Performance Measures of Students in Examinations: A Stochastic Approach

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Abstract

Data on Secondary and Higher Secondary examination (science stream) results from Tripura (North-East India) schools are analyzed to measure the performance of students based on tests and also the performance measures of schools based on final results and continuous assessment processes are obtained. The result variation in terms of grade points in the Secondary and Higher Secondary examinations are analysed using different sets of performance measures. The transition probabilities from one grade to another grade are calculated for individual students. The performance measures defined for students as well as for different categories are obtained numerically from real life data.

Keywords: Assessment, Performance, Transition Probabilities Matrix, Grade, Mean Waiting Time.

AMS Subject Classification: 60H30, 62-07, 97D60.

1. Introduction

Assessment of students' performance is one of the most challenging areas of teaching-learning process. Teaching-learning itself is a complex process which ensures communication between human minds contributing enormous wealth to the history of civilization. Assessment is an inseparable part of teaching-learning process which is not an end of the process but, is a tool for the improvement of the learning process. Power of assessment process is cumulative. Here point is to monitor students' performance and observe their progress towards prefixed goal in the spirit of continuous improvement. Hence, to understand the grading pattern and analyze the performance of the students even beyond classroom, scientific analysis of the assessment pattern is more than necessary.

Modeling of assessment pattern represents the behavior of teaching-learning process. Apart from students' individual performance, it seems that the nation has recognized the fact that its public high schools are not adequately preparing students' for college, careers, and life in the twenty-first-century global economy. National leaders and the education policy community have embraced the idea that the education system must establish - college and career readiness as the goal for all students'. There also has been wide-spread acknowledgement that addressing the problems in low-performing high schools is necessary if that goal is to be met. To improve the status of schools one has to define and analyze the performance measures of individual students' and also institutions in which they are enrolled.

Several important questions crop up with the continuation of assessment processes. If performance status of a student in the institute is determined by grade point achieved at the end of a test, then with what probability his / her level of performance changes in either direction may be answered only with proper probabilistic pattern of the grades. To describe the grading pattern, the Markov model with discrete time and discrete state space is adopted.

The next section of this article includes a brief review of literature on the subject. Section three is about the model adopted and description of performance measures considered for analysis. Data source and numerical results with real life data is incorporated in section four and five respectively.

2. Review of Literature:

Examination results are one of the indicators of students' success and school performance. A series of papers known as Black Papers (Cox and Dyson 1969a, 1969b and 1970) came into existence as a consequence of criticism of British Education system. Jencks et. al. (1972) argued that half the difference of students' academic outcomes could be due to their social background and type of schools they attended. In an article Wright and Wiese (1988) argued that Teachers judgment in students' evaluation is an important tool for comparing grading methods. Examination results remain always important tool for the young generation to get a better paid job. Stubbs (1998) performed data analysis of 3000 students for the year 1996. Kamath (2003) put forward a discussion on school performance where it was emphasized that low-performing public schools are responsible for high drop-out rate of school children in India. Dulwat and Rai (2005) identified students' level of satisfaction as performance measure. Khatoon and Mahmood (2010) analyzed data related to Mathematics anxiety of secondary school students based on parameters Gender, School Type and Score in Mathematics. Adenaike and Olaniyi (2010) stressed on Total Quality Measures on Secondary School Students' Academic performance in Ogun State. Besides this many more articles are available in literature which is beyond the scope of this article.

3. Model

3.1. Assumptions

3.1.a. A student is evaluated at regular interval of time and each time point the grade of the student is recorded.

3.1.b. The present grade of the student is dependent on immediate previous grade i.e. future course is decided by the immediate past position (Sarma, Sarmah 1999).

3.1.c. The range [0, 100] of scores "S" is partitioned into three non overlapping sets viz. S_0, S_1 and S_2 such that, $S_0 = [M_0, 100]$, $S_1 = [M_1, M_0)$, and $S_2 = (M_1, 0]$ where $0 \leq M_1 < M_0 \leq 100$.

3.1.d. The grade of a student is assumed to be i if $S \in S_i$ for i = 0, 1, 2.

Now let $X_n = i$ for i = 0, 1, 2 be the grade of a student at the n^{th} test, where n = 1, 2, 3 ..... The consequence of assumptions 3.1.a. to 3.1.d. states that

$\{X_n, n = 1, 2, \ldots\ldots\ldots\}$ follows a Markov chain with state space \$ = {0, 1, 2}, transition probability matrix $P = (P_{ij})$ with initial distribution (P_0, P_1, P_2), such that $P_0 + P_1 + P_2 = 1$ and $P_{ij} = P_r(X_n = j / X_{n-1} = i)$.

As far as the grades of individual students' are available at different point of time, they may be probabilistically analyzed by (Sarma, Sarmah 1999).

