agro areas the variability of the rain precipitation are very large, possibly affected by the climate change observed in the world in the last decades.

## 4. Concluding Remarks

As an important conclusion of our study is that for all months of the year, it is observed significant effects of time (years) except for the months June and August were the amount of rain are not affected. Also, it is observed significant effects of the covariates longitude (long), latitude (lat) and altitude (alt) in the response month rain precipitation. The rain precipitations have different behavior in the twelve months of the year considering the five countries (Kazakhstan, Kyrgyzstan, Tadjikistan, Turkmenistan and Uzbekistan). In addition, considering different geographical areas it is observed that rain precipitation means have small variability for the desert and semi-desert areas (small amount of rain with not great variability), while for the forest and agro areas the variability of the rain precipitation is very large. Finally, it is important to point out that other distributions for positive random variables could be assumed in the data analysis of rain precipitation data, as generalizations of the Weibull distribution as for example, the exponentiated-Weibull family (Muldholkar et al., 1995) but in our application the Weibull distribution was very well fitted by the data.

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## Appendix A. Figures

**Figure A.1. Central Asia countries (downloaded from http://www.sairamtourism.com/ca_today on 09/APR/12).**


**Figure A.2. Histograms of the monthly rain precipitations in Kazakhstan.**

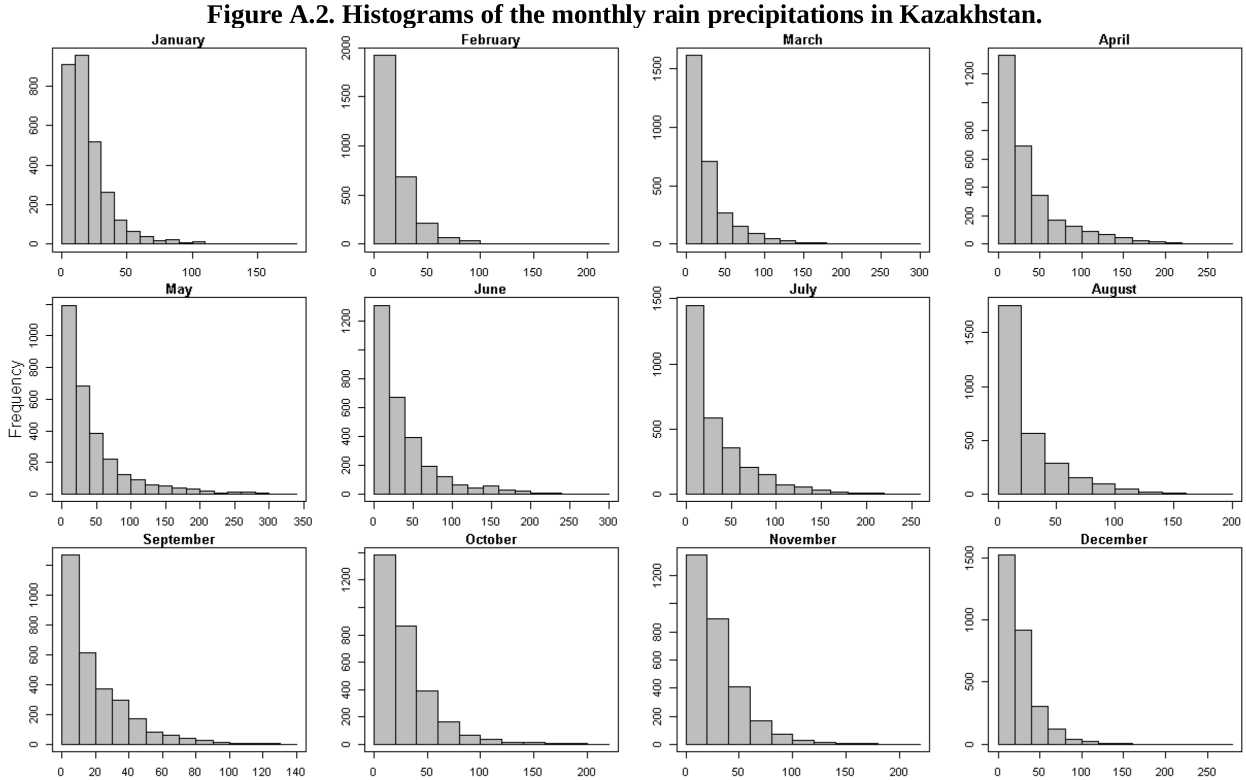


**Figure A.3. Histograms of the monthly rain precipitations in Kyrgystan.**

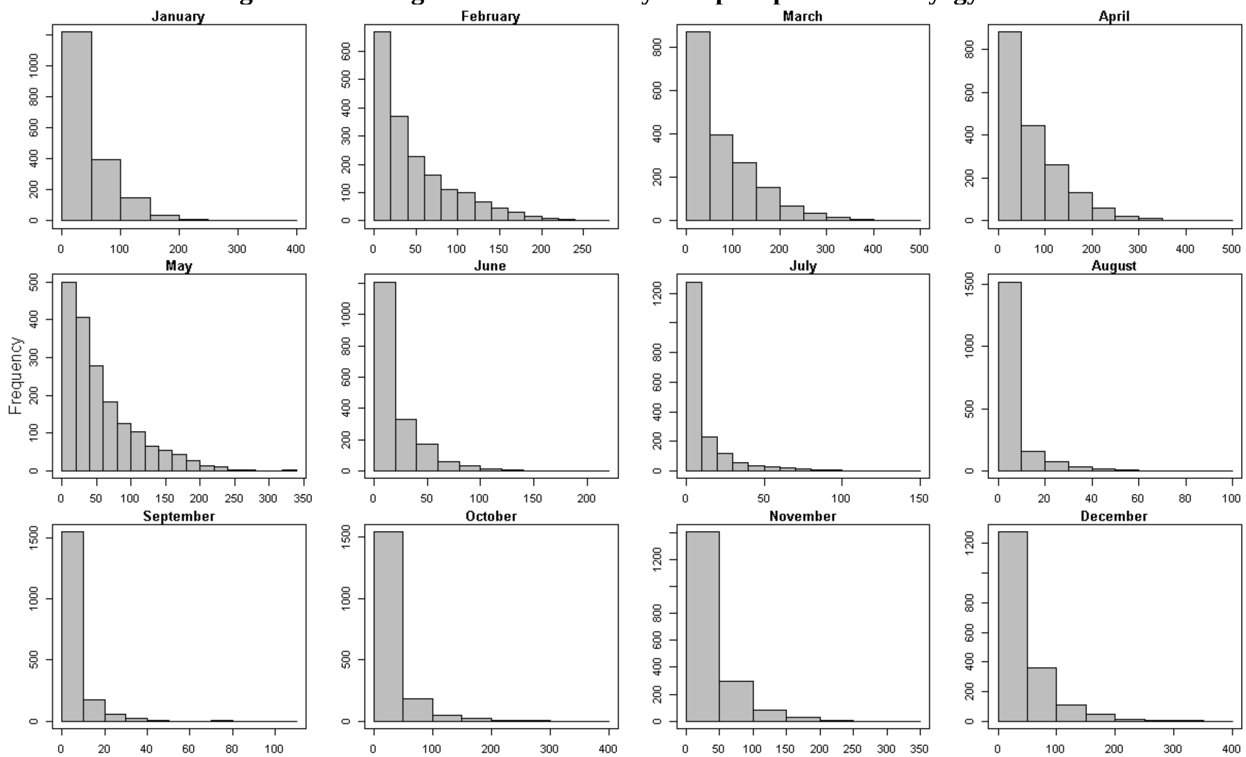



**Figure A.4. Histograms of the monthly rain precipitations in Tadjikistan.**

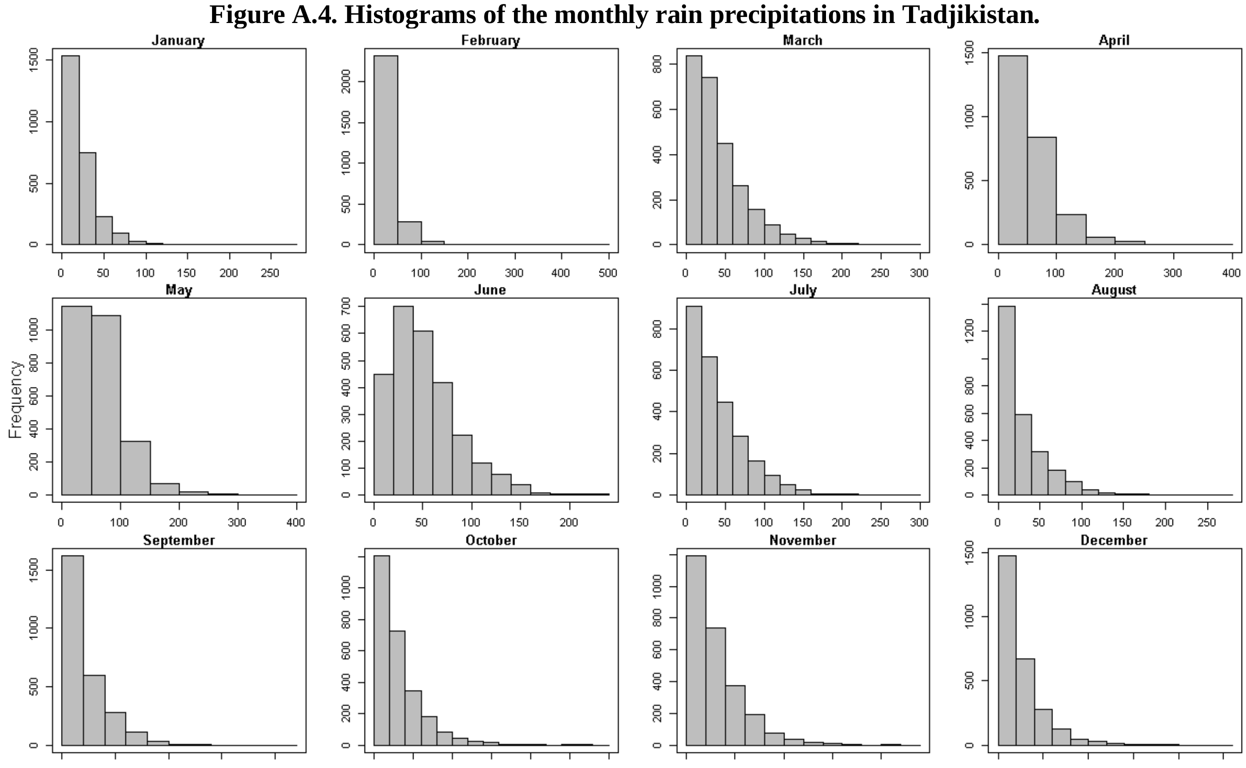


**Figure A.5. Histograms of the monthly rain precipitations in Turkmenistan.**

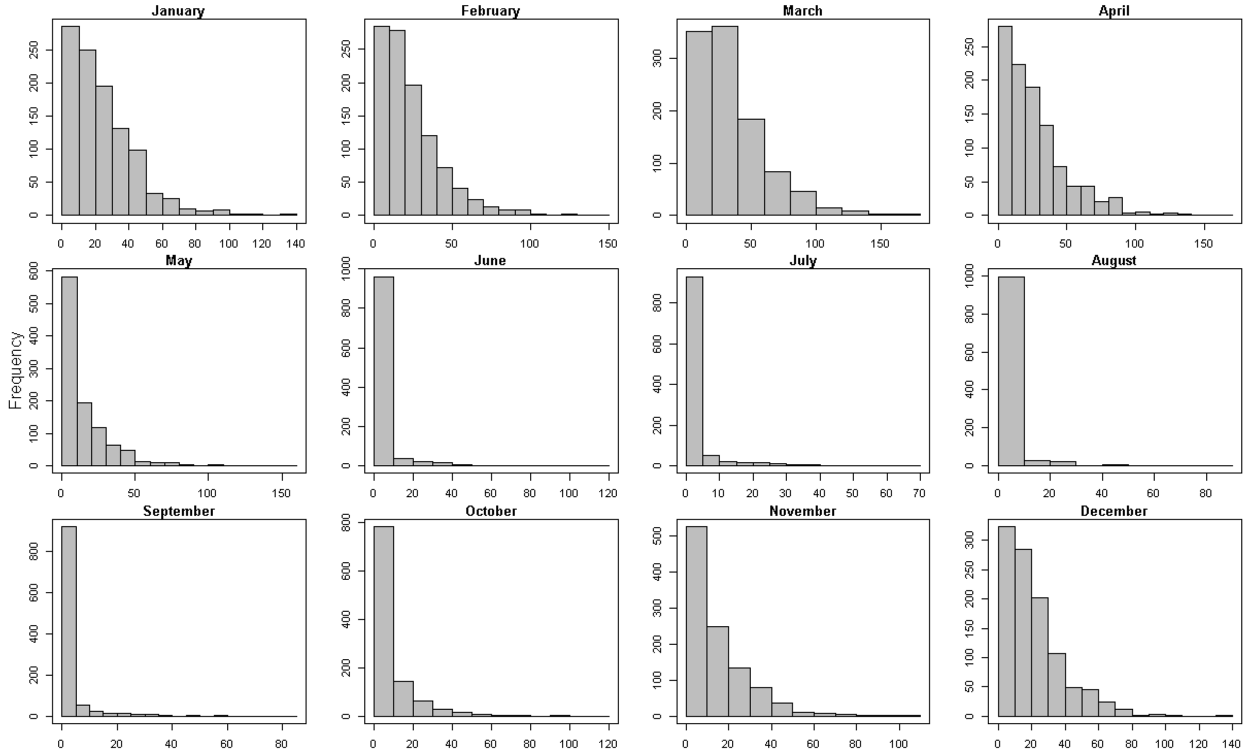
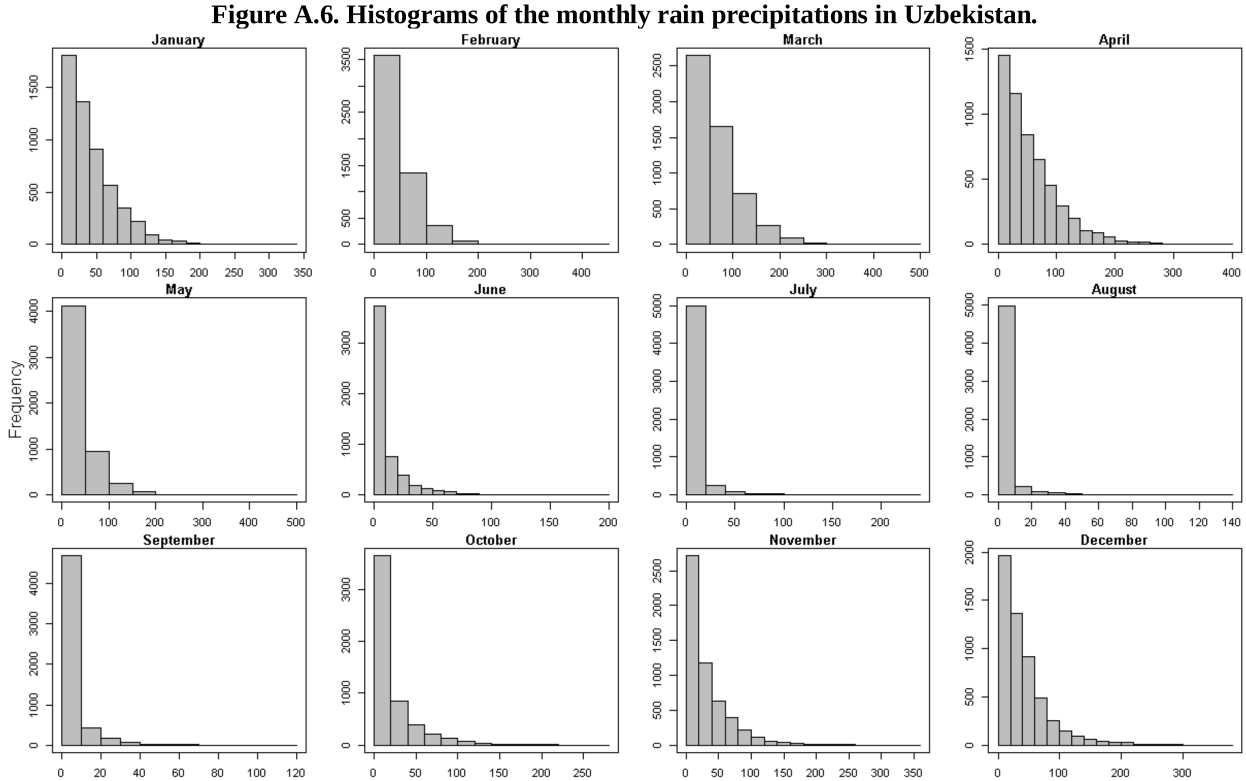



