

MONITORING THE PROCESS MEAN BASED ON QUALITY CONTROL CHARTS USING ON FOLDED RANKED SET SAMPLING

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Abstract

In this paper, we proposed a new quality control chart for the sample mean based on a cost free and anti wasting sampling unit's scheme known by the folded ranked set sampling (FRSS). The new control charts were compared with the classical control charts when the data obtained by using simple random sampling (SRS) and ranked set sampling (RSS). A simulation study showed that the FRSS based control charts are a good alternative to the RSS based charts and they have smaller average run length (ARL) compared with their counterpart charts using SRS.

Keywords: Average run length; Folded ranked set sampling; Quality control charts; Ranked set sampling; Simple random sampling.

1. Introduction

Control charts are significant statistical tools that are widely used for continuously monitoring the status of a manufacturing process. The vast applications of quality control charts in engineering, pharmaceutical companies as well as other industries motivate researchers to develop several techniques to monitor the mean and the variability of the process (Muttalak and Al-Sabah, 2003; Montgomery, 2005 and Riaz and Saghir, 2009). One of the main goals of control charts is to keep the process under control and to detect undesirable changes in the process that may lead to out-of-control status. Eventually, researchers need to investigate and search for suitable measures to detect the source of variability.

In spite of the early work on quality control theory during the nineteenth century, the first concrete work was suggested by Shewhart (1924). Later, numerous articles and several books have appeared in the literature discussing the various aspects of quality control charts based on simple random samples (SRS) drawn from the population of interest (for more details see Montgomery, 2005). The noticeable work of McIntyre (1952) is considered the backbone of the ranked set sampling (RSS) theory. The idea of RSS was introduced to estimate the mean pasture yields where the data collection is not only costly or burdensome but also time consuming and hard to achieve in certain situations. Accordingly, several articles were introduced in the literature to modify the RSS in order to increase the efficiency of the

parameters estimates (Takahasi and Wakimoto, 1968; Al-Saleh and Al-Omari, 2002; Jemain and Al-Omari, 2006 and Al-Omari and Jaber, 2007).

The use of ranked set sampling (RSS) to develop control charts for monitoring the process mean was first suggested by Salazar and Sinha (1997). They showed that the new charts were substantially better than those based on the traditional SRS. Later, Muttlak and Al-Sabah (2003) developed control charts based on different RSS schemes, and showed that all these charts dominates the classical SRS control charts for means. Al-Nasser and Al-Rawwash (2007) developed the \bar{X} -bar chart using robust L ranked set sampling method (Al-Nasser, 2007); and compared the results to those using different sampling methods. Al-Omari and Al-Nasser (2011) suggested a new quality control charts for the mean using robust extreme ranked set sampling (*RERSS*) method. They found that the *RERSS* charts perform better than all other charts based on *SRS* and *RSS* methods in terms of their average run length (*ARL*). Al-Omari and Abdul Haq (2012) were developed Shewhart-type control charts to improve the monitoring process mean by using the double quartile-ranked set sampling, quartile double-ranked set sampling, and double extreme-ranked set sampling method.

It could be noted that from the literature of the RSS sampling techniques and all of its modifications that they are a waste consuming sampling units; to overcome of this problem in data collections Bani Mustafa et al. (2011) proposed the folded ranked set sampling (FRSS) as another form of RSS techniques aiming to estimate the population mean and to save the amount of wasted sampling units.

In this article, we propose control charts of the sample mean assuming that the underlying distribution is normal with mean μ and variance σ^2 while the ranked set sampling schemes are carried out using perfect folded ranking.

The rest of this article is organized as follows. In section 2, we present an overview of the RSS and FRSS schemes. In Section 3, we outline the process of constructing the \bar{X} -bar control charts using SRS, RSS and FRSS. In section 4, we present the average run length (*ARL*) as our primary tool to investigate and compare the performance of the control charts in order to maintain the ability of detecting the out-of-control status; and we ends up this article with a conclusion section.

2. Sampling based on ranked data schemes

2.1 Ranked set sampling

The RSS proposed by McIntyre (1952) can be described via the following steps:

- Step 1:** Randomly select m^2 units from the target population of interest.
- Step 2:** Allocate the m^2 selected units as randomly as possible into m sets, each of size m .
- Step 3:** Rank the units within each set visually or by any inexpensive method with respect to the variable of interest.

