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New Exponential Ratio Estimator in Ranked Set Sampling

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Abstract
In this study, we adapted the families of estimators from Ünal and Kadilar (2021) using the exponential function for the population mean in case of non-response for simple random sampling for the estimation of the mean of the population with the RSS (ranked set sampling) method. The equations for the MSE (mean square error) and the bias of the adapted estimators are obtained for RSS and it in theory shows that the proposed estimator is additional efficient than the present RSS mean estimators in the literature. In addition, we support these theoretical results with real COVID-19 real data and conjointly the simulation studies with different distributions and parameters. As a result of the study, it was observed that the efficiency of the proposed estimator was better than the other estimators.

Key Words: Exponential ratio estimator; ranked set sampling; mean square error (MSE); efficiency.

Mathematical Subject Classification: 62D05, 94A17, 62F07.

1. Introduction

Ranked set sampling (RSS) is a popular method developed as an alternative to the commonly used simple random sampling (SRS) and is frequently used today because of the reason of it is known that the efficiency of the population mean is higher in this sampling method. It was proposed by McIntyre (1952) and subsequently, this sampling methodology has been examined in many ways to date such as, Takahasi and Wakimoto (1968) extended the idea of McIntyre (1952) and provided a necessary mathematical foundation to the theory of RSS. McIntyre (1952) and Takahasi and Wakimoto (1968) considered the case of the perfect ranking of units, whereas, Dell and Clutter (1972) considered the case of perfect and imperfect ranking and showed that the mean under RSS is an unbiased estimator of the population mean. Stokes (1977) utilized RSS to analyze the case when the ranking of the study variable is difficult. Muttlak and Mc Donald (1990) developed RSS for the case when units are selected with size-biased probability with respect to the concomitant variable. Kadilar, Unyazici, and Cingi (2009) suggested a ratio estimator for the estimation of population mean under RSS. Further, Singh, Tailor, and Singh (2014) introduced the general procedure for estimating the population mean under RSS. Mehta and Mandowara (2016) introduced modified ratio-cum-product estimators under RSS. Recently, Rather and Kadilar (2021) proposed a new exponential estimator under RSS.

Once the variable of interest is extremely costly or difficult to measure we tend to make use of RSS, whereas it can be easily ranked by a negligible or cost-free variable that is related to the variable of interest. In RSS, m random sets each of size m, are selected with equal probability and without replacement from the population. The members of each random set are ranked according to the auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. In this way, this procedure is continued until the unit with the largest rank is chosen from the m-th set. This cycle may be repeated r times to collect n sample size, so n=mr units have been collected in the sample during this process. In RSS, when we rank on the auxiliary variable, let $y_{[i]}$ denote an *i*-th judgment ordering in the *i*-th set for the study variable and $x_{(i)}$ the *i*-th order statistic in the *i*-th set for the auxiliary variable.

2. Mean estimators in literature

To estimate the population mean, there are ratio, product and regression estimators developed as alternatives to the simple mean estimator. These estimators are known to be more efficient than the simple mean estimator under certain condition.

The basic simple mean estimator of SRS is defined as,

$$\hat{\mu}_1 = \bar{y}_{SRS} \tag{1}$$

with the mean square error (MSE)

$$MSE(\hat{\mu}_1) \cong \gamma S_y^2 \tag{2}$$

where and it can be consider that *n=mr*. The traditional ratio estimator for the population mean in SRS is defined by

$$\hat{\mu}_2 = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \tag{3}$$

where it is assumed that the population mean $\bar{X}$ of the auxiliary variable x is known. Here $\bar{y}$ is the sample mean of the study variable and $\bar{x}$ is the sample mean of the auxiliary variable. The MSE of the traditional ratio estimator is as follows:

$$MSE(\hat{\mu}_2) \cong \gamma\left(R^2 S_x^2 - 2RS_{yx} + S_y^2\right) \tag{4}$$

Besides, $R = \bar{Y}/\bar{X}$ is the population ratio; S_x^2 is the population variance of auxiliary variable; S_y^2 is the population variance of study variable and S_{yx} is the population covariance between auxiliary variable and study variable (Cochran, 1977). N is the population size; n is the sample size, $\gamma = 1/n$ and note that $f = n/N$ is omitted in (4).

