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A Proposed Methodology for Modelling the Solvency of a National Pension Scheme

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Abstract

The main aim of this study is to fit a model for predicting pension liability. The study proposed a stochastic population model to determine the status of a pension scheme. By categorizing the members of the Social Security and National Insurance Trust (SSNIT) pension scheme of Ghana into five groups, (ie. contributors, contributors who die on the time of duty, retirees, pensioners and pensioner who die before age 72) the *birth and death process with emigration* and the *pure death process* coupled with assumption of the Yule's process, were combined to successfully formulate a model for forecasting the surplus of SSNIT to be used as a proxy for assessing the solvency status of the scheme. The reliability of the proposed model was corroborated by very high coverage probabilities of the estimates of expected surpluses produced. The study demonstrated how easy it is to use the proposed model to carry out sensitivity analysis which allows the exploration of various scenarios leading to formulation and implementation of policies to enhance the solvency of the scheme. The main advantage of the proposed model is that, it uses more information (variables) compared to others proposed elsewhere for the same purpose and hence improve precision. The model allows for the estimation of the expected values of the five population groups that play major roles in the solvency of the scheme. A key finding of the study is that SSNIT would have still been solvent had she increased pension by 50%.

Key Words: Solvency, Pension scheme, birth and death process, Yule's process, Stochastic population model

Mathematical Subject Classification: *37A50, 37H10*

1. Introduction

Social Security schemes are pension schemes imposed and controlled by government units for the purpose of providing social benefits to members of the community as a whole, or of particular sections of the community (Kwabla-King, 2017). In general, humans find ways to improve their economic security. In Ghana, the Social Security and National Insurance Trust (SSNIT) under the present law is mandated to run the first and second tier pension schemes which are compulsory National Pension Scheme. There is another pension scheme, (the third tier) which is voluntary, and it is managed by private fund managers.

A major cause of economic insecurity is the possible decrease of a person's earning power at a later stage of life (World Bank Report, 2010). The United Nations estimates that by 2050 there will be almost 2 billion people over 60 globally, with about 80% of whom will be in developing countries under which Ghana is no exception. Developing countries have demonstrated an increase in the old-age dependency ratio over the years (Zubair, 2016). According to the 2010, population census the old age dependency ratio in Ghana was 6.62 which signalled the slow rise from 5.18 since 1967 (Ghana Statistical Service, 2010). Some persons in the labour force do not prepare adequately towards their retirement and hence are unable to receive good pensions on retirement (Agblobi, 2011). The Ghanaian aging population is increasing in the sense that people are now living more than their anticipated ages (Aikins & Apt, 2016). However, there is no significant rise in the number of contributors to the scheme and even those who are contributors

are now refusing to pay their contributions (Ansah, 2016). There is also the lack of contribution by some private establishments, small and medium scale enterprises, hence the need to study the sustainability of the scheme to maintain the confidence of the people. Despite these risks, limited research has been done on the solvency of the scheme. This research, focused on analysing the solvency of the pension scheme by considering the enrolment rate and rate of beneficiary applications, which other studies have not intensively researched on.
A lot of Time Series models such as ARIMA, ARCH, GARCH, EGARCH have been used in modelling pension schemes across the globe. These are mostly univariate analysis, and hence do not include other variables that may have influence on the model. For example, Ansah (2016), in modelling the SSNIT pension fund used GARCH model to determine the performance of the scheme for the period 2006 to 2010. Zubair (2016), examine the impact of pension fund investments on the performance of capital market in Nigeria, using Autoregressive Integrated Moving Average (ARIMA) regression technique.
This study proposes a stochastic model known as Pure birth and death processes with emigration, which uses other variables to ascertain the sustainability of the Pension Scheme. The advantages of this model are that in addition to income and expenditure, it involves estimation of expected values of relevant variables such as population of contributors, contributors who die on the line of duty, retirees, pensioners and pensioners who die before age 72 years in the calculation of pension scheme surplus. This contributes to the precision of estimates from the model.

2. Methods and Materials

In this study, the various categories of the population in the pension scheme; namely contributors and pensioners on one side, and those who are paid lump sum at the end of the service or dying in the line of duty or pensioners who die before age 72 on the other side are modelled respectively by *birth and death processes with emigration* and *pure death process*. In each of these cases, the Yule's process is assumed. Also, in each case the resultant difference differential equations were not solved because our interest is on the expected values or estimates of the populations. For the *birth and death process with emigration*, we provided the proof of the resultant expected value from the corresponding difference differential equations. However, for *the pure death* process, the expected value which results from the difference differential equations is stated without proof. The proof for the later can be obtained from Bhat (1984) or any standard textbook on stochastic processes. The paper presents a brief theoretical review of the *birth and death process with emigration* and the *pure death process* in subsections 2.1 and 2.2, respectively.

