Shape Preserving Positive and Convex Data Visualization using Rational Bi-cubic Functions

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Abstract

This paper is concerned with the problem of positive and convex data visualization in the form of positive and convex surfaces. A rational bi-cubic partially blended function with eight free parameters in its description is applied to visualize the shape of positive surface data and convex surface data. The developed schemes in this paper have unique representations. Visual models of surfaces attained C^1smoothness.

Keywords: Data visualization, Rational cubic function, Rational bi-cubic partially blended function, Positive surface, Convex surface.

1. Introduction

Splines are basic tools for the visualization of shape preserving interpolating surface data. Although ordinary splines smoothly interpolate the given data points but do not fulfils the hidden shape properties of the data. Dealing with the different shape properties (monotone, positive and convex) of curves and surfaces data visualization has significant role. In this paper we concentrate on the visualization of positive surface data, and convex surface data. There are many fields where the entities only have meaning when their values are positive. For instance, in probability distributions, the representation of data is always positive. Similarly, when we deal with the samples of populations, the data are always in positive figures. Convexity is also an important shape feature and plays a significant role in nonlinear programming which occurs in engineering, scientific applications such as design, optimal control, parameter estimation and approximation of function. One can observe that in Figure 1, bi-cubic Hermite function interpolates the positive surface data points but inherent shape feature (i.e. positivity) is missing. Similarly in Figure 7, bi-cubic Hermite function interpolates the convex surface data points but inherent shape feature (i.e. convexity) is loses.

In recent years; researchers have published a significant number of articles in the field of shape persevering data visualization of curve and surface data. Asim and Brodlie (2003) discussed the problem of drawing positive curve through positive data. A piecewise cubic Hermite interpolation is used to fit a positive curve. In any interval where the positivity is lost authors added extra knots to cubic Hermiteinterpolant to obtain desire positive curve. Brodlie et al. (2005) adopted Modified Quadratic Shepard (MQS) method for the interpolation of scattered data of any dimension. They restricted the range of interpolating function between 0 and 1. Positivity of positive scattered data is achieved by MQS method but the graphical results are not visually pleasant. Brodlie et al. (1995) presented an algorithm for the visualization of positive surface data. They derived sufficient conditions in terms of first partial derivatives and mixed partial derivatives at the grid points. They also generalized the case of linearly constrained interpolation. Casciola and Romani (2003) constructed NURBS with tension parameters to control the shape of interpolatory surfaces. They presented some techniques to reconstruct the shape preserving bivariate NURBS, in which shapes of resulting surfaces can be modified by changing the values of tension parameters. Duan et al. (2006) constructed a bivariate rational interpolating function based on the function values and partial derivatives. They attached six parameters in the description of bivariate interpolating function to keep the interpolating surface in the original shape. Fangxum et al. (2010) presented an explicit expression of a weighted blending interpolator based on the function values. In this paper positive parameters and weight coefficients are freely selected according to the needs of practical design and the interpolator is C^1 in the whole interpolating region. Fujisawa et al. (2008) briefly described the implementation and visualization through some examples of applications to scientific arts such as archeology, sculpture, fine arts and information aesthetics. Hussain et al. (2007) and Hussain et al. (2008) discussed visualization 3D convex data. In the first paper, they used rational bi-cubic function while in second they used rational bi-quartic function to visualize convex data. They imposed conditions on free parameters in the description of rational functions to visualize convex data.

Kouibia and Pasadas (2008) presented an approximation problem of parametric curves and surfaces from the Lagrange or Hermite data set. They also discussed the interpolation problem of minimizing some functional on a Sobolev space that produced the new notion of interpolating variational spline. Piah et al. (2005) presented two algorithms for the positivity preservation of scattered data. The interpolating surface comprises cubic Bezier triangular patches with sufficient conditions imposed on the ordinates of the Bezier control points in each triangle to guarantee preservation of positivity. Renka (2004) discussed the construction of C^1 convex surface interpolation. For this purpose Renka introduced a Fortran-77 software package. Sarfraz et al. (2010) constructed a rational cubic function with two parameters in its representation and extend the rational cubic function to rational bi-cubic partially blended function. They visualized the shape of positive data in the view of positive curves and surface by deducing constraints on parameters. Wang and Tan (2006) generated a rational bi-quartic surface by using tensor product method. They developed an algorithm to preserve the shape of monotone data by piecewise rational bi-quartic function. Zhang et al. (2007) dealt out the problem of convexity control of interpolating bivariate surfaces. Sufficient and necessary conditions are derived to obtain convex surface through convex data based on the function values.

In this paper a rational bi-cubic function is used to visualize the shaped data. Two schemes are developed in this paper; one for the visualization of positive surface data and other for the visualization of convex surface data. Important features of this paper are:

- Developed schemes are designed in such a way that no additional knots are necessary to control the shape as developed by Asim et al. (2003).

- There is no limitation on the interpolating function of lying in a specified interval as in the article of Brodlie et al. (2005).

- In this paper, constraints are derived on free parameters which provide more control to the user to control the shape of interpolating surface as compared in the article by Brodlie et al. (1995).

- Developed schemes are equally applicable for the data with derivative or without derivatives while in schemes developed by Casciola et al. (2003) andDuan et al.(2006)work if partial derivatives at the knots are known.
- The rational bi-quartic function is used by Hussain et al. (2008)to preserve the shape of convex surface data while in this paper we used rational bi-cubic function to visualize the convex surface data.

- Developed schemes are useful for both equally spaced and unequally spaced data while Wang & Tan in 2006derived results for equally spaced data.
- Sarfraz et al. (2010)claimed that positive surfaces generated by their schemes are C^1 continuous and smooth but the visual models didn't depict that, while the surfaces generated by the schemes developed in this paper are visually smooth, C^1 continuous.
- Generated surfaces are unique in their representation.

This paper is directed as follows. Section 2 is review of the research article written by Hussain et al. (2011) and the extension of rational cubic function to the rational bicubic function is described in Section 3. Section 4 and Section 5 deal with the task of visualization of positive surface data and convex surface data respectively. Section 6 concludes the paper.

2. Rational Cubic Function

Let $\{(x_i, f_i),\ i=1,2,3,\dots,n\}$ be given set of data points where $x_1 < x_2 < \dots < x_n$. In each interval $[x_i, x_{i+1}]$, the rational cubic function $S(x)$ is defined as:

$$S(x) \equiv S(x_i) = \frac{\mu_i U_i (1-\theta)^3 + W_i \theta (1-\theta)^2 + T_i \theta^2 (1-\theta) + \upsilon_i V_i \theta^3}{\mu_i (1-\theta)^2 + (\mu_i + \upsilon_i)\theta(1-\theta) + \upsilon_i \theta^2} \tag{1}$$

where

$U_i = f_i$,

$W_i = \mu_i h_i d_i + (2\mu_i + \upsilon_i) f_i$,

$T_i = -\upsilon_i h_i d_{i+1} + (\mu_i + 2\upsilon_i) f_{i+1}$,

$V_i = f_{i+1}$,

and $\theta = \dfrac{x - x_i}{h_i}$, $h_i = x_{i+1} - x_i$, $\quad \forall\ i = 1,2,3,\dots,n-1$.

The rational cubic function (1) has following properties:

$$\left.\begin{aligned} S(x_i) &= f_i, & S(x_{i+1}) &= f_{i+1}, \\ S^{(1)}(x_i) &= d_i, & S^{(1)}(x_{i+1}) &= d_{i+1} \end{aligned}\right\} \tag{2}$$

where $S^{(1)}(x_i)$ denote the first order derivative with respect to x and d_i denotes derivative value at the knot x_i.

Remark: It is observed that when $\mu_i = \upsilon_i = 1$ the rational cubic function (1) reduces to the standard cubic Hermite polynomial $S(x) \in C^1[x_1, x_n]$.

