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Rhombus Ranked Set Sampling: Estimation and Efficiency Comparison

Mishal Choudri [1], Nadia Saeed[2], Kanwal Saleem[3*]

* Corresponding Author

1. Department of Decision and System Science, Saint Joseph's University -5400, USA, mchoudri@sju.edu
2. College of Statistical and Actuarial Sciences, University of the Punjab Lahore-54000, Pakistan, nadia.stat@pu.edu.pk
3. College of Statistical and Actuarial Sciences, University of the Punjab Lahore-54000, Pakistan, kanwalsaleem16@gmail.com

Abstract

A new scheme 'Rhombus Ranked Set Sampling' (RRSS) is developed in this research together with its properties for estimating the population means. Mathematical comparison with simulation is given. The proposed method is an addition to the family of different sampling methods and generalization of 'Folded Ranked Set Sampling' (FRSS). For the simulation process, nine probability distributions are considered for the efficiency comparison of proposed scheme from which four are symmetric and rest are asymmetric among which Weibull and beta distributions which are used twice, unlike parametric values. (Al-Naseer, 2007 and Bani-Mustafa, 2011). Through simulation processes, it is observed that RRSS is competent and more reliable relative to simple random sampling (SRS), ranked set sampling (RSS) and folded ranked set sampling (FRSS). It is noted that for all the underlying distributions, an increase in the efficiency of Rhombus Ranked Set Sampling (RRSS) is achieved via increasing the size of the sample 'p'. Besides the efficiency comparison, consistency of the proposed method is also valued by using Co-efficient of Variation (CV). Secondary data on zinc (Zn) concentration and lead (Pb) contamination in different parts and tissues of freshwater fish was collected to illustrate the evaluation of RRSS against SRS, RSS, FRSS and ERSS (extreme ranked set sampling). The results obtained through real life illustration defend the simulation study and hence indicates that the RRSS estimator is efficient substitute for existing methods (Al-Omari, 2011).

Key Words: Ranked Set Sampling (RSS), Extreme Ranked Set Sampling (ERSS), Folded Ranked Set Sampling (FRSS), Rhombus Ranked Set Sampling (RRSS), Relative Efficiency (RE).

1. Introduction

'Ranked set sampling' (RSS) is a non-parametric methodology of assembling data that improves estimation through sampler's belief or by using statistics of sampling units (Bohn. 1996; Presnell & Bohn, 1999; Sroka, 2008 and Barabesi & El-Sharaawi, 2001). It is amongst one of the most accepted and reliable method commenced by McIntyre (1952) that, over the years, have also been widely studied for estimations of parameters of different distributions (Muttlak & McDonald, 1992; Fei et al., 1994 and Lam et al., 1994). Recently, the properties and their impact have also been reviewed by Wolfe (2012). It is two stage sampling scheme that holds information about measurements on the variable of interest and the ranking method follows the concept of order statistics (Hogg & Craig, 1970). The representation of RSS measured units are by $Y_{1(1:p)}$, $Y_{2(2:p)}$, ..., $Y_{p(p:p)}$. The ranked set sample works on the basic rule of taking average of the sample observations as can be seen in equation (1) with variance given in equation (2), where $Y_{v[v:p]w}$ is the v^{th} order statistics of size 'p' in w^{th} cycle

$$\bar{Y}_{RRS} = \hat{\mu}_{RRS} = \frac{1}{pn}\left(\sum_{v=1}^{p}\sum_{w=1}^{n} Y_{v[v:p]w}\right) \qquad (1)$$

$$Var(\bar{X}_{RRS}) = \frac{\sigma^2}{n} - \frac{1}{m^2}\sum_{v=1}^{m}\left(\mu_{v[v:m]} - \mu\right)^2 \qquad (2)$$

'Extreme ranked set sampling' (ERSS) is one of the revised method of RSS introduced by Samawi et al. (1996) which is more robust to imperfect ranking errors as it only in-takes the first and last units for quantification (McIntyre,1952 and Samawi et al., 1996). 'Median Ranked Set Sampling' (Muttlak, 1997), 'Double-ranked set sampling' (Al-Saleh & Al-Kadiri, 2000) and 'Multi-stage ranked set sampling' (Al-Saleh & Al-Omari, 2002) are also proposed adjustment to RSS. Likewise RSS was also practiced on concomitant variables, namely as 'Two-layer ranked set sampling' (Chen & Shen, 2003). 'Moving extreme ranked set sampling' (Al-Saleh & Al-Hadrami, 2003), 'L ranked set sampling' (LRSS) based on L Statistics (Al-Naseer, 2007) have also been used as effective procedures. A further addition named as 'Folded ranked set sampling' has been introduced in recent years to overcome the problem of wastage of sampling units (Bani-Mustafa et al., 2011). By acknowledging the literature, a new tool 'Rhombus Ranked Set Sampling' (RRSS) is introduced, an extension to 'FRSS', whose working is healthier than almost all the advancements made in RSS in the preceding years. This paper is ordered as follows: In Section 2 FRSS is briefly defined, in Section 3 RRSS is introduced along with estimation of its population mean and expressions of mean and variance are introduced. Simulation results and comparisons along with graphs are given in Section 4. Application on real data sets is presented in Section 5, Section 6 briefly displays summary and conclusion of the findings and the final section directs attention to some future proposals.

2. Folded Ranked Set Sampling

FRSS is mapped when *'p'* samples are randomly chosen all of equal size i.e. *'p'*, where *'p'* is not large so that the ranking errors can be minimized. Underneath are the steps for FRSS: (Bani-Mustafa et al., 2011).

i. Chose $\left(\frac{p+1}{2}\right)$random samples all of size '*p'*.
ii. Through visual examination, rank each unit within the sample, w.r.t the variable of interest.
iii. From the first set of sample, chose $1^{st}$and p^{th} unit for actual calculations.
iv. Pick $2^{nd}$and $(p-1)^{th}$ unit from the second sample for the actual measurement.
v. This practice is repeated until the p^{th} unit is attained.
vi. The process could be iterated n*'* times to obtain the preferred sample size.

