Modified Regression Estimators for Improving Mean Estimation - Poisson Regression Approach

Zakir Hussain Wani⁺, S.E.H. Rizvi², Manish Sharma³, M. Iqbal Jeelani⁴, Saqib Mushtaq⁵

⁺ Corresponding Author

1. Division of Statistics and Computer Science, Main Campus SKUAST-J, Chatha Jammu-180009, India, wanizakir927@gmail.com
2. Division of Statistics and Computer Science, Main Campus SKUAST-J, Chatha Jammu-180009, India, sehririzvistat@gmail.com
3. Division of Statistics and Computer Science, Main Campus SKUAST-J, Chatha Jammu-180009, India, manshstat@gmail
4. Division of Statistics and Computer Science, Main Campus SKUAST-J, Chatha Jammu-180009, India, jeelani.miqbal@gmail.com
5. Department of Mathematics, Main Campus University of Kashmir Srinagar-190006-India, iamsaqib01@gmail.com

Abstract

In this article, a class of Poisson-regression based estimators has been proposed for estimating the finite population mean in simple random sampling without replacement (SRSWOR). The Poisson-regression model is the most common method used to model count responses in many studies. The expression for bias and mean square error (MSE) of proposed class of estimators are obtained up to first order of approximation. The proposed estimators have been compared theoretically with the existing estimators, and the condition under which the proposed class of estimators perform better than existing estimators have been obtained. Two real data sets are considered to assess the performance of the proposed estimators. Numerical findings confirms that the proposed estimators dominate over the existing estimators such as Koc (2021) and Usman et al. (2021) in terms of mean squared error.

Key Words: Ratio estimator; Poisson regression; Mean Square error; Bias; Efficiency; Auxiliary variable

Mathematical Subject Classification: 62D05

1. Introduction

Ratio type estimators take advantage of the correlation between the auxiliary variable $g$ and the study variable $y$. When data on the auxiliary variable is available, the ratio estimator is a good choice for estimating the population mean. In sampling theory, population information of the auxiliary variable, such as coefficient of variation or kurtosis, is frequently employed to improve the efficiency of the estimation for a population mean for the ratio estimator. However, the outlier problem, which occurs when data has extreme values, reduces efficiency because traditional estimators are sensitive to extreme values. In order to handle this problem, Kadilar, Candan, and Cingi (2007) adapted Huber-M estimate related to ratio estimator presented in Kadilar and Cingi (2004). They obtained the MSE equations in order to decrease the effect of outlier problem. Oral and Kadilar (2011a, 2011b) considered

Furthermore, it is problematic of using a linear regression model when the mean is large enough for the count data, this is the situation when Poisson distribution converges to normal distribution. The regression model is the most widely used method for modelling. The linear model links the estimated value with supplementary variables, prediction values are possible. The validity of hypothesis testing in linear regression is also dependent on the supposition of variables constant variance. These suppositions are erroneous for count data. As a result, in the applied sciences, modelling count data, the used technique is the Poisson regression. Koc (2021) and Usman et al. (2021) used the Poisson regression method for improving the population mean of the study variable. In this article, we attempted to propose estimators for estimating the population mean of the study variable \( Y \) using information on the auxiliary variable \( G \) using Poisson regression model. The mathematical properties of proposed estimators, such as bias and mean square error were examined using large sample approximation. The proposed estimators have been shown to outperform all other estimators tested in the literature. Numerical illustrations have also been done in support of current investigation.

In Poisson regression, the study variable \( Y_i = y_i = 0,1,2, \ldots \) is the number of events that occur at a particular period, with a Poisson distribution given by

\[
P(y_i, \theta_i) = \frac{e^{-\theta_i} \theta_i^{y_i}}{y_i!}, \theta > 0
\]  

(1)

The mean and variance are same in Poisson distribution and is given by

\[
E(Y) = Var(Y) = \theta_i
\]

The \( L(\theta, y) \) i.e., log-likelihood of \( P(y_i, \theta_i) \) is defined as;

\[
L(\theta, y) = \sum_{i=1}^{n} \left( y_i \ln(\theta_i) - \theta_i - \ln(y_i!) \right)
\]  

(2)

Let \( G \) be the matrix of order \( n \times (k + 1) \) of the auxiliary variable. Then, the association between \( Y_i \) and \( i \text{th} \) row of matrix, \( g_i \) associating through \( d(\theta_i) \), is

\[
\ln(\theta_i) = \xi_i = g_i^T B
\]  

(3)

Where, \( B = B_0, B_1, \ldots, B_k \) are the regression parameters.