2.1 Birth and death process with emigration

Consider a population (SSNIT contributors) subject to death (in this case retiring), birth (new contributors) and emigration (death during active service). When the current population size is n, let λ_n be the rate at which new members join the scheme, μ_n be the rate at which members retire and ξ be the rate at which active contributors die. In the manner of Bhat (1984), suppose the dynamics of the population are subject to the following assumptions: in a small time-interval of length Δt, the probability that the population will:

1. increase by one new member is $\lambda_n + o(\Delta t)$,
2. decrease by one retiree is $\mu_n + o(\Delta t)$,
3. decrease by one due to the death of an active contributor is $\xi + o(\Delta t)$,
4. increase or decrease by more than one is $o(\Delta t)$,

where $\lim_{t\to 0}\frac{0(\Delta t)}{\Delta t} = 0$. Also, births, deaths and emigration occur independently of each other of the population size.
Define,
$P_{i,n}(s,t) = P[X(t) = n|X(s) = i]$, $t > s$ and $P_n(t) = P_{in}(0,t) = P[X(t) = n|X(0) = i]$.
Then as a consequence of the assumptions and probabilities stated above, we get

$$\begin{aligned}P_{n,n-1}(t,t+\Delta t) &= [\mu_n\Delta t + o(\Delta t)][1-\lambda_n\Delta t + o(\Delta t)][1-\xi\Delta t + o(\Delta t)] + \\ &\quad [1-\mu_n\Delta t + o(\Delta t)][1-\lambda_n\Delta t + o(\Delta t)][\xi\Delta t + o(\Delta t)] \\ &= \mu_n\Delta t + \xi\Delta t + o(\Delta t) \\ &= (\mu_n + \xi)\Delta t + o(\Delta t) \end{aligned} \tag{1}$$

$$\begin{aligned}P_{n,n}(t,t+\Delta t) &= [\mu_n\Delta t + o(\Delta t)][\lambda_n\Delta t + o(\Delta t)][1-\xi\Delta t + o(\Delta t)] \\ &\quad +[1-\mu_n\Delta t + o(\Delta t)][\lambda_n\Delta t + o(\Delta t)][\xi\Delta t + o(\Delta t)] \\ &\quad +[1-\mu_n\Delta t + o(\Delta t)][1-\lambda_n\Delta t + o(\Delta t)][1-\xi\Delta t + o(\Delta t)] \\ &= 1-(\lambda_n + \mu_n + \varepsilon)\Delta t + o(\Delta t)\end{aligned} \tag{2}$$

$$\begin{aligned}P_{n,n+1}(t,t+\Delta t) &= [\lambda_n \Delta t + o(\Delta t)][1-\mu_n\Delta t + o(\Delta t)][1-\varepsilon\Delta t + o(\Delta t)]\\ &= \lambda_n \Delta t + o(\Delta t) \qquad (3)\end{aligned}$$

$$\sum_{i\neq n-1,n+1} P_{ni}(t,t+\Delta t) = 0(\Delta t) \qquad (4)$$

Now, let $P_n(t) = P[X(t) = n]$ be the probability that there are n contributors to the scheme within the time-interval $(0,t]$. Then for transitions occurring in non-overlapping intervals $(0,t]$ and $(t,t+\Delta t)$, based on equations (1) through (4), using the forward Chapman-Kolmogorov equations, we get;

$$P_n(t+\Delta t) = P_n(t)P_{n,n}(t,t+\Delta t) + P_{n+1}(t)P_{n+1,n}(t,t+\Delta t) + P_{n-1}(t)P_{n-1,n}(t,t+\Delta t)$$

$$P_n(t+\Delta t) = P_n(t)[1-(\lambda_n - \mu_n + \xi)\Delta t] + P_{n+1}(t)[\mu_{n+1}+\xi]\Delta t + P_{n-1}(t)[\lambda_{n-1}\Delta t] + o(\Delta t).$$

Which on rearranging and letting $\Delta t \to 0$ gives the difference differential equation

$$P'_n(t) = -(\lambda_n + \mu_n + \xi)P_n(t) + (\mu_{n+1}+\xi)P_{n+1}(t) + \lambda_{n-1}P_{n-1}(t), n = 0,1,2,3, \ . \ . \ . \qquad (5)$$

For the Yule's process (Bhat, 1984) in which $\lambda_n = n\lambda_1$ and $\mu_n = n\mu_1$, we get equation (5) to be

$$P'_n(t) = -(n\lambda_1 + n\mu_1 + \xi)P_n(t) + (n-1)\lambda_1 P_{n-1}(t) + [(n+1)\mu_1 + \xi]P_{n+1}(t) \qquad (6)$$