In the simple random sampling, Prasad (1989) proposed a ratio estimator as

$$\hat{\mu}_3 = \kappa\frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}_\kappa \bar{X} \tag{5}$$

where $\hat{R}_\kappa = \kappa(\bar{y}/\bar{x})$ and the MSE of this estimator was given by

$$\text{MSE}(\hat{\mu}_3) \cong \gamma\left(R^2 S_x^2 - 2R\kappa S_{yx} + \kappa^2 S_y^2\right) + \bar{Y}^2(\kappa - 1)^2 \tag{6}$$

$$\text{MSE}(\hat{\mu}_3) = \bar{Y}^2\left[\gamma\left(C_x^2 - 2\kappa\rho C_y C_x + \kappa^2 C_y^2\right) + (\kappa - 1)^2\right] \tag{7}$$

where the coefficient $\kappa = \dfrac{1+\gamma\rho C_y C_x}{1+\gamma C_y^2}$ makes the MSE in (7) minimum. Here, C_x and C_y are the population coefficients of variation of auxiliary variable and study variable, respectively and ρ is the correlation coefficient between y and x.

Comparing the equation (6) when κ is minimum in (4), Prasad (1989) showed that the ratio estimator, given in (5), was always more efficient than the classical ratio estimator, given in (3).

Samawi and Muttlak (1996) defined the estimator of the population ratio using RSS method as

$$\hat{R}_{RSS} = \bar{y}_{[n]} \big/ \bar{x}_{(n)} \tag{8}$$

where $\bar{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$ and $\bar{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$. As Samawi and Muttlak (1996) remind that this estimator can also be used for the population total and mean, we can write the following estimator for the population mean:

$$\mu_4 = \frac{\bar{y}_{[n]}}{\bar{x}_{(n)}} \bar{X} \tag{9}$$

and the MSE equation of this estimator can be written as,

$$\text{MSE}(\hat{\mu}_4) \cong \frac{1}{mr}\left(S_y^2 - 2RS_{yx} + R^2 S_x^2\right) - \frac{1}{m^2 r}\left(\sum_{i=1}^{m} \tau_{y[i]}^2 - 2R\sum_{i=1}^{m} \tau_{yx(i)} + R^2 \sum_{i=1}^{m} \tau_{x(i)}^2\right) \tag{10}$$

Bahl and Tuteja (1991) defined the exponential ratio estimator for the SRS which is adapted in RSS by Vishwakarma et al. (2017) as,

$$\hat{\mu}_5 = \bar{y}_{RSS} \exp\left(\frac{\bar{X} - \bar{x}_{RSS}}{\bar{X} + \bar{x}_{RSS}}\right) \tag{11}$$

and its MSE is given as follows:

$$MSE(\hat{\mu}_5) \cong \frac{1}{mr}\left(S_y^2 - 2RS_{yx} + \frac{R^2 S_x^2}{4}\right) - \frac{1}{m^2 r}\left(\sum_{i=1}^{m} \tau_{y[i]}^2 - 2R\sum_{i=1}^{m} \tau_{yx(i)} + \frac{R^2 \sum_{i=1}^{m} \tau_{x(i)}^2}{4}\right) \tag{12}$$

3. The Suggested Estimator

Adapting the ratio estimator for the population mean suggested by Unal and Kadilar (2021), we develop the following estimator under RSS method as,

$$\hat{\mu}_6 = \bar{y}_{[n]} \exp\left[\frac{\bar{X}}{\bar{X} + k\left(\bar{x}_{(n)} - \bar{X}\right)} - 1\right] \tag{13}$$

where k is a chosen constant to be determined that the MSE of $\hat{\mu}_6$ is minimum. We can write some equations to obtain the MSE and bias of $\hat{\mu}_6$ as follows:

$\bar{y}_{[n]} = \left(\bar{Y} + \bar{Y}e_0\right)$, $\bar{x}_{(n)} = \left(\bar{X} + \bar{X}e_1\right)$, $E(e_0) = E(e_1) = 0$, $V(\bar{y}_{RSS}) = \frac{1}{mr}\left(S_y^2 - \frac{1}{m}\sum_{i=1}^{m} \tau_{y[i]}^2\right)$, $V(\bar{x}_{RSS}) = \frac{1}{mr}\left(S_x^2 - \frac{1}{m}\sum_{i=1}^{m} \tau_{x(i)}^2\right)$ and

$\text{cov}(\bar{y}_{RSS}, \bar{x}_{RSS}) = \frac{1}{mr}\left(S_{yx} - \frac{1}{m}\sum_{i=1}^{m} \tau_{yx(i)}\right)$. Expressing the Eq. (13) in terms of and, we obtain

$$\hat{\mu}_6 = \bar{Y}\left(e_0 - ke_1 + \frac{3k^2 e_1^2}{2} - ke_0 e_1 + 1\right) \tag{14}$$