The transition probability matrix of the chain may be written as

$$P = \begin{pmatrix} 1-a(1-c) & a(1-c) & 0 \\ \frac{b}{2}(1-c) & 1-b(1-c) & \frac{b}{2}(1-c) \\ 0 & k(1-c) & 1-k(1-c) \end{pmatrix}$$

Statistical estimation of transition probabilities P_{ij} may also be obtained by using the method of maximum likelihood by (Anderson, Goodman, 1957).

3. 2. Performance Measures of Students Based on Tests.

As mentioned earlier the results on individual performances of students' are already obtained by (Sarma, Sarmah 1999), the expressions for performance measures are obtained by using the theory of Markov chain.

The mean time required for a student starting with the grade i at the beginning of the course to come out with grade j is

$$\mu_{ij} = \sum_n n f_{ij}^{(n)} \qquad i = j = 0, 1, 2.$$

Where $f_{ij}^{(n)}$ is the probability that a student visits the state j for the first time in 'n' tests provided, he / she enters the state i initially.

Hence, the mean time required for a student starting with initial grade '0' to reach the same grade '0' in a total number of 'k' tests is

$$\mu_{00} = \sum n f_{00}^{(n)}$$
$$= P_{00} + 2P_{01}P_{10} + 3P_{01}P_{11}P_{10} + 4P_{01}P_{11}^2P_{10} + \ldots\ldots\ldots\ldots + k\,P_{01}P_{11}^{k-2}P_{10}$$

Again the mean time required for a student starting with grade '1' to go back to grade '1' after 'k' tests is given by,

$$\mu_{11} = \sum n f_{11}^{(n)}$$
$$= [P_{11} + 2P_{01}P_{10} + 3P_{01}P_{00}P_{10} + 4P_{01}P_{00}^2P_{10} + \ldots\ldots\ldots + k\,P_{01}P_{00}^{k-2}P_{10}]$$
$$+ [P_{11} + 2P_{12}P_{21} + 3P_{12}P_{22}P_{21} + 4P_{12}P_{22}^2P_{21} + \ldots\ldots + k\,P_{12}P_{22}^{k-2}P_{21}]$$

Similarly, the mean time required for a student starting or entering the school initially with grade '2' to go back again to the same grade '2' at the end is,

$$\mu_{22} = \sum n f_{22}^{(n)}$$

$$= P_{22} + 2P_{21}P_{12} + 3P_{21}P_{11}P_{12} + 4P_{21}P_{11}^{2}P_{12} + \ldots\ldots\ldots\ldots + k\,P_{21}P_{11}^{k-2}P_{12}$$

3. 3. Expected Number of Times Students' Visits to Different States.

Let M_{ij} = Expected number of times a student visits the state j, provided the student was in the state i initially. That is, if the student visits the state j for the first time in k tests, then,

$$M_{ij} = \frac{\sum_{k=1}^{n} f_{ij}^{(k)}}{1-\sum_{k=1}^{n} f_{ij}^{(k)}} \quad ; i = j = 0, 1, 2.$$

In this context following three cases were discussed.

Case – I: If i = j = 0, then

$$M_{00} = \frac{\sum_{k=1}^{n} f_{00}^{(k)}}{1-\sum f_{00}^{(k)}}$$

$$= P_{00} + P_{01}P_{10}[1 / (1 - P_{11})] \qquad \text{(After Simplification)}$$

So, the expected number of times a student visits the grade '0' for a fixed number of tests 'n', can be easily obtained, provided the student starts with grade '0' initially.

Case – II: If i = j = 1, then

$$M_{11} = \frac{\sum_{k=1}^{n} f_{11}^{(k)}}{1-\sum f_{11}^{(k)}}$$

$$= P_{11} + P_{10}P_{01}[1/(1-P_{00})] + P_{12}P_{21}[1/(1-P_{22})] \quad \text{(After Simplification)}$$

Hence for i = j = 1, the expected number of times a student visits the grade '1', in a fixed number of tests can be obtained, if the student starts with grade '1' initially.

Case – III: If i = j = 2, then

$$M_{22} = \frac{\sum_{k=1}^{n} f_{22}^{(k)}}{1-\sum f_{22}^{(k)}}$$

$$= P_{22} + P_{12}P_{21}[1 / (1 - P_{11})] \qquad \text{(After Simplification)}$$

Thus M_{22} gives us the expected number of times, the student visits the grade '2', if the student starts with grade '2' initially.

3. 4. Performance Measures of Schools Based on Final Results.

Sarma, Sarmah (1999), described various sampling schemes that describe probabilistic structure of performance of educational institutions which are mentioned below:

Let n_i be the total number of students in grade i at the end of an assessment process for i = 0, 1, 2. Where n_i follows Trinomial distribution.

Hence the probability mass function (p.m.f.) of Trinomial distribution is,

$$P(n_0, n_1, n_2) = n\,!\,/\,n_0!\,n_1!\,n_2!\;P_0^{n_0}\,P_1^{n_1}\,P_2^{n_2}$$

Where $n = n_0 + n_1 + n_2$ and $P_0 + P_1 + P_2 = 1$.

Hence, the Expected value is $E(n_i) = n_i P_i$ for i = 0, 1, 2.