Differentiating (3) with respect to \( B \) yields \( \bar{B} \) which represents the maximum likelihood estimators of \( B \).

\[
\sum_{i=1}^{n} \left( y_i \exp(g_i^T \bar{B}) g_i \right)
\]  

(4)

For solving such \( k \) equations, we use iterative methods such as Newton-Raphson algorithms and Fisher Scoring algorithms (see Cameron and Trivedi (1998), Montgomery et al. (2006), and Koc (2021)).

2. Estimators from the literature

This section gives a brief introduction of some well-known estimators/ classes of estimators from the literature.

1. Kadilar and Cingi (2004) propose the following estimators for the estimation of population mean \( \bar{Y} \) in simple random sampling.

\[
\bar{y}_{KC1} = \frac{\bar{y} + b(\bar{G} - \bar{g})}{\bar{g}} \bar{G}
\]  

(5)

\[
\bar{y}_{KC2} = \frac{\bar{y} + b(\bar{G} - \bar{g})}{\bar{g} + C_g} (\bar{G} + \bar{C}_g)
\]  

(6)

\[
\bar{y}_{KC3} = \frac{\bar{y} + b(\bar{G} - \bar{x})}{\bar{g} + \beta_2(g)} (\bar{G} + \beta_2(g))
\]  

(7)

\[
\bar{y}_{KC4} = \frac{\bar{y} + b(\bar{G} - \bar{g})}{\bar{g} \beta_2(g) + C_g} (\bar{G} \beta_2(g) + C_g)
\]  

(8)
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\[ \bar{y}_{Kc5} = \frac{\bar{y} + b(\bar{g} - \bar{g})}{(\bar{g}c_g + \beta_2(g))} \left( \beta_2(g) \frac{\bar{g}}{\bar{g}c_g + \beta_2(g)} \right) \]  

(9)

Here, \( C_g \) and \( \beta_2(g) \) are the population coefficient of variation and the population coefficient of kurtosis respectively of the auxiliary variable. \( \bar{y} \) and \( \bar{g} \) are the sample means of the study and auxiliary variable respectively and it is assumed that the population mean \( \bar{G} \) of the auxiliary variable \( G \) is known. Here \( b = \frac{s_{gy}}{s_g^2} \) is obtained by the least square. Where \( s_{gy}^2 \) and \( s_g^2 \) are the sample variances of auxiliary and study variable respectively and \( s_{gy}^2 \) is the sample covariance between the auxiliary and study variable.

The \( MSE \) of the estimators 5-9 can be found using a first degree of approximation of the Taylor series expansion and is as follows:

\[ \text{MSE}(\bar{y}_{Kc1}) = \lambda \left[ R_{Kc1}^2 S_g^2 + 2B_R S_g^2 + B^2 S_g^2 - 2R_{Kc1} S_{gy} - 2BS_{gy} + S_y^2 \right] \]  

(10)

Where \( i=1, 2, \ldots, 5, B = \frac{s_{gy}}{s_g^2} \) is obtained by least square method \( f = \frac{n}{N} \); \( n \) is the sample size and \( N \) is the population size and \( \lambda = \frac{1}{n} - \frac{1}{\bar{y}} \)

Where:

\[ R_{Kc1} = R = \frac{\bar{y}}{G}, \quad R_{Kc2} = \frac{\bar{y}}{G + C_g}, \quad R_{Kc3} = \frac{\bar{y}}{G + \beta_2(g)} \]

\[ R_{Kc4} = \frac{\bar{y}c_g}{\beta_2(g) + C_g}, \quad R_{Kc5} = \frac{\bar{y}c_g}{\beta_2(g) + \beta_2(g)} \]

2. Motivated from Kadilar and Cingi (2004), Koc (2021) proposed a new way to improve the estimators by adding Poisson regression-based ratio estimators as follows;

\[ \bar{y}_1 = \frac{\bar{y} + b_p(\bar{G} - \bar{g})}{\bar{g}} \]  

(11)

\[ \bar{y}_2 = \frac{\bar{y} + b_p(\bar{G} - \bar{g})}{(\bar{g} + C_g)} \]  

(12)

\[ \bar{y}_3 = \frac{\bar{y} + b_p(\bar{G} - \bar{x})}{(\bar{g} + \beta_2(g))} \]  

(13)

\[ \bar{y}_4 = \frac{\bar{y} + b_p(\bar{G} - \bar{g})}{(\bar{g} + \beta_2(g)) (\beta_2(g) + C_g)} \]  