Now, the expected number of individuals at time t is $E[X(t)] = \sum_{n=0}^{\infty} nP_n(t)$ which when differentiated with respect to t becomes

$$\begin{aligned}E'[X(t)] &= \sum_{n=0}^{\infty} nP'_n(t)\\ &= \sum_{n=0}^{\infty} n[-(n\lambda_1 + n\mu_1 + \xi)P_n(t) + (n-1)\lambda_1 P_{n-1}(t) + [\mu_1(n+1)+\xi]P_{n+1}(t)]\\ &= -(\lambda_1+\mu_1)\sum_{n=0}^{\infty} n^2 P_n(t) - \xi\sum_{n=0}^{\infty} nP_n(t) + \lambda_1\sum_{n=0}^{\infty} n(n-1)P_{n-1}(t)\\ &\qquad +\mu_1\sum_{n=1}^{\infty}(n+1)nP_n(t) + \xi\sum_{n=1}^{\infty} nP_{n+1}(t) \qquad (7)\end{aligned}$$

Setting $n-1 = n$ and $n+1 = n$ in equation (7) gives;

$$\begin{aligned}E'[X(t)] &= -(\lambda_1+\mu_1)\sum_{n=0}^{\infty} n^2 P_n(t) - \xi\sum_{n=0}^{\infty} nP_n(t) + \lambda_1\sum_{n=0}^{\infty}(n+1)nP_n(t)\\ &\qquad +\mu_1\sum_{n=1}^{\infty} n(n-1)P_n(t) + \xi\sum_{n=1}^{\infty}(n-1)P_n(t)\\ &= (\lambda_1-\mu_1)\sum_{n=1}^{\infty} nP_n(t) - \xi\sum_{n=1}^{\infty} P_n(t)\\ &= (\lambda_1-\mu_1)E[X(t)] - \xi[1-P_0(t)]\\ &= (\lambda_1-\mu_1)E[X(t)] - \xi + P_0(t) \qquad (8)\end{aligned}$$

With initial conditions $P_n(0) = \begin{cases}1 \text{ if } n = i\\ 0 \text{ if } n \neq i\end{cases}$, $X(0) = i$ and $P_{-1}(t) = 0$, the differential equation (8) is easily solved to obtain

$$E[X(t)|X(0)=i] = \begin{cases}\left(\frac{\xi}{\lambda_1-\mu_1}\right)\left(\ell^{(\lambda_1-\mu_1)t}-1\right) + i\ell^{(\lambda_1-\mu_1)t}, & \lambda_1 \neq \mu_1\\ \xi t + i & , \quad \lambda_1 = \mu_1\end{cases} \qquad (9)$$

2.2 Pure Death Process

Given an initial size of the population of contributors to the scheme, say $i > 0$, contributors go for retirement or die at a certain rate, eventually reducing the size to zero. When the size is n, let μ_n be the rate of retirement or death and suppose the dynamics of the population are subject to the following assumptions; in the interval $(t,t+\Delta t)$ the probability that the population

1. reduces by one due to retirement or death is $\mu_n\Delta t + o(\Delta t)$,
2. does not reduce by one due to retirement or death is $1-\mu_n\Delta t + o(\Delta t)$,
3. reduces by more than one retirement or death is $o(\Delta t)$.

Also assume that the occurrence of retirement or death in the infinitesimal time interval $(t, t+\Delta t)$ is independent of time since last retirement or death. Suppose $P_n(t)$ is as previously defined and
let $P_{i,n}(s,t) = P[X(t) = n|X(s) = i], t > s$ and $P_n(t) = P_{in}(0,t) = P[X(t) = n|X(0) = i]$. Then from the assumptions and probabilities given above, we have

$$P_{n,n-1}(t, t+\Delta t) = \mu_n \Delta t + o(\Delta t) \quad (10)$$
$$P_{n,n}(t, t+\Delta t) = 1 - \mu_n \Delta t + o(\Delta t) \quad (11)$$
$$\sum_{i \neq n-1,n} P_{ni}(t, t+\Delta t) = o(\Delta t) \quad (12)$$

Based on equations (10) through (12) and by the Chapman-Kolmogorov equations for transitions in the non-overlapping time-intervals $(0, t\,]$ and $(t, t+\Delta t\,]$, we have

$$P_n(t+\Delta t) = P_n(t)[1 - \mu_n \Delta t + o(\Delta t)] + P_{n+1}(t)[\mu_{n+1}\Delta t + o(\Delta t)] + o(\Delta t). \quad (13)$$

Rearranging equation (13) and letting $\Delta t \to 0$ results in the difference differential equation
$$P'_n(t) = \mu_{n+1} P_{n+1}(t) - \mu_n P_n(t),\ n = 0,1,2,3,\ \ldots . \quad (14)$$

Considering the Yule's process for which $\mu_n = n\mu_3$, equation (14) becomes
$$P'_n(t) = (n+1)\mu_3 P_{n+1}(t) - n\mu_3 P_n(t),\ n = 0,1,2,3,\ \ldots . \quad (15)$$

with initial conditions $P_n(0) = \begin{cases} 1 & \text{if } n = i \\ 0 & \text{if } n \neq i \end{cases}$ and $P_{-1}(t) = 0$.