Theorem 1: The rational cubic function (1) is positive in each interval $[x_i, x_{i+1}]$ if the shape parameters μ_i, υ_i satisfy the following constraints:

$\upsilon_i > 0$,

$$\mu_i = \kappa_i + \max\left\{0, \frac{-\upsilon_i f_i}{2f_i + h_i d_i}, \frac{(h_i d_{i+1} - 2f_i)\upsilon_i}{f_{i+1}}\right\}, \qquad \kappa_i > 0.$$

Theorem 2: The rational cubic function (1) is convex in each interval $[x_i, x_{i+1}]$ if the shape parameters μ_i, υ_i satisfy the following constraints:

$\upsilon_i > 0$,

$$\mu_i = \delta_i + \max\left\{0, \frac{(d_{i+1} - d_i)\upsilon_i}{(\Delta_i - d_i)}, \frac{(\Delta_i - d_{i+1})\upsilon_i}{(d_i - \Delta_i)}\right\}, \quad \delta_i > 0.$$

3. Rational Bi-cubic Function

A piecewise rational cubic function (1) is extended to rational bi-cubic partially blended function $S(x,y)$ over the rectangular region $\Omega=[a,b]\times[c,d]$. Let $\pi: a=x_0<x_1<...<x_n=b$ be partition of $[a,b]$ and $\tilde{\pi}: c=y_0<y_1<...<y_m=d$ be the partition of $[c,d]$, the rational bi-cubic partially blended function is defined over each rectangular patch $[x_i,x_{i+1}]\times[y_j,y_{j+1}]$, $i=0,1,2,...,n-1;\ j=0,1,2,...,m-1$ as:

$$S(x,y)=-PFQ^T \tag{3}$$

where

$$F=\begin{pmatrix} 0 & S(x,y_j) & S(x,y_{j+1}) \\ S(x_i,y) & S(x_i,y_j) & S(x_i,y_{j+1}) \\ S(x_{i+1},y) & S(x_{i+1},y_j) & S(x_{i+1},y_{j+1}) \end{pmatrix},$$

$$P=\begin{bmatrix}-1 & p_0(\theta) & p_1(\theta)\end{bmatrix},\ Q=\begin{bmatrix}-1 & q_0(\phi) & q_1(\phi)\end{bmatrix},$$

with $p_0=(1-\theta)^2(1+2\theta)$, $p_1=\theta^3(3-2\theta)$, $q_0=(1-\phi)^2(1+2\phi)$, $q_1=\phi^2(3-2\phi)$.
$\theta=(x-x_i)/h_i$, $h_i=x_{i+1}-x_i$, $\phi=(y-y_j)/\hat{h}_j$, $\hat{h}_j=y_{j+1}-y_j$, $0\le\theta,\phi\le 1$.

$S(x,y_j)$, $S(x,y_{j+1})$, $S(x_i,y)$ and $S(x_{i+1},y)$ are rational cubic function (1) defined over the boundary of rectangular patch $[x_i,x_{i+1}]\times[y_j,y_{j+1}]$ as:

$$S(x,y_j)=\frac{\sum_{i=1}^{4} A_{i,j}(1-\theta)^{4-i}\theta^{i-1}}{q_{i,j}(\theta)} \tag{4}$$

with

$$A_{1,j}=\mu_{i,j}^2 F_{i,j},$$
$$A_{2,j}=(2\mu_{i,j}+\upsilon_{i,j})F_{i,j}+\mu_{i,j}h_i F_{i,j}^x,$$
$$A_{3,j}=(\mu_{i,j}+2\upsilon_{i,j})F_{i+1,j}-\upsilon_{i,j}h_i F_{i+1,j}^x,$$
$$A_{4,j}=\upsilon_{i,j}^2 F_{i+1,j},$$
$$q_{i,j}(\theta)=\mu_{i,j}(1-\theta)^2+(\mu_{i,j}+\upsilon_{i,j})\theta(1-\theta)+\upsilon_{i,j}\theta^2,$$

$$S(x,y_{j+1})=\frac{\sum_{i=1}^{4} B_{i,j}(1-\theta)^{4-i}\theta^{i-1}}{q_{i,j+1}(\theta)} \tag{5}$$

with

$$B_{1,j}=\mu_{i,j+1}^2 F_{i,j+1},$$

$$B_{2,j} = \left(2\mu_{i,j+1} + \upsilon_{i,j+1}\right) F_{i,j+1} + \mu_{i,j+1} h_i F^x_{i,j+1},$$
$$B_{3,j} = \left(\mu_{i,j+1} + 2\upsilon_{i,j+1}\right) F_{i+1,j+1} - \upsilon_{i,j+1} h_i F^x_{i+1,j+1},$$
$$B_{4,j} = \upsilon^2_{i,j+1} F_{i+1,j},$$
$$q_{i,j+1}(\theta) = \mu_{i,j+1}(1-\theta)^2 + \left(\mu_{i,j+1} + \upsilon_{i,j+1}\right)\theta(1-\theta) + \upsilon_{i,j+1}\theta^2,$$

$$S(x_i, y) = \frac{\sum_{i=1}^{4} C_{i,j} (1-\phi)^{4-i} \phi^{i-1}}{\hat{q}_{i,j}(\phi)} \tag{6}$$

with

$$C_{1,j} = \hat{\mu}^2_{i,j} F_{i,j},$$
$$C_{2,j} = \left(2\hat{\mu}_{i,j} + \hat{\upsilon}_{i,j}\right) F_{i,j} + \hat{\mu}_{i,j} \hat{h}_j F^y_{i,j},$$
$$C_{3,j} = \left(\hat{\mu}_{i,j} + 2\hat{\upsilon}_{i,j}\right) F_{i,j+1} - \hat{\upsilon}_{i,j} \hat{h}_j F^y_{i,j+1},$$
$$C_{4,j} = \hat{\upsilon}^2_{i,j} F_{i,j+1},$$
$$\hat{q}_{i,j}(\phi) = \hat{\mu}_{i,j}(1-\phi)^2 + \left(\hat{\mu}_{i,j} + \hat{\upsilon}_{i,j}\right)\phi(1-\phi) + \hat{\upsilon}_{i,j}\phi^2,$$

$$S(x_{i+1}, y) = \frac{\sum_{i=1}^{4} D_{i,j} (1-\phi)^{4-i} \phi^{i-1}}{\hat{q}_{i+1,j}(\phi)} \tag{7}$$

with

$$D_{1,j} = \hat{\mu}^2_{i+1,j} F_{i,j},$$
$$D_{2,j} = \left(2\hat{\mu}_{i+1,j} + \hat{\upsilon}_{i+1,j}\right) F_{i+1,j} + \hat{\mu}_{i+1,j} \hat{h}_j F^y_{i+1,j},$$
$$D_{3,j} = \left(\hat{\mu}_{i+1,j} + 2\hat{\upsilon}_{i+1,j}\right) F_{i+1,j+1} - \hat{\upsilon}_{i+1,j} \hat{h}_j F^y_{i+1,j+1},$$
$$D_{4,j} = \hat{\upsilon}^2_{i+1,j} F_{i+1,j+1},$$
$$\hat{q}_{i+1,j}(\phi) = \hat{\mu}_{i+1,j}(1-\phi)^2 + \left(\hat{\mu}_{i+1,j} + \hat{\upsilon}_{i+1,j}\right)\phi(1-\phi) + \hat{\upsilon}_{i+1,j}\phi^2.$$

3.1. Choice of Derivatives

For most of the applications, the derivative parameters are unknown. There are many methods for the approximation of these derivative parameters. In this article they are calculated by Arithmetic Mean Method. The description of this method is as follow:

Let $F^x_{i,j}$ and $F^y_{i,j}$ denote the first derivative with respect to x and y respectively and $F^{xy}_{i,j}$ be the mixed derivative at the data point $F_{i,j}$.