(14)

\[ \bar{y}_5 = \frac{\bar{y} + b_p(\bar{G} - \bar{g})}{(\bar{g} + \beta_2(g)) (\alpha_2 + \beta_2(g))} \]  

(15)

The mean square error of \( \bar{y}_i, i = 1, 2, \ldots, 5 \) is given as;

\[ \text{MSE} (\bar{y}_i) = \lambda \left[ R_{Kc1}^2 S_g^2 + 2B_p R_{Kc1} S_g^2 + B_p^2 S_g^2 - 2R_{Kc1} S_{gy} - 2B_p S_{gy} + S_y^2 \right], i = 1, 2, \ldots, 5 \]  

(16)

The value of \( B_p \) can be obtained by using Poisson regression model.

3. Usman et al. (2021) proposed a regression estimator using Poisson regression model and is given as

\[ \bar{y}_u = \bar{y} + b_p(\bar{G} - \bar{g}) \]  

(17)

The mean square of \( \bar{y}_u \) is given as;

\[ \text{MSE} (\bar{y}_u) = \lambda \left[ S_y^2 - 2B_p S_{gy} + B_p^2 S_g^2 \right] \]  

(18)

3. Proposed Estimators

In this section, a new efficient class of ratio-cum-regression type estimator for estimation the population mean under SRSWOR based on Poisson regression method is proposed. Some members of the family of estimators are written in Table 1. Expressions for bias and \( MSE \) are obtained up to first degree of approximation.

\[ Z_{pi} = \bar{y} \left( \frac{\alpha_i \bar{G} + \beta_i}{\alpha_i \bar{G} + \beta_i} \right)^{\alpha_i \bar{G} + \beta_i} + b_p \left( 1 - \frac{\bar{g}}{\bar{G}} \right) \]  

(19)

Where, \( \alpha_i \neq 0 \) and \( \beta_i \) may be any constant or functions of some known parameters of auxiliary variable \( G \), which are determined such that the \( MSE \) of \( Z_{pi}, i = 1, 2, \ldots, 15 \) is minimum.