3. 5. Performance Measures of Schools Based on Continuous Assessment.

In this section, we are trying to analyze the performance of schools on the basis of sets of students' falling under different categories. The performances are measured on the basis of number of tests and total number of students falling in three classes S_0, S_1 and S_2 defined in section 3.1.

Let us define performance measure $R_j(k, k-1)$ for k^{th} test measured against $(k\text{-}1)^{th}$ test under j^{th} state for j = 0, 1, 2 and k > 1. The performance measure $R_j(k, k-1)$ may be obtained for every parameter mentioned above clearly. Hence, we can write,

$$R_j(k, k-1) = \frac{y_j(k)}{y_j(k-1)} \quad \text{for j = 0, 1, 2 and } y_j(k-1) \neq 0$$

$$= \left(n_j(k) \Big/ n_j(k-1)\right).\left(N_{k-1} \Big/ N_k\right)$$

Where, $n_j(k)$ = No. of students getting grade j in the k^{th} test and

N_k = Total No. of students' appearing in the k^{th} test with k > 1 and k ∈ l.

In general, $\left(N_{k-1} \Big/ N_k\right) \sim 1$.

The range of R_j may be determine by taking lower limit of $R_j(k,k-1)$ is

$$R_{jl}(k,k-1) = \frac{Min\{\eta_j(2),.....,\eta_j(k)\}}{Max\{\eta_j(1),.....,\eta_j(k-1)\}}$$

And the upper limit of $R_j(k,k-1)$ is

$$R_{ju}(k,k-1) = \frac{Max\{\eta_j(2),.....,\eta_j(k)\}}{Min\{\eta_j(1),.....,\eta_j(k-1)\}}$$

that is, $R_{jl}(k,k-1) < R_j(k,k-1) < R_{ju}(k,k-1)$

Now it is clear that,

$$\prod_{j=1}^{m} R_j(k,k-1) = \eta_j(m)/\eta_j(1) = R_j(m,1) \text{ (Say)}$$

is performance measure of mth test in comparison to the 1st test under a defined category for state j, where j = 0, 1, 2.

$$\left\{\sum_{j=1}^{m} \eta_j(k)/\eta_j(k-1)\right\}/m = R_j(m) \text{ (Say)}$$

is the average performance measure of all 'm' tests under a defined category for state j, where j = 0, 1, 2.

4. Data Source

This set of data has been collected from a Minor Research Project (MRP) entitled "*Prospects and Problems of Educational development (Higher Secondary Stage) in Tripura - An in-depth Study*" sponsored by University Grants Commission, New Delhi, India, Ref. No.F.5-338/2009-10 (MRP/NERO)/5799 - 5800, Dated: 14/12/2009. Tripura is one of the smallest states of North-East India. It is bounded by Bangladesh on the North, West, South and South-East, and shares common boundary with Assam and Mizoram in the East. Tripura is a tribal state, dominated by Bengali population and rich in culture. Tripura has made a comprehensive progress in the field of education since launching of first Five-Year plan, but limited economic resources and backward geographical position creates some problem in growing the literacy rate in the state.

The state government has given highest priority for universalization of elementary education and it can be evident from the census - 2011 data. According to the 2011 census, Tripura become the 4th most literate state in India with 87.75% literacy rate and the 2nd most literate state in North-East region after Mizoram (literacy rate 92.49%) by [10]. Out of total literacy rate of Tripura in 2011 census, the male and female literacy rates are 92.18% and 83.15% respectively by some basic statistics of Tripura (2006) and statistical abstracts of Tripura (2007). With these information's in view our interest is to analyze the performance of the students' according to (i) medium of instructions, (ii) gender, (iii) board of examinations (CBSE, ICSE and State Board of Examination i.e., TBSE) and (iii) different types of schools.

5. Numerical Analysis:

5.1. Performance Measures of Students Based on Tests.

5. 1. a. Estimation of Transition Probabilities:

Estimation of transition probabilities P_{ij} i, j = 0, 1, 2, in (Saha, Sarmah, 2010) obtained these transition probabilities for the above mentioned categories with the above set of data. Performance measures in terms of transition probabilities, which are functions of Maximum Likelihood Estimation, are presented as Table 1 in the appendix.

5. 1. b. Calculation of the Limiting Probabilities:

The transition probability matrix for a finite state irreducible Markov chain satisfies the result $LimP^n = \begin{pmatrix} \alpha & \alpha & \alpha \end{pmatrix}'$, where $\alpha = \begin{pmatrix} P_0 & P_1 & P_2 \end{pmatrix}$, with P_i representing the probability system remains in state 1 as a consequence of 'n' transitions.

The values of 'n' and the limiting probabilities of the different matrices discussed in (Saha, Sarmah, 2010) are shown in Table 2 in the appendix.

