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Reliability Estimation in a Rayleigh Pareto Model with Progressively Type-II Right Censored Data

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Abstract

In this paper, we consider inference problems including estimation for a Rayleigh Pareto distribution under progressively type-II right censored data. We use two approaches, the classical maximum likelihood approach and the Bayesian approach for estimating the distribution parameters and the reliability characteristics. Bayes estimators and corresponding posterior risks (PR) have been derived using different loss functions (symmetric and asymmetric). The estimators cannot be obtained explicitly, so we use the method of Monte Carlo. Also we use the integrated mean square error (IMSE) and the Pitman closeness criterion to compare the results of the two methods. Finally, a real data set has been analyzed to investigate the applicability and the usefulness of the proposed model.

Key Words: Rayleigh Pareto Distribution; Progressive Censoring; Bayesian Estimation; Maximum Likelihood Estimation; Monte Carlo Methods.

Mathematical Subject Classification: 60N01, 62F15, 62N02, 62N05.

1. Introduction

In a context of survival analysis, the Rayleigh and Pareto distributions play a very important role in the modeling of certain random phenomena. Each of these distributions has a specificity that characterizes it, however in the adjustment of certain data generally with heavy tails, it is wise to use a composition of laws. This process has been used by various authors, we can cite among the most recent Boumaraf et al. (2020) who studied the maximum likelihood estimators of Beta-Pareto distribution; Maurya et al. (2019) who studied the estimation of unknown parameters of the inverted exponentiated Rayleigh model; Kim et al. (2011) who were interested in the Bayesian estimation of the parameters of an Exponentiated -Weibull distribution or else, Valiollahi et al. (2018) who were interested in the estimation and prediction for the Power-Lindley model. All of these studies were performed in the context of progressively right-censored data. Progressive right-censored data were defined by Balakrishnan et al. (2000) , Balakrishnan (2007). They gave the algorithm to generate progressively censored observations and Balakrishnan and Cramer (2014) where they applied this process in an industrial context for classical distributions namely, Weibull, Log-normal... . Several authors have used progressively right-censored data in the case of classical distributions; we can cite Chadli et al. (2017) for a Rayleigh distribution or Yadav et al. (2019) for Hjorth distribution.
In this paper, We are interested in the Rayleigh Pareto (RP) model introduced by Al-Kadim and Mohammed (2018),

they gave the main statistical characteristics of this distribution and approached the estimation of the parameters of the law by maximum likelihood. The purpose of this article is twofold; on the one hand we are interested in the study of the estimators of the parameters, the reliability function and the hazard rate function using a Bayesian approach under different loss functions symmetric (quadratic loss function) and asymmetric (Linex and Entropy) by taking informative prior laws of the Gamma type for the parameters of the Pareto distribution and a non-informative prior law for the parameter of the Rayleigh distribution; on the other hand, we do an analysis of real data to show the relevance of the RP model as an alternative to a classical model. Simulations were carried out using the Balakrishnan algorithm to generate progressively right-censored data on the one hand and on the other hand, the Markov chain Monte Carlo (MCMC) methods and the Metropolis-Hastings algorithm were used for the calculation of estimators.
The rest of this article is arranged as the following. After briefly introducing the progressive type-II censoring in section 2, we present the model of RP in section 3. In section 4, we drive the maximum likelihood estimators (MLEs) of the parameters α, β ,γ and the reliability characteristics under progressive type-II censored data. In section 5, we discuss the Bayesian estimators under different loss functions. In section 6, a simulation study is conducted for obtaining the Bayes estimators and the MLEs. A comparison between the performance of the two methods by using the closeness Pitman criterion and the IMSE is provided in section 7. In section 8, we provide an application of the RP distribution to The Floyd River flood data. Finally, a conclusion is given in Section 9.

2. Progressive Type-II Right Censored Data

In survival tests or reliability, it often happens that data are lost for different reasons, so we say that the data are censored. When the observation time is prefixed and the number of failures is random, we say that the data are censored of type-I. If the number of observations is prefixed for obvious cost reasons and the duration of life-test is random, therefore we say that the data are censored of type-II. These two censoring schemes (type-I and type-II) are the most popular.

The loss of units at points other than the final termination point may be unavoidable, as in the case of the accidental breakage of experimental units or the loss of contact with individuals under experiment. These reasons lead us to the progressive censoring.

A progressive censoring scheme can be described as follows, suppose n identical units are placed on a life-testing experiment at time zero, with the corresponding lifetimes $X_1, X_2, ..., X_n$ being independent and identically distributed. After the first failure R_1 surviving items are removed randomly from further observation, after the second failure, R_2 surviving items are randomly removed too. This experiment stops at the time when the $m\ th$ failure is observed and the remaining $R_m = n - R_1 - R_2 - ... - R_{m-1} - m$ surviving units are withdrawn. So the set of an observed lifetime $X_{1:m:n}, X_{2:m:n}, ..., X_{m:m:n}$ is progressively type-II right censored sample.

It's clear that when $R_1 = R_2 = ... = R_{m-1} = 0$ and $R_m = n - m$, this scheme includes the conventional type-II right censoring scheme, and complete sampling scheme (no censoring) when $n = m$ and $R_1 = R_2 = ... = R_m = 0$. The probability density function (pdf) of progressive type-II right censored data is given by

$$f_{X_{1:m:n},X_{2:m:n},...,X_{m:m:n}}(x_1, x_2, ..., x_m) = C\prod_{i=1}^{m} f(x_i)[1 - F(x_i)]^{R_i}, \tag{1}$$

Where $C = n(n - R_1 - 1)(n - R_1 - R_2 - 2)...(n - \sum_{i=1}^{m-1}(R_i + 1))$.

3. The Model

In this study, we consider Rayleigh Pareto distribution, it's proposed and studied recently by Al-Kadim and Mohammed (2018). The cumulative distribution function (cdf) is given by

$$F(x) = \int_0^{\frac{1}{1-F^{'}(x)}} f^{''}(t)dt, \tag{2}$$

where $F^{'}(x)$ is the cdf of Pareto distribution, $F^{'}(x) = 1 - (\frac{\gamma}{x})^{\theta}$ and $f^{''}(t)$ is the probability density function of the Rayleigh distribution, $f^{''}(t) = \frac{x}{\beta^2}\exp(-\frac{1}{2}(\frac{x^2}{\beta^2}))$.