Expanding the right hand side of (14) and neglecting the terms involving powers of e_0 and e_1 greater than two, we have

$$\left(\hat{\mu}_6 - \bar{Y}\right) = \bar{Y}\left(e_0 - ke_1 + \frac{3k^2 e_1^2}{2} - ke_0 e_1\right) \tag{15}$$

We take expectation on both side of (15), we obtain the bias of the proposed estimator $B(\hat{\mu}_6)$, as

$$B(\hat{\mu}_6) = \bar{Y}\left[\frac{3R^2 k^2}{2}\left(\frac{1}{mr}\left(S_x^2 - \frac{1}{m}\sum_{i=1}^{m} \tau_{x[i]}^2\right)\right) - \frac{kR}{mr}\left(S_{yx} - \frac{1}{m}\sum_{i=1}^{m} \tau_{yx(i)}\right)\right] \tag{16}$$

Firstly, we take square of (15) and then expectation, we derive the $\text{MSE}(\hat{\mu}_6)$ of the proposed estimator as follows:

$$\text{MSE}(\hat{\mu}_6) = \bar{Y}^2\left(\frac{1}{mr}\left(S_y^2 - \frac{1}{m}\sum_{i=1}^{m} \tau_{y[i]}^2\right) + R^2 k^2\left(\frac{1}{mr}\left(S_x^2 - \frac{1}{m}\sum_{i=1}^{m} \tau_{x[i]}^2\right)\right) - \frac{2kR}{mr}\left(S_{yx} - \frac{1}{m}\sum_{i=1}^{m} \tau_{yx(i)}\right)\right) \tag{17}$$

The optimum value of k (k^*) to minimize the MSE of $\hat{\mu}_6$ can easily be found as follows:

$$\frac{\partial \text{MSE}(\hat{\mu}_6)}{\partial k} = 0$$

and

$$k^{*}=\frac{E\left(e_0e_1\right)}{E\left(e_1^2\right)}=\frac{1}{R}\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x(i)}^2\right)} \tag{18}$$

When we replace k^* substituted for k in (17), we obtain the minimum MSE of the proposed estimator as follows:

$$\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)=\bar{Y}^2\left[\frac{1}{mr}\left(S_y^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right)-\frac{\frac{1}{mr}\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)}\right] \tag{19}$$

When we do not have the population information, we can estimate optimal value of k^* from sample by

$$k^{*}=\frac{1}{\hat{R}}\frac{\left(s_{yx}-\frac{1}{m}\sum_{i=1}^{m}\hat{\tau}_{yx(i)}\right)}{\left(s_x^2-\frac{1}{m}\sum_{i=1}^{m}\hat{\tau}_{x(i)}^2\right)} \tag{20}$$

Here s_y and s_x are the sample standard deviations of study and auxiliary variables, respectively.

4. Efficiency Comparisons

In this section, the performances of the proposed estimator have been demonstrated over the traditional ratio estimator in the SRS and RSS estimators as follows:

- $\mathrm{MSE}\left(\hat{\mu}_1\right)-\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)>0$

$$C_y^2>\left[\left(S_y^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right)-\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)}\right] \tag{21}$$

- $\mathrm{MSE}\left(\hat{\mu}_2\right)-\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)>0$

$$\left(C_x^2-2C_{yx}+C_y^2\right)>\left[\left(S_y^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right)-\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)}\right] \tag{22}$$

- $\mathrm{MSE}\left(\hat{\mu}_3\right)-\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)>0$

$$\left(C_x^2-2\kappa C_{yx}+\kappa^2C_y^2\right)+\left(\kappa-1\right)^2>\left[\left(S_y^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{y[i]}^2\right)-\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)}\right] \tag{23}$$

- $\mathrm{MSE}\left(\hat{\mu}_4\right)-\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)>0$

$$\left(2R\left(\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}+S_{yx}\right)+R^2\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)\right)>\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{\left(S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2\right)} \tag{24}$$

- $\mathrm{MSE}\left(\hat{\mu}_5\right)-\mathrm{MSE}_{\min}\left(\hat{\mu}_6\right)>0$

$$\frac{R^2S_x^2}{4}-2RS_{yx}+\frac{1}{m}\left(2R\sum_{i=1}^{m}\tau_{yx(i)}-\frac{R^2\sum_{i=1}^{m}\tau_{x(i)}^2}{4}\right)>\frac{\left(S_{yx}-\frac{1}{m}\sum_{i=1}^{m}\tau_{yx(i)}\right)^2}{S_x^2-\frac{1}{m}\sum_{i=1}^{m}\tau_{x[i]}^2} \qquad (25)$$