5. 1. c. Calculation of Mean Recurrence Times:

The mean number of steps required for a student starting with the grade i at the beginning of the course to achieve grade j is

$$\mu_{ij} = \sum_{n=1}^{\infty} n f_{ij}^{(n)} \quad ; \ i, j = 0, 1, 2.$$

Section 3.2 provides the expressions for μ_{ij} and they are numerically obtained as functions of MLE with the above set of data and are presented in the appendix as Table 3:

5. 1. d. Calculation of Expected No. of Times Students' Visit to Different States:

Expected number of times students' visits to different states are presented in section 3.3 and numerically they are obtained for same set of data, shown in Table 4 in the appendix.

5. 2. Performance Measures of Schools Based on Final Results.

In this section, expected value of n_i, the number of students' in grade 'i' in final examination for i = 0, 1, 2 are estimated with the same set of data, which is shown in the appendix as Table 5.

Ratio of observed to expected number of states corresponding to different categories are calculated by using the formula:

$$r_{kj} = n_{kj} \Big/ E_j \quad \text{for j = 0, 1, 2 and k = 0 and 1.} \qquad .$$

Where, k = 0 indicates initial stage and k = 1 indicates final stage.

Where, Table 6 shows the Ratio of observed to expected number of states corresponding to different categories in the appendix.

5.2.1. Comparing r_{00} and r_{10}:

Comparison between r_{00} and r_{10} by column diagram is shown as Figure 1 in the appendix.

5.2.2. Comparing r_{01} and r_{11}:

Comparison between r_{01} and r_{11} by column diagram is shown as Figure 2 in the appendix.

5.2.3. Comparing r_{02} and r_{12}:

Comparison between r_{02} and r_{12} by column diagram is shown as Figure 3 in the appendix.

Discussion

From the Table 5, it has been observed that, the observed values of n_i for i = 0, 1, 2 at the end of the course for all the categories are closer to the expected values. This shows that, there is somewhat a noticeable difference between the number of observed states at the entry level and at the exit point. It is to be noted that, $E(n_1)/E(n_0)$ for all categories except (ICSE , Bengali medium and boys' school) lies approximately in (9, 10) which indicates that expected number of students getting grade 1 is almost 10 times larger than the expected number of students getting grade 0.

Similarly, the expected number of students' getting grade 2 is almost 2 times larger than the expected number of students' getting grade 1. The situation in which expected number of students whose score falls in [0, 50] is tweenty times larger than the expected number of students' whose score falls in [75, 100] is somewhat alarming.

Hence, Educationists, Policy Makers and Human Resource Ministry should join hands to face the situation. From Table 6 and the corresponding graphs, it is clear that, the ratio (i) $r_{00} > r_{10}$ (ii) $r_{01} > r_{11}$ and (iii) $r_{02} < r_{12}$. So, this is a clear indication of sharp raise in number of students falling in the range [0, 50] at final examination per unit of expectation.

5.3. **Performance Measures of Schools Based on Continuous Assessment**

The three performance measures viz.

(1) Test to test performance $R_j(k, k-1)$

(2) Performance against 1st test $R_j(k,1)$

(3) Performance against average $R_j(k, m)$

are calculated based on the basis of number of students' included in grade j, j = 0, 1, 2 in different tests for all the four parameters.

The numerical values of performance measures are presented as Table 7, 8, 9, 10, 11 and 12 in the appendix only for one parameter viz. Medium of instructions (English and Bengali medium) for demonstration purpose only.

Discussion

From Table 7, 8, 9, 10, 11, 12 and from the corresponding graphs (i.e., Figures 4, 5, 6, 7, 8 and 9), it may be observed that, the range of performance measure $R_j(k, k-1)$ corresponding to grade "0" for both the categories (i.e., English and Bengali medium) are much greater than that of performance measures $R_j(k,1)$ and $R_j(k, m)$. However for grade "1" variability among the measures is less. Again there is noticeable variability corresponding to grade "2" for both the categories. This indicates that, performance measures corresponding to grade "1" for both the categories are somewhat remain uniform whereas grade "0" and grade "2" are more sensitive.

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APPENDIX

Table 1: Measurement of Transition Probabilities

Probability → Categories ↓	P_{00}	P_{01}	P_{02}	P_{10}	P_{11}	P_{12}	P_{20}	P_{21}	P_{22}
English	0.305	0.475	0.220	0.036	0.417	0.548	0.000	0.229	0.771
Bengali	0.208	0.427	0.365	0.081	0.406	0.513	0.000	0.343	0.657
Male	0.212	0.471	0.318	0.071	0.394	0.536	0.000	0.359	0.642
Female	0.286	0.414	0.300	0.064	0.429	0.508	0.000	0.245	0.755
Boys'	0.278	0.472	0.250	0.035	0.388	0.577	0.000	0.292	0.708
Girls'	0.267	0.333	0.400	0.070	0.423	0.507	0.000	0.179	0.821
Co-Edu	0.225	0.472	0.303	0.088	0.416	0.496	0.000	0.380	0.620
TBSE	0.206	0.412	0.381	0.080	0.388	0.532	0.000	0.304	0.696
CBSE	0.107	0.643	0.250	0.100	0.267	0.333	0.000	0.308	0.692
ICSE	0.500	0.367	0.133	0.000	0.400	0.600	0.000	0.300	0.700