Hence, the cdf of the RP distribution is given by

$$F_{R.P}(x;\alpha,\beta,\gamma) = 1 - \exp(-\frac{1}{2\beta^2}(\frac{x}{\gamma})^{\alpha}); \alpha,\beta,\gamma > 0. \tag{3}$$

The probability density function is defined by

$$f_{R.P}(x;\alpha,\beta,\gamma) = \frac{\alpha}{2\beta^2\gamma}(\frac{x}{\gamma})^{\alpha-1}\exp\{-\frac{1}{2\beta^2}(\frac{x}{\gamma})^{\alpha}\}. \tag{4}$$

When $\alpha = 1$, the RP distribution reduces to the exponential distribution with parameter $\lambda = \frac{1}{2\beta^2\gamma}$.

When $\beta = \sqrt{1/2}$, the RP distribution reduces to the Weibull distribution $W(x;\alpha,\gamma)$.

The Reliability and the hazard rate function are respectively given by

$$R(t) = \exp(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^{\alpha}),\ t > 0, \tag{5}$$

and

$$h(t) = \frac{\alpha}{2\beta^2\gamma}(\frac{t}{\gamma})^{\alpha-1},\ t > 0. \tag{6}$$

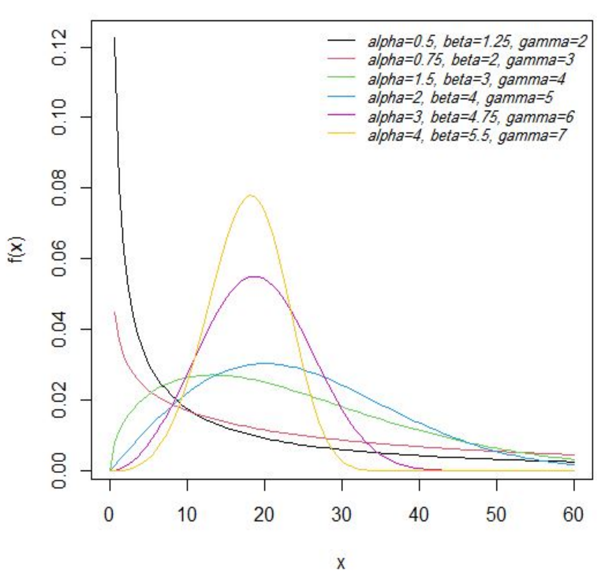

Figure 1: The pdf of RP distribution for various values of parameters.

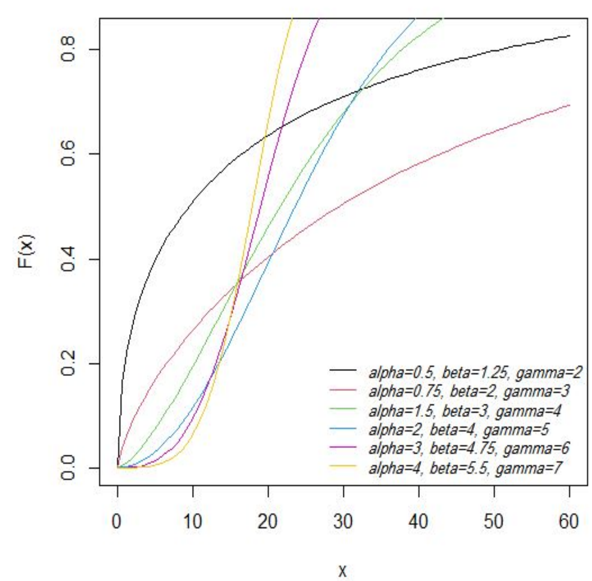

Figure 2: The cdf of RP distribution for various values of parameters.

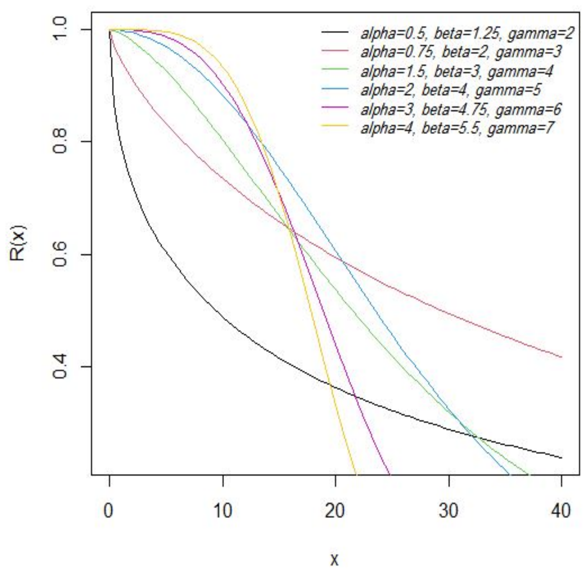

Figure 3: The reliability function of RP distribution for various values of parameters.

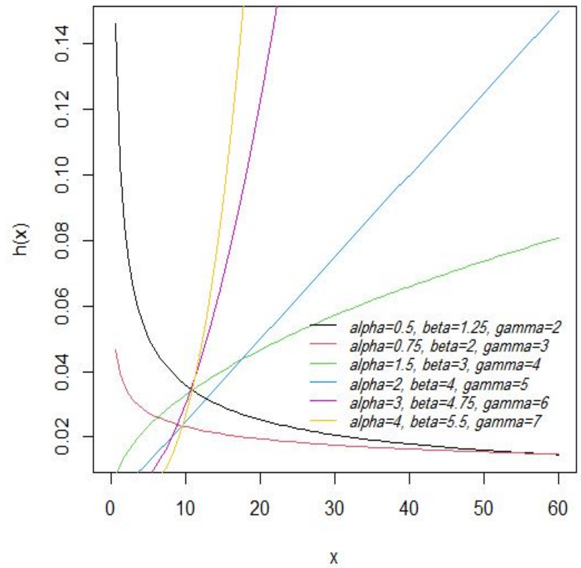

Figure 4: The hazard rate function of RP distribution for various values of parameters.