Step 4: From the first set of m units, select the smallest ranked unit for actual measurement. The second smallest ranked unit is selected for actual measurement from the second set. The process is continued until the m^{th} smallest ranked unit is measured from the last set.

Step 5: Repeat Steps 1 through 4 n cycles, if necessary to obtain a sample of size mn .

To this end, we let $X_{i[i:m]j}$ to be the i^{th} ordered observation from the i^{th} set of size m for a given cycle j . Accordingly, the RSS estimator of the population mean is given by

$$\bar{X}_{RSS} = \frac{1}{mn} \sum_{j=1}^n \sum_{i=1}^m X_{i[i:m]j}, \quad (1)$$

with variance

$$\text{Var}(\bar{X}_{RSS}) = \frac{\sigma^2}{m} - \frac{1}{m^2} \sum_{i=1}^m (\mu_{i[i:m]} - \mu)^2, \quad (2)$$

where $\mu_{i[i:m]}$ is the mean of the i^{th} order statistics of a sample of size m given by

$$\mu_{i[i:m]} = \int_{-\infty}^{\infty} xf_{[i:m]}(x)dx \text{ and } f_{[i:m]}(x) = m \binom{m-1}{i-1} F^{i-1}(x) [1-F(x)]^{m-i} f(x).$$

2.2. Folded ranked set sampling

In order to plan a FRSS design as proposed by Bani Mustafa et al. (2011), m random samples should be selected each of size m , where m is typically small to reduce ranking error. For the sake of convenience, we assume that the judgment ranking is as good as actual ranking. Accordingly, the folded ranked set sampling can be described according to the following steps:

Step 1: Select $\left[\frac{m+1}{2} \right]$ random samples each of size m from the target population;

where $[\cdot]$ is the integer operator.

Step 2: Rank the units within each sample with respect to the variable of interest via visual inspection or any cost free method.

Step 3: Select the 1^{st} and the m^{th} units from the first sample for actual measurement.

Step 4: Select the 2^{nd} and the $(m-1)^{\text{th}}$ units from the second sample for actual measurement.

Step 5: we continue the process until the $\left[\frac{m+1}{2} \right]^{\text{th}}$ unit is selected from the $\left[\frac{m+1}{2} \right]$ sample.

We may repeat the cycle n times if needed to obtain the desired sample of size mn . Figure 1 illustrates the mechanism of the FRSS scheme.

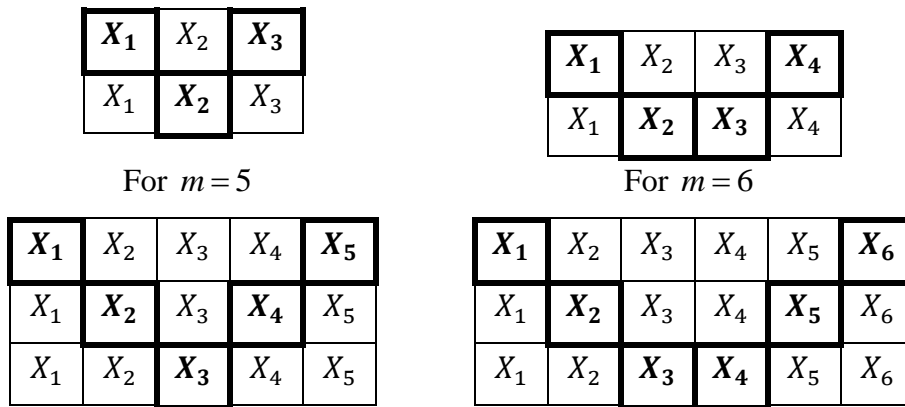


Figure.1 Selected Schemes of FRSS for different sample sizes

Bani Mustafa et al. (2011) showed that the FRSS estimator of the population mean is

$$\bar{X}_{FRSS} = \frac{1}{nm} \sum_{j=1}^n \sum_{\substack{i=1 \\ i < m-i+1}}^{\lfloor \frac{m+1}{2} \rfloor} (X_{i[i:m]j} + X_{i[m-i+1:m]j}), \tag{3}$$

and

$$E(\bar{X}_{FRSS}) = \frac{1}{m} \sum_{i=1}^m \int_{-\infty}^{\infty} x_{i[i:m]} dF(x_{i[i:m]}),$$

where

$$dF(x_{(i:m)}) = f(x_{(i:m)}) = \frac{m!}{(i-1)!(m-i)!} [F(x)]^{i-1} [1-F(x)]^{m-i} f(x).$$