Table 2: The values of 'n' and the Limiting Probabilities

Parameters	Categories	n	Limiting probabilities corresponding to state 0, 1, 2 respectively
Medium of Instruction	English	10	(0.0147 0.2860 0.6992)
	Bengali	8	(0.0379 0.3697 0.5924)
Gender of Students	Male	8	(0.0338 0.3753 0.5906)
	Female	10	(0.0272 0.3056 0.6672)
Type of Schools	Boys	10	(0.0159 0.3260 0.6580)
	Girls	10	(0.0231 0.2408 0.7356)
	Co-Educational	9	(0.0452 0.3985 0.5563)
Board of Examinations	TBSE	10	(0.0337 0.3361 0.6302)
	CBSE	10	(0.0489 0.4374 0.5136)
	ISCE	10	(0.0000 0.3333 0.6666)
Over all Results	All students	10	(0.0309 0.3446 0.6246)

Table 3: Calculation of Mean Waiting Time

Mean Waiting Time → Categories ↓	μ_{00}	μ_{11}	μ_{22}
English	0.36405	2.76815	1.45067
Bengali	0.41633	2.73742	1.35090
Male	0.35746	2.66239	2.42011
Female	0.41270	2.95736	1.35274
Boys'	0.34904	3.04037	1.43252
Girls'	0.37728	2.92191	1.25041
Co-Educational	0.41841	3.34827	1.49228
TBSE	0.34810	2.89917	1.39150
CBSE	0.31364	1.94030	1.02170
ICSE	0.50000	3.07790	1.49960

Table 4: Expected No. of Times Students' Visit Different States

Expected No. of Visits → Categories ↓	M_{00}	M_{11}	M_{22}
English	0.33433	0.98960	0.98625
Bengali	0.26623	0.96267	0.95322
Male	0.26718	0.97395	0.95953
Female	0.33240	0.97410	0.97296
Boys' school	0.30499	0.98788	0.98330
Girls' school	0.30738	0.96180	0.97828
Co-Educational schools	0.29612	0.96559	0.94273
TBSE	0.25985	0.96151	0.96026
CBSE	0.19472	0.67220	0.83192
ICSE	0.50000	1.00000	1.00000

Table 5: No. of States at Different Stages

States → Category ↓	Observed No. of Initial States at			Observed No. of Final States at			Expected No. of States		
	n_0	n_1	n_2	n_0	n_1	n_2	E_0	E_1	E_2
English	59	84	35	21	71	86	7	66	105
Bengali	96	197	67	36	144	180	5	103	252
Male	85	155	53	29	120	144	10	110	173
Female	70	126	49	28	95	122	7	75	163
Boys'	36	85	24	13	57	75	2	47	95
Girls'	30	71	28	13	45	71	3	31	95
Co-Edu	89	125	50	31	113	120	12	105	147
TBSE	97	201	69	36	139	192	12	123	231
CBSE	28	30	13	6	39	26	3	31	36
ICSE	30	50	20	15	37	48	0	33	67

Table 6: Ratio of Observed to Expected No. of States

States → Category ↓	Observed No. of States at Initial Stage			Observed No. Of States at Final Stage		
	r_{00}	r_{01}	r_{02}	r_{10}	r_{11}	r_{12}
English	8.43	1.27	0.33	3.00	1.08	0.82
Bengali	19.2	1.91	0.27	7.20	1.40	0.71
Male	8.50	1.41	0.31	2.90	1.09	0.83
Female	10.0	1.68	0.30	4.00	1.27	0.75
Boys'	18.0	1.81	0.25	6.50	1.21	0.79
Girls'	10.0	2.29	0.29	4.33	1.45	0.75
Co-Edu.	7.41	1.19	0.34	2.58	1.08	0.81
TBSE	8.08	1.63	0.30	3.00	1.13	0.83
CBSE	9.33	0.97	0.36	2.00	1.25	0.72
ICSE	0.00	1.15	0.30	0.00	1.12	0.72