4. Maximum Likelihood Estimation

Let $X = (X_{1:m:n}, X_{2:m:n}, ..., X_{m:m:n})$ be the ordered m observed failures under type-II progressively censored scheme from the Rayleigh Pareto distribution, obtained from a sample of size n with the censoring scheme $R_i = (R_1, ..., R_m)$.
The likelihood function is then given by

$$L(x \mid \alpha, \beta, \gamma) \propto \frac{\alpha^m}{\beta^{2m}\gamma^{m\alpha}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp\{-\frac{1}{2\beta^2} \sum_{i=1}^{m} (\frac{x_i}{\gamma})^{\alpha}(1 + R_i)\}. \tag{7}$$

We set $\ln L(x \mid \alpha, \beta, \gamma) = l(x \mid \alpha, \beta, \gamma)$, so the log-likelihood function is

$$l(x \mid \alpha, \beta, \gamma) \propto m \ln \alpha - 2m \ln \beta - m\alpha \ln \gamma + (\alpha - 1) \sum_{i=1}^{m} \ln(x_i) - \frac{1}{2\beta^2} \sum_{i=1}^{m} (\frac{x_i}{\gamma})^{\alpha}(1 + R_i). \tag{8}$$

Therefore, we obtain the maximum likelihood estimators of α, β and γ by solving the following non-linear equations
$\frac{\partial l}{\partial \alpha} = \frac{m}{\alpha} - m \ln \gamma + \sum_{i=1}^{m} \ln(x_i) - \frac{1}{2\beta^2} \sum_{i=1}^{m} (\frac{x_i}{\gamma})^{\alpha}(1 + R_i) \ln(\frac{x_i}{\gamma}) = 0,$

$\frac{\partial l}{\partial \beta} = \frac{-2m}{\beta} + \frac{1}{\beta^3} \sum_{i=1}^{m} (\frac{x_i}{\gamma})^{\alpha}(1 + R_i) = 0,$

$\frac{\partial l}{\partial \gamma} = -\frac{m\alpha}{\gamma} + \frac{1}{2\beta^2} \sum_{i=1}^{m} x_i^{\alpha} (\frac{\alpha}{\gamma^{\alpha+1}})(1 + R_i) = 0.$

We see that there is no analytical solution of this system then we use the numerical methods to obtain the estimators $\hat{\alpha}_{MLE}, \hat{\beta}_{MLE}$ and $\hat{\gamma}_{MLE}$.
when we obtain the parameters simultaneously, the reliability characteristics can be written as

$$\hat{R}(t)_{MLE} = \exp(-\frac{1}{2\widehat{\beta}^2}(\frac{t}{\widehat{\gamma}})^{\widehat{\alpha}}) \tag{9}$$

and

$$\hat{h}(t)_{MLE} = \frac{\widehat{\alpha}}{2\widehat{\beta}^2\widehat{\gamma}}(\frac{t}{\widehat{\gamma}})^{\widehat{\alpha}-1}. \tag{10}$$

5. Bayesian Estimation

In this section, using the progressive censored data, we provide the Bayes estimators of the unknown parameters, their corresponding posterior risks and the reliability characteristics under the three loss functions: quadratic loss function($L_Q(\hat{\phi},\phi)=(\hat{\phi}-\phi)^2$), the entropy loss function ($L_E(\hat{\phi},\phi)\propto(\frac{\phi}{\hat{\phi}})^p-p\ln(\frac{\phi}{\hat{\phi}})-1$) and Linex loss function($L_L(\hat{\phi},\phi)\propto \exp(r(\hat{\phi}-\phi))-r(\hat{\phi}-\phi)-1$).

5.1. The prior density

Here we assume that the unknown parameters are independent. We propose for the priors of γ and α : $G(a,b)$ and $G(c,d)$ distributions, respectively with the following densities

$$\pi(\gamma)=\frac{b^a}{\Gamma(a)}\gamma^{a-1}\exp(-b\gamma),$$

$$\pi(\alpha)=\frac{d^c}{\Gamma(c)}\alpha^{c-1}\exp(-d\alpha)$$

and the non-informative prior for β

$$\pi(\beta)\propto\frac{1}{\beta}.$$

So, the prior density is

$$\pi(\alpha,\beta,\gamma)\propto\frac{b^a d^c}{\Gamma(a)\Gamma(c)}\frac{\alpha^{c-1}\gamma^{a-1}}{\beta}\exp(-d\alpha-b\gamma). \quad (11)$$

5.2. The posterior density

The joint posterior density function of α,β and γ is given by

$$\pi(\alpha,\beta,\gamma\mid x)=A^{-1}\frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}\prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}], \quad (12)$$

where

$$A = \int_0^\infty\int_0^\infty\int_0^\infty \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}\prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma.$$

5.3. The Bayes estimators

In this subsection, we estimate the parameters of the Rayleigh Pareto distribution with the Bayesian method under three loss functions (quadratic, entropy and Linex).

- Under quadratic loss function, the Bayesian estimators of α,β and γ are

$$\hat{\alpha}_{BQ}=A^{-1}\iiint\frac{\alpha^{m+c}}{\beta^{2m+1}\gamma^{\alpha m-a+1}}\prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma,$$

$$\hat{\beta}_{BQ}=A^{-1}\iiint\frac{\alpha^{m+c-1}}{\beta^{2m}\gamma^{\alpha m-a+1}}\prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma,$$

$$\hat{\gamma}_{BQ}=A^{-1}\iiint\frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a}}\prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BQ}) = E_\pi(\alpha^2) - 2\hat{\alpha}_{BQ} E_\pi(\alpha) + \hat{\alpha}_{BQ}^2,$$

$$PR(\hat{\beta}_{BQ}) = E_\pi(\beta^2) - 2\hat{\beta}_{BQ} E_\pi(\beta) + \hat{\beta}_{BQ}^2,$$

$$PR(\hat{\gamma}_{BQ}) = E_\pi(\gamma^2) - 2\hat{\gamma}_{BQ} E_\pi(\gamma) + \hat{\gamma}_{BQ}^2.$$

- Under the entropy loss function, the Bayesian estimators of α, β and γ are

$$\hat{\alpha}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-p-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma]^{-1/p},$$
$$\hat{\beta}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+p+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma]^{-1/p},$$
$$\hat{\gamma}_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-p-1}}{\beta^{2m+1}\gamma^{\alpha m-a+p+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma]^{-1/p}$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BE}) = pE(\ln(\alpha) - \ln(\hat{\alpha}_{BE})),$$

$$PR(\hat{\beta}_{BE}) = pE(\ln(\beta) - \ln(\hat{\beta}_{BE})),$$

$$PR(\hat{\gamma}_{BE}) = pE(\ln(\gamma) - \ln(\hat{\gamma}_{BE})).$$

- The Bayesian estimators of α, β and γ under Linex loss function are

$$\hat{\alpha}_{BL} = -\frac{1}{r} \ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + r\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma,$$
$$\hat{\beta}_{BL} = -\frac{1}{r} \ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + r\beta + d\alpha + b\gamma\}] d\alpha d\beta d\gamma,$$
$$\hat{\gamma}_{BL} = -\frac{1}{r} \ln(A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + r\gamma + d\alpha + b\gamma\}] d\alpha d\beta d\gamma$$

and the corresponding posterior risks are

$$PR(\hat{\alpha}_{BL}) = r(\hat{\alpha}_{BQ} - \hat{\alpha}_{BL}),$$
$$PR(\hat{\beta}_{BL}) = r(\hat{\beta}_{BQ} - \hat{\beta}_{BL}),$$
$$PR(\hat{\gamma}_{BL}) = r(\hat{\gamma}_{BQ} - \hat{\gamma}_{BL}).$$