**Figure A.6. Histograms of the monthly rain precipitations in Uzbekistan.**



 DOI: http://dx.doi.org/10.18187/pjsor.v18i2.3976

**Figure A.7. Observed and the estimated Weibull means for the rain precipitations (211-Kazakhstan, 213-Kyrgyzstan, 227-Tadjikistan, 229-Turkmenistan and 231-Uzbekistan)**

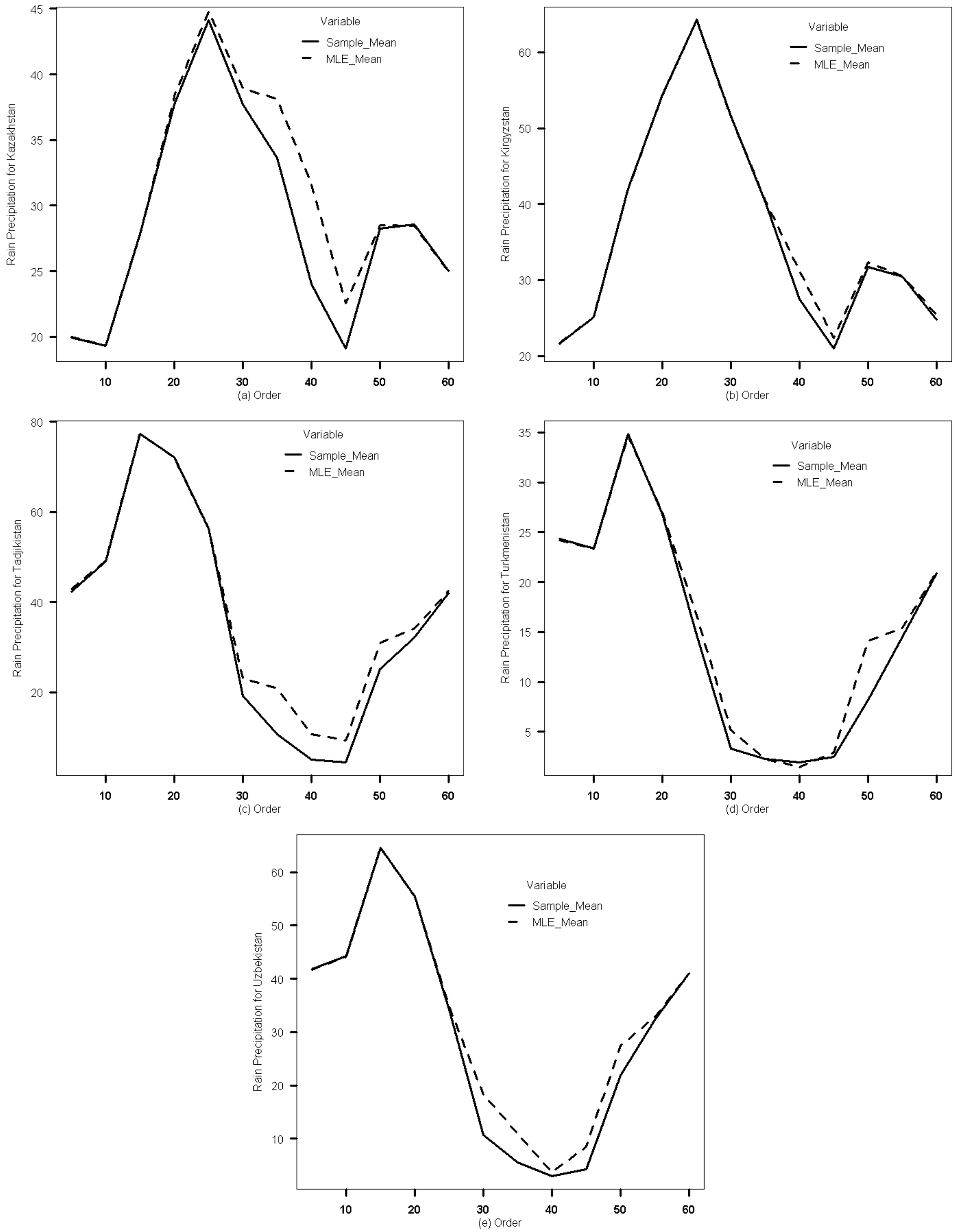


**Figure A.8. Histograms for the rain monthly precipitations in different areas (agro, desert, forest, semi-desert and urban areas)**