Figure 1: Comparison between r_{00} and r_{10}

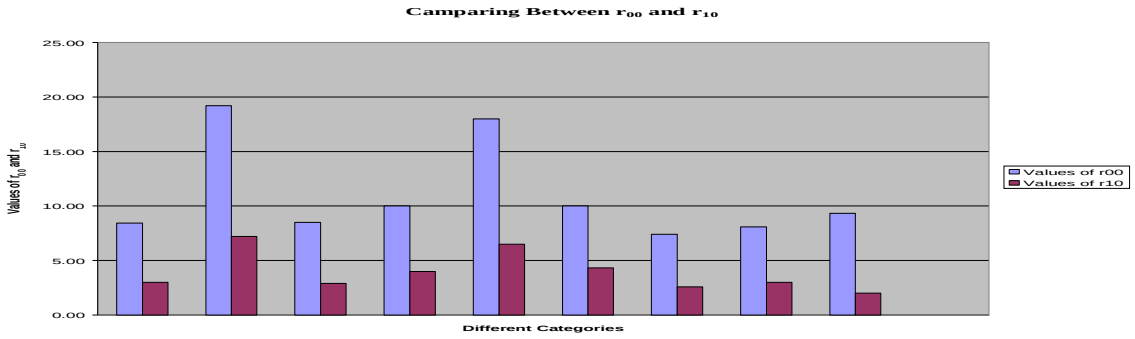


Figure 2: Comparison between r_{01} and r_{11}

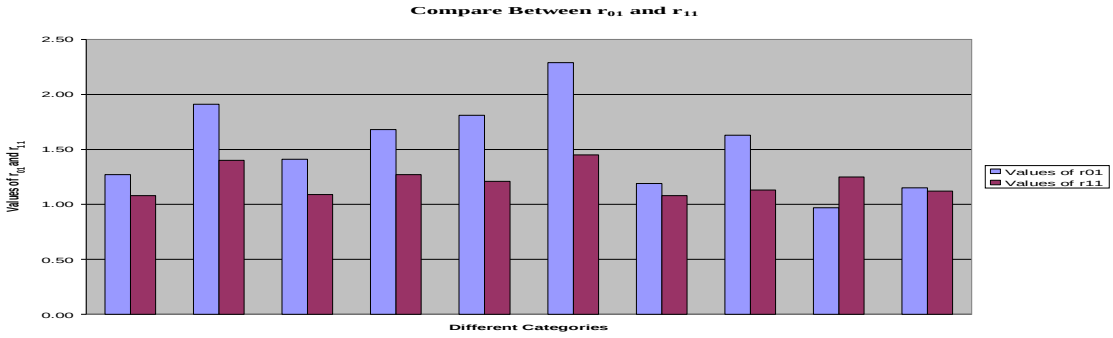
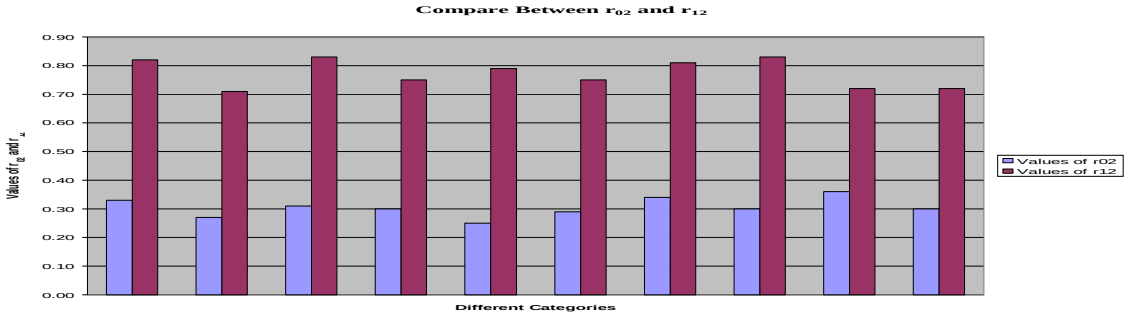


Figure 3: Comparison Between r_{02} and r_{12}



Performance measures of English medium students for Grade 0, 1 and 2.

Table 7: Performance Measures of English Medium Schools for Grade '0'

Test	No. of Students	$R_j(k, k-1)$	$R_j(k, 1)$	$R_j(k, \text{m}^*)$
1	96	0.000	1.000	1.096
2	48	0.500	0.500	0.548
3	197	4.104	2.052	2.249
4	159	0.807	1.656	1.816
5	107	0.673	1.115	1.221
6	36	0.336	0.375	0.411
7	29	0.806	0.302	0.331
8	85	2.931	0.885	0.970
9	55	0.647	0.573	0.628
10	64	1.164	0.667	0.731

*m = mean = Total no. students in Grade '0' / 10 = 876/10 = 87.60.

Table 8: Performance Measures of English Medium Schools for Grade '1'

Test	No. of Students	$R_j(k, k-1)$	$Rj(k, 1)$	$R_j(k, \mathrm{m})$
3	197	0.000	1.000	1.431
2	150	0.761	0.802	1.089
3	85	0.567	0.455	0.617
4	134	1.576	0.717	0.973
5	163	1.216	0.872	1.184
6	144	0.883	0.770	1.046
7	127	0.882	0.679	0.922
8	100	0.787	0.535	0.726
9	145	1.450	0.775	1.053
10	132	0.910	0.706	0.959

Table 9: Performance Measures of English Medium Schools for Grade '2'

Test	No. of Students	$R_j(k, k-1)$	$R_j(k, 1)$	$R_j(k, \mathrm{m})$
1	65	0.000	1.000	0.490
2	160	2.462	2.462	1.207
3	76	0.475	1.169	0.573
4	65	0.856	1.000	0.490
5	88	1.354	1.354	0.664
6	178	2.023	2.738	1.342
7	202	1.135	3.108	1.523
8	173	0.856	2.662	1.305
9	157	0.908	2.416	1.184
10	162	1.032	2.492	1.222