With mean and variance as follows:

$$\bar{Y}_{FRRS} = \frac{1}{pn}\sum_{v=1}^{\frac{p+1}{2}}\sum_{w=1}^{n} \left(Y_{v[v:p]w} + Y_{v[p-v+1:p]w}\right) \; and\; for\; v < p - v + 1 \qquad (3)$$

$$Var(\bar{Y}_{FRRS}) = \frac{1}{p^2 n}\sum_{w=1}^{n}\left\{\sum_{v=1}^{p} Var\left(Y_{[v:m]}\right) + 2\sum_{v=1}^{\frac{p+1}{2}} Cov\left(Y_{[v:p]}, Y_{[p-v+1:p]}\right)\right\} \qquad (4)$$

It is interesting to note that when the observations are observed from their own row, they will be dependent and as a result covariance term exists whereas, the observations will be independent when studied from different rows.

3. Rhombus Ranked Set Sampling

The proposed procedure is fairly inspired by FRSS. The application will become more suitable when the strongly correlated variables of interest are effortlessly ranked. The execution of the process begins with the selection of *'p'* random samples each of size *'p'*, where *'p'* must not be large so that the ranking errors can be minimized (Patil et al., 2002).

3.1 Proposed Algorithm

Following are the steps to carry out RRSS scheme:

Step 1: Select *'p'* samples each of size *'p'*, randomly from the target population.

Step 2: Within each set, rank the units with accordance to the variable of interest through visual investigation.

Step 3: Sample size i.e.'*p*'will be either even or odd

- For even sample size:

  Initiating from $(p/2+1)^{th}$ sample set chose 1^{st} and p^{th} ranked unit, from $(p/2+2)^{th}$ set select 2^{nd} and $(p-1)^{th}$ ranked unit and continue this process till v^{th} and $(p-w)^{th}$ ranked units are selected from $(p/2+v)^{th}$ sample set where $v= 1, \ldots, (p/2)$ and $w< v$ for every sample.

  Repeat the above procedure for the remaining 1^{st} to $(p/2)^{th}$ samples vice versa, for the sample set $(p/2)^{th}$ pick 1^{st} and p^{th} ranked unit, for $(p/2-1)^{th}$ sample chose 2^{nd} and $(p-1)^{th}$ unit, perform the same process till from $(p/2-v)^{th}$ sample set w^{th} and $(p-v)^{th}$ ranked units are selected where $w= 1, \ldots, (p/2)$ and $v < w$ for every sample.

- For odd sample size:

  Starting from $((p+1)/2)^{th}$ sample set select 1^{st} and p^{th} ranked unit, select 2^{nd} and $(p-1)^{th}$ ranked unit from $(((p+1)/2)+1)^{th}$ set and repeat until from $(((p+1)/2)+v)^{th}$ sample set w^{th} and $(p-v)^{th}$ ranked units are selected where $w= 1, \ldots, ((p+1)/2)$ and $v < w$ for every sample.

  From remaining 1^{st} to $(((p+1)/2)-1)^{th}$ samples repeat the same process but vice versa, for $(((p+1)/2)-1)^{th}$ sample select 2^{nd} and $(p-1)^{th}$ unit, repeat this process until from $(((p+1)/2)-v)^{th}$ sample set w^{th} and $(p-v)^{th}$ ranked units are selected where $w= 2, \ldots,(((p+1)/2)-1)$ and $v < w$ for every sample.

Step 4: The process could be repeated *'n'* times to obtain the preferred sample sizei.e. N.

Without loss of generality, assume that the cycle repeats only once, $n=1$.Samples are ranked critically since it is supposed that the judgment ranking provides accurate and precise details as the actual ranking.Observed values whether from the same rows or different rows are assumed to be independently distributed.

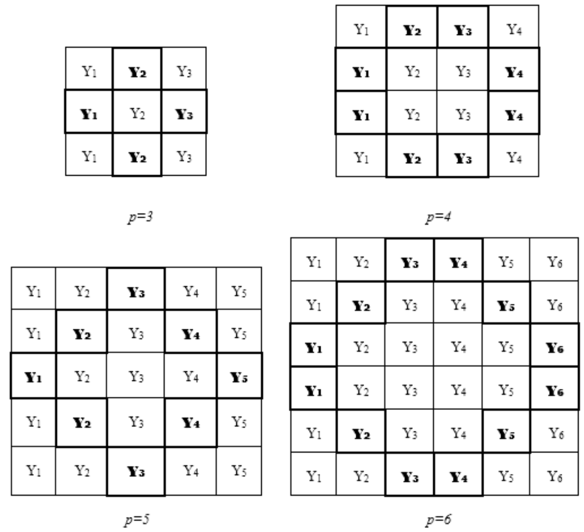


Figure 1: Selection scheme of RRSS for different sample size

If *'p'* is even, for the v^{th} sample $(v = 1, \dots, p)$, if it is $((p/2)-v) \geq 0$ then let $Y^*_{v\left(\frac{p}{2}-v+1:p\right)w}$ and $Y^*_{v\left(v+\frac{p}{2}:p\right)w}$ be the $((p/2)-v+1)^{th}$ and $(v+(p/2))^{th}$ selected ranked units respectively for the w^{th} cycle $(w = 1, \dots, n)$. If the v^{th} sample is $((p/2)-v) < 0$ then let $Y^*_{v\left(v-\frac{p}{2}:p\right)w}$ and $Y^*_{v\left(\frac{3p}{2}-v+1:p\right)w}$ be the $(v-(p/2))^{th}$ and $((3p/2)-v+1)^{th}$ selected ranked units respectively for the w^{th} cycle.