Table 1: Some members of the proposed family of estimators using different values of \( \alpha \) and \( \beta \)
<table>
<thead>
<tr>
<th>S.No.</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
<th>Estimators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \alpha_1 = 1 )</td>
<td>( \beta_1 = S_g )</td>
<td>( Z_{p1} = \bar{y} \left( \frac{\alpha_1 \bar{G} + \beta_1}{\alpha_1 \bar{G} + \beta_1} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( \alpha_2 = C_g )</td>
<td>( \beta_2 = S_g )</td>
<td>( Z_{p2} = \bar{y} \left( \frac{\alpha_2 \bar{G} + \beta_2}{\alpha_2 \bar{G} + \beta_2} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>3</td>
<td>( \alpha_3 = \rho_{gy} )</td>
<td>( \beta_3 = S_g )</td>
<td>( Z_{p3} = \bar{y} \left( \frac{\alpha_3 \bar{G} + \beta_3}{\alpha_3 \bar{G} + \beta_3} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>4</td>
<td>( \alpha_4 = 1 )</td>
<td>( \beta_4 = fS_g )</td>
<td>( Z_{p4} = \bar{y} \left( \frac{\alpha_4 \bar{G} + \beta_4}{\alpha_4 \bar{G} + \beta_4} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>5</td>
<td>( \alpha_5 = \rho_{gy} )</td>
<td>( \beta_5 = fS_g )</td>
<td>( Z_{p5} = \bar{y} \left( \frac{\alpha_5 \bar{G} + \beta_5}{\alpha_5 \bar{G} + \beta_5} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>6</td>
<td>( \alpha_6 = \rho_{gy} C_g )</td>
<td>( \beta_6 = fS_g )</td>
<td>( Z_{p6} = \bar{y} \left( \frac{\alpha_6 \bar{G} + \beta_6}{\alpha_6 \bar{G} + \beta_6} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>7</td>
<td>( \alpha_7 = 1 )</td>
<td>( \beta_7 = f(1 - f)S_g )</td>
<td>( Z_{p7} = \bar{y} \left( \frac{\alpha_7 \bar{G} + \beta_7}{\alpha_7 \bar{G} + \beta_7} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>8</td>
<td>( \alpha_8 = 1 )</td>
<td>( \beta_8 = 2f(1 - f)S_g )</td>
<td>( Z_{p8} = \bar{y} \left( \frac{\alpha_8 \bar{G} + \beta_8}{\alpha_8 \bar{G} + \beta_8} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>9</td>
<td>( \alpha_9 = 1 )</td>
<td>( \beta_9 = \left( \frac{f}{1 - f} \right)S_g )</td>
<td>( Z_{p9} = \bar{y} \left( \frac{\alpha_9 \bar{G} + \beta_9}{\alpha_9 \bar{G} + \beta_9} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>10</td>
<td>( \alpha_{10} = 1 )</td>
<td>( \beta_{10} = \left( \frac{f}{1 + f} \right)S_g )</td>
<td>( Z_{p10} = \bar{y} \left( \frac{\alpha_{10} \bar{G} + \beta_{10}}{\alpha_{10} \bar{G} + \beta_{10}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>11</td>
<td>( \alpha_{11} = 1 )</td>
<td>( \beta_{11} = \left( \frac{f}{1 + 2f} \right)S_g )</td>
<td>( Z_{p11} = \bar{y} \left( \frac{\alpha_{11} \bar{G} + \beta_{11}}{\alpha_{11} \bar{G} + \beta_{11}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>12</td>
<td>( \alpha_{12} = 1 )</td>
<td>( \beta_{12} = \left( \frac{2f}{1 + 2f} \right)S_g )</td>
<td>( Z_{p12} = \bar{y} \left( \frac{\alpha_{12} \bar{G} + \beta_{12}}{\alpha_{12} \bar{G} + \beta_{12}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>13</td>
<td>( \alpha_{13} = C_g )</td>
<td>( \beta_{13} = \left( \frac{2f}{1 - f} \right)S_g )</td>
<td>( Z_{p13} = \bar{y} \left( \frac{\alpha_{13} \bar{G} + \beta_{13}}{\alpha_{13} \bar{G} + \beta_{13}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>14</td>
<td>( \alpha_{14} = \rho_{gy} )</td>
<td>( \beta_{14} = \left( \frac{2f}{1 - f} \right)S_g )</td>
<td>( Z_{p14} = \bar{y} \left( \frac{\alpha_{14} \bar{G} + \beta_{14}}{\alpha_{14} \bar{G} + \beta_{14}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
<tr>
<td>15</td>
<td>( \alpha_{15} = 1 )</td>
<td>( \beta_{15} = \left( \frac{1 - 2f}{1 + 2f} \right)S_g )</td>
<td>( Z_{p15} = \bar{y} \left( \frac{\alpha_{15} \bar{G} + \beta_{15}}{\alpha_{15} \bar{G} + \beta_{15}} \right) + b_p \left( 1 - \frac{\bar{y}}{\bar{G}} \right) )</td>
</tr>
</tbody>
</table>
The Bias of the proposed estimator $Z_{pi}, i = 1, 2, ..., 15$ up to first order of approximation is given as,

$$Bias(Z_{pi}) = \lambda \gamma \left[ \frac{\varphi_1(q_1 + 1)}{2} C_g^2 - (\varphi_1^2 + B_g)C_{gy} \right]$$  \hspace{1cm} (20)

The MSE of the proposed estimator $Z_{pi}, i = 1, 2, ..., 15$ up to first order of approximation is given as

$$MSE(Z_{pi}) = \lambda \left[ S_g^2 + R^2(\varphi_1^2 + B_g)S_{gy}^2 - 2R(\varphi_1^2 + B_g)S_{gy} \right]$$  \hspace{1cm} (21)

Where:

- $\varphi_1 = \frac{a_i G}{G + S_g}$, $i = 1, 2, 3, ..., 15$
- $\varphi_2 = \frac{G C_g}{G + S_g}$, $\varphi_3 = \frac{G \rho_g G}{G + S_g}$, $\varphi_4 = \frac{G}{G + f S_g}$
- $\varphi_5 = \frac{\rho_{gy} G}{\rho_{gy} G + f S_g}$, $\varphi_6 = \frac{G \rho_{gy}}{G + f S_g}$, $\varphi_7 = \frac{G + f (1-f) S_g}{G + f (1-f) S_g}$, $\varphi_8 = \frac{G + 2f(1-f) S_g}{G + 2f(1-f) S_g}$
- $\varphi_9 = \frac{G + \left(\frac{f}{1-f}\right) S_g}{G}$, $\varphi_{10} = \frac{G + \left(\frac{f}{1+f}\right) S_g}{G}$, $\varphi_{11} = \frac{G + \left(\frac{1}{1+2f}\right) S_g}{G}$, $\varphi_{12} = \frac{G + \left(\frac{1-2f}{1+2f}\right) S_g}{G}$
- $\varphi_{13} = \frac{C_g G}{C_g G + \left(\frac{2f}{1-f}\right) S_g}$, $\varphi_{14} = \frac{G \rho_{gy}}{G + \left(\frac{2f}{1-f}\right) S_g}$, $\varphi_{15} = \frac{G + \left(\frac{2f}{1+2f}\right) S_g}{G}$