As in the other processes, the conditional expected number of contributors less those who retire or die within time interval $(0, t]$, can be obtained based on equation (15) and the corresponding initial conditions, as
$$E[X(t)|X(0) = i] = i\ell^{-\mu_2 t} \quad (16)$$

2.3 Model Specification

The main objective of this study is to propose a model for predicting the pension scheme one period ahead based on data from the immediate past T periods. Suppose there are g sub-periods in the period after the past T periods. The estimates of the expected population sizes of contributors and pensioners based on equations (9), are given respectively by

$$N_{i1} = \left(\frac{\xi}{\lambda_i - \mu_i}\right)\left(\ell^{(\lambda_i - \mu_i)g} - 1\right) + j_i \ell^{(\lambda_i - \mu_i)g}, \qquad i = 1, 2. \quad (17)$$

where j_i is the number of contributors or number of pensioners in the last period of the past T periods, with $i = 1$ representing contributors and $i = 2$ representing pensioners.
The corresponding estimates for the number of contributors and pensioners who received lumpsums based on equation (16) are given by

$$N_{i2} = j_i(1 - \ell^{-\mu_i^* t}),\ i = 1, 2, 3. \quad (18)$$

where $i = 1$ corresponds to contributors who are paid lump-sum after dying on the line of duty, $i = 2$ corresponds to retirees who are paid lump-sum and $i = 3$ corresponds to pensioners who are paid lump-sum after dying before age 72.

Suppose ν_{i1} ($i = 1,2$) is the average contribution made per person or average pension paid per person, where $i = 1$ corresponds contribution and $i = 2$ corresponds pension. Also, let ν_{i2} $(i = 1, 2, 3)$ average lump sum paid to a contributor who dies or retires, or a pensioner who dies before age 72, where $i = 1$ corresponds to a contributor, $i = 2$ corresponds a pensioner and $i = 3$ corresponds a pensioner who dies before age 72. Then the total expected contribution to be made, total pensions and total lump-sum expected to be paid one period ahead are respectively given as
$$C_g = N_1 \nu_{11}\,, \quad P_g = N_{21}\nu_{21} \quad \text{and} \quad L_g = N_{12}\nu_{12} + N_{22}\nu_{22} + N_{32}\nu_{32} \quad (19)$$

An estimate of the surplus of the scheme one period ahead is proposed to be

$$S_g = C_g - L_g - P_g + (I_g - E_g) \tag{20}$$

where $I_g = \frac{I}{T}$ and $E_g = \frac{E}{T}$ with I and E being the income from investment and expenditure of the scheme for the previous T periods respectively.
The proposed estimates of the model parameters $\hat{v}_{ij} = \bar{y}_{ij}$ $(i = 1, 2; j = 1, 2, 3)$, $\hat{\lambda}_i$ $(i = 1, 2)$, $\hat{\mu}_i (i = 1, 2)$, $\hat{\xi}_i (i = 1, 2)$ and $\hat{\mu}_i^*$ $(i = 1, 2, 3)$ are discussed as follows.

Suppose y_{11t}, y_{12t}, y_{13t} and n_{1t} are the total contribution made by contributors, total lumpsum paid to contributors who die, total lumpsum paid to those who retire and number of contributors in period t $(t = 1,2,3,\ldots,T)$ respectively with T being the number of periods under consideration. The average contribution made per contributor or lumpsum paid per person in period t are given by

$$\bar{y}_{1it} = \frac{y_{1it}}{n_{1t}} \quad, i = 1, 2, 3 \,; t = 1,2,\ldots,T \tag{21}$$

where $i = 1$ corresponds to contributors, $i = 2$ correspond to those who died on the line of duty and $i = 3$ corresponds to those who retired in period t.
The corresponding rates of contribution or lumpsum payment for all the T periods under consideration are given by

$$\bar{y}_{1i} = \frac{1}{T} \sum_{t=1}^{T} \bar{y}_{1it}, \; i = 1, 2, 3. \tag{22}$$