The estimator of the population mean using RRSS is given by

$$\bar{Y}^*_{RRSS(E)} = \frac{1}{2pn}\left(\sum_{v=1}^{2p}\sum_{w=1}^{n} f_{vw}\right) \tag{5}$$

where

$$f_{vw} = \begin{cases} Y^*_{v\left(\frac{p}{2}-v+1:m\right)w} + Y^*_{v\left(v+\frac{p}{2}:p\right)w} & if \quad \left(\frac{\mathrm{p}}{2}-\mathrm{v}\right) \geq 0 \\ Y^*_{v\left(v-\frac{p}{2}:p\right)w} + Y^*_{v\left(\frac{3p}{2}-v+1:p\right)w} & if \quad \left(\frac{\mathrm{p}}{2}-\mathrm{v}\right) < 0 \end{cases}$$

with variance given by

$$Var\left(\bar{Y}^*_{RRSS(E)}\right) = \frac{1}{4p^2n^2}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(var\left(Y^*_{v\left(\frac{p}{2}-v+1:p\right)w}\right) + var\left(Y^*_{v\left(v+\frac{p}{2}:p\right)w}\right)\right) + \sum_{v=\left(\frac{p}{2}\right)+1}^{p}\left(var\left(Y^*_{v\left(v-\frac{p}{2}:p\right)w}\right) + var\left(Y^*_{v\left(\frac{3p}{2}-v+1:p\right)w}\right)\right)\right\}\right] \tag{6}$$

In the case of odd sample size *'p'*, if the v^{th} sample $(v = 1, \ldots, p)$ is $(2 \leq v \leq ((p+1)/2))$ then let $Y^*_{v\left(\frac{p+1}{2}-v+1:p\right)w}$ and $Y^*_{v\left(v+\frac{p-1}{2}:p\right)w}$ be the $(((p+1)/2)-v+1)^{th}$ and $(v+((p-1)/2))^{th}$ ranked units respectively for the w^{th} cycle $(w = 1, \ldots, n)$. If the v^{th} sample is $(((p+1)/2) < v \leq (p-1))$ then let $Y^*_{v\left(v-\frac{p+1}{2}+1:p\right)w}$ and $Y^*_{v\left(\frac{3p+1}{2}-v:p\right)w}$ be the $(v-((p+1)/2)+1)^{th}$ and $(((3p+1)/2)-v)^{th}$ ranked units for the w^{th} cycle $(w = 1, \ldots, n)$ and if the v^{th} sample is $v=p$ then let and be the $((v+1)/2)^{th}$ ranked units from the 1^{st} and last sample, respectively. The estimator of the population mean using RRSS is given by

$$\bar{Y}^*_{RRSS(O)} = \frac{1}{2(p-1)n}\left(\sum_{v=1}^{2(p-1)}\sum_{w=1}^{n} f_{vw}\right) \tag{7}$$

where

$$f_{vw} = \begin{cases} Y^*_{v\left(\frac{p+1}{2}-v+1:p\right)w} + Y^*_{v\left(v+\frac{p-1}{2}:p\right)w} & if \quad 2 \leq v \leq \dfrac{p+1}{2} \\ Y^*_{v\left(v-\frac{p+1}{2}+1:p\right)w} + Y^*_{v\left(\frac{3p+1}{2}-v:p\right)w} & if \quad \dfrac{p+1}{2} < v \leq p-1 \\ Y^*_{\frac{v}{p}\left(\frac{v+1}{2}\right)w} + Y^*_{v\left(\frac{v+1}{2}\right)w} & if \ v = p \end{cases}$$

with variance given by

$$\begin{aligned} Var\left(\bar{Y}^*_{RRSS(O)}\right) = \frac{1}{4(p^2-1)n^2}\Bigg[\sum_{w=1}^{n}\Bigg\{ & var\left(Y^*_{v\left(\frac{p+1}{2}:p\right)w}\right) + var\left(Y^*_{p\left(\frac{p+1}{2}:p\right)w}\right) + var\left(Y^*_{\frac{p+1}{2}(1:p)w}\right) + var\left(Y^*_{\frac{p+1}{2}(p:p)w}\right) \\ & + \sum_{v=2}^{\frac{p-1}{2}}\left(var\left(Y^*_{v\left(\frac{p+1}{2}-v+1:p\right)w}\right) + var\left(Y^*_{v\left(v+\frac{p-1}{2}:p\right)w}\right)\right) \\ & + \sum_{v=\left(\frac{p+1}{2}\right)+1}^{p-1}\left(var\left(Y^*_{v\left(v-\frac{p+1}{2}+1:p\right)w}\right) + var\left(Y^*_{v\left(\frac{3p+1}{2}-v:p\right)w}\right)\right)\Bigg\}\Bigg] \end{aligned} \tag{8}$$

Let that the measured units for even and odd sample sizes are mutually independent with same probability distribution. Let $\mu^*_{(k:p)}=E\left(Y^*_{v(k:p)w}\right)$ and $\sigma^{2*}_{(k:p)}= var\left(Y^*_{v(k:p)w}\right)$ where $k = ((p/2)-v+1), (v+(p/2)), (v-(p/2)), ((3p/2)-v+1), (((p+1)/2)-v+1), (v+((p-1)/2)), (v-((p+1)/2)+1), (((3p+1)/2)-v), (1), (p), ((p+1)/2))$ (Al-Omari, 2011). Keeping these notations in view, the RRSS variance can also be stated as follows.