$$F^x_{0,j} = \hat{\Delta}_{0,j} + \frac{\left(\hat{\Delta}_{0,j} - \hat{\Delta}_{1,j}\right) h_0}{\left(h_0 + h_1\right)},$$

$$F^x_{m,j} = \hat{\Delta}_{m-1,j} + \frac{\left(\hat{\Delta}_{m-1,j} - \hat{\Delta}_{m-2,j}\right)h_{m-1}}{h_{m-1} + h_{m-2}},$$

$$F^x_{i,j} = \frac{\Delta_{i,j} - \Delta_{i-1,j}}{2},\ i = 1,2,...,m-1;\ j = 0,1,2,...,n$$

$$F^y_{i,n} = \hat{\Delta}_{i,n-1} + \frac{\left(\hat{\Delta}_{i,n-1} - \hat{\Delta}_{i,n-2}\right)\hat{h}_{n-1}}{\hat{h}_{n-1} + \hat{h}_{n-2}},$$

$$F^y_{i,j} = \frac{\hat{\Delta}_{i,j} - \hat{\Delta}_{i,j-1}}{2}, \qquad i = 0,1,2,...,m;\ j = 1,2,...,n-1$$

$$F^{xy}_{i,j} = \frac{1}{2}\left\{\frac{F^x_{i,j+1} - F^x_{i,j-1}}{\hat{h}_{j-1} + \hat{h}_j} + \frac{F^x_{i+1,j} - F^x_{i-1,j}}{h_{i-1} + h_i}\right\}, \qquad i = 1,2,...,m-1;\ j = 1,2,...,n-1$$

$$\Delta_{i,j} = \frac{F_{i+1,j} - F_{i,j}}{h_i},\ \hat{\Delta}_{i,j} = \frac{F_{i,j+1} - F_{i,j}}{\hat{h}_j}.$$

4. Positive Rational Bi-cubic Function

Let $\left\{\left(x_i, y_j, F_{i,j}\right): i = 0,1,2,...,n\ ;\ j = 0,1,2,...,m\right\}$ be the set of positive data points defined over rectangular grid $I_{i,j} = \left[x_i, x_{i+1}\right] \times \left[y_j, y_{j+1}\right],\ i = 0,1,2,...,n-1$; $j = 0,1,2,...,m-1$.

Casciola et al. (2003) derived the following result:

Bi-cubic partially blended surface patch inherits all the properties of network of boundary curves.

According to this result bi-cubic partially blended surface patch defined in (3) will be positive in each rectangular patch $I_{i,j} = \left[x_i, x_{i+1}\right] \times \left[y_j, y_{j+1}\right]$, if each of the boundaries curve $S\left(x, y_j\right)$, $S\left(x, y_{j+1}\right)$, $S\left(x_i, y\right)$ and $S\left(x_{i+1}, y\right)$, is positive i.e.

Now $S\left(x, y_j\right) > 0$ if

$\upsilon_{i,j} > 0$, $\mu_{i,j} > 0$, $\mu_{i,j} > \dfrac{-\upsilon_{i,j}F_{i,j}}{2F_{i,j} + h_i F^x_{i,j}}$ and $\mu_{i,j} > \dfrac{\left(h_i F^x_{i+1,j} - 2F_{i,j}\right)\upsilon_{i,j}}{F_{i+1,j}}$

$S\left(x, y_{j+1}\right) > 0$ if

$\upsilon_{i,j+1} > 0$, $\mu_{i,j+1} > 0$, $\mu_{i,j+1} > \dfrac{-\upsilon_{i,j+1}F_{i,j+1}}{2F_{i,j+1} + h_i F^x_{i,j+1}}$ and $\mu_{i,j+1} > \dfrac{\left(h_i F^x_{i+1,j+1} - 2F_{i,j+1}\right)\upsilon_{i,j+1}}{F_{i+1,j+1}}$

$S\left(x_i, y\right) > 0$ if

$\hat{\upsilon}_{i,j} > 0$, $\hat{\mu}_{i,j} > 0$, $\hat{\mu}_{i,j} > \dfrac{-\hat{\upsilon}_{i,j} F_{i,j}}{2F_{i,j} + \hat{h}_j F^y_{i,j}}$ and $\hat{\mu}_{i,j} > \dfrac{\left(\hat{h}_j F^y_{i,j+1} - 2F_{i,j}\right)\hat{\upsilon}_{i,j}}{F_{i,j+1}}$

$S\left(x_{i+1}, y\right) > 0$ if

$\hat{\upsilon}_{i+1,j} > 0$, $\hat{\mu}_{i+1,j} > 0$, $\hat{\mu}_{i+1,j} > \dfrac{-\hat{\upsilon}_{i+1,j} F_{i+1,j}}{2F_{i+1,j} + \hat{h}_j F^y_{i+1,j}}$ and $\hat{\mu}_{i+1,j} > \dfrac{\left(\hat{h}_j F^y_{i+1,j+1} - 2F_{i+1,j}\right)\hat{\upsilon}_{i+1,j}}{F_{i+1,j+1}}$.

All the above discussion can be summarized as:

Theorem 3: The rational bi-cubic partially blended function defined in (3) visualizes positive surface data in the view of positive surface in each rectangular patch $I_{i,j} = \left[x_i, x_{i+1}\right] \times \left[y_j, y_{j+1}\right]$, if the shape parameters $\mu_{i,j}, \upsilon_{i,j}, \mu_{i,j+1}, \upsilon_{i,j+1}, \hat{\mu}_{i,j}, \hat{\upsilon}_{i,j}, \hat{\mu}_{i+1,j}$ and $\hat{\upsilon}_{i+1,j}$ satisfy the following conditions:

$\upsilon_{i,j} > 0$, $\upsilon_{i,j+1} > 0$, $\hat{\upsilon}_{i,j} > 0$, $\hat{\upsilon}_{i+1,j} > 0$,

$$\mu_{i,j} = \delta_{i,j} + \max\left\{0, \frac{-\upsilon_{i,j} F_{i,j}}{2F_{i,j} + h_i F^x_{i,j}}, \frac{\left(h_i F^x_{i+1,j} - 2F_{i,j}\right)\upsilon_{i,j}}{F_{i+1,j}}\right\}$$

$$\mu_{i,j+1} = \eta_{i,j} + \max\left\{0, \frac{-\upsilon_{i,j+1} F_{i,j+1}}{2F_{i,j+1} + h_i F^x_{i,j+1}}, \frac{\left(h_i F^x_{i+1,j+1} - 2F_{i,j+1}\right)\upsilon_{i,j+1}}{F_{i+1,j+1}}\right\}$$

$$\hat{\mu}_{i,j} = \hat{\delta}_{i,j} + \max\left\{0, \frac{-\hat{\upsilon}_{i,j} F_{i,j}}{2F_{i,j} + \hat{h}_j F^y_{i,j}}, \frac{\left(\hat{h}_j F^y_{i,j+1} - 2F_{i,j}\right)\hat{\upsilon}_{i,j}}{F_{i,j+1}}\right\}$$

$$\hat{\mu}_{i+1,j} = \hat{\eta}_{i,j} + \max\left\{0, \frac{-\hat{\upsilon}_{i+1,j} F_{i+1,j}}{2F_{i+1,j} + \hat{h}_j F^y_{i+1,j}}, \frac{\left(\hat{h}_j F^y_{i+1,j+1} - 2F_{i+1,j}\right)\hat{\upsilon}_{i+1,j}}{F_{i+1,j+1}}\right\},$$

where $\delta_{i,j}, \eta_{i,j}, \hat{\delta}_{i,j}, \hat{\eta}_{i,j} > 0$.

Algorithm 1

Step 1. Enter the $(n+1)\times(m+1)$ positive data points$\left(x_i, y_j, F_{i,j}\right), i = 0,1,2,...,n;$ $j = 0,1,2,...,m.$

Step 2. Estimate the derivatives $F^x_{i,j}, F^y_{i,j}$ and $F^{xy}_{i,j}$ at knots.

Step 3. Calculate the values of shape parameters $\mu_{i,j}, \upsilon_{i,j}, \mu_{i,j+1}, \upsilon_{i,j+1}, \hat{\mu}_{i,j}, \hat{\upsilon}_{i,j}, \hat{\mu}_{i+1,j}$ and $\hat{\upsilon}_{i+1,j}$ using Theorem 3.