5. Simulation

In the simulation coded by the R and general parameters are defined as the correlation coefficient between variable of interest and the auxiliary variable ρ_{XY} = 0.65, 0.75, 0.85; set size m = 3, 4, 5 and cycle r= 4, 5 and 6. The mean estimators have been obtained for the number of trials of 10000. The population size 1000 is derivated from bivariate normal distribution with random parameter 5 and 1 as mean and standart deviation respectively and bivariate beta distribution with parameters 4 and 1. The relative efficiency (RE) values, for which the reference estimator is simple mean estimator of RSS as $\hat{\mu}_0=\overline{y}_{RSS}$, are calculated over the complete simulation through proportioning the mean square errors as follows:

$$RE_a = MSE\left(\hat{\mu}_0\right)/MSE\left(\hat{\mu}_a\right),\ a=1,2,3,4,5,6 \qquad (26)$$

Using (26), Calculated RE results are given in Table 1 and Table 2, for the normal and beta distributions, respectively.

Table 1: Relative efficiency values under N(5,1) distribution

	ρ	0.65			0.75			0.85		
	r	4	5	6	4	5	6	4	5	6
m=3	RE_1	0.7893	0.8371	0.8288	0.7356	0.7448	0.7221	0.6564	0.6684	0.6388
	RE_2	1.1234	1.1769	1.1647	1.5723	1.5870	1.5725	2.2308	2.2560	2.2024
	RE_3	1.1265	1.1794	1.1666	1.5755	1.5901	1.5741	2.2339	2.2576	2.2039
	RE_4	1.2969	1.2561	1.2901	1.6670	1.6434	1.6799	2.3445	2.3527	2.2711
	RE_5	1.3570	1.3599	1.3808	1.6206	1.6066	1.6265	1.9759	1.9507	1.9319
	RE_6	**1.3710**	**1.3657**	**1.4013**	**1.7231**	**1.7211**	**1.7658**	**2.3849**	**2.3988**	**2.3464**
m=4	RE_1	0.7709	0.7893	0.7747	0.6725	0.6552	0.6654	0.5799	0.5737	0.5804
	RE_2	1.0948	1.0920	1.1230	1.4108	1.4376	1.4081	1.9883	1.9916	2.0013
	RE_3	1.0974	1.0939	1.1250	1.4131	1.4398	1.4092	1.9908	1.9928	2.0019
	RE_4	1.2331	1.2331	1.2426	1.5403	1.5471	1.5411	2.0215	2.0339	2.1013
	RE_5	1.3136	1.3034	1.3073	1.5022	1.5164	1.5039	1.7869	1.7804	1.8055
	RE_6	**1.3139**	**1.3154**	**1.3274**	**1.5881**	**1.6137**	**1.6090**	**2.0731**	**2.0874**	**2.1553**
m=5	RE_1	0.7443	0.7727	0.7746	0.6350	0.6343	0.632	0.5362	0.5374	0.5525
	RE_2	1.0496	1.0903	1.0846	1.3617	1.3681	1.3905	1.8379	1.8453	1.9343
	RE_3	1.0519	1.0912	1.0858	1.3627	1.3691	1.3918	1.8393	1.8462	1.9347
	RE_4	1.2107	1.1946	1.2093	1.4609	1.4356	1.4845	1.9018	1.9314	1.9224
	RE_5	1.2638	1.2505	1.2592	1.4373	1.4188	1.4438	1.6832	1.6917	1.7010
	RE_6	**1.2692**	**1.2614**	**1.2765**	**1.5112**	**1.4914**	**1.5387**	**1.9344**	**1.9673**	**1.9731**