For even case:

$$\sigma^{2*}_{RRSS(E)} = \frac{1}{4p^2n^2}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(\sigma^{2*}_{v\left(\frac{p}{2}-v+1:p\right)w} + \sigma^{2*}_{v\left(v+\frac{p}{2}:p\right)w}\right) + \sum_{v=\left(\frac{p}{2}\right)+1}^{p}\left(\sigma^{2*}_{v\left(v-\frac{p}{2}:p\right)w} + \sigma^{2*}_{v\left(\frac{3p}{2}-v+1:p\right)w}\right)\right\}\right] \tag{9}$$

For odd case:

$$\sigma^{2*}_{RRSS(O)} = \frac{1}{4(p^2-1)n^2}\left[\sum_{w=1}^{n}\left\{\sigma^{2*}_{v\left(\frac{p+1}{2}:p\right)w} + \sigma^{2*}_{p\left(\frac{p+1}{2}:p\right)w} + \sigma^{2*}_{\frac{p+1}{2}(1:p)w} + \sigma^{2*}_{\frac{p+1}{2}(p:p)w} + \sum_{v=2}^{\frac{p-1}{2}}\left(\sigma^{2*}_{v\left(\frac{p+1}{2}-v+1:p\right)w} + \sigma^{2*}_{Y^*_{v\left(v+\frac{p-1}{2}:p\right)w}}\right) + \sum_{v=\left(\frac{p+1}{2}\right)+1}^{p-1}\left(\sigma^{2*}_{v\left(v-\frac{p+1}{2}+1:p\right)w} + \sigma^{2*}_{v\left(\frac{3p+1}{2}-v:p\right)w}\right)\right\}\right] \quad (10)$$

Mean Square Error of RRSS for even and odd sample size when underlying distribution is asymmetric is given respectively in the following equations:

$$MSE\left(\bar{Y}^*_{RRSS(E)}\right) = \frac{1}{4p^2n^2}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(\sigma^{2*}_{v\left(\frac{p}{2}-v+1:p\right)w} + \sigma^{2*}_{v\left(v+\frac{p}{2}:p\right)w}\right) + \sum_{v=\left(\frac{p}{2}\right)+1}^{p}\left(\sigma^{2*}_{v\left(v-\frac{p}{2}:p\right)w} + \sigma^{2*}_{v\left(\frac{3p}{2}-v+1:p\right)w}\right)\right\}\right] + \left(E\left(\bar{Y}^*_{RRSS(E)}\right) - \mu^*\right)^2 \quad (11)$$

$$MSE\left(\bar{Y}^*_{RRSS(O)}\right) = \frac{1}{4(p^2-1)n^2}\left[\sum_{w=1}^{n}\left\{\sigma^{2*}_{v\left(\frac{p+1}{2}:p\right)w} + \sigma^{2*}_{p\left(\frac{p+1}{2}:p\right)w} + \sigma^{2*}_{\frac{p+1}{2}(1:p)w} + \sigma^{2*}_{\frac{p+1}{2}(p:p)w} + \sum_{v=2}^{\frac{p-1}{2}}\left(\sigma^{2*}_{v\left(\frac{p+1}{2}-v+1:p\right)w} + \sigma^{2*}_{Y^*_{v\left(v+\frac{p-1}{2}:p\right)w}}\right) + \sum_{v=\left(\frac{p+1}{2}\right)+1}^{p-1}\left(\sigma^{2*}_{v\left(v-\frac{p+1}{2}+1:p\right)w} + \sigma^{2*}_{v\left(\frac{3p+1}{2}-v:p\right)w}\right)\right\}\right] + \left(E\left(\bar{Y}^*_{RRSS(odd)}\right) - \mu^*\right)^2 \quad (12)$$

Theorem: Prove $\bar{Y}^*_{RRSS(even)}$ and $\bar{Y}^*_{RRSS(even)}$ are unbiased estimators of the population mean if the underlying distribution about μ is symmetric.

Consider equation (5) for when sample size is even

$$\bar{Y}^*_{RRSS(E)} = \frac{1}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(Y^*_{v\left(\frac{p}{2}-v+1:p\right)w} + Y^*_{v\left(v+\frac{p}{2}:p\right)w}\right) + \sum_{v=\left(\frac{p}{2}\right)+1}^{p}\left(Y^*_{v\left(v-\frac{p}{2}:p\right)w} + Y^*_{v\left(\frac{3p}{2}-v+1:p\right)w}\right)\right\}\right] \quad (13)$$

Under symmetrical distribution about μ, let assume on the base of literature that $\bar{Y}^*{}_{(v:p)} = \bar{Y}^*{}_{(p-v+1:p)}$ and $\mu_{(v:p)} + \mu_{(p-v+1:p)} = 2\mu$ also $\sigma^2{}_{(v:p)} = \sigma^2{}_{(p-v+1:p)}$ (David & Nagaraja, 2003). So,

$$Y^*_{\left(\frac{p}{2}-v+1:p\right)} + Y^*_{\left(v+\frac{p}{2}:p\right)} = Y^*_{\left(v-\frac{p}{2}:p\right)} + Y^*_{\left(\frac{3p}{2}-v+1:p\right)}$$

$$\bar{Y}^*_{RRSS(E)} = \frac{2}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(Y^*_{v\left(\frac{p}{2}-v+1:p\right)w} + Y^*_{v\left(v+\frac{p}{2}:p\right)w}\right)\right\}\right] \quad (14)$$