Step 4. Substitute the values of $F_{i,j}, F^x_{i,j}, F^y_{i,j}, F^{xy}_{i,j}$, $i = 0,1,2,...,n;$ $j = 0,1,2,...,m$ and $\mu_{i,j}, \upsilon_{i,j}, \mu_{i,j+1}, \upsilon_{i,j+1}, \hat{\mu}_{i,j}, \hat{\upsilon}_{i,j}, \hat{\mu}_{i+1,j}$ and $\hat{\upsilon}_{i+1,j}$ $i = 0,1,2,...,n-1;$ $j = 0,1,2,...,m-1$ in rational bi-cubic function (3) to obtain positive rational bi-cubic function.

4.1. Demonstration:

A positive data set is taken in Table 1, which is generated by the function $F_1(x,y)=0.0012+\dfrac{1.25+\cos(5.4y)}{6+(3x-1)^2}$, correct to four decimal places defined over the rectangular grid $[1,300]\times[1,300]$.

Figure 1 is generated using bi-cubic Hermite function, which interpolates the data points but did not visualize the shape of data. Figure 2 and Figure 3 are the yz-view and xz- view of Figure 1 respectively. Figure 4 is generated by the positive rational bi-cubic function developed in Section 4 with $\upsilon_{i,j}=\upsilon_{i,j+1}=\hat{\upsilon}_{i,j}=\hat{\upsilon}_{i+1,j}=0.5$. Figure 5 and Figure 6 are the yz-view and xz-view of Figure 4 respectively. It is noted that Figure 4 preserved the shape of data taken in Table 1.

Table 1:

y_j/x_i	1	100	200	300
1	0.1897	0.0012	0.0012	0.0012
100	0.2200	0.0012	0.0012	0.0012
200	0.2022	0.0012	0.0012	0.0012
300	0.1749	0.0012	0.0012	0.0012

Table 2: Numerical results of Figure 4

(x_i, y_j)	$F_x(i,j)$	$F_y(i,j)$	$\mu_{i,j}$	$\mu_{i,j+1}$	$\hat{\mu}_{i,j}$	$\hat{\mu}_{i+1,j}$
(1,1)	-0.0029	0.5475×10^{-3}	0.0025	0.0030	0.0040	0.0050
(1,100)	-0.0033	0.0641×10^{-3}	0.0025	0.0030	0.0040	0.0050
(1,200)	-0.0030	-0.2254×10^{-3}	0.0025	0.0030	0.0040	0.0050
(1,300)	-0.0026	-0.3195×10^{-3}	0.0025	0.0030	0.0040	0.0050
(100,1)	-0.0010	0.0001×10^{-3}	0.0025	0.0030	0.0040	0.0050
(100,100)	-0.0011	0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(100,200)	-0.0010	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(100,300)	-0.0009	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(200,1)	-0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(200,100)	-0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(200,200)	-0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(200,300)	-0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(300,1)	0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(300,100)	0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(300,200)	0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050
(300,300)	0.0000	-0.0000×10^{-3}	0.0025	0.0030	0.0040	0.0050

Graphical Comparison between Bi-cubic Hermite Function and Rational Bi-cubic Function

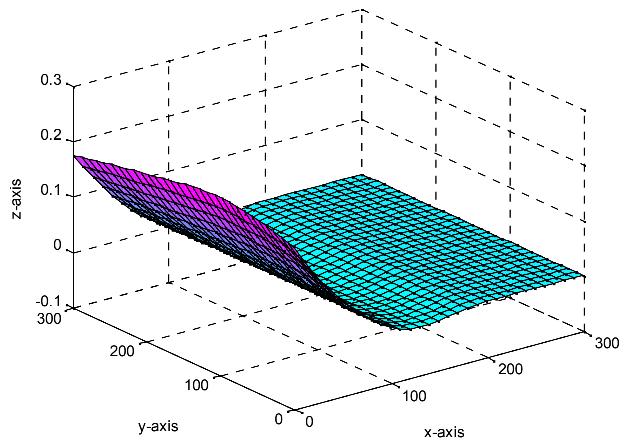
<table>
<tr><th>Bi-cubic Hermite Function</th><th>Rational Bi-cubic Function</th></tr>
<tr><td>

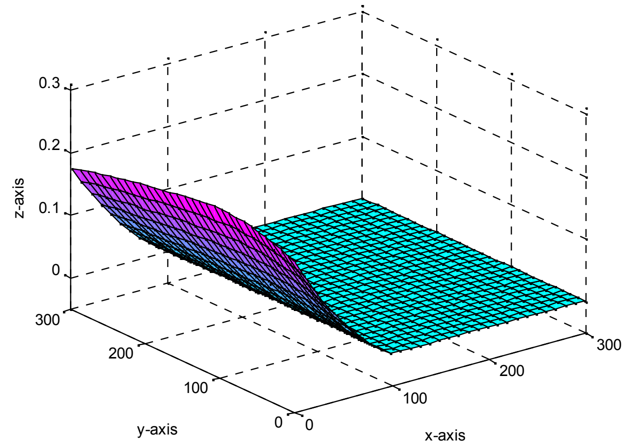
Figure 1</td><td>

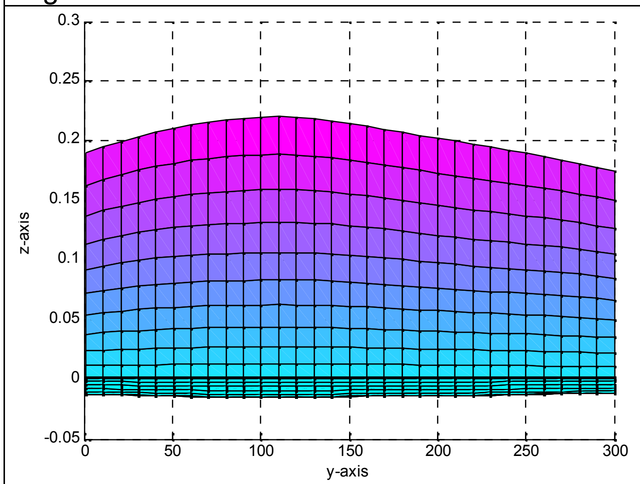
Figure 4</td></tr>
<tr><td>

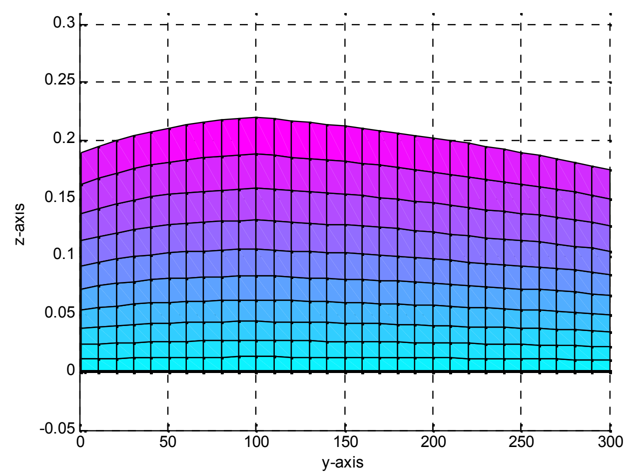
Figure 2: yz-view of Figure 1</td><td>

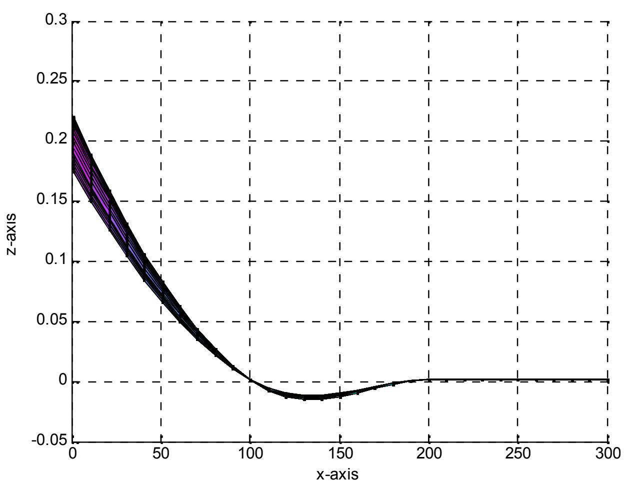
Figure 5: yz-view of Figure 4</td></tr>
<tr><td>