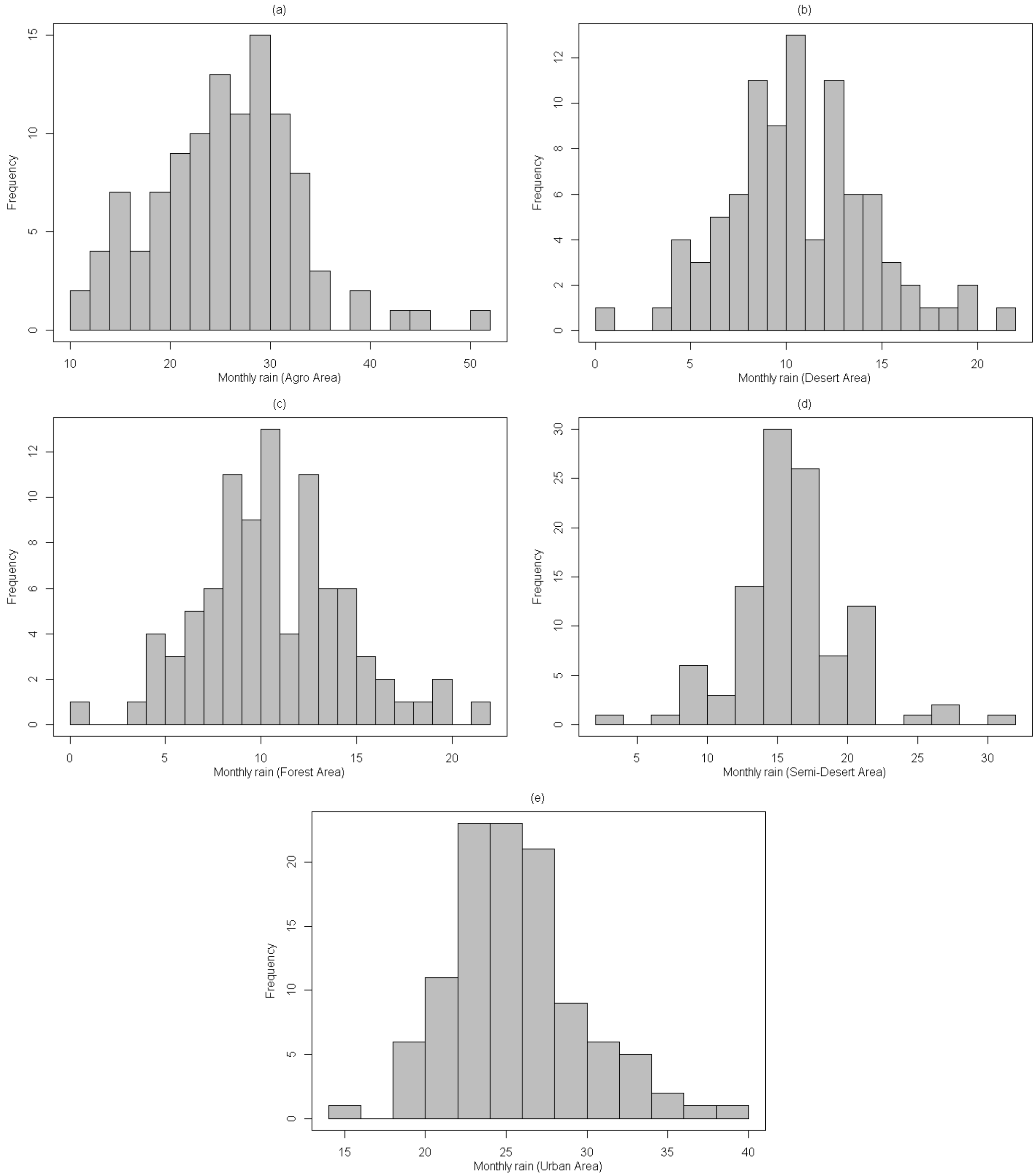
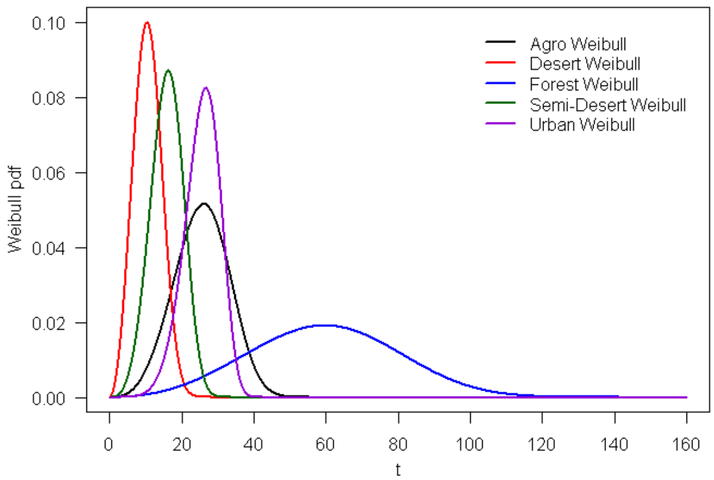


**Figure A.9. Fitted Weibull distributions for the rain precipitation in each area (agro, desert, forest, semi-desert, and urban areas)**