Assume that $q = (p/2 - v + 1), (v + p/2)$ then the above equation becomes

$$\bar{Y}^*_{RRSS(E)} = \frac{2}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{\frac{p}{2}}\left(Y^*_{v(q:p)w} + Y^*_{v(q:p)w}\right)\right\}\right]$$

$$\bar{Y}^*_{RRSS(E)} = \frac{2}{2pn}\left[\sum_{w=1}^{n}\left\{2\sum_{v=1}^{\frac{p}{2}}\left(Y^*_{v(q:p)w}\right)\right\}\right]$$

$$\bar{Y}^*_{RRSS(E)} = \frac{2}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{p}\left(Y^*_{v(q:p)w}\right)\right\}\right]$$

$$\bar{Y}^*_{RRSS(E)} = \frac{1}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{2p}\left(Y^*_{v(q:p)w}\right)\right\}\right]$$

Applying expectation,

$$\bar{Y}^*_{RRSS(E)} = \frac{1}{2pn}\left[\sum_{w=1}^{n}\left\{\sum_{v=1}^{p}E\left(Y^*_{v(q:p)w}\right)\right\}\right]$$

$$\bar{Y}^*_{RRSS(E)} = \mu \qquad (15)$$

Hence it has been proved that for even number of samples that the mean of proposed scheme is an unbiased estimator of population mean under symmetrical distribution, likewise for the odd sample size.

4. Simulation Study

The efficiency of RRSS relative to SRS (for symmetric distributions) can be attained through the following formula (Al-Omari, 2011)

$$eff\left(\bar{Y}^*_{RRSS(E)},\bar{Y}_{SRS}\right) = \frac{Var(\bar{Y}_{SRS})}{Var\left(\bar{Y}^*_{RRSS(E)}\right)} \qquad (16)$$

$$eff\left(\bar{Y}^*_{RRSS(O)},\bar{Y}_{SRS}\right) = \frac{Var(\bar{Y}_{SRS})}{Var\left(\bar{Y}^*_{RRSS(O)}\right)} \qquad (17)$$

For skewed distributions, effectiveness can be assessed by incorporating the following definition:

$$eff\left(\bar{Y}^*_{RRSS(E)},\bar{Y}_{SRS}\right) = \frac{Var(\bar{Y}_{SRS})}{MSE\left(\bar{Y}^*_{RRSS(E)}\right)} \qquad (18)$$

$$eff\left(\bar{Y}^*_{RRSS(O)},\bar{Y}_{SRS}\right) = \frac{Var(\bar{Y}_{SRS})}{MSE\left(\bar{Y}^*_{RRSS(O)}\right)} \qquad (19)$$

The Tables inserted below summarized the increased efficiency by using various sampling scheme. The simulations are carried out through R language. The study took 10,000 cycles for efficiency test of the process. Furthermore, by varying the sample sizes from 3 to 8, observations were generated from symmetric and skewed distributions.

4.1 Efficiency Comparison

The following tables explain the gain in efficiency by means of several sampling procedures i.e RSS, FRSS and RRSS. The reduced amount of variance is directly proportional to perfectly ranked items in each set. It is evident from the following distributions (discussed in the Table 1-Table 6) that RRSS delivers significant outcomes as according to the texts greater the efficiency, superior the estimate (Al-Naseer, 2007 and Bani-Mustafa et al., 2011).

Table 1: The Efficiency relative to SRS for estimating the population mean with $p = 3$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	1.900689	1.586927	2.274733
Uniform (0, 1)	1.938468	1.619806	2.210398
Student-t (5)	1.808032	1.529049	2.228187
Logistic (0, 1)	1.840119	1.520984	2.216770
Exponential (1)	1.748657	1.511075	2.290616
Weibull (2, 1)	1.815457	1.563754	2.192410
Weibull (1, 3)	1.625265	1.428479	2.115486
Gamma (0.5, 1)	1.474751	1.386893	2.138804
Beta (7, 4)	1.992553	1.669410	2.320844
Beta (2, 9)	1.851630	1.574827	2.272376
Log N (0, 1)	1.369955	1.296123	2.054768

Table 2: The Efficiency relative to SRS for estimating the population mean with $p = 4$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	2.375616	1.720148	3.399560
Uniform (0, 1)	2.535309	1.708118	3.350705
Student-t (5)	2.117512	1.594729	3.264217
Logistic (0, 1)	2.217024	1.720836	3.337768
Exponential (1)	2.086292	1.595515	3.261677
Weibull (2, 1)	2.396377	1.701187	3.467639
Weibull (1, 3)	1.967228	1.655908	3.335833
Gamma (0.5, 1)	1.619501	1.568024	3.204246
Beta (7, 4)	2.393648	1.688905	3.404205
Beta (2, 9)	2.276412	1.667624	3.337134
Log N (0, 1)	1.482573	1.369666	2.887037

Table 3: The Efficiency relative to SRS for estimating the population mean with $p = 5$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	2.724755	2.146754	3.764511
Uniform (0, 1)	2.978888	2.373142	3.587909
Student-t (5)	2.351561	2.109256	3.621224
Logistic (0, 1)	2.540010	2.087141	3.677086
Exponential (1)	2.144132	1.908045	3.615522
Weibull (2, 1)	2.884309	2.280303	3.728871
Weibull (1, 3)	2.303079	1.986898	3.707177
Gamma (0.5, 1)	1.884225	1.740036	3.622538
Beta (7, 4)	2.816648	2.179442	3.742866
Beta (2, 9)	2.705152	2.206215	3.679316
Log N (0, 1)	1.450402	1.413952	3.133893

Table 4: The Efficiency relative to SRS for estimating the population mean with $p = 6$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	3.214739	2.344427	4.690370
Uniform (0, 1)	3.533365	2.431436	4.812852
Student-t (5)	2.733802	2.200724	4.493076
Logistic (0, 1)	3.059141	2.331085	4.599552
Exponential (1)	2.590381	2.149536	4.379441
Weibull (2, 1)	3.243132	2.343701	4.899566
Weibull (1, 3)	2.499491	2.074259	4.124868
Gamma (0.5, 1)	1.979120	1.856055	3.697573
Beta (7, 4)	3.256671	2.426500	4.995697
Beta (2, 9)	3.006295	2.254875	4.608488
Log N (0, 1)	1.809860	1.670231	3.269977