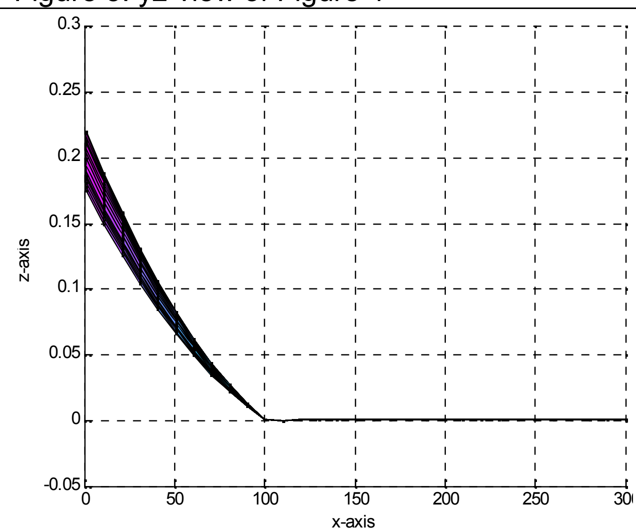
Figure 3: xz-view of Figure 1</td><td>

Figure 6: xz-view of Figure 4</td></tr>
</table>

5. Convex Rational Bi-cubic Function

Let $\left\{\left(x_i, y_j, F_{i,j}\right): i=0,1,2,...,n\ ;\ j=0,1,2,...,m\right\}$ be the collection of convex data points defined over rectangular grid $I_{i,j}=\left[x_i, x_{i+1}\right]\times\left[y_j, y_{j+1}\right],\ i=0,1,2,...,n-1;$ $j=0,1,2,...,m-1$. The data will be convex if it satisfies the following necessary conditions:

$F_{i,j} < F_{i,j+1}$, $\Delta_{i,j} > 0$, $F_{i,j} < F_{i+1,j}$, $\hat{\Delta}_{i,j} > 0$, $F^x_{i,j} > 0$, $F^y_{i,j} > 0$,
$F^x_{i,j} < \Delta_{i,j} < F^x_{i+1,j}$, $F^y_{i,j} < \hat{\Delta}_{i,j} < F^y_{i,j+1}$, $F^x_{i,j} < F^x_{i+1,j}$, $F^y_{i,j} < F^y_{i,j+1}$, $\Delta_{i,j} < \Delta_{i+1,j}$, $\hat{\Delta}_{i,j} < \hat{\Delta}_{i,j+1}$, $\forall\ i,\ j$

where

$$\Delta_{i,j} = \frac{F_{i+1,j} - F_{i,j}}{h_i},\quad \hat{\Delta}_{i,j} = \frac{F_{i,j+1} - F_{i,j}}{\hat{h}_j}.$$

The bi-cubic partially blended surface patch defined in (3) will be convex in each rectangular patch $I_{i,j}=\left[x_i, x_{i+1}\right]\times\left[y_j, y_{j+1}\right]$, if each of the boundaries curve $S\left(x, y_j\right)$, $S\left(x, y_{j+1}\right)$, $S\left(x_i, y\right)$ and $S\left(x_{i+1}, y\right)$ is convex i.e.

$S^{(2)}\left(x, y_j\right) > 0$, $S^{(2)}\left(x, y_{j+1}\right) > 0$, $S^{(2)}\left(x_i, y\right) > 0$ and $S^{(2)}\left(x_{i+1}, y\right) > 0\ \forall\ i, j$

The second ordered derivatives of the boundaries curves $S\left(x, y_j\right)$, $S\left(x, y_{j+1}\right)$, $S\left(x_i, y\right)$ and $S\left(x_{i+1}, y\right)$ are as follows:

$$S^{(2)}\left(x, y_j\right) = \frac{\sum_{i=1}^{6} T_{i,j}\theta^{i-1}\left(1-\theta\right)^{6-i}}{\left(q_{i,j}\left(\theta\right)\right)^3} \qquad (8)$$

where

$$T_{1,j} = 2\mu^2_{i,j}\left\{\left(\mu_{i,j}+\upsilon_{i,j}\right)\left(\Delta_{i,j}-F^x_{i,j}\right)-\upsilon_{i,j}\left(F^x_{i+1,j}-\Delta_{i,j}\right)\right\}/h_i$$
$$T_{2,j} = 2\upsilon^2_{i,j}\left\{\left(2\mu_{i,j}+5\upsilon_{i,j}\right)\left(\Delta_{i,j}-F^x_{i,j}\right)-2\upsilon_{i,j}\left(F^x_{i+1,j}-\Delta_{i,j}\right)\right\}/h_i$$
$$T_{3,j} = 2\mu_{i,j}\left\{\mu_{i,j}\left(\mu_{i,j}+7\upsilon_{i,j}\right)\left(\Delta_{i,j}-F^x_{i,j}\right)+\upsilon_{i,j}\left(F^x_{i+1,j}-\Delta_{i,j}\right)\left(-\mu_{i,j}+3\upsilon_{i,j}\right)\right\}/h_i$$
$$T_{4,j} = 2\upsilon_{i,j}\left\{\mu_{i,j}\left(3\mu_{i,j}-\upsilon_{i,j}\right)\left(\Delta_{i,j}-F^x_{i,j}\right)+\upsilon_{i,j}\left(F^x_{i+1,j}-\Delta_{i,j}\right)\left(7\mu_{i,j}+\upsilon_{i,j}\right)\right\}/h_i$$
$$T_{5,j} = 2\mu^2_{i,j}\left\{\left(5\mu_{i,j}+2\upsilon_{i,j}\right)\left(F^x_{i+1,j}-\Delta_{i,j}\right)-2\upsilon_{i,j}\left(\Delta_{i,j}-F^x_{i,j}\right)\right\}/h_i$$
$$T_{6,j} = 2\upsilon^2_{i,j}\left\{\left(\mu_{i,j}+\upsilon_{i,j}\right)\left(F^x_{i+1,j}-\Delta_{i,j}\right)-\mu_{i,j}\left(\Delta_{i,j}-F^x_{i,j}\right)\right\}/h_i$$

$S^{(2)}\left(x, y_j\right) > 0$, if $T_{i,j}{}'s > 0,\quad i=1,2,...,6$ and $q_{i,j}(\theta) > 0$

$q_{i,j}(\theta) > 0$ if $\mu_{i,j} > 0$, $\upsilon_{i,j} > 0$ and $T_{i,j}{}'s > 0$, if