- The Bayesian estimators of the reliability and the hazard rate function under the three loss functions are given by

$$\hat{R}(t)_{BQ} = A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} e^{(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^\alpha)} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}],$$
$$\hat{h}(t)_{BQ} = A^{-1} \iiint \frac{\alpha^{m+c}}{2\beta^{2m+3}\gamma^{\alpha m-a+2}} (\frac{t}{\gamma})^{\alpha-1} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}],$$
$$\hat{R}(t)_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} e^{(\frac{p}{2\beta^2}(\frac{t}{\gamma})^\alpha)} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma]^{-1/p},$$
$$\hat{h}(t)_{BE} = [A^{-1} \iiint \frac{\alpha^{m+c-p-1}}{2\beta^{2m-2p+1}\gamma^{\alpha m-a-p+1}} (\frac{t}{\gamma})^{\alpha-p-1} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma]^{-1/p},$$
$$\hat{R}(t)_{BL} = -\frac{1}{r} \ln(A^{-1} \iiint e^{-r\exp(-\frac{1}{2\beta^2}(\frac{t}{\gamma})^\alpha)} \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m} (x_i)^{\alpha-1} \exp[-\{\frac{1}{2\beta^2} \sum_{i=1}^{m} (R_i+1)(\frac{x_i}{\gamma})^\alpha + d\alpha + b\gamma\}] d\alpha d\beta d\gamma)$$

and

$$\hat{h}(t)_{BL} = -\frac{1}{r}\ln(A^{-1}\iiint e^{-r\frac{\alpha}{2\beta^2\gamma}(\frac{t}{\gamma})^{\alpha-1}} \frac{\alpha^{m+c-1}}{\beta^{2m+1}\gamma^{\alpha m-a+1}} \prod_{i=1}^{m}(x_i)^{\alpha-1}\exp[-\{\frac{1}{2\beta^2}\sum_{i=1}^{m}(R_i+1)(\frac{x_i}{\gamma})^{\alpha}+d\alpha+b\gamma\}]d\alpha d\beta d\gamma).$$

To calculate these estimators, we use the MCMC numerical methods, such as the Metropolis-Hastings algorithm in the simulation study section.

6. Simulation Study

In this section, we perform a Monte Carlo simulation study using different sample sizes ”n” (10, 30, 50), different censoring schemes R_i (see Table1), and under different loss functions (symmetric and asymmetric). We simulated 10000 samples of the Rayleigh Pareto distribution, by assuming that $\alpha = 2$, $\beta = 0.25$ and $\gamma = 1.5$ with the hyper-parameters $a = 3, b = 2, c = 4$ and $d = 2$.

Table 1: Progressively Censoring Schemes.

(n, m)	R_i	censoring scheme
$(10, 10)$	$R_1(complete)$	$(0 * 10) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$
$(10, 5)$	$R_2(type - II)$	$(0 * 4, 5)$
$(10, 5)$	R_3	$(5, 0 * 4)$
$(30, 30)$	$R_4(complete)$	$(0 * 30)$
$(30, 20)$	$R_5(type - II)$	$(0 * 19, 10)$
$(30, 20)$	R_6	$(10, 0 * 19)$
$(50, 50)$	$R_7(complete)$	$(0 * 50)$
$(50, 40)$	$R_8(type - II)$	$(0 * 39, 10)$
$(50, 40)$	R_9	$(10, 0 * 39)$

6.1. Maximum likelihood estimation

In this study, we use the R package BB (Varadhan and Gilbert (2009)) which has high capacities for solving a non-linear system of equations, in order to obtain the MLEs of the parameters (α, β and γ) and the reliability characteristics $R(t)$ and $h(t)$ where $t = 0.25$ ($R(0.25) = 0.8007$, $h(0.25) = 1.7777$). The results are presented in Table 2 and Table 3.

From Table 2, we observe that the estimator of β is close to the real value more than the other estimators.

We notice also from Table 3 that the estimators of $R(t)$ and $h(t)$ are closer to the real values as the sample size n increases.

Table 2: Maximum Likelihood Estimation and Quadratic Errors of the RP Parameters.

(n, m)	R_i	$\hat{\alpha}_{MLE}$	$\hat{\beta}_{MLE}$	$\hat{\gamma}_{MLE}$
$(10, 10)$	R_1	1.9906(0.0081)	0.2317(0.0013)	1.6131(0.0174)
$(10, 5)$	R_2	1.9649(0.0269)	0.2503(0.0024)	1.6492(0.0323)
$(10, 5)$	R_3	1.8320(0.1498)	0.2200(0.0045)	1.7090(0.0775)
$(30, 30)$	R_4	1.9990(0.0074)	0.2350(0.0005)	1.5933(0.0126)
$(30, 20)$	R_5	1.9780(0.0070)	0.2361(0.0006)	1.6216(0.0181)
$(30, 20)$	R_6	1.9225(0.0293)	0.1728(0.0067)	1.6725(0.0396)
$(50, 50)$	R_7	2.0030(0.0072)	0.2365(0.0003)	1.5858(0.0113)
$(50, 40)$	R_8	1.9918(0.0083)	0.2359(0.0003)	1.6010(0.0136)
$(50, 40)$	R_9	2.9190(0.0116)	0.1888(0.0041)	1.6509(0.0256)

Table 3: MLEs with Quadratic Errors of the Reliability Function and the Hazard Rate Function

(n, m)	R_i	$\widehat{R}(t)_{MLE}$	$\widehat{h}(t)_{MLE}$
$(10, 10)$	R_1	$0.7823(0.0055)$	$1.9693(0.5413)$
$(10, 5)$	R_2	$0.7998(0.0078)$	$1.7684(0.6533)$
$(10, 5)$	R_3	$0.6900(0.0300)$	$2.6696(2.1197)$
$(30, 30)$	R_4	$0.7949(0.0019)$	$1.8329(0.1516)$
$(30, 20)$	R_5	$0.8021(0.0020)$	$1.8089(0.1921)$
$(30, 20)$	R_6	$0.6329(0.0331)$	$3.5540(3.8524)$
$(50, 50)$	R_7	$0.7978(0.0013)$	$1.8059(0.0921)$
$(50, 40)$	R_8	$0.7959(0.0015)$	$1.8129(0.1053)$
$(50, 40)$	R_9	$0.6815(0.0159)$	$2.9577(1.6362)$