For the detailed derivation of the expression of Bias and MSE of the proposed estimator $Z_{pi}, i = 1, 2, ..., 15$ one can see the Appendix-A.

4. Efficiency Comparison:

In this section, we present the comparison of the proposed estimators ($Z_{pi}, i = 1, 2, ..., 15$) with Koc (2021) ($\bar{y}_i, i = 1, 2, ..., 5$) and Usman et al. (2021) ($\tilde{y}_U$)

4.1. Comparing the MSE of proposed estimator $Z_{pi}, i = 1, 2, ..., 15$ with Koc (2021) $\bar{y}_i, i = 1, 2, ..., 5$

Z$_{pi}, i = 1, 2, ..., 15$ Perform better than $\bar{y}_i, i = 1, 2, ..., 5$ if

$$MSE(\bar{y}_i) > MSE(Z_{pi})$$

$$\lambda \left[ R_{KCI}^2 S_g^2 + 2B_g R_{KCI} S_g^2 + B_g^2 S_g^2 - 2 R_{KCI} S_{gy} - 2 B_g S_{gy} + S_{gy}^2 \right] > \lambda \left[ S_g^2 + R^2(\varphi_1^2 + B_g)S_{gy} - 2R(\varphi_1^2 + B_g)S_{gy} \right]$$

4.1. Comparing the MSE of proposed estimator $Z_{pi}, i = 1, 2, ..., 15$ with Usman et al. (2021) $\tilde{y}_U$

Z$_{pi}, i = 1, 2, ..., 15$ Perform better than $\tilde{y}_U$ if

$$MSE(\tilde{y}_U) > MSE(Z_{pi})$$

$$\lambda \left[ S_g^2 - 2B_g S_{gy} + B_g^2 S_g^2 \right] > \lambda \left[ S_g^2 + R^2(\varphi_1^2 + B_g)S_{gy} - 2R(\varphi_1^2 + B_g)S_{gy} \right]$$

5. Empirical Study:

To examine the performance of the proposed class of estimator $Z_{pi}, i = 1, 2, ..., 15$ over the other well-known estimators, two real data sets have been considered. The descriptions of the populations along with the values of various parameters are listed in Table 2 and Table 3.

Population: 1

We consider the dataset collected between 2006 and 2010 from the Afyon Respiratory Disease Hospital and the Afyon Environmental Department Air Pollution Unit, which was used by Koc (2021). The number of patients admitted to the hospital on a weekly basis was taken as the dependent variable Y, and PM10 was taken as the explanatory variable G.
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Table: 2 The descriptive statistics of population 1 are

<table>
<thead>
<tr>
<th></th>
<th>Population 1</th>
<th></th>
<th>Population 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>213</td>
<td>n</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{Y}$</td>
<td>4.676056338</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{G}$</td>
<td>116.1915</td>
<td></td>
</tr>
<tr>
<td>$C_y$</td>
<td>0.633555159</td>
<td>$C_g$</td>
<td>0.439846</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\hat{P}_2(g)$</td>
<td>$-0.6908$</td>
<td></td>
</tr>
<tr>
<td>$S_g$</td>
<td>51.10636</td>
<td>$S_{gy}$</td>
<td>53.51706</td>
<td></td>
</tr>
<tr>
<td>$B_p$</td>
<td>0.004</td>
<td>$\rho_{gy}$</td>
<td>0.353</td>
<td></td>
</tr>
<tr>
<td>$S_y$</td>
<td></td>
<td></td>
<td>$2.962539616$</td>
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</tr>
<tr>
<td>$S_g$</td>
<td></td>
<td></td>
<td></td>
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<td>$S_{gy}$</td>
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<td>$\rho_{gy}$</td>
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</tr>
<tr>
<td>$B_l$</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Population: 2
We consider the dataset obtained from TUIK of 81 provinces in 2019 used by Koc (2021). The number of people who died due to traffic accidents was taken as a dependent variable Y, and the number of motor vehicles was taken as explanatory variable G.