The variance of \bar{X}_{FRSS} is given by

$$\sigma_{\bar{X}_{FRSS}}^2 = \frac{1}{m^2} \left(\sum_{i=1}^m Var(X_{i[i:m]}) + 2 \sum_{i=1}^{\lfloor \frac{m+1}{2} \rfloor} Cov(X_{i[i:m]}, X_{i[m-i+1:m]}) \right). \tag{4}$$

2.3. Merits of the FRSS

The ranked data methodologies have shown several evidences to conclude the superiority of such methods over the traditional sampling schemes while estimating the population mean. Several articles pointed out this vital issue through the relative efficiency of the population mean estimates using the ranked data approach. The relative efficiency (RE) of the population mean estimator is defined by

$$RE = \frac{Var(\bar{X}_{SRS})}{Var(\bar{X}_{*RSS})}$$

where *RSS could be, naïve RSS or FRSS. To illustrate the efficiency of the two sampling schemes and without loss of generality, we assume that $n = 1$. Accordingly, for a set of size $m = 4, 5$ and 6 ; from different symmetric (Standard Uniform, Standard Normal, Student T with degrees of freedom 3 and a beta distribution with parameters 3 and 3); and asymmetric distributions (Exponential, Chi-squared, Log Normal, Gamma, Weibull and Beta); in this comparisons we are adopting of using the different distributions as suggested by Bani Mustafa *et al* (2011); the exact relative efficiencies (RE) were computed by using Mathematica.6 and the results are presented in Tables 1 and Table 2.

Table 1: Results for symmetric distributions with $m = 4, 5, 6$

Distribution	Sample size	RSS	FRSS
Uniform (0,1)	$m = 4$	2.5000	1.6667
	$m = 5$	3.0000	2.3333
	$m = 6$	3.5000	2.3333
Normal (0,1)	$m = 4$	2.3469	1.6767
	$m = 5$	2.7702	2.2190
	$m = 6$	3.1856	2.3306
Student T (3)	$m = 4$	1.7049	1.4367
	$m = 5$	1.8647	1.6654
	$m = 6$	1.8793	1.7596
Beta (3,3)	$m = 4$	2.4433	1.6548
	$m = 5$	2.9145	1.8783
	$m = 6$	3.3829	2.3564

Table 2: Results for asymmetric distributions with $m = 4, 5, 6$.

Distribution	$m = 4$		$m = 5$		$m = 6$	
	RSS	FRSS	RSS	FRSS	RSS	FRSS
Exp (1)	1.92	1.5652	2.1898	1.9468	2.4489	2.0678
Log N (0,1)	1.4711	1.3518	1.5891	1.5191	1.6971	1.6017
Gamma (0.5,1)	1.6963	1.4873	1.8979	1.7754	2.0908	1.8974
Weibull (0.5,1)	1.3345	1.2809	1.4249	1.4002	1.5094	1.4755
Gamma (2,1)	2.0958	1.6152	2.4244	2.0653	2.7423	2.1825
Weibull (2,1)	2.3251	1.6694	2.7436	2.2167	3.1551	2.3177
Beta (2,9)	2.2667	1.6577	3.6310	2.1869	3.0551	2.2910
Chi (1)	2.2393	1.6489	2.6284	2.1761	3.0100	2.2722

The results indicate that the *RE* increases as the sample size increases. For all distributions, the RSS is more efficient than the proposed FRSS method. In spite of the efficient estimators produced using the RSS as well as several recent modifications, the cost of such sampling schemes is considered a major pitfall against these approaches compared to the traditional ones (Mode, et al., 1999; Buchanan, et al., 2005 and Bani Mustafa, et al., 2011). In fact, the selection of a RSS of size *m* will force the researcher to dispose $m(m - 1)$ units which represents, in certain situations, an unacceptable waste especially when the sampling is costly. However, to reduce the cost involved with such wasting of sampling units we adopt a new sampling procedure developed by Bani Mustafa et al. (2011) based on folded range (quasi range) of the data. This new procedure improves the efficiency of the estimator of the population mean especially for skewed distributions. Moreover, it reduces the number of wasted sampling units which is an advantage of the proposed sampling method (FRSS) over its counterparts. It is known that in RSS for a set of size *m* we usually dispose $m^2 - m$ units, while in FRSS we discard only $\left[\frac{m+1}{2} \right] \times m - m$, where $[\cdot]$ is the integer operator. Table 3 shows the number of wasted sampling units in RSS and FRSS and their sampling unit's saving ratio.