Similarly, suppose y_{21t}, y_{22t} and n_{2t} are the total pension paid, total lumpsum paid to pensioners who died before age 72 and number of pensioners in period t $(t = 1,2,3,\ldots,T)$ respectively with T as defined previously. Then the average pension paid per person or lumpsum paid per person in period t are given by

$$\bar{y}_{2it} = \frac{y_{2it}}{n_{2t}} \quad, i = 1, 2 \,; t = 1,2,\ldots,T \tag{23}$$

where $i = 1$ corresponds to pensioners and $i = 2$ correspond to pensioners who received lumpsum in period t.
The corresponding rates of contribution or lumpsum payment for all the T periods under consideration are given by

$$\bar{y}_{2i} = \frac{1}{T} \sum_{t=1}^{T} \bar{y}_{2it}, \; i = 1, 2. \tag{24}$$

To estimate the parameters in the population models discussed in the previous section, consider the following definitions.
Define λ_{it} $(i = 1, 2)$ to be the number of people who join the scheme and the number of contributors who join the population of pensioners in period t $(t = 1,2,\ldots,T)$ respectively. Then the rates at which people join the scheme and contributors join the population of pensioners in period t per sub-period are given respectively by

$$\hat{\lambda}_{it} = \frac{\lambda_{it}}{n_{it} m_t} \quad, t = 1,2,\ldots,T \text{ and } i = 1, 2. \tag{25}$$

where n_{it} $(i = 1, 2)$ and m_t $(t = 1,2,\ldots,T)$ is the number of equally spaced sub-periods in period t; with $i = 1$ corresponding to new contributors and $i = 2$ corresponding to new retirees.
The corresponding rates over the Tperiods under investigation which are also estimates of the corresponding model parameters are given as

$$\hat{\lambda}_i = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_{it}, \; i = 1, 2. \tag{26}$$

Here, $\hat{\lambda}_{it}$ is an estimate of the average rate at which workers join the scheme per sub-period and $\hat{\lambda}_i$ is an estimate of the average rate at which contributors are retiring per sub-period. For this paper, the sub-periods are days and the periods are months with $T = 12$.
Also let $\mu_{it} (i = 1, 2)$ be the number of contributors who retire or pensioners who die after age 72 in period $t (t = 1,2,\ldots,T)$. Then the rate at which contributors retire or the rate at which pensioners die after age 72 in period t per sub-period are given as

$$\hat{\mu}_{it} = \frac{\mu_{it}}{n_{it}m_t} \quad , t = 1,2,\dots,T \; ; i = 1,2. \tag{27}$$

where $i = 1$ corresponds to contributors who retire and $i = 2$ corresponds to pensioners who die after age 72. The corresponding rates over the T periods under investigation which are estimates of the model parameters are given as

$$\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\mu}_{it} \; , i = 1,2. \tag{28}$$

It is worth noting at this juncture that $\hat{\lambda}_2 = \hat{\mu}_1$.

Finally, define ε_{it} $(i = 1,2)$ to be the number of contributors who died or number of pensioners who died before age 72 in period t. The rate at which contributors die or pensioners die after age 72 in period t per sub-period are given as

$$\hat{\xi}_{it} = \frac{\xi_{it}}{n_{it}m_t} \; , \quad t = 1,2,\dots,T \; ; i = 1,2. \tag{29}$$

where $i = 1$ corresponds to contributors who die and $i = 2$ corresponds to pensioners who die before age 72. The corresponding estimates for the rates over the T periods under investigation which also double as estimates of the corresponding model parameters are given as

$$\hat{\xi}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\xi}_{it} \; , \; i = 1,2. \tag{30}$$

Estimates of the parameters in model (18) may be obtained as follows.

$$\hat{\mu}_1^* = \hat{\xi}_1, \quad \hat{\mu}_2^* = \hat{\mu}_1 \quad \text{and} \quad \hat{\mu}_3^* = \hat{\xi}_2 \tag{31}$$

Here $\hat{\mu}_1$, $\hat{\xi}_1$ and $\hat{\xi}_2$ are the estimates of the average rates at which contributors are retiring, contributors are dying in the line of duty and pensioners are dying before age 72 years respectively, for sub period. As indicated earlier, sub-periods are days and periods are months with $T = 12$.

The paper estimated the model parameters based on periods of months and sub-periods of days because the available data were monthly data for only two years 2015, 2016. However, if data were available for several years, then T will be the number of years for which data is available with sub-period and period being month and year respectively. In this case, estimates of the model parameters will be monthly average for one year ahead $(g = 12)$ or two years ahead $(g = 24)$. Here the annual quantitative I and E will be used in equation (20) instead of I_g and E_g. Also, the quantities v_{ij} in equation (19) are the corresponding annual averages.