Table: 3 The descriptive statistics of population 2 are

<table>
<thead>
<tr>
<th></th>
<th>Population 1</th>
<th></th>
<th>Population 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>81</td>
<td>n</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{Y}$</td>
<td>82.40740741</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\bar{G}$</td>
<td>274308</td>
<td></td>
</tr>
<tr>
<td>$C_y$</td>
<td>1.025504149</td>
<td>$C_g$</td>
<td>1.90331</td>
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<tr>
<td></td>
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<td>$\hat{P}_2(g)$</td>
<td>35.77509</td>
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<tr>
<td>$S_g$</td>
<td>522093</td>
<td>$S_{gy}$</td>
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<tr>
<td>$B_p$</td>
<td>6.131E-07</td>
<td>$\rho_{gy}$</td>
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</tr>
<tr>
<td>$S_y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_g$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{gy}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{gy}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_l$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table: 4 Mean square error of the existing and proposed estimators for population 1 and population 2

<table>
<thead>
<tr>
<th>Estimators</th>
<th>Population 1</th>
<th>Population 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{y}_1$</td>
<td>0.18587</td>
<td>334.73926</td>
</tr>
<tr>
<td>$\bar{y}_2$</td>
<td>0.18549</td>
<td>334.73236</td>
</tr>
<tr>
<td>$\bar{y}_3$</td>
<td>0.18648</td>
<td>334.60966</td>
</tr>
<tr>
<td>$\bar{y}_4$</td>
<td>0.18643</td>
<td>334.73906</td>
</tr>
<tr>
<td>$\bar{y}_5$</td>
<td>0.18728</td>
<td>334.67116</td>
</tr>
<tr>
<td>$\bar{y}_U$</td>
<td>0.17037</td>
<td>267.16300</td>
</tr>
<tr>
<td>$Z_{P1}$</td>
<td>0.15599</td>
<td>179.62946</td>
</tr>
<tr>
<td>$Z_{P2}$</td>
<td>0.161537</td>
<td>111.18094</td>
</tr>
<tr>
<td>$Z_{P3}$</td>
<td>0.164036</td>
<td>193.63367</td>
</tr>
<tr>
<td>$Z_{P4}$</td>
<td>0.166353</td>
<td>68.14308</td>
</tr>
<tr>
<td>$Z_{P5}$</td>
<td>0.157918</td>
<td>70.09819</td>
</tr>
<tr>
<td>$Z_{P6}$</td>
<td>0.156482</td>
<td>86.12029</td>
</tr>
<tr>
<td>$Z_{P7}$</td>
<td>0.1679</td>
<td>74.05904</td>
</tr>
<tr>
<td>$Z_{P8}$</td>
<td>0.162312</td>
<td>82.08443</td>
</tr>
<tr>
<td>$Z_{P9}$</td>
<td>0.164673</td>
<td>75.05373</td>
</tr>
<tr>
<td>$Z_{P10}$</td>
<td>0.167644</td>
<td>71.68617</td>
</tr>
<tr>
<td>$Z_{P11}$</td>
<td>0.168662</td>
<td>79.98185</td>
</tr>
<tr>
<td>$Z_{P12}$</td>
<td>0.163265</td>
<td>75.44276</td>
</tr>
<tr>
<td>$Z_{P13}$</td>
<td>0.156066</td>
<td>77.54750</td>
</tr>
<tr>
<td>$Z_{P14}$</td>
<td>0.15685</td>
<td>150.51009</td>
</tr>
<tr>
<td>$Z_{P15}$</td>
<td>0.159036</td>
<td>76.67033</td>
</tr>
</tbody>
</table>
Also, from Table 5, the proposed estimator $Z_p$, $i=1,2,\ldots,15$ have minimum MSE than Koc (2021) $\bar{y}_i$, $i=1,2,\ldots,5$ and Usman et al. (2021) $\bar{y}_U$ for both population 1 and population 2.

Table 5 presents the efficiency comparison based on percent relative efficiency of our proposed estimators $Z_p$, $i=1,2,\ldots,15$ with $\bar{y}_i$, $i=1,2,\ldots,5$ and $\bar{y}_U$ using population 1. It can be seen that our proposed estimators are more efficient than $\bar{y}_i$, $i=1,2,\ldots,5$ and $\bar{y}_U$. Among all the proposed estimators we can also see from Table 5 that the proposed estimator $Z_p$, $i=1$ is more efficient using population 1.