## Appendix B. Tables

**Table B.1. MLE, SE and p-values of Weibull regression model for the 12 months**

| **January** | MLE | SE | p-value | **February** | MLE | SE | p-value |
|---|---|---|---|---|---|---|---|
| Intercept | 3.24759 | 0.732515 | < 0.001 | Intercept | 2.13626 | 0.742636 | 0.004 |
| Long | 0.014535 | 0.001644 | < 0.001 | Long | 0.015329 | 0.001639 | < 0.001 |
| Lat | -0.11618 | 0.005091 | < 0.001 | Lat | -0.13722 | 0.005029 | < 0.001 |
| Alt | 4.88E-05 | 1.27E-05 | < 0.001 | Alt | 5.16E-05 | 1.27E-05 | < 0.001 |
| Years | 0.002056 | 0.000351 | < 0.001 | Years | 0.003063 | 0.00036 | < 0.001 |
| Kyrgyzstan | -0.52434 | 0.03717 | < 0.001 | Kyrgyzstan | -0.43304 | 0.03668 | < 0.001 |
| Tadjikistan | -0.12921 | 0.049453 | 0.009 | Tadjikistan | -0.10012 | 0.048675 | 0.040 |
| Turkmenistan | -0.53372 | 0.058006 | < 0.001 | Turkmenistan | -0.67419 | 0.057304 | < 0.001 |
| Uzbekistan | 0.089959 | 0.038781 | 0.020 | Uzbekistan | 0.064284 | 0.038234 | 0.093 |
| Shape | 1.05691 | 0.007127 | | Shape | 1.06365 | 0.007149 | |
| **March** | MLE | SE | p-value | **April** | MLE | SE | p-value |
| Intercept | 8.05906 | 0.697847 | < 0.001 | Intercept | 4.38317 | 0.758461 | < 0.001 |
| Long | 0.014241 | 0.001529 | < 0.001 | Long | 0.020741 | 0.001693 | < 0.001 |
| Lat | -0.17125 | 0.004496 | < 0.001 | Lat | -0.13783 | 0.005046 | < 0.001 |
| Alt | 8.77E-05 | 1.18E-05 | < 0.001 | Alt | 0.0002 | 1.33E-05 | < 0.001 |
| Years | 0.001027 | 0.000337 | 0.002 | Years | 0.001947 | 0.000368 | < 0.001 |
| Kyrgyzstan | -0.37026 | 0.031807 | < 0.001 | Kyrgyzstan | -0.3465 | 0.034147 | < 0.001 |
| Tadjikistan | -0.18022 | 0.042709 | < 0.001 | Tadjikistan | -0.34299 | 0.046648 | < 0.001 |
| Turkmenistan | -0.84901 | 0.051642 | < 0.001 | Turkmenistan | -0.90794 | 0.057404 | < 0.001 |
| Uzbekistan | -0.02875 | 0.033683 | 0.393 | Uzbekistan | -0.21643 | 0.036944 | < 0.001 |
| Shape | 1.17442 | 0.007764 | | Shape | 1.05879 | 0.007217 | |
| **May** | MLE | SE | p-value | **June** | MLE | SE | p-value |
| Intercept | -2.01748 | 0.821702 | 0.014 | Intercept | -9.00574 | 1.5399 | < 0.001 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Long | 0.02775 | 0.001848 | < 0.001 | Long | 0.021715 | 0.003393 | < 0.001 |
| Lat | -0.06208 | 0.005756 | < 0.001 | Lat | 0.226768 | 0.011357 | < 0.001 |
| Alt | 0.000368 | 1.51E-05 | < 0.001 | Alt | 0.000846 | 2.75E-05 | < 0.001 |
| Years | 0.003157 | 0.000396 | < 0.001 | Years | -0.0002 | 0.000728 | 0.780 |
| Kyrgyzstan | -0.18817 | 0.038122 | < 0.001 | Kyrgyzstan | 0.568895 | 0.070371 | < 0.001 |
| Tadjikistan | -0.32028 | 0.052121 | < 0.001 | Tadjikistan | 0.063426 | 0.096453 | 0.511 |
| Turkmenistan | -0.8336 | 0.064833 | < 0.001 | Turkmenistan | -0.7288 | 0.123077 | < 0.001 |
| Uzbekistan | -0.40092 | 0.041737 | < 0.001 | Uzbekistan | -0.23833 | 0.075869 | 0.002 |