Table 3: Wasted sampling units in RSS and FRSS and their ratio

Set Size	Wasted sampling units in RSS	Wasted sampling units in FRSS	Ratio FRSS/RSS
3	6	3	0.500
4	12	4	0.333
5	20	10	0.500
6	30	12	0.400
7	42	21	0.500

The results in Table 3 indicate that we reduce the number of wasted measurements by half if the set size is even and at least a half $(1 - Ratio)$ if the set size is odd; by using FRSS comparing to RSS.

3. Quality control charts

3.1 Shewhart Control Charts Based on SRS

Suppose X_{ij} , ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) are *n* independent samples each of size *m* such that X_{ij} is the *i*th measured unit in the *j*th sample. Quality control classical literature assumes that the probability density function is normal with mean μ and variance σ^2 . Therefore, \bar{X}_j is normally distributed and the probability that any sample mean \bar{X}_j will fall between the two limits $\mu + Z_{1-(\alpha/2)}\sigma_{\bar{X}}$ and $\mu - Z_{1-(\alpha/2)}\sigma_{\bar{X}}$ is $(1 - \alpha)$ where $Z_{\alpha/2}$ denotes the $(\alpha/2)$ th percentile of the standard normal distribution.

Consequently, if μ and σ^2 are known, then the Shewhart control chart of the sample mean

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^m X_{ij}, j=1,2,\dots,n \text{ is given by}$$

$$\begin{aligned} UCL &= \mu + 3 \frac{\sigma}{\sqrt{m}}, \\ CL &= \mu \\ LCL &= \mu - 3 \frac{\sigma}{\sqrt{m}} \end{aligned} \tag{5}$$

where UCL, CL and LCL represent the upper control limit, center line and the lower control limit, respectively.

It is noteworthy to point out that one or both population parameters are expected to be unknown in practical situations, therefore the control limits in (5) will depend on estimate values of these parameters based on the collected data X_{ij} which implies that

$$\begin{aligned} UCL &= \bar{\bar{X}} + 3\hat{\sigma}_{\bar{X}}, \\ CL &= \bar{\bar{X}}, \\ LCL &= \bar{\bar{X}} - 3\hat{\sigma}_{\bar{X}}, \end{aligned}$$

where $\bar{\bar{X}}$ and $\hat{\sigma}_{\bar{X}}$ are the grand mean and the standard error of the sample mean, respectively that happen to be unbiased estimators of μ and σ provided that

$$\bar{\bar{X}} = \frac{1}{n} \sum_{j=1}^n \bar{X}_j, \hat{\sigma}_{\bar{X}} = \frac{\Gamma\left(\frac{m-1}{2}\right)}{n\sqrt{m}\left(\frac{2}{m-1}\right)^2 \Gamma(m-2)} \sum_{j=1}^n \sqrt{\frac{1}{m-1} \sum_{i=1}^m (X_{ij} - \bar{X}_j)^2},$$

and $\Gamma(t) = \int_0^{\infty} y^{t-1} e^{-y} dy$ for $t > 0$, (Montgomery, 2005).

3.2 Shewhart Control Charts Based on RSS

The ranked set sampling estimates of μ and σ^2 given in (1) and (2) will be used to construct the control limits and the center line assuming that these parameters are unknown. In fact, Salazar and Sinha (1997) studied the behavior of quality control charts based on RSS and they derived the UCL, CL and LCL for the sample mean when μ and σ^2 are known as follows

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{X}_{RSS}}, \\ CL &= \mu, \\ LCL &= \mu - 3\sigma_{\bar{X}_{RSS}}. \end{aligned} \tag{6}$$

However, if μ and σ^2 are unknown, the control limits in (6) can be obtained as follows

$$\begin{aligned} UCL &= \bar{X}_{RSS} + 3\hat{\sigma}_{\bar{X}_{RSS}}, \\ CL &= \bar{X}_{RSS}, \\ LCL &= \bar{X}_{RSS} - 3\hat{\sigma}_{\bar{X}_{RSS}}. \end{aligned} \tag{7}$$