Table 6 presents the efficiency comparison based on percent relative efficiency of our proposed estimators $Z_p$, $i=1,2,\ldots,15$ with $\bar{y}_i$, $i=1,2,\ldots,5$ and $\bar{y}_U$ using population 2. It can be seen that our proposed estimators are more efficient than $\bar{y}_i$, $i=1,2,\ldots,5$ and $\bar{y}_U$. Among all the proposed estimators we can also see from Table 6 that the proposed estimator $Z_p$, $i=15$ is more efficient using population 2.

Also, from Table 5 and Table 6 we see that our proposed estimators $Z_p$, $i=1,2,\ldots,15$ perform much better in population 2 than population 1.
6. Conclusion:
In this paper, we have proposed a class of estimators $Z_{p_i}$, $i = 1, 2, ..., 15$ by using simple random sampling for estimation of population mean based on the Poisson regression model. The proposed estimator was found to generate more efficient results than estimators suggested by Koc (2021) $\tilde{y}_i$ $i = 1, 2, ..., 5$ and Usman (2021) $\tilde{y}_i$. A comparative study of the proposed estimators with the existing estimators has also been presented. To evaluate the benefits of the proposed estimator over others, a real data set was investigated in support of the current study, and it is clear that the proposed class of estimators is more competent as compared to existing estimators in terms of percent relative efficiency, as all values exceed 100. As a result, for count data analysis, we strongly indorse choosing proposed class of estimators instead of existing estimator.

Acknowledgement:
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References
Appendix-A: A detailed derivation of Bias and MSE for the proposed class of estimators in the present study.

To obtain the bias and MSE of the proposed estimator \( Z_{pi}, i = 1, 2, ..., 15 \), we write

\[
\eta_0 = \frac{\bar{y} - \bar{y}'}{\bar{y}'}, \quad \text{and} \quad \eta_1 = \frac{\bar{g} - \bar{G}}{\bar{G}}
\]

Such that \( E(\eta_0) = E(\eta_1) = 0 \) and to first degree of approximation

\[
E(\eta_0^2) = \lambda C_y^2, \quad E(\eta_1^2) = \lambda C_y^2, \quad E(\eta_0\eta_1) = \lambda C_{gy}
\]

Now expressing the (19) in terms of \( \eta \)'s, we have

\[
Z_{pi} = \bar{y}(1 + \eta_0) \left( \frac{\alpha_i \bar{G} + \beta_i}{\alpha_i \bar{G}(1 + \eta_1) + \beta_i} \right) \left( 1 - \frac{\bar{G}(1 + \eta_1)}{\bar{G}} \right) + b_p \left( \frac{1}{1 + \frac{\alpha_i \bar{G}}{\alpha_i \bar{G} + \beta_i \eta_1}} \right)
\]

(22)

Now expanding the right-hand side of \( Z_{pi}, i = 1, 2, ..., 15 \) at (22) and neglecting terms of \( \eta \)'s having power greater than two we have

\[
Z_{pi} = \bar{y}(1 + \eta_0) [(1 + \varphi_1 \eta_1)^{\varphi_1} - b_p \eta_1]
\]

Where:

\[
\varphi_i = \frac{\alpha_i \bar{G}}{\alpha_i \bar{G} + \beta_i}, \quad i = 1, 2, 3, \ldots, 15
\]

\[
\begin{align*}
\varphi_1 &= \frac{\bar{G}}{\bar{G} + S_g}, & \varphi_2 &= \frac{C_g \bar{G} + S_g}{C_g \bar{G} + S_g}, & \varphi_3 &= \frac{\rho_{gy} \bar{G}}{\rho_{gy} \bar{G} + S_g}, & \varphi_4 &= \frac{\bar{G}}{\bar{G} + \bar{G} S_g}, \\
\varphi_5 &= \frac{\rho_{gy} \bar{G}}{\rho_{gy} \bar{G} + S_g}, & \varphi_6 &= \frac{\rho_{gy} C_g \bar{G}}{\rho_{gy} C_g \bar{G} + S_g}, & \varphi_7 &= \frac{\bar{G}}{\bar{G} + f(1 - f) S_g}, & \varphi_8 &= \frac{\bar{G}}{\bar{G} + 2f(1 - f) S_g}, \\
\varphi_9 &= \frac{\bar{G}}{\bar{G} + \left( \frac{f}{1-f} \right) S_g}, & \varphi_{10} &= \frac{\bar{G}}{\bar{G} + \left( \frac{f}{1-f} \right) S_g}, & \varphi_{11} &= \frac{\bar{G}}{\bar{G} + \left( \frac{f}{1+f} \right) S_g}, & \varphi_{12} &= \frac{\bar{G}}{\bar{G} + \left( \frac{f}{1+f} \right) S_g}, \\
\varphi_{13} &= \frac{C_g \bar{G}}{C_g \bar{G} + \left( \frac{2f}{1-f} \right) S_g}, & \varphi_{14} &= \frac{\rho_{gy} \bar{G}}{\rho_{gy} \bar{G} + \left( \frac{2f}{1-f} \right) S_g}, & \varphi_{15} &= \frac{\bar{G}}{\bar{G} + \left( \frac{1-f}{1+f} \right) S_g}
\end{align*}
\]