$\mu_{i,j} > 0$, $\upsilon_{i,j} > 0$, $\mu_{i,j} > \dfrac{\left(F^x_{i+1,j} - F^x_{i,j}\right)\upsilon_{i,j}}{\left(\Delta_{i,j} - F^x_{i,j}\right)}$ and $\mu_{i,j} > \dfrac{\left(\Delta_{i,j} - F^x_{i+1,j}\right)\upsilon_{i,j}}{\left(\Delta_{i,j} - F^x_{i,j}\right)}$

$$S^{(2)}\left(x, y_{j+1}\right) = \frac{\sum_{i=1}^{6} S_{i,j}\theta^{i-1}\left(1-\theta\right)^{6-i}}{\left(q_{i,j+1}\left(\theta\right)\right)^3} \tag{9}$$

where

$$S_{1,j} = 2\mu^2_{i,j+1}\left\{\left(\mu_{i,j+1} + \upsilon_{i,j+1}\right)\left(\Delta_{i,j+1} - F^x_{i,j+1}\right) - \upsilon_{i,j+1}\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right)\right\}\Big/h_i$$

$$S_{2,j} = 2\upsilon^2_{i,j+1}\left\{\left(2\mu_{i,j+1} + 5\upsilon_{i,j+1}\right)\left(\Delta_{i,j+1} - F^x_{i,j+1}\right) - 2\upsilon_{i,j+1}\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right)\right\}\Big/h_i$$

$$S_{3,j} = 2\mu_{i,j+1}\left\{\mu_{i,j+1}\left(\mu_{i,j+1} + 7\upsilon_{i,j+1}\right)\left(\Delta_{i,j+1} - F^x_{i,j+1}\right) + \upsilon_{i,j+1}\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right)\left(-\mu_{i,j+1} + 3\upsilon_{i,j+1}\right)\right\}\Big/h_i$$

$$S_{4,j} = 2\upsilon_{i,j+1}\left\{\mu_{i,j+1}\left(3\mu_{i,j+1} - \upsilon_{i,j+1}\right)\left(\Delta_{i,j+1} - F^x_{i,j+1}\right) + \upsilon_{i,j+1}\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right)\left(7\mu_{i,j+1} + \upsilon_{i,j+1}\right)\right\}\Big/h_i$$

$$S_{5,j} = 2\mu^2_{i,j+1}\left\{\left(5\mu_{i,j+1} + 2\upsilon_{i,j+1}\right)\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right) - 2\upsilon_{i,j+1}\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)\right\}\Big/h_i$$

$$S_{6,j} = 2\upsilon^2_{i,j+1}\left\{\left(\mu_{i,j+1} + \upsilon_{i,j+1}\right)\left(F^x_{i+1,j+1} - \Delta_{i,j+1}\right) - \mu_{i,j+1}\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)\right\}\Big/h_i$$

$S^{(2)}\left(x, y_{j+1}\right) > 0$, if $S_{i,j}{}'s > 0, \quad i = 1,2,\ldots,6$ and $q_{i,j+1}(\theta) > 0$

$q_{i,j+1}(\theta) > 0$ if $\mu_{i,j+1} > 0$, $\upsilon_{i,j+1} > 0$ and $S_{i,j}{}'s > 0$, if

$\mu_{i,j+1} > 0$, $\upsilon_{i,j+1} > 0$, $\mu_{i,j+1} > \dfrac{\left(F^x_{i+1,j+1} - F^x_{i,j+1}\right)\upsilon_{i,j+1}}{\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)}$ and $\mu_{i,j+1} > \dfrac{\left(\Delta_{i,j+1} - F^x_{i+1,j+1}\right)\upsilon_{i,j+1}}{\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)}$

$$S^{(2)}\left(x_i, y\right) = \frac{\sum_{i=1}^{6} U_{i,j}\phi^{i-1}\left(1-\phi\right)^{6-i}}{\left(\hat{q}_{i,j}\left(\phi\right)\right)^3} \tag{10}$$

where

$$U_{1,j} = 2\hat{\mu}^2_{i,j}\left\{\left(\hat{\mu}_{i,j} + \hat{\upsilon}_{i,j}\right)\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right) - \hat{\upsilon}_{i,j}\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right)\right\}\Big/\hat{h}_j$$

$$U_{2,j} = 2\hat{\upsilon}^2_{i,j}\left\{\left(2\hat{\mu}_{i,j} + 5\hat{\upsilon}_{i,j}\right)\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right) - 2\hat{\upsilon}_{i,j}\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right)\right\}\Big/\hat{h}_j$$

$$U_{3,j} = 2\hat{\mu}_{i,j}\left\{\hat{\mu}_{i,j}\left(\hat{\mu}_{i,j} + 7\hat{\upsilon}_{i,j}\right)\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right) + \hat{\upsilon}_{i,j}\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right)\left(-\hat{\mu}_{i,j} + 3\hat{\upsilon}_{i,j}\right)\right\}\Big/\hat{h}_j$$

$$U_{4,j} = 2\hat{\upsilon}_{i,j}\left\{\hat{\mu}_{i,j}\left(3\hat{\mu}_{i,j} - \hat{\upsilon}_{i,j}\right)\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right) + \hat{\upsilon}_{i,j}\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right)\left(7\hat{\mu}_{i,j} + \hat{\upsilon}_{i,j}\right)\right\}\Big/\hat{h}_j$$

$$U_{5,j} = 2\hat{\mu}^2_{i,j}\left\{\left(5\hat{\mu}_{i,j} + 2\hat{\upsilon}_{i,j}\right)\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right) - 2\hat{\upsilon}_{i,j}\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)\right\}\Big/\hat{h}_j$$

$$U_{6,j} = 2\hat{\upsilon}^2_{i,j}\left\{\left(\hat{\mu}_{i,j} + \hat{\upsilon}_{i,j}\right)\left(F^y_{i,j+1} - \hat{\Delta}_{i,j}\right) - \hat{\mu}_{i,j}\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)\right\}\Big/\hat{h}_j$$

$S^{(2)}\left(x_i, y\right) > 0$, if $U_{i,j}{}'s > 0, \quad i = 1,2,\ldots,6$ and $\hat{q}_{i,j}(\phi) > 0$

$\hat{q}_{i,j}(\phi) > 0$ if $\hat{\mu}_{i,j} > 0$, $\hat{\upsilon}_{i,j} > 0$ and $U_{i,j}{}'s > 0$, if

$\hat{\mu}_{i,j} > 0$, $\hat{\upsilon}_{i,j} > 0$, $\hat{\mu}_{i,j} > \frac{\left(F^y_{i,j+1} - F^y_{i,j}\right)\hat{\upsilon}_{i,j}}{\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)}$ and $\hat{\mu}_{i,j} > \frac{\left(\hat{\Delta}_{i,j} - F^y_{i,j+1}\right)\hat{\upsilon}_{i,j}}{\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)}$

$$S^{(2)}\left(x_{i+1}, y\right) = \frac{\sum_{i=1}^{6} W_{i,j} \phi^{i-1} \left(1-\phi\right)^{6-i}}{\left(\hat{q}_{i+1,j}\left(\phi\right)\right)^3} \qquad (11)$$

where

$$W_{1,j} = 2\hat{\mu}^2_{i+1,j} \left\{\left(\hat{\mu}_{i+1,j} + \hat{\upsilon}_{i+1,j}\right)\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right) - \hat{\upsilon}_{i+1,j}\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$$W_{2,j} = 2\hat{\upsilon}^2_{i+1,j} \left\{\left(2\hat{\mu}_{i+1,j} + 5\hat{\upsilon}_{i+1,j}\right)\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right) - 2\hat{\upsilon}_{i+1,j}\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$$W_{3,j} = 2\hat{\mu}_{i+1,j} \left\{\hat{\mu}_{i+1,j}\left(\hat{\mu}_{i+1,j} + 7\hat{\upsilon}_{i+1,j}\right)\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right) + \hat{\upsilon}_{i+1,j}\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right)\left(-\hat{\mu}_{i+1,j} + 3\hat{\upsilon}_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$$W_{4,j} = 2\hat{\upsilon}_{i+1,j} \left\{\hat{\mu}_{i+1,j}\left(3\hat{\mu}_{i+1,j} - \hat{\upsilon}_{i+1,j}\right)\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right) + \hat{\upsilon}_{i+1,j}\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right)\left(7\hat{\mu}_{i+1,j} + \hat{\upsilon}_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$$W_{5,j} = 2\hat{\mu}^2_{i+1,j} \left\{\left(5\hat{\mu}_{i+1,j} + 2\hat{\upsilon}_{i+1,j}\right)\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right) - 2\hat{\upsilon}_{i+1,j}\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$$W_{6,j} = 2\hat{\upsilon}^2_{i+1,j} \left\{\left(\hat{\mu}_{i+1,j} + \hat{\upsilon}_{i+1,j}\right)\left(F^y_{i+1,j+1} - \hat{\Delta}_{i+1,j}\right) - \hat{\mu}_{i+1,j}\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right)\right\} \Big/ \hat{h}_j$$