3. Results and Discussion

Monthly data on number of SSNIT contributors, new SSNIT contributors, contributors who die before retirement, contributors who retire, pensioners who die before age 72 and after age 72, as well as amount contributors paid, lumpsums beneficiaries received and monthly pension paid. These data were obtained from the Actuarial and Research Department of SSNIT for 2015 and 2016. The annual income and expenditure data were also obtained from SSNIT Financial Statement Reports for 2015 and 2016.

For the data analysis, months are the periods whiles the sub-periods are days with the number of periods T equal to 12.

Estimates of the average monthly contribution, lumpsum and pension per person based on equations (22) and (24) for 2015 and 2016 are shown in Table 1.

Table 1: Average monthly contribution, lumpsum and pension for 2015 and 2016

Year	**Contribution** ($\hat{v}_{11}$)	**Lumpsum**			**Pension** ($\hat{v}_{21}$)
		New Retirees ($\hat{v}_{22}$)	Active workers who die($\hat{v}_{12}$)	Pensioners who died before age 72 ($\hat{v}_{23}$)	
2015	142.94	13,683.58	8,347.27	3,930.99	514.49
2016	161.13	14,523.56	7,286.15	4,794.05	531.12

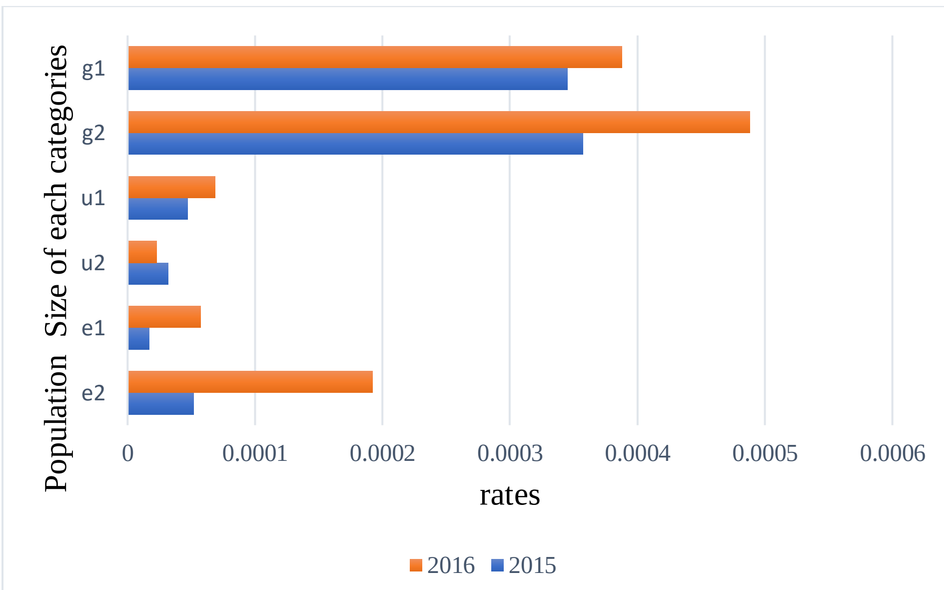
From Table 1, apart from lumpsum paid to contributors who die on the line of duty which recorded a decrease from 8347.27 in 2015 to 7286.15 in 2016 (13%), all the other categories recorded a rise which translated into a maximum of 22% for lumpsum paid to pensioners who died before age 72 and a minimum of 3% for pensions.

For each of the years, daily rates at which new contributors join the scheme, active contributors die, contributors retire, pensioners die before age 72 and pensioners die after age 72 are computed based on equations (25), (27) and (29) and presented in tables A1 and A2 respectively in the appendix. The resultant corresponding estimates based on equations (26), (28) and (30) for the whole period, which also serve as the model parameter estimates are presented for each year in table 2.

Table 2: Estimates of model parameters (rates per day) by year

Year	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\xi}_1$	$\hat{\xi}_2$
2015	0.000344936	0.000357082	0.000047247	0.000031729	0.000016877	0.000051698
2016	0.000387848	0.000488237	0.000068672	0.000022707	0.000057202	0.000192070

It is obvious from table 2 that except for the rate at which pensioners die after age 72 ($\hat{\mu}_2$) which recorded a reduction from 2015 to 2016, there were increases in all the rates for the other categories of populations in the scheme. Also, the rates at which contributors ($\hat{\lambda}_1$) and pensioners ($\hat{\lambda}_2$) increase are far higher than those of the other population categories of the scheme for each year. Figure 1 provides a pictorial display of the rates by year.



g1: new contributors, g*2*: new pensioners, *u1*: retiring contributors
u2: death after age 72, e*1*: death in active service, e*2*: death before age 72

Figure 1: Pictorial display of rates of categories of populations in the SSNIT scheme

In Table 3, estimates of the sizes of the various categories of the population in the SSNIT scheme expected a month ($g = 30$) ahead of the data by appropriately substituting the estimates of the parameters in equations (17) and (18) are presented for 2015 and 2016.