| Shape | 0.944881 | 0.006535 | | Shape | 0.507737 | 0.00374 | |
| **July** | MLE | SE | p-value | **August** | MLE | SE | p-value |
| Intercept | -29.6909 | 2.13402 | < 0.001 | Intercept | -31.8302 | 2.37305 | < 0.001 |
| Long | 0.025072 | 0.0046 | < 0.001 | Long | 0.021018 | 0.00507 | < 0.001 |
| Lat | 0.492529 | 0.016213 | < 0.001 | Lat | 0.647153 | 0.017978 | < 0.001 |
| Alt | 0.00127 | 0.000038 | < 0.001 | Alt | 0.001561 | 4.18E-05 | < 0.001 |
| Years | 0.003668 | 0.001002 | < 0.001 | Years | 0.000889 | 0.001128 | 0.431 |
| Kyrgyzstan | 1.06991 | 0.098628 | < 0.001 | Kyrgyzstan | 1.29148 | 0.10857 | < 0.001 |
| Tadjikistan | 0.366697 | 0.13512 | 0.007 | Tadjikistan | 0.271521 | 0.149872 | 0.070 |
| Turkmenistan | 0.422353 | 0.172853 | 0.015 | Turkmenistan | 1.53434 | 0.192511 | < 0.001 |
| Uzbekistan | - 0.1841 | 0.105557 | 0.081 | Uzbekistan | -0.23541 | 0.117654 | 0.045 |
| Shape | 0.366611 | 0.002568 | | Shape | 0.328926 | 0.002185 | |
| **September** | MLE | SE | p-value | **October** | MLE | SE | p-value |
| Intercept | -44.4029 | 2.13289 | < 0.001 | Intercept | -9.69988 | 1.35753 | < 0.001 |
| Long | 0.010007 | 0.004712 | 0.034 | Long | 0.020514 | 0.002997 | < 0.001 |
| Lat | 0.420884 | 0.016273 | < 0.001 | Lat | 0.07973 | 0.009608 | < 0.001 |
| Alt | 0.001084 | 3.83E-05 | < 0.001 | Alt | 0.000348 | 2.34E-05 | < 0.001 |
| Years | 0.013259 | 0.000999 | < 0.001 | Years | 0.003789 | 0.000646 | < 0.001 |
| Kyrgyzstan | 0.586624 | 0.100466 | < 0.001 | Kyrgyzstan | 0.009618 | 0.063313 | 0.879 |
| Tadjikistan | -0.72912 | 0.137843 | < 0.001 | Tadjikistan | -0.08294 | 0.085749 | 0.333 |
| Turkmenistan | 0.193518 | 0.174033 | 0.266 | Turkmenistan | -0.40399 | 0.10542 | < 0.001 |
| Uzbekistan | -0.40382 | 0.108063 | < 0.001 | Uzbekistan | 0.015114 | 0.067105 | 0.822 |
| Shape | 0.357941 | 0.00251 | | Shape | 0.579933 | 0.004195 | |
| **November** | MLE | SE | p-value | **December** | MLE | SE | p-value |
| Intercept | -1.83605 | 0.919812 | 0.046 | Intercept | -0.60266 | 0.813177 | 0.459 |
| Long | 0.023503 | 0.002023 | < 0.001 | Long | 0.014865 | 0.00181 | < 0.001 |
| Lat | -0.02295 | 0.006313 | < 0.001 | Lat | -0.06769 | 0.005637 | < 0.001 |
| Alt | 0.000158 | 1.56E-05 | < 0.001 | Alt | 9.63E-05 | 0.000014 | < 0.001 |
| Years | 0.002217 | 0.000443 | < 0.001 | Years | 0.002949 | 0.000392 | < 0.001 |
| Kyrgyzstan | -0.22353 | 0.043158 | < 0.001 | Kyrgyzstan | -0.4569 | 0.040123 | < 0.001 |
| Tadjikistan | -0.17385 | 0.058404 | 0.003 | Tadjikistan | -0.07308 | 0.053731 | 0.174 |
| Turkmenistan | -0.41281 | 0.070467 | < 0.001 | Turkmenistan | -0.46032 | 0.063428 | < 0.001 |
| Uzbekistan | 0.058594 | 0.045371 | 0.197 | Uzbekistan | 0.121883 | 0.041747 | 0.004 |
| Shape | 0.857381 | 0.005926 | | Shape | 0.954871 | 0.006472 | |