6.2. Bayesian estimation

In this subsection, we obtain the Bayesian estimators of the unknown parameters and the reliability characteristics of RP distribution by using MCMC methods and Metropolis-Hastings algorithm in R software. In Table 4, we present the Bayes estimators $\hat{\alpha}_{BQ}$, $\hat{\beta}_{BQ}$ and $\hat{\gamma}_{BQ}$ under quadratic loss function with their corresponding posterior risks. The Bayes estimators of the distribution parameters under the entropy loss function and their corresponding posterior risks where p=-0.5,1,2 are given in Table 5. And under the Linex loss function (with r=-0.5,1,2) the results are summarized in Table 6. Then, the Bayesian estimators of $R(t)$ and $h(t)$ under the three loss functions are presented in Table 7.

It's noticed from Table 4 that the best estimators of α, β and γ are in the case of n=10.

From Table 5, it's observed that p=-0.5 gives the best estimators of the distribution parameters under entropy loss function, and from Table 6, r=-0.5 gives the best estimators of the parameters under the Linex loss function.

We remark from the Table 7, that the good results for the reliability function R(t) are under the Linex loss function. However, for the hazard rate function h(t), the estimators under the entropy loss function perform better than the other loss functions.

From the results in Table 8, we find that the Bayesian estimation of the reliability characteristics performs better than the classical maximum likelihood approach.

Table 4: Bayesian Estimation under Quadratic Loss Function with Posterior Risks between brackets.

(n, m)	R	$\widehat{\alpha}_{BQ}$	$\widehat{\beta}_{BQ}$	$\widehat{\gamma}_{BQ}$
$(10, 10)$	R_1	$2.0203(0.0735)$	$0.2773(0.0504)$	$1.5223(0.0200)$
$(10, 5)$	R_2	$2.0109(0.0011)$	$0.3064(0.0029)$	$1.5239(0.0012)$
$(10, 5)$	R_3	$2.0247(0.0014)$	$0.3132(0.0033)$	$1.5357(0.0016)$
$(30, 30)$	R_4	$2.0887(0.5475)$	$0.3492(0.3185)$	$1.5917(0.1378)$
$(30, 20)$	R_5	$2.0207(0.0017)$	$0.3162(0.0028)$	$1.5337(0.0008)$
$(30, 20)$	R_6	$1.9808(0.0016)$	$0.2938(0.0028)$	$1.4988(0.0008)$
$(50, 50)$	R_7	$1.8324(0.8387)$	$0.5716(0.4744)$	$1.5113(0.2129)$
$(50, 40)$	R_8	$2.0382(0.0062)$	$0.3232(0.0072)$	$1.5482(0.0026)$
$(50, 40)$	R_9	$2.0196(0.0061)$	$0.2941(0.0071)$	$1.5266(0.0026)$

Table 5: Bayesian Estimation under Entropy Loss Function with PR between brackets

(n, m)	R_i	$parameters$	$p = -0.5$	$p = 1$	$p = 2$
		$\hat{\alpha}_{BE}$	$1.9323(0.0036)$	$1.8818(0.0192)$	$1.8313(0.0927)$
$(10, 10)$	R_1	$\hat{\beta}_{BE}$	$0.3682(0.0154)$	$0.3415(0.0441)$	$0.3319(0.1456)$
		$\hat{\gamma}_{BE}$	$1.4940(0.0014)$	$1.4795(0.0068)$	$1.4670(0.0306)$
		$\hat{\alpha}_{BE}$	$2.0152(2.94e-05)$	$2.0149(0.0001)$	$2.01467(0.0004)$
$(10, 5)$	R_2	$\hat{\beta}_{BE}$	$0.2986(7.17e-04)$	$0.2973(0.0027)$	$0.2965(0.0107)$
		$\hat{\gamma}_{BE}$	$1.5249(3.10e-05)$	$1.5246(0.0001)$	$1.5244(0.0004)$
		$\hat{\alpha}_{BE}$	$2.0208(3.16e-05)$	$2.0204(0.0001)$	$2.0201(0.0005)$
$(10, 5)$	R_3	$\hat{\beta}_{BE}$	$0.3016(8.33e-04)$	$0.3002(0.0031)$	$0.2993(0.0124)$
		$\hat{\gamma}_{BE}$	$1.5297(3.74e-05)$	$1.5294(0.0001)$	$1.5291(0.0005)$
		$\hat{\alpha}_{BE}$	$1.4146(1.50e-02)$	$1.2967(0.0570)$	$1.2347(0.2119)$
$(30, 30)$	R_4	$\hat{\beta}_{BE}$	$0.6883(2.88e-02)$	$0.5745(0.1230)$	$0.5135(0.4705)$
		$\hat{\gamma}_{BE}$	$1.2335(5.89e-03)$	$1.1915(0.0228)$	$1.1665(0.0881)$
		$\hat{\alpha}_{BE}$	$2.0003(6.98e-05)$	$1.9993(0.0003)$	$1.9984(0.0016)$
$(30, 20)$	R_5	$\hat{\beta}_{BE}$	$0.2923(9.91e-04)$	$0.2907(0.0034)$	$0.2898(0.0131)$
		$\hat{\gamma}_{BE}$	$1.5125(3.94e-05)$	$1.5122(0.0001)$	$1.5119(0.0007)$
		$\hat{\alpha}_{BE}$	$2.0020(6.76e-05)$	$2.0010(0.0003)$	$2.0001(0.0016)$
$(30, 20)$	R_6	$\hat{\beta}_{BE}$	$0.2925(9.27e-04)$	$0.2910(0.0032)$	$0.2901(0.0123)$
		$\hat{\gamma}_{BE}$	$1.5138(3.88e-05)$	$1.5134(0.0001)$	$1.5131(0.0007)$
		$\hat{\alpha}_{BE}$	$1.1489(9.79e-03)$	$1.0928(0.0304)$	$1.0699(0.1032)$
$(50, 50)$	R_7	$\hat{\beta}_{BE}$	$0.8794(1.49e-02)$	$0.7837(0.0852)$	$0.6967(0.4060)$
		$\hat{\gamma}_{BE}$	$1.0943(4.44e-03)$	$1.0684(0.0149)$	$1.0560(0.0533)$
		$\hat{\alpha}_{BE}$	$2.0105(0.0002)$	$2.0061(0.0016)$	$2.0012(0.0081)$
$(50, 40)$	R_8	$\hat{\beta}_{BE}$	$0.3115(0.0021)$	$0.3080(0.0068)$	$0.3063(0.0244)$
		$\hat{\gamma}_{BE}$	$1.5259(0.0001)$	$1.5245(0.0006)$	$1.5233(0.0028)$
		$\hat{\alpha}_{BE}$	$2.0118(0.0002)$	$2.0075(0.0015)$	$2.0027(0.0079)$
$(50, 40)$	R_9	$\hat{\beta}_{BE}$	$0.3117(0.0021)$	$0.3083(0.0067)$	$0.3066(0.0241)$
		$\hat{\gamma}_{BE}$	$1.5269(0.0001)$	$1.5255(0.0006)$	$1.5244(0.0027)$