Table 3: Estimates of expected size of population categories in the SSNIT scheme by year

Year	Contributors ($\hat{N}_{11}$)	New Retirees($\hat{N}_{22}$)	Active workers who die ($\hat{N}_{12}$)	Pensioners who died before age 72 ($\hat{N}_{23}$)	Pensioners ($\hat{N}_{21}$)
2015	1,163,721	1,634	584	241	156,606
2016	1,222,032	2,491	2,075	1,007	177,782

It can be observed from table 3 that there is an expected increase in the sizes of all population categories from 2016 to 2017. These translate into a maximum increase of 318% for pensioners who died before age 72, followed by contributors (Active workers) who die before retirement (255%) with the least increase of 5% recorded by the contributors' population.

Using equations (19) and (20), estimates of total contributions, total lumpsums to be paid to beneficiaries and surplus expected one month ahead of the period of the data used were computed for each year. The results together with expected percentage changes from 2016 to 2017 are displayed in table 4.

Table 4: Expected total Contribution, total lumpsums and total pension, income, expenditure and surplus by years together with percentage changes

Item	**2015 (GH₵)**	**2016 (GH₵)**	**Percentage Change (%)**
Contribution (*Cg*)	166,338,350.09	96,904,568.54	18.4
Lumpsum (*Lg*)	28,173,110.59	48,346,730.44	71.6
Pensions (*Pg*)	80,572,819.61	94,422,533.76	17.2
Income (*Ig*)	186,104,666.70	208,447,166.70	12.0
Expenditure (*Eg*)	135,043,583.30	181,884,333.30	34.7
Surplus (*Sg*)	108,653,503.28	80,698,137.74	(25.7)

A quick glance at the results in Table 4 reveals that, apart from the surplus which was expected to reduce by 25.7%, all the others were expected to increase from 2016 to 2017. For the rest, the largest expected percentage increase (71.6%) is recorded by lumpsum, followed by expenditure (34.7%), contribution (18.4%) and pensions (17.2%) with income recording the least percentage increase of 12.0%.

3.1 Sensitivity Analysis

In order to demonstrate the usefulness of the proposed method, some sensitivity analyses were carried out. For example, based on the 2016 data, if it was desired to achieve in 2017 the expected surplus observed in 2016 based on the 2015 data, the management should have target a reduction of 15.4% in the expenditure in 2017. Also, using the model and based on the data in 2015 and 2016, one would have expected surpluses of GH₵ 68,367,093.49 and GH₵ 33,486,870.86 respectively for 2016 and 2017 if the pensions were increased by 50%. Implying that based on these data, SSNIT would have still been solvent in 2016 and 2017 if pensions increased by 50% holding all others constant.

3.2 Reliability of proposed model

To assess the reliability of the proposed method of estimating the surplus, bootstrap samples were selected from the monthly estimates of the model parameters in tables A1 and A2 in the appendix to generate 100 bootstrap samples each of size 100 for each of the two years (2015 and 2016) under consideration.

Coverage probabilities of the estimates of the surpluses based on the bootstrap samples were then computed for each year. These resulted in coverage probabilities of 0.94 and 0.95 respectively for 2015 and 2016; implying a very reliable model performance.

Again, using the bootstrap samples, 100 independent sample *t*-tests were carried out at 5% significance level to check whether the surpluses in the two years are significantly different. The results of these tests were used to compute what this paper calls the '*confirmatory test probability*' p_c given by

$$p_c = \frac{\sum_{i=1}^{n} \delta_i}{n} \; ; \quad \delta_i = \begin{cases} 1 & \text{if the } i^{th} \text{ test is significant} \\ 0 & \text{otherwise} \end{cases} \; , \; i = 1,\ 2,\ \ldots, n.$$

The analysis ended up in a confirmatory test probability of unity. It can therefore be inferred from table 4 that, the surplus obtained from the 2015 data is significantly higher than that of the 2016 data.

4. Conclusion and Recommendation

As stated earlier, the study sorted to propose a model for forecasting the surplus of SSNIT one month ahead based on monthly data from a previous year to determine the expected solvency status of SSNIT the following year. This objective was achieved, and the proposed model was successfully applied to data obtained from the Actuarial and

Research Department of SSNIT for 2015 and 2016. Coverage probabilities (0.94 and 0.95) of estimates of the expected surpluses in 2016 and 2017 based on data from 2015 and 2016 respectively were very high. This indicate that the proposed model is reliable.
The paper also discussed a method for testing statistically, the existence of a difference between surpluses for two years. In the process, the expected surplus based on the 2015 data was found to be significantly higher than that based on the 2016 data. This may largely be due to an increase of 34.7% in expenditure.