**Table B.2. MLE of mean of the Weibull distribution for the 12 months and the five countries**

| Row month | Country sample | Sample mean | sd | MLE mean | lower 95% | Upper 95% |
|---|---|---|---|---|---|---|
| 1 January | 211 | 19.956 | 18.149 | 19.957 | 19.347 | 20.586 |
| 2 January | 213 | 21.569 | 21.348 | 21.723 | 20.832 | 22.652 |
| 3 January | 227 | 42.330 | 43.950 | 42.903 | 40.570 | 45.370 |
| 4 January | 229 | 24.332 | 20.361 | 24.219 | 22.957 | 25.550 |
| 5 January | 231 | 41.790 | 36.425 | 41.693 | 40.692 | 42.720 |
| 6 February | 211 | 19.294 | 18.529 | 19.248 | 18.609 | 19.984 |
| 7 February | 213 | 25.124 | 24.830 | 25.140 | 24.286 | 26.132 |
| 8 February | 227 | 48.960 | 48.250 | 49.103 | 46.715 | 51.614 |
| 9 February | 229 | 23.414 | 20.101 | 23.330 | 22.057 | 24.676 |
| 10 February | 231 | 44.299 | 38.613 | 44.172 | 43.111 | 45.260 |
| 11 March | 211 | 27.842 | 29.998 | 27.841 | 26.828 | 28.892 |
| 12 March | 213 | 41.996 | 37.052 | 41.912 | 40.504 | 43.369 |
| 13 March | 227 | 77.350 | 73.360 | 77.366 | 73.862 | 81.035 |
| 14 March | 229 | 34.871 | 26.720 | 34.653 | 33.018 | 36.369 |
| 15 March | 231 | 64.613 | 53.781 | 64.552 | 63.136 | 65.999 |
| 16 April | 211 | 37.698 | 42.048 | 35.312 | 36.618 | 40.085 |
| 17 April | 213 | 54.353 | 43.454 | 54.333 | 52.721 | 55.994 |
| 18 April | 227 | 72.090 | 65.790 | 72.037 | 68.929 | 75.286 |
| 19 April | 229 | 26.830 | 24.072 | 26.986 | 25.299 | 28.785 |
| 20 April | 231 | 55.502 | 50.315 | 55.460 | 54.026 | 56.932 |
| 21 May | 211 | 44.138 | 51.182 | 44.820 | 42.809 | 46.926 |
| 22 May | 213 | 64.240 | 42.151 | 64.338 | 62.792 | 65.921 |
| 23 May | 227 | 56.370 | 51.940 | 56.346 | 53.847 | 58.962 |
| 24 May | 229 | 14.710 | 18.357 | 16.709 | 14.978 | 18.640 |
| 25 May | 231 | 34.171 | 36.731 | 34.912 | 33.757 | 36.107 |
| 26 June | 211 | 37.723 | 42.753 | 38.891 | 37.044 | 40.831 |
| 27 June | 213 | 51.615 | 34.788 | 51.454 | 50.138 | 52.804 |
| 28 June | 227 | 19.160 | 23.025 | 22.912 | 20.905 | 25.111 |
| 39 June | 229 | 3.312 | 8.958 | 5.188 | 3.875 | 6.945 |
| 30 June | 231 | 10.714 | 18.058 | 18.302 | 16.721 | 20.033 |
| 31 July | 211 | 33.641 | 37.601 | 38.129 | 35.793 | 40.616 |
| 32 July | 213 | 40.287 | 35.117 | 40.475 | 38.881 | 42.134 |
| 33 July | 227 | 10.628 | 18.147 | 20.870 | 17.469 | 24.934 |
| 34 July | 229 | 2.290 | 6.861 | 2.298 | 1.687 | 3.129 |
| 35 July | 231 | 5.581 | 14.016 | 10.996 | 9.659 | 12.518 |
| 36 August | 211 | 24.017 | 28.723 | 31.590 | 29.085 | 34.310 |
| 37 August | 213 | 27.447 | 29.768 | 31.155 | 29.160 | 33.287 |
| 38 August | 227 | 4.999 | 9.951 | 10.640 | 8.574 | 13.205 |
| 39 August | 229 | 1.931 | 7.137 | 1.428 | 1.055 | 1.932 |
| 40 August | 231 | 2.955 | 9.652 | 3.752 | 3.279 | 4.294 |
| 41 September | 211 | 19.087 | 20.175 | 22.529 | 21.074 | 24.086 |
| 42 September | 213 | 20.947 | 22.038 | 22.337 | 21.124 | 23.620 |
| 43 September | 227 | 4.394 | 8.316 | 9.305 | 7.590 | 11.408 |
| 44 September | 229 | 2.474 | 7.287 | 2.931 | 2.152 | 3.990 |
| 45 September | 231 | 4.228 | 8.890 | 8.508 | 7.528 | 9.615 |
| 46 October | 211 | 28.266 | 26.602 | 28.482 | 27.382 | 39.627 |
| 47 October | 213 | 31.742 | 33.983 | 32.299 | 30.841 | 33.825 |
| 48 October | 227 | 25.145 | 39.040 | 30.928 | 17.801 | 34.406 |
| 49 October | 229 | 8.177 | 13.190 | 14.115 | 11.491 | 17.338 |
| 50 October | 231 | 21.837 | 32.634 | 27.464 | 25.805 | 29.230 |
| 51 November | 211 | 28.562 | 24.797 | 28.462 | 27.546 | 29.410 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 52 November | 213 | 30.413 | 28.802 | 30.552 | 29.335 | 31.820 |
| 53 November | 227 | 32.113 | 39.328 | 34.072 | 31.691 | 36.633 |
| 54 November | 229 | 14.520 | 15.132 | 15.415 | 14.129 | 16.817 |
| 55 November | 231 | 32.243 | 37.142 | 32.765 | 31.680 | 33.887 |
| 56 December | 211 | 24.981 | 22.312 | 24.966 | 24.208 | 25.749 |
| 57 December | 213 | 24.724 | 26.805 | 25.410 | 24.197 | 26.684 |
| 58 December | 227 | 41.970 | 47.350 | 42.415 | 40.088 | 44.878 |
| 59 December | 229 | 20.923 | 17.858 | 21.033 | 19.735 | 22.416 |
| 60 December | 231 | 41.049 | 41.352 | 41.100 | 39.976 | 42.256 |

**Table B.3. MLE for the shape, scale and rain precipitation mean of the Weibull distribution in different areas (agro, desert, forest, semi-desert, and urban areas)**

| | **MLE** | **SE** | **Lower 95 %** | **Upper 95 %** |
|---|---|---|---|---|
| Agro Area | | | | |
| α | 3.8046 | 0.2661 | 3.3171 | 4.3637 |
| λ | 28.2129 | 0.7505 | 26.7796 | 29.7229 |
| mean | 25.5002 | 0.7167 | 24.1334 | 26.9443 |
| Desert Area | | | | |
| α | 3.0292 | 0.2444 | 2.5860 | 3.5484 |
| λ | 11.8500 | 0.4338 | 11.0296 | 12.7315 |
| mean | 10.5864 | 0.4010 | 9.8288 | 11.4024 |
| Forest Area | | | | |
| α | 3.2806 | 0.2937 | 2.7525 | 3.9099 |
| λ | 66.5811 | 2.5508 | 61.7647 | 71.7730 |
| mean | 59.7066 | 2.3886 | 55.2039 | 64.5766 |
| Semi-Desert Area | | | | |
| α | 3.9927 | 0.2743 | 3.4896 | 4.5682 |
| λ | 17.4643 | 0.4534 | 16.5978 | 18.3761 |
| mean | 15.8281 | 0.4349 | 14.9981 | 16.7040 |
| Urban Area | | | | |
| α | 6.0713 | 0.4099 | 5.3187 | 6.9305 |
| λ | 27.4490 | 0.4594 | 26.5630 | 28.3645 |
| mean | 25.4816 | 0.4669 | 24.5827 | 26.4133 |