Table 2: Relative efficiency values under Beta(4,1) distribution

	ρ	0.65			0.75			0.85		
	r	4	5	6	4	5	6	4	5	6
m=3	RE_1	0.8241	0.8069	0.7939	0.7565	0.8096	0.7579	0.6796	0.6456	0.6569
	RE_2	1.1115	1.0788	1.1364	1.3532	1.4352	1.3854	2.2423	2.2153	2.2679
	RE_3	1.1149	1.0810	1.1391	1.3560	1.4371	1.3881	2.2455	2.2176	2.2714
	RE_4	1.2152	1.1996	1.2233	1.4735	1.4712	1.4401	**2.3198**	2.3060	2.3012
	RE_5	**1.3577**	**1.3547**	**1.3696**	1.5276	1.5123	1.4948	1.9699	1.9870	1.9792
	RE_6	1.2920	1.3253	1.3630	**1.5354**	**1.5575**	**1.5460**	2.2879	**2.3623**	**2.3833**
m=4	RE_1	0.7577	0.7673	0.7523	0.7072	0.7255	0.6873	0.5825	0.5786	0.6032
	RE_2	1.0532	1.0812	1.0629	1.2860	1.3055	1.2686	1.9288	1.9409	2.1118
	RE_3	1.0558	1.0833	1.0641	1.2892	1.3077	1.2699	1.9314	1.9417	2.1122
	RE_4	1.1626	1.2125	1.1538	1.4000	1.3727	1.3772	2.0801	1.9895	2.1478
	RE_5	**1.2835**	**1.3156**	**1.2862**	1.4395	1.4248	1.4365	1.8560	1.7822	1.8396
	RE_6	1.2547	1.3091	1.2802	**1.4575**	**1.4593**	**1.4779**	**2.1006**	**2.0430**	**2.1965**

m=5	RE_1	0.7246	0.7265	0.7095	0.6547	0.6679	0.6741	0.5553	0.5468	0.5613
	RE_2	0.9886	1.0077	1.0457	1.2252	1.1959	1.2374	1.8867	1.8669	1.9012
	RE_3	0.9903	1.0090	1.0474	1.2267	1.1969	1.2388	1.8881	1.8675	1.9021
	RE_4	1.1519	1.0967	1.1319	1.3053	1.3126	1.3196	1.9105	1.9331	1.9082
	RE_5	**1.2561**	**1.2214**	**1.2369**	1.3473	1.3589	1.3612	1.7064	1.7109	1.7091
	RE_6	1.2363	1.2045	1.2343	**1.3653**	**1.3884**	**1.3992**	**1.9265**	**1.9677**	**1.9622**

It is shown that the estimator suggested in Tables 1 and 2 gave better results with the normal distribution than the skewed beta distribution in the simulation study performance. In general, it can be said that the RE value of the $\hat{\mu}_6$ estimator is directly related to the correlation and the number of cycles. As can be seen from Table 1, the proposed estimator is that the best under the normal distribution, however within the skewed beta distribution, from Table 2, the efficiency of the proposed estimator will increase once the number of repetitions and correlation increase when the set size is smaller.

6. Real Data

We use data of Kocyigit and Kadilar (2020) in this section. Real data set includes confirmed cases (as a study variable) and suspected cases (as a auxiliary variable) of COVID-19 acute respiratory disease reported by 34 provinces, regions and cities in China in 2020.

Table 3. RE values of real data set (N=34, ρ =0.842)

m	2			3	
r	1	2	3	1	2
RE_1	1.0226	1.0618	1.1645	1.0291	1.1340
RE_1	-	-	-	-	-
RE_2	-	-	-	-	-
RE_3	-	-	-	-	-
RE_4	4.2763	3.1442	2.6199	3.4889	2.5318
RE_5	**4.4109**	**4.8581**	**3.6101**	**4.9620**	**4.7612**
RE_6	1.0226	1.0618	1.1645	1.0291	1.1340

From Table 3, similar to the results in Kocyigit and Kadilar (2020), the classical ratio estimators could not be calculated because of the zero values. Although the distribution of the real data set was not normal or symmetrical distributed, the efficiency of the proposed estimator gave the best results thanks to the exponential part in proposed estimator.

7. Conclusion

In this study, we propose a new estimator using the RSS technique. The proposed estimator is compared with the present estimators conjointly the numerical comparison is supported with the simulation and real data study. In line with the obtained results, the proposed estimator can be used for estimating the population mean under the RSS method.

With this study, the most recent suggested mean estimator for non-response in SRS has been adjoined to the RSS estimators. Since the proposed estimator can be better than the other estimators under certain conditions, a simulation study was carried out considering the non-symmetrical population, which is one of the most common situations in practice. It has been observed that the proposed estimator is superior to all other estimators under the normal distribution, but its performance is lower in the case of low correlation in the skewed beta distribution. In future studies, this estimator can be modified to ensure that it always gives good results even with skewed distributions.

Since there are situations where ratio estimators cannot be calculated in some real data studies in the literature, the proposed estimator was also calculated on a real data set and it was seen that it gave better results than other estimators.

This study also shows that the RSS method can be utilized in the ratio estimator for the population mean and using the RSS method is improved the efficiency of ratio estimators. Therefore, we hope to develop a new ratio estimator using a stratified ranked set sampling method in the forthcoming studies, utilizing the methods within the papers Samawi and Siam (2003) and Kadilar and Cingi (2005).

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