Sensitivity analysis carried out using the model, revealed that the surplus expected in 2017 based on the 2016 data could have been the same as that expected in 2016 based on the 2015 data if the expenditure had been reduced by 15.4%. It was also shown that SSNIT could have still been solvent even if pensions were increased by 50%.

The results show that the rates at which contributors and pensioners increase are far higher than the rates of the other population categories and this culminated into increases in the estimated sizes of all the population categories in the scheme. A finding which is consistent with the assertion of decreasing number of contributors by Kwabla-King (2017). The increasing sizes of population categories translated into corresponding increase in average contribution paid and average lumpsum received per person except for average lumpsum received by representatives of contributors who died on the line of duty. It can be inferred from this result that most of the contributors who died in active service were low-income earners.

We recommend the use of the proposed model to assess the impact of COVID 19 on the solvency of SSNIT. Broadly, the proposed model can be used to assess the solvency of any pension scheme which has a structure similar to that of SSNIT.

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Conflict of Interest Statement
The authors declare that, there is no conflict of interest.

Appendix

Table A1: Monthly parameter estimates from 2015 data

Month	lamda1	lamda2	mui1	epsilon1	mui3	epsilon2	mui2
JAN	0.000193987	0.000338128	0.000043819	0.000019289	0.000075984	0.000101461	0.000063108
FEB	0.000310753	0.000542967	0.000071022	0.000002949	0.000063215	0.000019357	0.000073970
MAR	0.000295922	0.000140459	0.000018250	0.000000374	0.000011963	0.000000000	0.000018625
APR	0.000351557	0.000328672	0.000042950	0.000012971	0.000050652	0.000062009	0.000055921
MAY	0.000354810	0.000319273	0.000042197	0.000027289	0.000075852	0.000111713	0.000069485
JUN	0.000363141	0.000500278	0.000066615	0.000036756	0.000076148	0.000135031	0.000103371
JUL	0.000380265	0.000313264	0.000041776	0.000000000	0.000020416	0.000000000	0.000041776
AUG	0.000407318	0.000104172	0.000013827	0.000009699	0.000000000	0.000008308	0.000023526
SEP	0.000435464	0.000459139	0.000060782	0.000033064	0.000000221	0.000009929	0.000093846
OCT	0.000485345	0.000351337	0.000046721	0.000010006	0.000000211	0.000046367	0.000056727
NOV	0.000316168	0.000319999	0.000042656	0.000017605	0.000001516	0.000053477	0.000060261
DEC	0.000244500	0.000567294	0.000076354	0.000032527	0.000004572	0.000072730	0.000108881
Average	0.000344936	0.000357082	0.000047247	0.000016877	0.000031729	0.000051698	0.000064125

Table A2: Monthly parameter estimates from 2016 data

Month	lamda1	lamda2	mui1	epsilon1	mui3	epsilon2	mui2
JAN	0.000470189	0.000361176	0.000049196	0.000031469	0.000012073	0.000067324	0.000080664
FEB	0.000388889	0.000370594	0.000050484	0.000032919	0.000008115	0.000114965	0.000083403
MAR	0.000386459	0.000371618	0.000050921	0.000033579	0.000013121	0.000112434	0.000084500
APR	0.000466430	0.000605182	0.000083493	0.000026862	0.000023951	0.000141023	0.000110355
MAY	0.000415533	0.000672905	0.000093298	0.000054964	0.000024343	0.000198903	0.000148262
JUN	0.000419980	0.000673472	0.000094447	0.000081764	0.000040999	0.000291014	0.000176211
JUL	0.000356316	0.000504339	0.000071459	0.000047295	0.000025957	0.000172279	0.000118754
AUG	0.000327750	0.000563081	0.000080951	0.000095677	0.000029347	0.000309941	0.000176627
SEP	0.000284050	0.000470359	0.000067769	0.000064876	0.000017349	0.000213250	0.000132645
OCT	0.000452428	0.000425257	0.000060896	0.000070738	0.000036792	0.000219445	0.000131634
NOV	0.000443440	0.000510950	0.000073370	0.000084827	0.000021673	0.000231692	0.000158196
DEC	0.000242711	0.000329911	0.000047785	0.000061457	0.000018768	0.000232575	0.000109243
Average	0.000387848	0.000488237	0.000068672	0.000057202	0.000022707	0.000192070	0.000125874