Muttalak and Al-Sabah (2003) proposed an estimator of $\sigma_{\bar{X}_{RSS}}$ as follows

$$\hat{\sigma}_{\bar{X}_{RSS}} = \sqrt{\frac{1}{m} \left(\hat{\sigma}_{RSS}^2 - \frac{1}{m} \sum_{i=1}^m (\bar{X}_{[i]} - \bar{X}_{RSS})^2 \right)},$$

where

$$\hat{\sigma}_{RSS}^2 = \frac{1}{mn-1} \sum_{i=1}^m \sum_{j=1}^n (X_{i[i]j} - \bar{X}_{RSS})^2 \quad \text{and} \quad \bar{X}_{[i]} = \frac{1}{n} \sum_{j=1}^n X_{i[i]j}.$$

3.3 Shewhart Control Charts Based on FRSS

The FRSS scheme will be used to construct the quality control limit of the sample mean. Once again, if μ and σ^2 are known, the UCL, CL and LCL control limits of a control chart for the sample mean based on FRSS are given by

$$\begin{aligned} UCL &= \mu + 3\sigma_{\bar{X}_{FRSS}}, \\ CL &= \mu, \\ LCL &= \mu - 3\sigma_{\bar{X}_{FRSS}}. \end{aligned} \tag{8}$$

However, in practice μ and σ^2 may happen to be unknown which means that the control limits in (8) will be estimated as

$$\begin{aligned} UCL &= \bar{X}_{FRSS} + 3\hat{\sigma}_{\bar{X}_{FRSS}}, \\ CL &= \bar{X}_{FRSS}, \\ LCL &= \bar{X}_{FRSS} - 3\hat{\sigma}_{\bar{X}_{FRSS}}. \end{aligned} \tag{9}$$

4. Comparison for the proposed control charts

In this section, we compare the FRSS control charts with the SRS control charts based on the average run length (ARL). In fact, if we define W to be the number of observations plotted on the chart until the first observation gets out-of-control limits, then W has a geometric distribution and the mean of W is called the ARL. In this section, we use type I error (α) to define the ARL when the process is under control such that

$$ARL = \frac{1}{\alpha}. \tag{10}$$

However, if the process gets out of control, then the ARL is written in terms of type II error (β) as follows

$$ARL = \frac{1}{1 - \beta}. \quad (11)$$

Based on the ARL criterion, the process remains in control with mean μ_0 and standard deviation σ_0 and sometimes it may get out of control in terms of a mean shift of the amount $\delta\sigma_0/\sqrt{m}$, where δ is nonnegative and selected to dominate the shift in the mean μ .

To carry on our task, we use a simulation study to illustrate the quality control mechanism via SRS, RSS as well as FRSS schemes. The simulation study is conducted under the normality assumption with mean μ_0 and variance σ_0^2 assuming the ranking is perfect.

The program codes were prepared by the authors using the FORTRAN power station environment programs linked to IMSL library. Note that under the SRS procedure, the ARL of the \bar{X} chart will be 370. This represents the reciprocal of the probability that a single point falls outside the control limits when the process is in fact under control. In other words, the out-of-control signal will flash once every 370 observed samples even though the process is already under control.

We followed the same procedure of Muttlak and Al-Sabah (2003) to simulate one million iterations for each value of δ and for all sampling methodologies. At each iteration, we simulate a sample of size $m = 3, 4, 5, 6$ which represent the most recommended sample size in the RSS literature.

As another simulation option, we set the shift-in-mean δ to vary between 0 and 3.4 to cover the "under control" process as well as the "out of control" process; the comparisons between the three sampling methods are given in Figure 2.