\[
Z_{pi} = \bar{y}(1 + \eta_0) \left[ 1 - \varphi_1^2 \eta_1 + \frac{\varphi_1^2 (\varphi_1 + 1)}{2} \eta_1^2 - b_p \eta_1 \right]
\]

\[
Z_{pi} = \bar{y} \left[ 1 - \varphi_1^2 \eta_1 + \frac{\varphi_1^2 (\varphi_1 + 1)}{2} \eta_1^2 - b_p \eta_1 + \eta_0 - \varphi_1 \eta_0 \eta_1 - b_p \eta_0 \eta_1 \right]
\]

(23)

Taking expectation of both sides of (23) we get the bias of \( Z_{pi}, i = 1, 2, ..., 15 \) up to first order of approximation as
\[
E(Z_{pl} - \bar{Y}) = \bar{Y} \left[ E(\eta_o) - \varphi_i^2 E(\eta_1) + \frac{\varphi_i^3 (\varphi_i + 1)}{2} E(\eta_1^2) - b_p E(\eta_1) - \varphi_i^2 E(\eta_0 \eta_1) - b_p E(\eta_0 \eta_1) \right]
\]
\[
E(Z_{pl} - \bar{Y}) = \bar{Y} \left[ E(\eta_o) - \varphi_i^2 E(\eta_1) + \frac{\varphi_i^3 (\varphi_i + 1)}{2} E(\eta_1^2) - b_p E(\eta_1) - \varphi_i^2 E(\eta_0 \eta_1) - b_p E(\eta_0 \eta_1) \right]
\]
\[
E(Z_{pl} - \bar{Y}) = \bar{Y} \left[ \frac{\varphi_i^3 (\varphi_i + 1)}{2} E(\eta_1^2) - (\varphi_i^2 + b_p) E(\eta_0 \eta_1) \right]
\]
\[
Bias(Z_{pl}) = \lambda \bar{Y} \left[ \frac{\varphi_i^3 (\varphi_i + 1)}{2} C_{\eta_1}^2 - (\varphi_i^2 + b_p) C_{\eta_{0\eta_1}} \right]
\]

Squaring both sides (23) and neglecting terms of \( \eta \)'s having power greater than two, we have

\[
(Z_{pl} - \bar{Y})^2 = \bar{Y}^2 \left[ \eta_o - \varphi_i^2 \eta_1 + \frac{\varphi_i^3 (\varphi_i + 1)}{2} \eta_1^2 - b_p \eta_1 - \varphi_i^2 \eta_0 \eta_1 - b_p \eta_0 \eta_1 \right]^2
\]

\[
(Z_{pl} - \bar{Y})^2 = \bar{Y}^2 \left[ \eta_o^2 + (\varphi_i^4 + b_p^2 + 2b_p \varphi_i^3) \eta_o \eta_1 - 2(\varphi_i^2 + b_p) \eta_0 \eta_1 \right]
\]

Taking expectation of both sides (25), we get the MSE of the proposed class of estimator \( Z_{pl} \), \( i = 1, 2, ..., 15 \) to the first degree of approximation.

\[
E(Z_{pl} - \bar{Y})^2 = MSE(Z_{pl}) = \bar{Y}^2 \left[ E(\eta_o^2) + (\varphi_i^2 + b_p)^2 E(\eta_1^2) - 2(\varphi_i^2 + b_p) E(\eta_0 \eta_1) \right]
\]

\[
MSE(Z_{pl}) = \lambda \left[ S_{\eta_o}^2 + R^2 (\varphi_i^2 + b_p)^2 S_{\eta_1}^2 - 2R(\varphi_i^2 + b_p) S_{\eta_{0\eta_1}} \right]
\]