Table 5: The Efficiency relative to SRS for estimating the population mean with $p = 7$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	3.636909	2.903593	5.283566
Uniform (0, 1)	3.990101	3.054665	4.824234
Student-t (5)	2.926974	2.736542	5.389158
Logistic (0, 1)	3.299143	2.707349	5.198378
Exponential (1)	2.914194	2.538131	5.184560
Weibull (2, 1)	3.708990	2.809191	5.038000
Weibull (1, 3)	2.833727	2.573972	5.067725
Gamma (0.5, 1)	2.407013	2.177907	4.848202
Beta (7, 4)	3.726049	2.903047	5.201646
Beta (2, 9)	3.576307	2.907880	5.110221

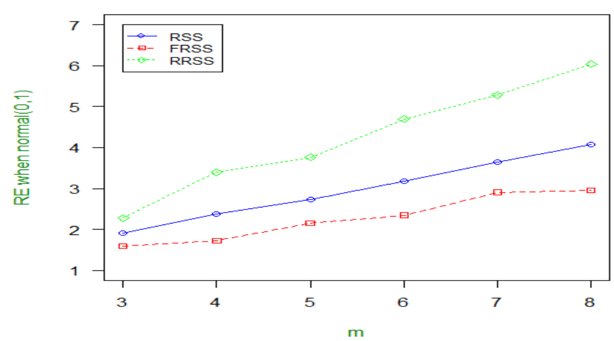
Log N (0, 1)	2.271208	2.076661	4.651208

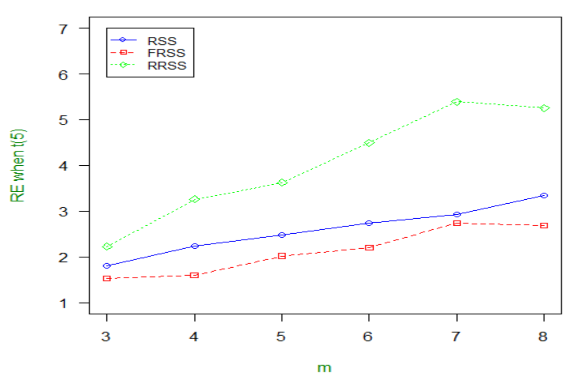
Table 6: The Efficiency relative to SRS for estimating the population mean with $p = 8$

Distribution	RSS	FRSS	RRSS
Normal (0, 1)	4.068623	2.956947	6.034384
Uniform (0, 1)	4.548384	3.060433	6.230850
Student-t (5)	3.335561	2.682162	5.257967
Logistic (0, 1)	3.590705	2.834324	5.541082
Exponential (1)	2.948848	2.478356	5.211668
Weibull (2, 1)	3.912778	2.926632	6.055105
Weibull (1, 3)	2.979072	2.617919	5.351596
Gamma (0.5, 1)	2.567787	2.386182	4.937070
Beta (7, 4)	4.302969	3.040364	6.266706
Beta (2, 9)	3.943365	3.063744	6.149940
Log N (0, 1)	2.076824	1.816431	3.859022

4.2 Graphical Illustrations

All the discussed mean estimators are found to be precise but RRSS has more precision in comparison to the others as seen in the Tables 1- Table 6. Based off different sample sizes, RE is graphically demonstrated for symmetrical (Figure 2) and asymmetrical (Figure 3) distributions for comprehension of the study. Rise in efficiency can be demonstrated effortlessly through the Figures as *'p'* increases, rise in effectiveness is seen and approximately for all the distributions a linear trend is observed for the proposed estimator