$S^{(2)}\left(x_{i+1}, y\right) > 0$, if $W_{i,j}{}^{'}s > 0, \quad i = 1, 2, ..., 6$ and $\hat{q}_{i+1,j}(\phi) > 0$

$\hat{q}_{i+1,j}(\phi) > 0$ if $\hat{\mu}_{i+1,j} > 0$, $\hat{\upsilon}_{i+1,j} > 0$ and $W_{i,j}{}^{'}s > 0,$ if

$\hat{\mu}_{i+1,j} > 0$, $\hat{\upsilon}_{i+1,j} > 0$, $\hat{\mu}_{i+1,j} > \frac{\left(F^y_{i+1,j+1} - F^y_{i+1,j}\right)\hat{\upsilon}_{i+1,j}}{\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right)}$ and $\hat{\mu}_{i+1,j} > \frac{\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j+1}\right)\hat{\upsilon}_{i+1,j}}{\left(\hat{\Delta}_{i+1,j} - F^y_{i+1,j}\right)}$.

All the above discussion can be summarized in the form of following theorem:

Theorem 4: The rational bi-cubic partially blended function defined in (3) preserves the shape of convex data in the view of convex surface, in each rectangular patch $I_{i,j} = \left[x_i, x_{i+1}\right] \times \left[y_j, y_{j+1}\right]$, if the shape parameters $\mu_{i,j}, \upsilon_{i,j}, \mu_{i,j+1}, \upsilon_{i,j+1}, \hat{\mu}_{i,j}, \hat{\upsilon}_{i,j}, \hat{\mu}_{i+1,j}$ and $\hat{\upsilon}_{i+1,j}$ satisfy the following conditions:

$\upsilon_{i,j} > 0$, $\upsilon_{i,j+1} > 0$, $\hat{\upsilon}_{i,j} > 0$, $\hat{\upsilon}_{i+1,j} > 0$,

$$\mu_{i,j} = \alpha_{i,j} + \max\left\{0, \frac{\left(F^x_{i+1,j} - F^x_{i,j}\right)\upsilon_{i,j}}{\left(\Delta_{i,j} - F^x_{i,j}\right)}, \frac{\left(\Delta_{i,j} - F^x_{i+1,j}\right)\upsilon_{i,j}}{\left(\Delta_{i,j} - F^x_{i,j}\right)}\right\}$$

$$\mu_{i,j+1} = \beta_{i,j} + \max\left\{0, \frac{\left(F^x_{i+1,j+1} - F^x_{i,j+1}\right)\upsilon_{i,j+1}}{\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)}, \frac{\left(\Delta_{i,j+1} - F^x_{i+1,j+1}\right)\upsilon_{i,j+1}}{\left(\Delta_{i,j+1} - F^x_{i,j+1}\right)}\right\}$$

$$\hat{\mu}_{i,j} = \hat{\alpha}_{i,j} + \max\left\{0, \frac{\left(F^y_{i,j+1} - F^y_{i,j}\right)\hat{\upsilon}_{i,j}}{\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)}, \frac{\left(\hat{\Delta}_{i,j} - F^y_{i,j+1}\right)\hat{\upsilon}_{i,j}}{\left(\hat{\Delta}_{i,j} - F^y_{i,j}\right)}\right\}$$

$$\hat{\mu}_{i+1,j}=\hat{\beta}_{i,j}+\max\left\{0,\ \frac{\left(F^{y}_{i+1,j+1}-F^{y}_{i+1,j}\right)\hat{\upsilon}_{i+1,j}}{\left(\hat{\Delta}_{i+1,j}-F^{y}_{i+1,j}\right)},\ \frac{\left(\hat{\Delta}_{i+1,j}-F^{y}_{i+1,j+1}\right)\hat{\upsilon}_{i+1,j}}{\left(\hat{\Delta}_{i+1,j}-F^{y}_{i+1,j}\right)}\right\},$$

where $\alpha_{i,j},\ \beta_{i,j},\ \hat{\alpha}_{i,j},\ \hat{\beta}_{i,j}>0$.

Algorithm 2:

Step 1. Enter the $(n+1)\times(m+1)$ convex data points$\left(x_i,y_j,F_{i,j}\right), i=0,1,2,...,n;$ $j=0,1,2,...,m.$

Step 2. Estimate the derivatives $F^{x}_{i,j},F^{y}_{i,j}$ and $F^{xy}_{i,j}$ at knots.

Step 3. Calculate the values of shape parameters $\mu_{i,j},\upsilon_{i,j},\mu_{i,j+1},\upsilon_{i,j+1},\hat{\mu}_{i,j},\hat{\upsilon}_{i,j},\hat{\mu}_{i+1,j}$ and $\hat{\upsilon}_{i+1,j}$ using Theorem 4.

Step 4. Substitute the values of $F_{i,j},\ F^{x}_{i,j},F^{y}_{i,j}\ ,F^{xy}_{i,j},\ \forall\, i=0,1,2,...,n;\ j=0,1,2,...,m$ and $\mu_{i,j},\ \upsilon_{i,j},\ \mu_{i,j+1},\ \upsilon_{i,j+1},\hat{\mu}_{i,j},\hat{\upsilon}_{i,j},\hat{\mu}_{i+1,j},\ \hat{\upsilon}_{i+1,j}\ \forall\ i=0,1,2,...,n-1;\ j=0,1,2,...,m-1$ in rational bi-cubic function (3) to obtain the convex rational bi-cubic function.

5.1. Demonstration:

A convex data set is taken in Table 3, which is generated by the function $F_2\left(x,y\right)=x^4+y^2$, defined over the rectangular grid $\left[-2,2\right]\times\left[-2,2\right]$.

Figure 7 is generated by using bi-cubic Hermite function, which interpolate the data points but did not preserve the shape of convex data. Figure 8 and Figure 9 are the yz-view and xz-view of Figure 7 respectively. Figure 10 is generated by the convex rational bi-cubic function developed in Section 5 with $\upsilon_{i,j}=5$, $\upsilon_{i,j+1}=4$, $\hat{\upsilon}_{i,j}=3$ and $\hat{\upsilon}_{i+1,j}=2$. Figure 11 and Figure 12 are the yz-view and xz-view of Figure 10 respectively. It is noted that Figure 10 preserved the shape of data taken in Table 2.

Table 3:

y_j/x_i	-2	-1	0	1	2
-2	20	5	4	5	20
-1	17	2	1	2	17
0	16	1	0	1	16
1	17	2	1	2	17
2	20	5	4	5	20

Graphical Comparison between Bi-cubic Hermite Function and Rational Bi-cubic Function