Figure 4: Bar Diagram of Performance Measures of English Medium Schools for Grade '0'

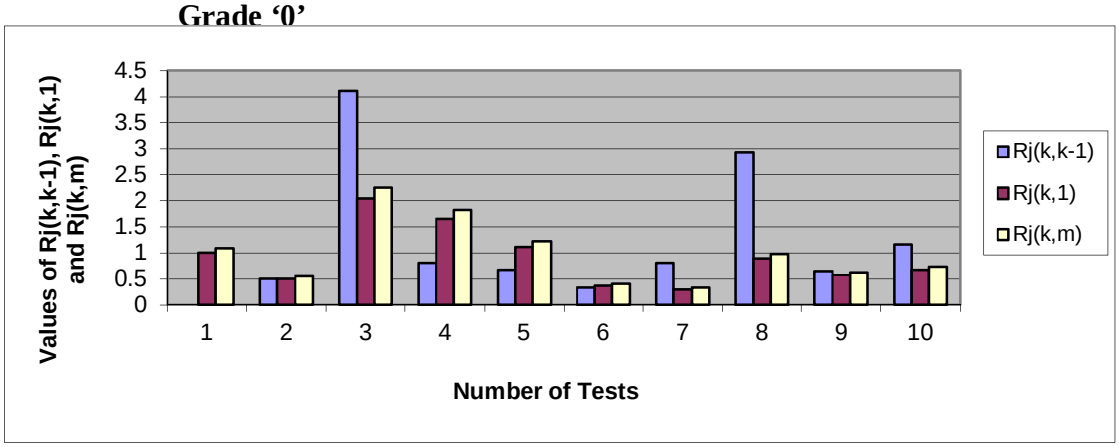


Figure 5: Bar Diagram of Performance Measures of English Medium Schools for Grade '1'

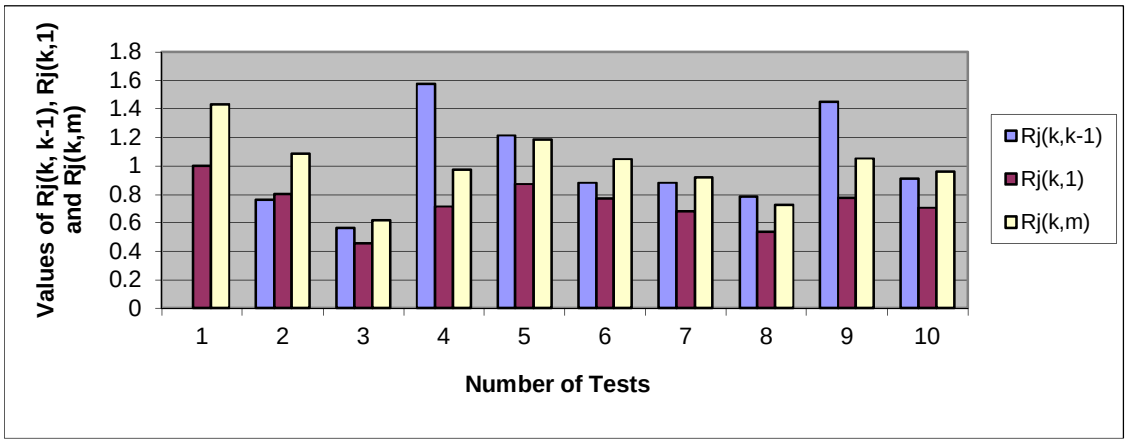
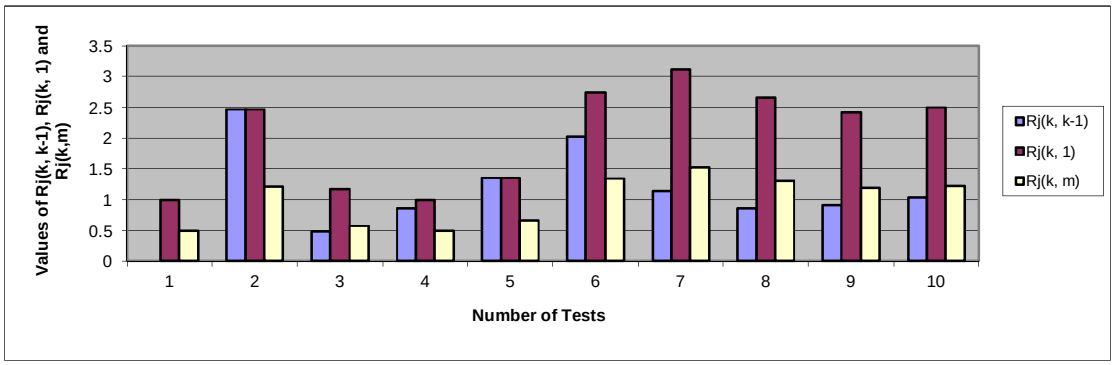


Figure 6: Bar Diagram of Performance Measures of English Medium Schools for Grade '2'



Performance measures of Bengali medium students for Grade 0, 1 and 2.