Table 7: Bayesian Estimation and the Quadratic Errors of the Reliability and Hazard Rate Functions.

(n, m)	R_i	$\widehat{R}(t)_{BQ}$	$\widehat{h}(t)_{BQ}$	$\widehat{R}(t)_{BE}$	$\widehat{h}(t)_{BE}$	$\widehat{R}(t)_{BL}$	$\widehat{h}(t)_{BL}$
(10,10)	R_1	0.8402 (0.0068)	1.4081 (0.6727)	0.8751 (0.0001)	1.0056 (0.0174)	0.8758 (0.0001)	1.0697 (0.0156)
(10,5)	R_2	0.8611 (0.0030)	1.2052 (0.2619)	0.8507 $(8.28e-05)$	1.2976 $(3.54e-03)$	0.8510 $(6.14e-05)$	1.3156 $(4.77e-03)$
(10,5)	R_3	0.8685 (0.0036)	1.1442 (0.3110)	0.8552 $(1.02e-04)$	1.2551 $(4.84e-03)$	0.8556 $(7.72e-05)$	1.2780 $(5.96e-03)$
(30,30)	R_4	0.9163 (0.0091)	0.7286 (1.3834)	0.8942 $(6.17e-05)$	0.6054 $(9.17e-03)$	0.8944 $(4.98e-05)$	0.6283 $(5.13e-03)$
(30,20)	R_5	0.8730 (0.0021)	1.0986 (0.1891)	0.8406 $(8.63e-05)$	1.3862 $(3.14e-03)$	0.8409262 $(6.21e-05)$	1.4047 $(5.13e-03)$
(30,20)	R_6	0.8326 (0.0021)	1.4601 (0.1959)	0.8415 $(8.77e-05)$	1.3779 $(3.21e-03)$	0.8418 $(6.32e-05)$	1.3967 $(5.20e-03)$
(50,50)	R_7	0.9301 (0.0081)	0.4527 (1.5825)	0.8892 $(4.02e-05)$	0.5198 $(1.74e-03)$	0.8893 $(3.30e-05)$	0.5233 $(7.11e-04)$
(50,40)	R_8	0.8855 (0.0046)	0.9906 (0.4083)	0.8628 (0.0001)	1.1755 (0.0058)	0.8632 $(8.93e-05)$	1.2042 $(7.70e-03)$
(50,40)	R_9	0.8521 (0.0047)	1.2975 (0.4160)	0.8635 (0.0001)	1.1699 (0.0059)	0.8639 $(9.19e-05)$	1.1994 $(7.90e-03)$

Table 6: Bayesian Estimation with PR between brackets under Linex Loss Function.

(n, m)	R_i	$parameters$	$r = -0.5$	$r = 1$	$r = 2$
		$\hat{\alpha}_{BL}$	1.9594(0.0077)	1.8980(0.0459)	1.8258(0.2363)
$(10, 10)$	R_1	$\hat{\beta}_{BL}$	0.3915(0.0044)	0.3686(0.0140)	0.3583(0.0486)
		$\hat{\gamma}_{BL}$	1.5025(0.0023)	1.4864(0.0114)	1.4717(0.0524)
		$\hat{\alpha}_{BL}$	$2.0156(1.18e-04)$	2.0149(0.0004)	2.0144(0.0019)
$(10, 5)$	R_2	$\hat{\beta}_{BL}$	$0.2992(6.93e-05)$	0.2987(0.0002)	0.2985(0.0010)
		$\hat{\gamma}_{BL}$	$1.5251(7.32e-05)$	1.5247(0.0002)	1.5244(0.0011)
		$\hat{\alpha}_{BL}$	$2.0211(1.29e-04)$	2.0204(0.0005)	2.0198(0.0020)
$(10, 5)$	R_3	$\hat{\beta}_{BL}$	$0.3023(8.28e-05)$	0.3018(0.0003)	0.3015(0.0012)
		$\hat{\gamma}_{BL}$	$1.5300(8.92e-05)$	1.5295(0.0003)	1.5291(0.0013)
		$\hat{\alpha}_{BL}$	$1.5217(3.20e-02)$	1.3405(0.1170)	1.2519(0.4114)
$(30, 30)$	R_4	$\hat{\beta}_{BL}$	$0.7476(1.14e-02)$	0.6775(0.0472)	0.6317(0.1861)
		$\hat{\gamma}_{BL}$	$1.2670(9.39e-03)$	1.2122(0.0360)	1.1800(0.1364)
		$\hat{\alpha}_{BL}$	$2.0009(1.99e-04)$	1.9995(0.0010)	1.9980(0.0051)
$(30, 20)$	R_5	$\hat{\beta}_{BL}$	$0.2932(1.30e-04)$	0.2925(0.0004)	0.2921(0.0016)
		$\hat{\gamma}_{BL}$	$1.5128(7.91e-05)$	1.5123(0.0003)	1.5119(0.0014)
		$\hat{\alpha}_{BL}$	$2.0026(1.88e-04)$	2.0012(0.0009)	1.9997(0.0050)
$(30, 20)$	R_6	$\hat{\beta}_{BL}$	$0.2933(1.23e-04)$	0.2927(0.0004)	0.2923(0.0015)
		$\hat{\gamma}_{BL}$	$1.5141(7.75e-05)$	1.5136(0.0003)	1.5132(0.0014)
		$\hat{\alpha}_{BL}$	$1.2202(2.24e-02)$	1.1138(0.0615)	1.0784(0.1938)
$(50, 50)$	R_7	$\hat{\beta}_{BL}$	$0.9119(6.05e-03)$	0.8698(0.0300)	0.8318(0.1360)
		$\hat{\gamma}_{BL}$	$1.1205(7.68e-03)$	1.0807(0.0244)	1.0630(0.0842)
		$\hat{\alpha}_{BL}$	$2.0129(6.80e-04)$	2.0074(0.0041)	1.9998(0.0234)
$(50, 40)$	R_8	$\hat{\beta}_{BL}$	$0.3140(4.41e-04)$	0.3118(0.0014)	0.3107(0.0049)
		$\hat{\gamma}_{BL}$	$1.5267(2.33e-04)$	1.5251(0.0011)	1.5237(0.0051)
		$\hat{\alpha}_{BL}$	$2.0141(6.56e-04)$	2.0088(0.0039)	2.0014(0.0228)
$(50, 40)$	R_9	$\hat{\beta}_{BL}$	$0.3142(4.30e-04)$	0.3120(0.0013)	0.3109(0.0048)
		$\hat{\gamma}_{BL}$	$1.5277(2.28e-04)$	1.5261(0.0010)	1.5247(0.0050)