DOI: http://dx.doi.org/10.18187/pjsor.v17i1.3651

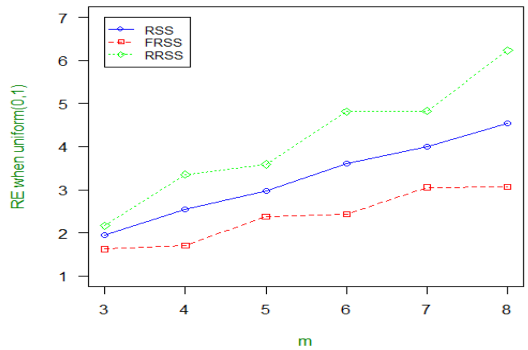
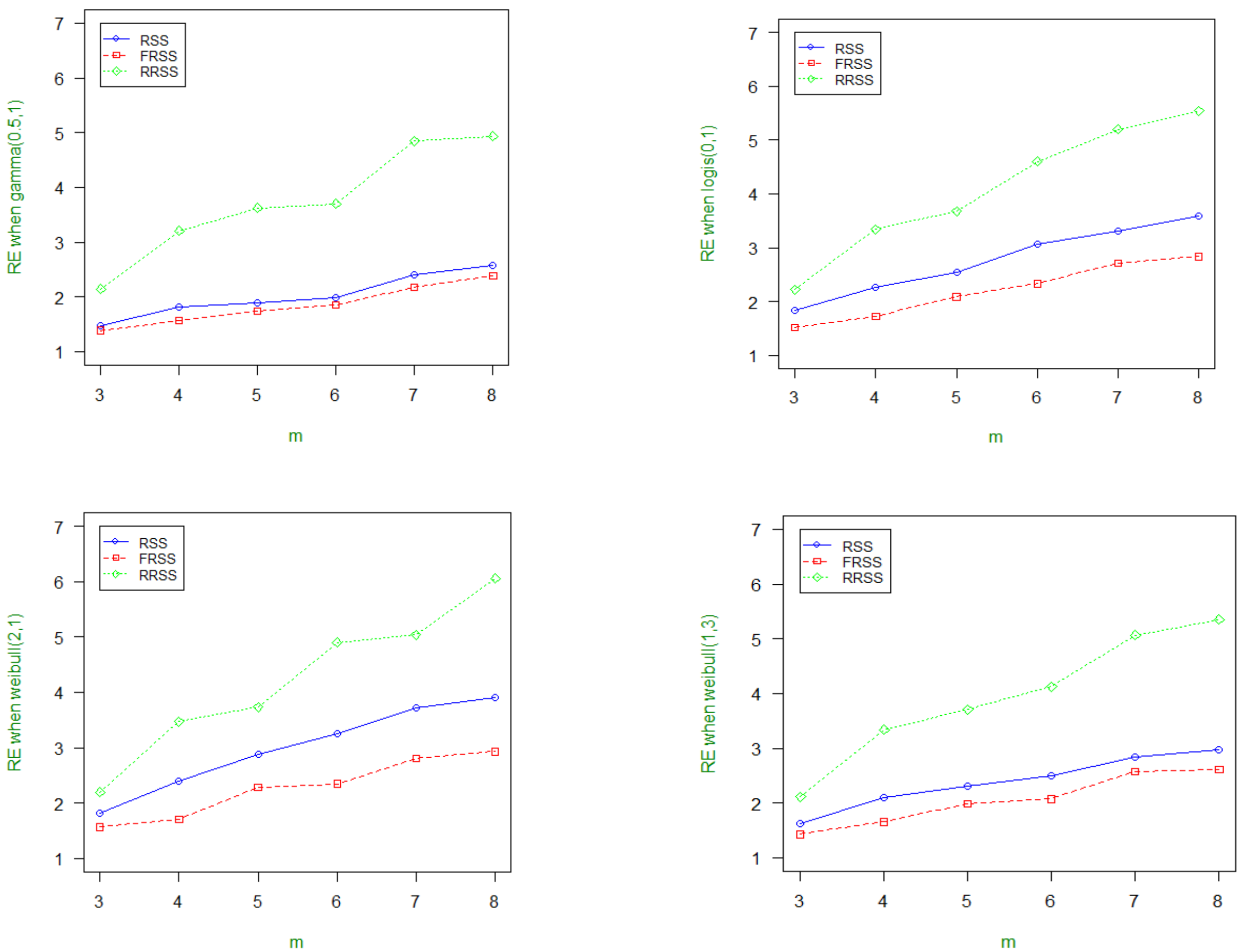