Table 10: Performance Measures of Bengali Medium Schools for Grade '0'

Test	No. of Students	$R_j(k, k-1)$	$R_j(k, 1)$	$R_j(k, \mathrm{m})$
1	23	0.000	1.000	0.619
2	87	3.783	3.783	2.343
3	66	0.759	2.869	1.778
4	21	0.318	0.913	0.566
5	6	0.286	0.261	0.162
6	45	0.750	1.957	1.212
7	24	0.533	1.043	0.646
8	25	1.042	1.087	0.673

Table 11: Performance Measures of Bengali Medium Schools for Grade '1'

Test	No. of Students	$R_j(k, k-1)$	$R_j(k, 1)$	$R_j(k, m)$
1	74	0.000	1.000	0.959
2	64	0.865	0.865	0.829
3	82	1.281	1.108	1.063
4	79	0.963	1.068	1.024
5	77	0.975	1.041	0.999
6	66	0.857	0.892	0.856
7	96	1.455	1.297	1.245
8	79	0.823	1.068	1.024

Table 12: Performance Measures of Bengali Medium Schools for Grade '2'

Test	No. of Students	$R_j(k, k-1)$	$R_j(k, 1)$	$R_j(k, m)$
1	91	0.000	1.000	1.234
2	37	0.406	0.381	0.502
3	40	1.081	0.412	0.542
4	88	2.200	0.907	1.193
5	105	1.193	1.082	1.424
6	77	0.733	0.794	1.04
7	68	0.883	0.701	0.922
8	84	1.235	0.865	1.139

Figure 7: Bar Diagram of Performance Measures of Bengali Medium Schools for Grade '0'

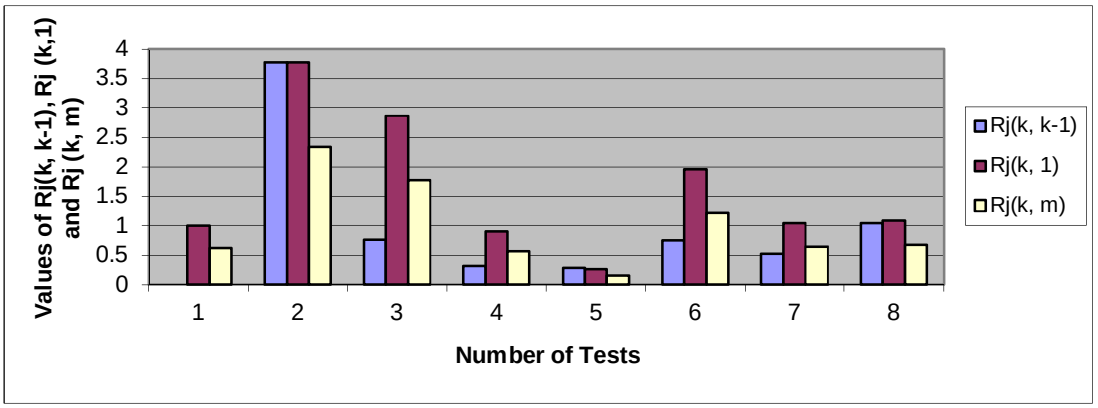


Figure 8: Bar Diagram of Performance Measures of Bengali Medium Schools for Grade '1'

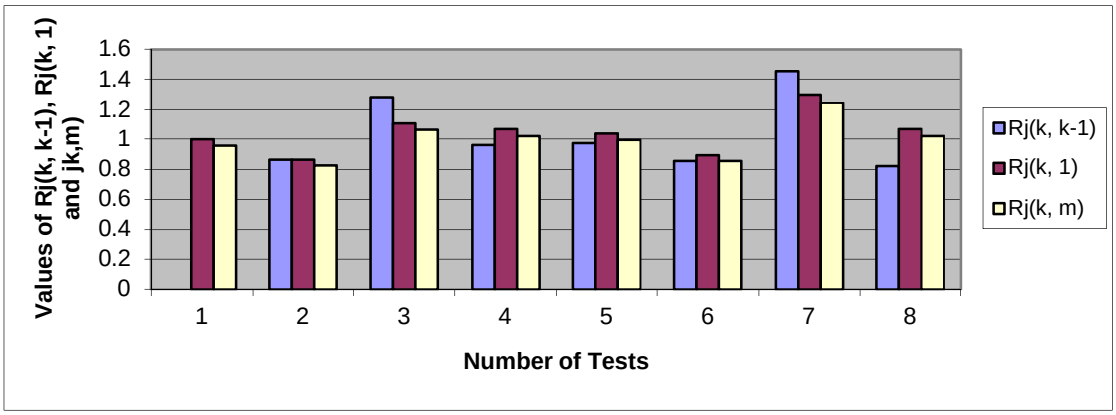


Figure 9: Bar Diagram of Performance Measures of Bengali Medium Schools for Grade '2'