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A New Method to Solve Interval Transportation Problems

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Abstract

The present paper aims to propose an alternative solution approach in obtaining the optimal interval to an interval transportation problem (ITP) in which the cost-coefficients of the objective function, source and destination parameters are all in the form of interval. In this paper, the single objective interval transportation problem is transformed into an equivalent crisp bi-objective transportation problem where the left-limit and width of the interval are to be minimized. The solution to this bi-objective model is then obtained with the help of fuzzy programming technique. A set of twenty random numerical examples has been solved using the proposed approach. A comparative study has also been made between the proposed solution method and the method proposed by Das et al.(1999) which shows that the proposed method provides better solutions for eleven out of twenty problems.

Key Words: Interval Transportation Problem; Fuzzy Programming; Interval numbers.

Mathematical Subject Classification: 90B06, 90B50, 90C05, 90C08, 90C29, 90C70, 90C90, 65K05.

1. Introduction

In a classical transportation problem, a homogeneous product is to be transported from $'m'$ sources to $'n'$ destinations in such a way that the overall transportation cost becomes minimum. The availability of the product at source i is denoted by a_i, $i = 1, 2 \dots m$ and the demand of the destination j is b_j, $j = 1, 2 \dots n$. C_{ij} is the cost of transporting one unit of product from source i to destination j.

In the past several methods have been developed for solving transportation problems in which the cost coefficients, source and destination parameters are precisely defined but in many practical situations it is not always possible. In such situations, the cost of transportation, the supply and demand parameters may reflect imprecise behaviour. To deal with imprecise parameters in transportation problems, fuzzy and interval programming techniques are often used [see (Inuiguchi and Kume,1991), (Alefeld and Herzberger, 1983), (Bitran, 1980), (Chanas and Kuchta, 1996), (Tanaka and Asai, 1984), (Soyster, 1973), (Moore, 1979)]. Using the method developed by Ishibuchi and Tanaka(1990) , one can compare two interval numbers. For example, in a problem where the objective function is to be minimized, A is better than B, i.e. $A \leq_{MW} B$ if and only if $a_m \leq b_m$(lower expected cost) and $a_w \leq b_w$ (less uncertainty). Das et al.(1999) proposed a method to solve the ITP by considering the right-limit and mid-point of the interval. Sengupta and Pal (2009) developed a new fuzzy orientation method for solving ITP. In this method, they have considered the mid-point and width of the interval. Natarajan(2010) proposed a new separation method based on the zero point method for finding an optimal solution for the interval integer transportation problem. Pandian and Anuradha(2011) applied split

and bound approach for finding an optimal solution to a fully integer ITP with additional impurity constraints. Güzel et al.(2012) proposed two solution procedures for the interval fractional transportation problem. Panda and Das(2013) proposed a model for two vehicle cost varying ITP in which they have considered the right-limit and mid-point of the interval. Nagarajan et al.(2014) suggested a solution procedure for the multiobjective solid transportation problem with interval cost in source and demand parameters under stochastic environment. Henriques and Coelho(2017) provided a short review of some interval programming techniques. Akilbasha et al.(2018) proposed an innovative exact method for solving fully interval integer transportation problem. In this method they have considered mid-point and width of the interval. Habiba and Quddoos(2020) considered multiobjective ITP with stochastic supply and demand.
In this paper, we have proposed a new solution approach for finding the optimal solution to an ITP in which the cost-coefficients of the objective function, source and destination parameters have been represented in the form of interval numbers. The single objective ITP is converted into an equivalent crisp bi-objective transportation problem where the left-limit and width of the interval are to be minimized. To obtain the solution of the equivalent bi-objective problem, fuzzy programming technique [see(Bit et al.1992)] is used. To demonstrate the efficiency of the proposed method we have considered a set of twenty numerical examples. A comparative study has also been made between the proposed method and the method suggested by Das et al.(1999)

2. Preliminaries

Let the lower case letters e.g. a, b etc. denote real numbers and upper case letters e.g. A, B etc. denote the closed intervals on the real line $\mathbb{R}$.

2.1. Definition

$$A = [a_L, a_R] = \{\ a:\ a_L \leq\ a \leq\ a_R, a \in \mathbb{R}\},$$

where a_L and a_R are the left-limit and right-limit of the interval A on the real line $\mathbb{R}$.