Figure 2: *Relative Efficiency for Symmetrical Distributions*



DOI: http://dx.doi.org/10.18187/pjsor.v17i1.3651

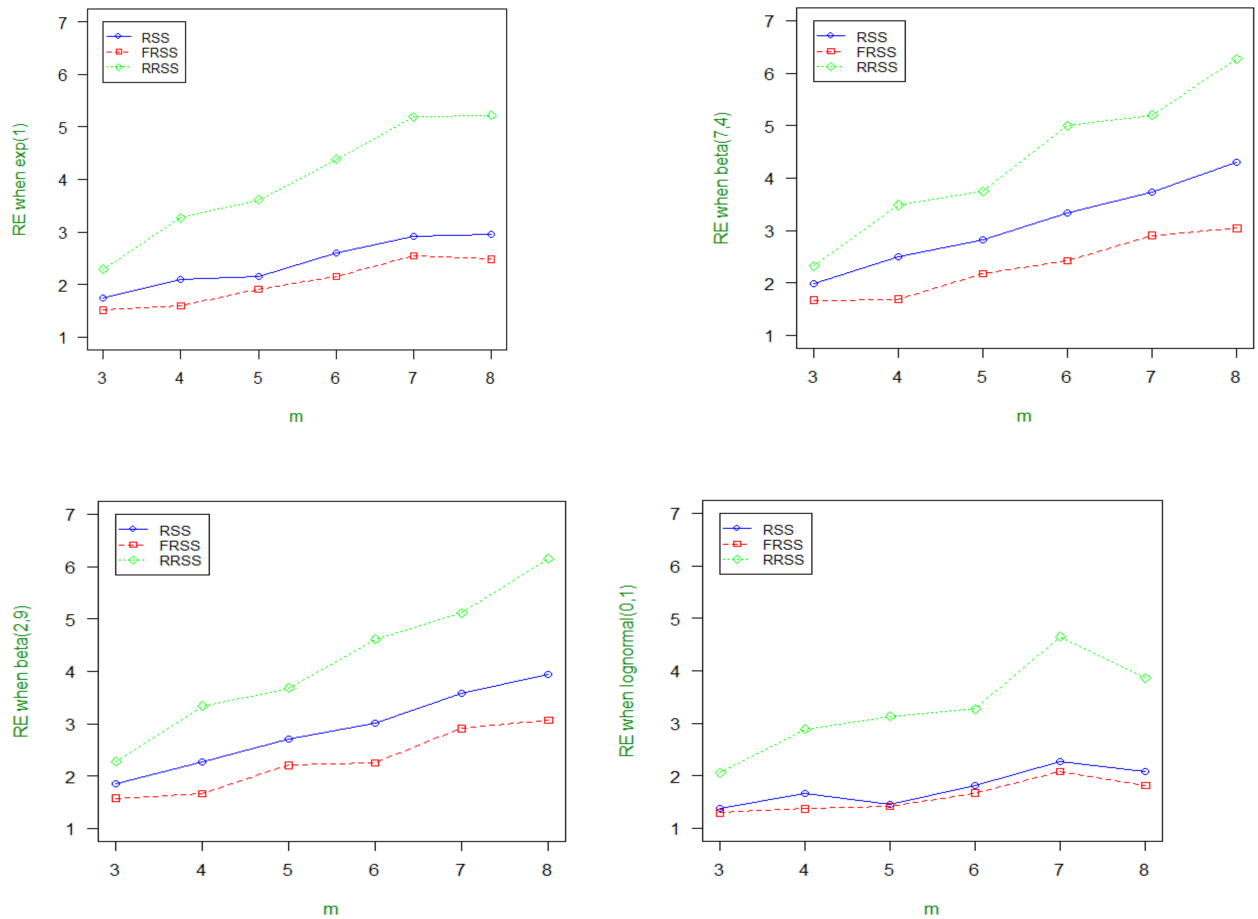


Figure 3: *Relative Efficiency for Asymmetrical Distributions*

4.3 Consistency Comparison

Larger the co-efficient of variation, greater the variability and smaller the value signifies less variability. It is manifested from Table 7 that less inconsistency is achieved with increase in sample size, this explains sure beneficial outcomes through RRSS with increase in the numbers of elements selected from portion of population, and thereby study pronounces less risk and more striking results.

Table 7: The Co-efficient of Variation for the RRSS for asymmetric and symmetric distributions

Distribution	**Sample size**					
	p= 3	*p= 4*	*p= 5*	*p= 6*	*p= 7*	*p= 8*
Normal (0, 1)	21.929	18.601	15.149	9.480	4.826	3.010
Uniform (0, 1)	0.230	0.159	0.128	0.106	0.094	0.079
Student-t (5)	27.526	17.617	10.948	8.224	2.999	2.560
Exponential (1)	0.381	0.269	0.246	0.192	0.168	0.152
Weibull (2, 1)	0.193	0.147	0.118	0.098	0.091	0.074
Weibull (1, 3)	0.391	0.277	0.248	0.185	0.182	0.153
Gamma (0.5, 1)	0.579	0.417	0.355	0.279	0.260	0.221
Beta (7, 4)	0.084	0.060	0.053	0.041	0.037	0.029
Beta (2, 9)	0.225	0.167	0.146	0.114	0.105	0.086

Log N (0, 1)	0.568	0.394	0.367	0.227	0.243	0.229

5. Application

To further establish the credibility of the RRSS, the method was applied to the data of freshwater fish to check its practical implementation. For quantification, a sample of treated fresh water fish was considered to further investigate the working of the proposed scheme. The records of mean concentration of heavy metals such as zinc (Zn) and lead (Pb) accumulated in different parts and tissues of fish were studied (Afshan et al., 2013).

Example: Mean concentration of zinc (Zn) and lead (Pb)

The efficiency of the proposed plan for $p= 3, 4, 5, 6, 7$was calculated by means of the attained data on zinc (Zn) and lead (Pb). From Table 8, it is observed that the increase in sample size gives rise to the Relative Efficiency, hence providing competent effectiveness. RRSS provides enhanced results as comparative to all the discussed plans therefore justifying the simulation study. In Table 9, 'Extreme Ranked Set Sampling' is also discussed to value the execution of RRSS. Working of ERSS in this framework is demonstrated for highlighting the process of the proposed plan. ERSS is found to be less proficient than SRS for many of skewed distributions.

Table 8: The RE for estimating the average zinc (Zn) concentrated freshwater fish

Sample Size	RSS	FRSS	RRSS
3	2.132455	1.687155	2.575721
4	2.649796	1.741772	4.283374
5	3.243720	2.495261	4.820145
6	4.354396	2.860048	8.405086
7	5.854871	4.027556	12.31199

Table 9: The RE for estimating lead (Pb) contaminated freshwater fish

Sample Size	ERSS	FRSS	RRSS
3	0.899672	1.140856	1.571429
4	0.565171	1.176044	2.195381
5	0.611542	1.187171	2.606006
6	0.545894	1.314159	2.876818
7	0.710573	1.608054	3.484297

Table 8 and 9 verifies that the improvement in Relative Efficiency is directly proportional to size of sample. Observing the outturns, for all sample sizes RRSS generates better results. Hence proving that ERSS, FRSS and SRS are less fit than 'Rhombus Ranked Set Sampling' and thus recommended to be used as an enhanced alternative.

In this paper, evidence based on real data set of fish tissue is provided that defends the literature of ERSS being least reliable than all the considered plans. It can also be observed that it is less competent when compared to SRS (Bani-Mustafa et al., 2011).

6. Conclusion

A new procedure 'Rhombus Ranked Set Sampling' has been introduced. The proposed method is divided into even and odd sample size. The number of total samples selected from even sample set are more than the odd i.e. for even, the number of samples included are '*2p'*whereas for odd sample set '*2(p-1)'* samples are integrated. Both symmetric and asymmetric distributions are used to demonstrate the effective outcomes. The results of General Monte Carlo simulations encourage the use of the proposed procedure as it has shaped attractive conclusions i.e., relative efficiency is greater than those of RSS and FRSS (Al-Naseer, 2007 and Bani-Mustafa et al., 2011) and it enhances with the increase in sample size. The achieved values of co-efficient of variation explains reduced amount of dispersion with increase in the size of *'p'* for almost all probability distributions, hence signifying consistent outcomes.With underlying symmetrical distribution RRSS is also proved as an unbiased estimator of population mean for both cases(even and odd).

The idea functioned to assess RRSS through two sets of real-lifesecondary data of zinc (Zn) and lead (Pb) concentration in fresh water fish was encouraged by literature and the observed variances justifies the proposed theory of exhibiting improved estimates. This study verifies the fact that ERSS is less capable while dealing with skewed distributions based on real data set (Bani-Mustafa et al., 2011). Therefore, it is deduced that the proposed estimators came out with better precision relative to the existing estimators based on RSS, FRSS and ERSS schemes.

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