Bi-cubic Hermite Function

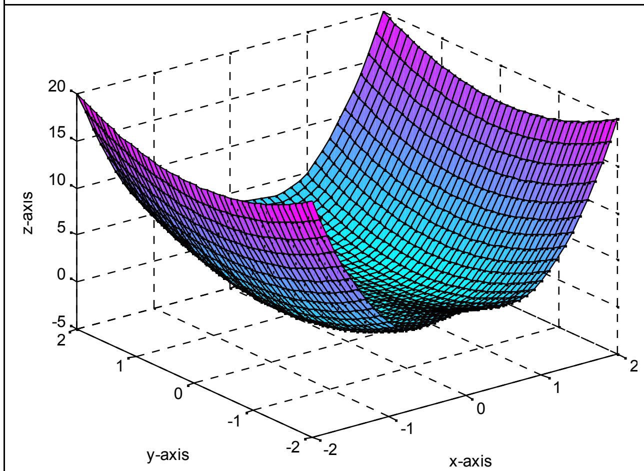


Figure 7

Rational Bi-cubic Function

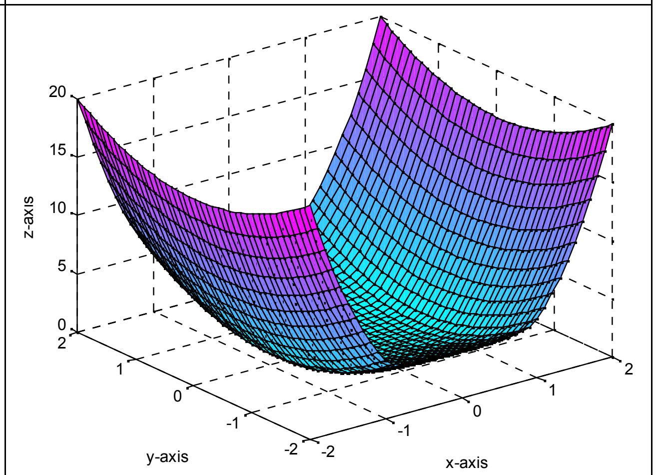


Figure 10

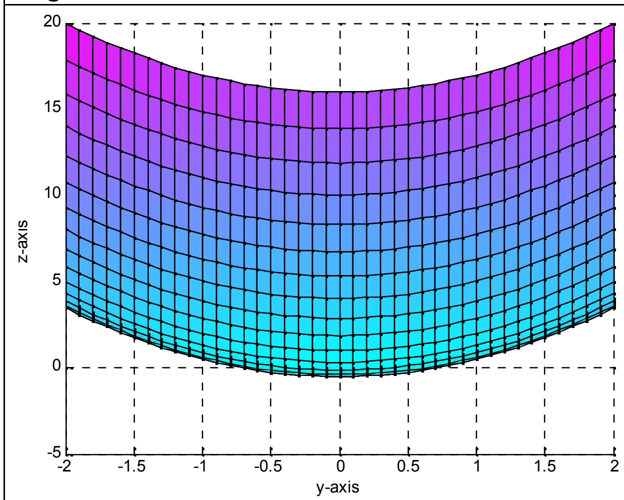


Figure 8: yz-view of Figure 7

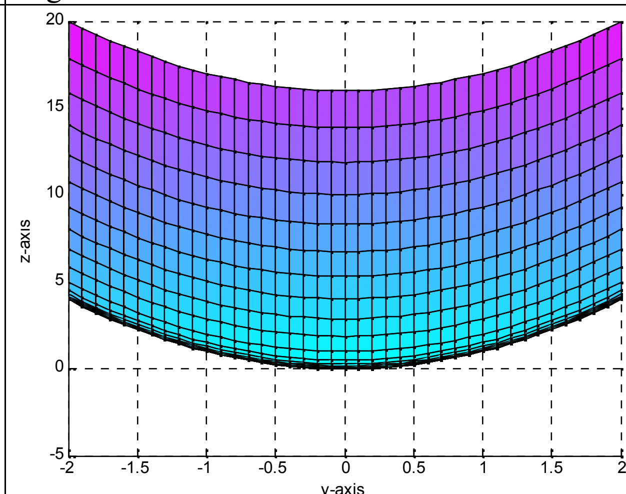


Figure 11: yz-view of Figure 10

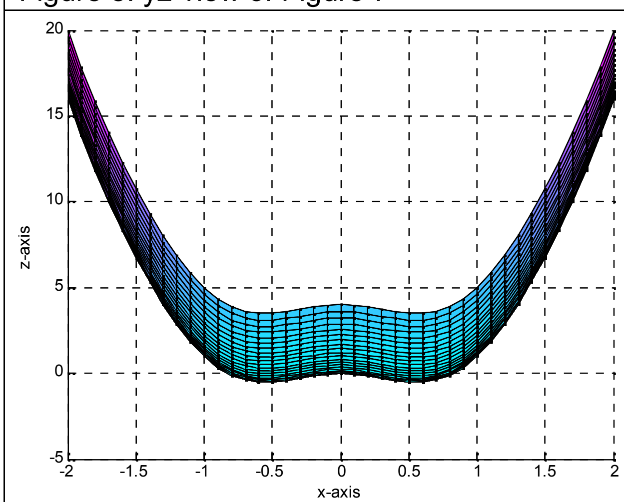


Figure 9: xz-view of Figure 7

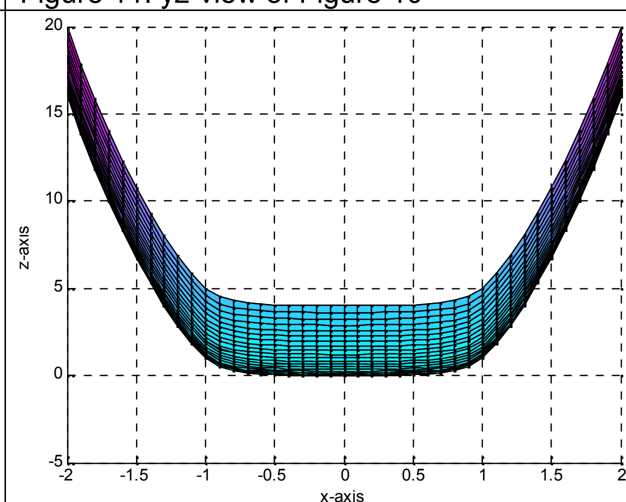


Figure 12: xz-view of Figure 10

Table 4: Numerical results of Figure 10.

(x_i, y_j)	$F_x(i, j)$	$F_y(i, j)$	$\mu_{i,j}$	$\mu_{i,j+1}$	$\hat{\mu}_{i,j}$	$\hat{\mu}_{i+1,j}$
(-2,-2)	-22	-4	5.0027	4.0032	3.0041	2.0051
(-2,-1)	-22	-2	5.0027	4.0032	3.0041	2.0051
(-2,0)	-22	0	5.0027	4.0032	3.0041	2.0051
(-2,1)	-22	2	5.0027	4.0032	3.0041	2.0051
(-2,2)	-22	4	5.0027	4.0032	3.0041	2.0051
(-1,-2)	-8	-4	5.0027	4.0032	3.0041	2.0051
(-1,-1)	-8	-2	5.0027	4.0032	3.0041	2.0051
(-1,0)	-8	0	5.0027	4.0032	3.0041	2.0051
(-1,1)	-8	2	5.0027	4.0032	3.0041	2.0051
(-1,2)	-8	4	5.0027	4.0032	3.0041	2.0051
(0,-2)	0	-4	5.0027	4.0032	3.0041	2.0051
(0,-1)	0	-2	5.0027	4.0032	3.0041	2.0051
(0,0)	0	0	5.0027	4.0032	3.0041	2.0051
(0,1)	0	2	5.0027	4.0032	3.0041	2.0051
(0,2)	0	4	5.0027	4.0032	3.0041	2.0051
(1,-2)	8	-4	5.0027	4.0032	3.0041	2.0051
(1,-1)	8	-2	5.0027	4.0032	3.0041	2.0051
(1,0)	8	0	5.0027	4.0032	3.0041	2.0051
(1,1)	8	2	5.0027	4.0032	3.0041	2.0051
(1,2)	8	4	5.0027	4.0032	3.0041	2.0051
(2,-2)	22	-4	5.0027	4.0032	3.0041	2.0051
(2,-1)	22	-2	5.0027	4.0032	3.0041	2.0051
(2,0)	22	0	5.0027	4.0032	3.0041	2.0051
(2,1)	22	2	5.0027	4.0032	3.0041	2.0051
(2,2)	22	4	5.0027	4.0032	3.0041	2.0051

6. Conclusion

A rational bi-cubic partially blended function (cubic/quadratic) is used, to preserve the shape of positive and convex surface data. Eight free parameters are attached in the description of rational bi-cubic partially blended function. Simple data dependent constraints are derived on free parameters to preserve the shape of positive and convex data in the view of positive and convex surfaces respectively. Developed schemes work for both equally and unequally spaced data. In the developed schemes there is no constraint on derivatives, they are equally useful for the data with or without derivatives. Developed schemes are local and C^1.

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