Table 8: MLEs and Bayes Estimators of the Reliability Characteristics with Quadratic Errors between brackets.

(n, m)	R_i	$\widehat{R}(t)_{MLE}$	$\widehat{h}(t)_{MLE}$	$\widehat{R}(t)_{BL}$	$\widehat{h}(t)_{BE}$
$(10, 10)$	R_1	$0.7969(0.0040)$	$1.9197(0.5254)$	$0.8758(0.0001)$	$1.0056(0.0174)$
$(10, 5)$	R_2	$0.7994(0.0096)$	$1.8688(1.3082)$	$0.8510(6.14e-05)$	$1.2976(3.54e-03)$
$(10, 5)$	R_3	$0.6855(0.0367)$	$3.2914(8.4523)$	$0.8556(7.72e-05)$	$1.2551(4.84e-03)$
$(30, 30)$	R_4	$0.8077(0.0011)$	$1.7795(0.1240)$	$0.8944(4.98e-05)$	$0.6054(9.17e-03)$
$(30, 20)$	R_5	$0.8021(0.0020)$	$1.8089(0.1921)$	$0.8409262(6.21e-05)$	$1.3862(3.14e-03)$
$(30, 20)$	R_6	$0.6451(0.0301)$	$3.5749(4.1357)$	$0.8418(6.32e-05)$	$1.3779(3.21e-03)$
$(50, 50)$	R_7	$0.8101(0.0007)$	$1.7490(0.0694)$	$0.8893(3.30e-05)$	$0.5198(1.74e-03)$
$(50, 40)$	R_8	$0.8074(0.0008)$	$1.7800(0.0883)$	$0.8632(8.93e-05)$	$1.1755(0.0058)$
$(50, 40)$	R_9	$0.8075(0.0008)$	$1.7797(0.0896)$	$0.8639(9.19e-05)$	$1.1699(0.0059)$

7. Comparison Study for the Parameter Estimators

In this section, we compare the performance of the two methods for different censoring schemes, by using the Pitman closeness criterion (Pitman (1937) and Jozani et al. (2012)) and the integrated mean square error defined (respectively) as follow.

Definition 7.1. *Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be two different estimators of a parameter θ, we say that $\hat{\theta}_1$ is Pitman closer estimate than $\hat{\theta}_2$ if, for all values of the $\theta \in \Theta$*

$$P_\theta(\left|\hat{\theta}_1 - \theta\right| < \left|\hat{\theta}_2 - \theta\right|) > \frac{1}{2}.$$

Definition 7.2. *The integrated mean square error is defined as*

$$IMSE = \frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i - \theta)^2.$$

For this comparison, we select the best Bayesian estimators of α, β and γ under the three loss function (quadratic, entropy (p=-0.5), and Linex (r=-0.5)) to compare it with the MLEs .

In Table 9, we compare the MLEs with the Bayes estimators by using the Pitman criterion, so we say that the Bayesian estimator is better than the MLEs when the Pitman probability is greater than 0.5. In term of this criterion, we notice that the Bayesian estimators of α and γ seem to be more efficient than the maximum likelihood method, especially when the data are progressive censored.

In Table 10, we compare the Bayesian estimators with the MLEs by using IMSE criterion. It's observed from the results that the Bayesian method performs well, but in the complete data (R_1, R_4 and R_7) the MLEs are better than the Bayesian estimators.

Table 9: Comparison of MLE and Bayesian Estimation under the Three Loss Functions using Pitman Criterion.

n	m	R	parameters	quadratic	Entropy(p=-0.5)	Linex(a=-0.5)
			α	0.6043	0.6055	0.5982
10	10	R_1	β	0.1982	0.2187	0.1880
			γ	0.7548	0.7566	0.7520
			α	0.7297	0.7304	0.7227
10	5	R_2	β	0.3036	0.3526	0.2778
			γ	0.9614	0.9629	0.9596
			α	0.7405	0.7445	0.7289
10	5	R_3	β	0.4822	0.5125	0.4624
			γ	0.9592	0.9599	0.9579
			α	0.2175	0.2182	0.2108
30	30	R_4	β	0.0135	0.0160	0.0125
			γ	0.2932	0.2952	0.2927
			α	0.8199	0.8092	0.8311
30	20	R_5	β	0.2209	0.2649	0.1999
			γ	0.9826	0.9827	0.9823
			α	0.7827	0.7684	0.7969
30	20	R_6	β	0.8208	0.8518	0.7980
			γ	0.9932	0.9932	0.9928
			α	0.0834	0.0840	0.0826
50	50	R_7	β	$9e-04$	$9e-04$	$8e-04$
			γ	0.1148	0.1164	0.1136
			α	0.7817	0.7816	0.7770
50	40	R_8	β	0.1061	0.1275	0.0938
			γ	0.8645	0.8705	0.8563
			α	0.7727	0.7697	0.7700
50	40	R_9	β	0.5346	0.5775	0.5056
			γ	0.9782	0.9793	0.9776

Table 10: Comparison of the MLE and the Bayesian Estimation using the IMSE.