For $m = 3$

For $m = 4$

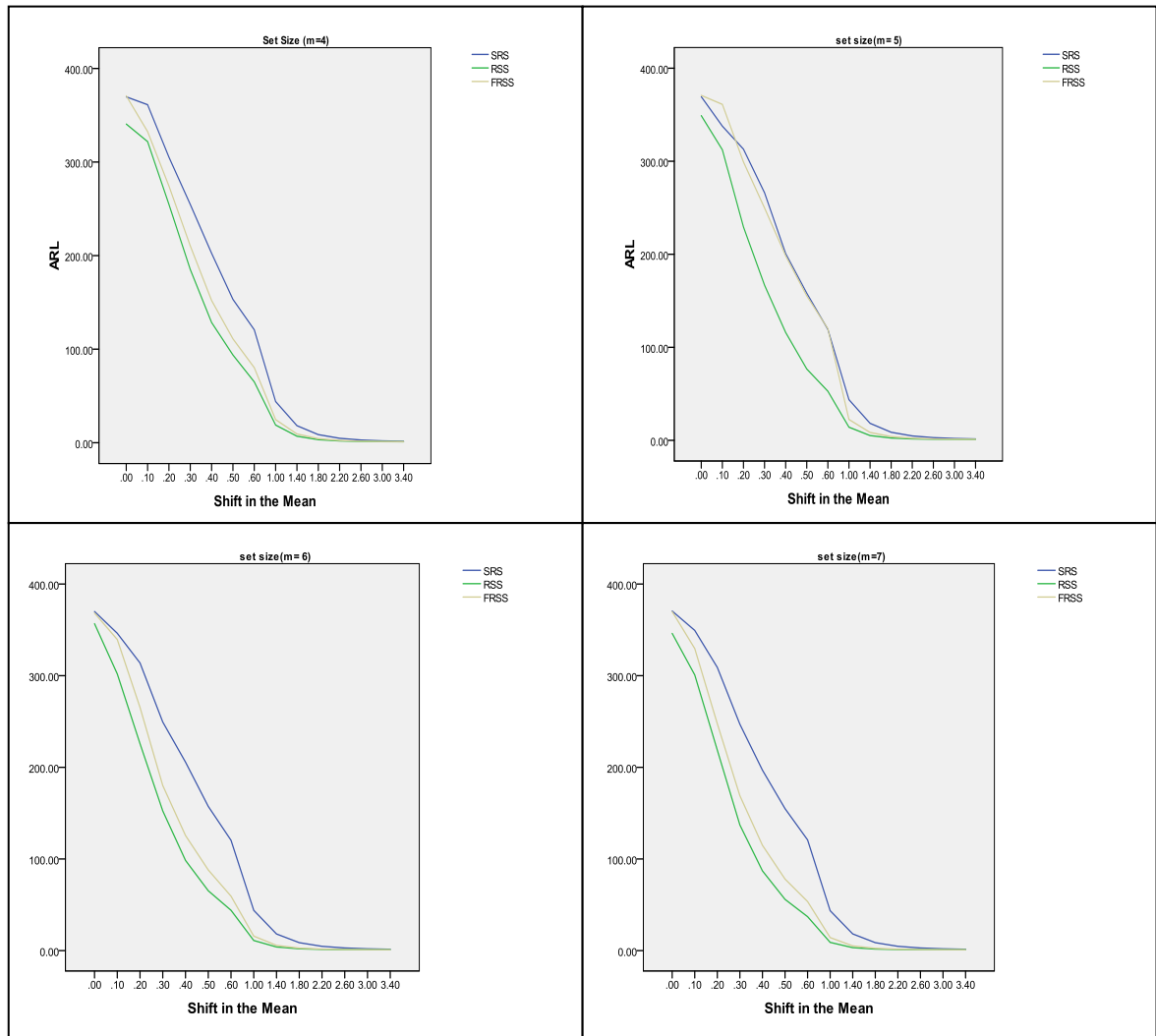


Figure.2 ARL Comparisons between SRS, FRSS and RSS

The main criterion ARL is computed for all combinations of m , δ and the sampling method of interest (for more details, see Harter and Balakrishnan, 1996 and Montgomery, 2005); the numerical values of the ARL are given in Table 4 and Table 5.

Table 4: Average run length using SRS, RSS and FRSS when $m = 3, 4$

δ	$m = 3$			$m = 4$		
	SRS	RSS	FRSS	SRS	RSS	FRSS
0.0	369.6858	340.5995	370.7504	369.4126	349.0401	370.7898
0.1	361.2717	321.8539	332.5574	337.7238	312.3048	361.1412
0.2	305.2503	254.7771	274.4237	312.9890	229.4104	299.2220
0.3	254.7122	185.1852	210.4377	266.0990	166.7500	249.7502
0.4	202.4701	128.5017	152.0219	200.7226	115.9420	198.3733
0.5	153.2332	93.7910	111.0124	158.1778	76.7048	155.3036
0.6	120.7146	65.0280	80.2504	119.2890	52.7816	120.0192
1.0	44.0393	18.8929	24.6944	43.7101	14.1495	22.4175
1.4	18.2282	6.9544	9.3480	18.3006	5.1341	8.4971
1.8	8.6675	3.2767	4.3358	8.6781	2.4803	3.9722
2.2	4.7417	1.9340	2.4552	4.7293	1.5504	2.2734
2.6	2.9033	1.3782	1.6504	2.9022	1.1932	1.5542
3.0	1.9985	1.1425	1.2815	1.9999	1.0584	1.2314
3.4	1.5246	1.0461	1.1122	1.5244	1.0138	1.0871

Table 5: Average run length using SRS, RSS and FRSS; when $m = 5, 6$

δ	$m = 5$			$m = 6$		
	SRS	RSS	FRSS	SRS	RSS	FRSS
0.0	370.0238	356.7606	368.3704	370.5163	346.1405	369.9010
0.1	346.2604	301.9324	339.4433	349.4060	300.8423	329.5979
0.2	313.8732	225.8356	265.3928	309.0235	218.7705	247.5860
0.3	249.4388	152.4623	180.1802	247.0356	137.1178	169.0331
0.4	205.6767	98.4252	125.5966	196.8891	87.0019	114.9029
0.5	157.3812	65.3339	88.0204	154.9427	55.9503	78.1128
0.6	120.4094	44.0238	59.4177	120.8021	37.0508	53.5332
1.0	43.9638	11.0552	15.8471	43.5749	9.0035	14.2637
1.4	18.1831	3.9908	5.7379	18.2435	3.2467	5.1605
1.8	8.6989	2.0078	2.7176	8.6944	1.7118	2.5013
2.2	4.7118	1.3390	1.6650	4.7149	1.2144	1.5616
2.6	2.9120	1.1008	1.2453	2.9026	1.0530	1.1987
3.0	2.0007	1.0237	1.0797	1.9961	1.0095	1.0605
3.4	1.5254	1.0040	1.0209	1.5244	1.0012	1.0147

Based on the results given in Table 4, Table 5 and Figure 2 we can conclude that:

- When $\delta = 0$, i.e., if the process is under control, the ARL values obtained by FRSS are about 370.
- In most cases, the FRSS control charts are more efficient than the SRS based control charts in terms of ARL specially when the shift in the mean gets to be large.
- The ARL values based on FRSS method are less than the corresponding values using SRS for different set size except for some small values of δ . For example, for $m = 6$, the ARL value using FRSS is 247.5860 compared to 309.0235 based on SRS when $\delta = 0.2$
- If δ gets larger than zero, i.e., the process starts to get out of control, the ARL values decrease using all sampling methods. Setting $m = 5$ and $\delta = 0, 0.1, 0.2, 0.3, 0.4$, then the ARL values using FRSS are 368.3704, 339.4433, 265.3928, 180.1802, 125.5966, respectively.
- In general; both RSS and FRSS are more efficient than SRS in terms of the ARL values for all cases considered in this article.

In summation, the FRSS is recommended for estimating the population mean and to built mean charts.

5. Conclusions

Ranked set sampling has been demonstrated to be an efficient cost free sampling method. However, it is a cost wasting sampling unit's method. To overcome of this problem; in this article we used the FRSS to develop X-bar quality control chart. The proposed chart is compared with the RSS and SRS based control charts. It is clear that the proposed FRSS chart is more efficient than the SRS control chart, and less efficient than the RSS charts.

It is worthy to conclude that by using FRSS we are saving wasting sampling units by at least half of $(1 - \text{Ratio})$ comparing to RSS; where the saving Ratio of a set of size m is equal to $\frac{\lfloor \frac{m+1}{2} \rfloor - 1}{m-1}$; and this value in even set size is equal to 50% of the wasting sampling units by using RSS. Moreover, for any set size, the FRSS charts dominate the SRS charts; if the process starts to get out of control then the FRSS chart reduced the average run length (ARL) substantially.

Finally we recommend of using FRSS to construct the X-bar quality control chart. Since it reduce the ARL compared SRS, save wasting sampling units comparing to RSS; and it is more easily to be implemented in the data collection for any field study.

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