n	m	R	parameters	MLE	quadratic	Entropy(p=-0.5)	Linex(a=-0.5)
			α	0.0088	0.0759	0.0778	0.0735
10	10	R_1	β	0.0013	0.0520	0.0498	0.0536
			γ	0.0176	0.0204	0.0206	0.0230
			α	0.0241	0.0012	0.0012	0.0011
10	5	R_2	β	0.0023	0.0029	0.0024	0.0033
			γ	0.0311	0.0012	0.0011	0.0012
			α	0.1445	0.0015	0.0015	0.0015
10	5	R_3	β	0.0044	0.0034	0.0029	0.0037
			γ	0.0741	0.0016	0.0015	0.0017
			α	0.0075	0.5412	0.5440	0.5375
30	30	R_4	β	0.0005	0.3150	0.3125	0.3169
			γ	0.0128	0.1363	0.1366	0.1359
			α	0.0072	0.0015	0.0017	0.0013
30	20	R_5	β	0.0006	0.0026	0.0021	0.0031
			γ	0.0183	0.0007	0.0007	0.0007
			α	0.0270	0.0016	0.0018	0.0015
30	20	R_6	β	0.0068	0.0027	0.0022	0.0031
			γ	0.0385	0.0008	0.0008	0.0008
			α	0.0073	0.8052	0.8067	0.8033
50	50	R_7	β	0.0003	0.4584	0.4571	0.4594
			γ	0.0114	0.2018	0.2019	0.2016
			α	0.0082	0.0045	0.0046	0.0043
50	40	R_8	β	0.0003	0.0062	0.0054	0.0068
			γ	0.0136	0.0022	0.0022	0.0023
			α	0.0114	0.0053	0.0056	0.0051
50	40	R_9	β	0.0041	0.0067	0.0059	0.0073
			γ	0.0255	0.0024	0.0024	0.0025

8. Real Data Analysis

In this section, we compare the fit of the proposed model (RP) with competitive and classical models such as Rayleigh,Pareto, Beta Pareto, Log-normal. We Provide this application to real data set by using the exceedances of flood peaks (for the years 1935-1973) of the Floyd River (James, Iowa, USA), for more details see Mudholkar and Hutson (1996).

Table 11: The Consecutive Annual Flood Discharge Rates of the Floyd River at James, Iowa.

Years	Flood discharge in (ft^3/s)
1935 − 1944	1460, 4050, 3570, 2060, 1300, 1390, 1720, 6280, 1360, 7440
1945 − 1954	5320, 1400, 3240, 2710, 4520, 4840, 8320, 13900, 71500, 6250
1955 − 1964	2260, 318, 1330, 970, 1920, 15100, 2870, 20600, 3810, 726
1965 − 1973	7500, 7170, 2000, 829, 17300, 4740, 13400, 2940, 5660

To test the goodness of fit of the above models, we consider some goodness-of-fit measures including; Akaike information criterion (AIC), the corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC) given by

$$AIC = -2l + 2p\log(log(n)),$$

$$CAIC = -2l + \frac{2p}{n-p-1},$$

$$BIC = -2l + p\log(n)$$

and

$$HQIC = -2l + 2p\log(\log(n)),$$

where l is the log-likelihood, n is the size sample and p the number of parameters. In general, the smaller the values of these statistics, the better the fit to the data. The following required computations are carried out using the R software.

Table 12: The statistics AIC, CAIC, BIC and HQIC for Floyd River flood data.

Distribution	AIC	CAIC	BIC	HQIC
Rayleigh Pareto	788.2277	788.9134	793.2183	790.0183
Rayleigh	436627347	436627347	436627348	436627347
Pareto	953.1139	953.4467	956.4405	954.3072
Beta Pareto	4618.009	4619.185	4624.663	4620.396
Log-normal	901.1985	901.5319	904.5257	902.3923

In Table 12, we note that the RP model has the smallest values of the AIC, CAIC, BIC and HQIC statistics among all the tested models. So, the RP distribution could be chosen as the best model for the data set.

9. Conclusions

In this paper, we studied the Rayleigh Pareto parameters and reliability characteristics with Bayesian and maximum likelihood methods under the progressive type-II censoring. We used numerical methods to obtain the Bayes estimators and MLEs, because they are not in their explicit form. The Bayesian estimators are obtained under three loss functions by the use of MCMC method. It has been checked that we can obtain the results under complete data ($R_1 = R_2 = ... = R_m = 0$) and type-II censoring ($R_1 = R_2 = ... = R_{m-1} = 0$ and $R_m = n - m$) from the progressively censored scheme. For the reliability characteristics, we found that the method of Bayes performs better than the classical maximum likelihood method. For the parameters of the RP distribution, we used two criterions (Pitman and IMSE) to compare the performance of the two approaches and we showed that the Bayesian estimation under progressive censored data is more efficient than the maximum likelihood estimation. the application to real data of Floyd River floods shows the superiority of the proposed model than some other models.

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Notations and abbreviations

PR: posterior risk.
RP: Rayleigh Pareto model.
IMSE: integrated mean square error.
MCMC: Markov chain Monte Carlo.
MLEs: maximum likelihood estimators.
pdf: probability density function.
cdf: cumulative distribution function.
$\hat{\alpha}_{MLE}$: maximum likelihood estimator of α.
$\hat{\beta}_{MLE}$: maximum likelihood estimator of β.
$\hat{\gamma}_{MLE}$: maximum likelihood estimator of γ.
$\hat{R}(t)_{MLE}$: maximum likelihood estimator of $R(t)$.
$\hat{h}(t)_{MLE}$: maximum likelihood estimator of $h(t)$.
$\pi(.)$: prior density
$\hat{\alpha}_{BQ}$: Bayesian estimator of α under quadratic loss function.
$\hat{\beta}_{BQ}$: Bayesian estimator of β under quadratic loss function.
$\hat{\gamma}_{BQ}$: Bayesian estimator of γ under quadratic loss function.
$\hat{\alpha}_{BE}$: Bayesian estimator of α under entropy loss function.
$\hat{\beta}_{BE}$: Bayesian estimator of β under entropy loss function.
$\hat{\gamma}_{BE}$: Bayesian estimator of γ under entropy loss function.
$\hat{\alpha}_{BL}$: Bayesian estimator of α under Linex loss function.
$\hat{\beta}_{BL}$: Bayesian estimator of β under Linex loss function.
$\hat{\gamma}_{BL}$: Bayesian estimator of γ under Linex loss function.
$\hat{R}(t)_{BQ}$: Bayesian estimator of $R(t)$ under quadratic loss function.
$\hat{h}(t)_{BQ}$: Bayesian estimator of $h(t)$ under quadratic loss function.
$\hat{R}(t)_{BE}$: Bayesian estimator of $R(t)$ under entropy loss function.
$\hat{R}(t)_{BE}$: Bayesian estimator of $h(t)$ under entropy loss function.
$\hat{R}(t)_{BL}$: Bayesian estimator of $R(t)$ under Linex loss function.
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