2.2. Definition

$$A = \langle a_m, a_w \rangle = \{\ a:\ a_m - a_w \leq\ a \leq\ a_m + a_w, a \in \mathbb{R}\},$$

where a_m and a_w are the mid-point and half-width (or simply known as 'width') of interval A on the real line $\mathbb{R}$, i.e.

$$a_m = (\frac{a_R + a_L}{2})$$
$$a_w = (\frac{a_R - a_L}{2})$$

2.3. Definition

If $A = [a_L, a_R]$ and $B = [b_L, b_R]$ are two closed interval then,

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$$
$$A + B = \langle a_m, a_w \rangle + \langle b_m, b_w \rangle = \langle a_m + b_m, a_w + b_w \rangle$$
$$\lambda A = \lambda[a_L, a_R] = [\lambda a_L, \lambda a_R] \text{ if } \lambda \geq 0$$
$$\lambda A = \lambda[a_L, a_R] = [\lambda a_R, \lambda a_L] \text{ if } \lambda < 0$$
$$\lambda A = \lambda \langle a_m, a_w \rangle = \langle \lambda a_m, |\lambda| a_w \rangle$$

where λ is a real number.

3. Definition of order relations between intervals

The present section is devoted to the study of decision maker's preferences in the minimization problem. The preference can be decided with the help of an order relation $\leq_D$ which is defined as follows:

3.1. Definition

Let A and B be two intervals which represent uncertain costs from two alternatives. Consider the cost of each alternative lie in the corresponding interval.
The order relation $\leq_D$ between $A = \langle a_m, a_w \rangle$ and $B = \langle b_m, b_w \rangle$ is defined as:

$$A \leq_D B \text{ if } d_{IA} \leq d_{IB}$$
$$A <_D B \text{ if } A \leq_D B \text{ and } A \neq B$$

where $I = \langle i_m, i_w \rangle$ represent the ideal expected value and ideal uncertainty.

$$d_{IA} = \sqrt{(a_m - i_m)^2 + (a_w - i_w)^2}$$
$$d_{IB} = \sqrt{(b_m - i_m)^2 + (b_w - i_w)^2}$$

If $A \leq_D B$, then A is preferred over B.

4. Mathematical Model of Interval Transportation Problem

The generalized mathematical model of the ITP is written as Problem-I:

Problem-I:

$$\text{Minimize}: \ Z = [z_L, z_R] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{L_{ij}}, c_{R_{ij}}]x_{ij} \tag{1}$$

Subject to;

$$\sum_{j=1}^{n} x_{ij} = [a_{L_i}, a_{R_i}], \ i = 1, 2, \ldots, m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = [b_{L_j}, b_{R_j}], \ j = 1, 2, \ldots, n \tag{3}$$

$$x_{ij} \geq 0, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n \tag{4}$$

with

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \tag{5}$$

The notations and assumptions used in the above Problem-I are listed below.
Notations and Assumptions

z_L	:	the left-limit of interval valued objective function
z_R	:	the right-limit of interval valued objective function
$[c_{L_{ij}}, c_{R_{ij}}]$	:	an interval representing the uncertain cost components for the transportation problem; it can represent transportation cost
$c_{L_{ij}}$	:	lowest possible cost of transporting one unit of product from source i to destination j
$c_{R_{ij}}$	:	highest possible cost of transporting one unit of product from source i to destination j
$[a_{L_i}, a_{R_i}]$	:	interval availability of source i
$[b_{L_j}, b_{R_j}]$	:	interval demand of destination j
x_{ij}	:	quantity transported from source i to destination j

5. Formulation of the crisp constraints and crisp objective function

The objective function and constraints (1)-(3) contains the interval quantities which are not easy to deal, so it is better to obtain an equivalent crisp problem for the ease of complex mathematical calculations. For this purpose we describe the procedures for obtaining equivalent crisp constraints and objective function in the following subsections (5.1) and (5.2), respectively.

5.1. Formulation of crisp constraints

Let us consider the interval constraint (2) of Problem-I which can be represented in the form of two crisp constraints as follows:

$$\sum_{j=1}^{n} x_{ij} \leq a_{R_i},\ i = 1, 2, \ldots, m \tag{6}$$

and

$$\sum_{j=1}^{n} x_{ij} \geq a_{L_i},\ i = 1, 2, \ldots, m \tag{7}$$

Similarly, the equivalent crisp constraints of (3) may also be written as:

$$\sum_{i=1}^{m} x_{ij} \leq b_{R_j},\ j = 1, 2, \ldots, n \tag{8}$$

and

$$\sum_{i=1}^{m} x_{ij} \geq b_{L_j},\ j = 1, 2, \ldots, n \tag{9}$$

5.2. Formulation of crisp objective function

In (1) of Problem-I, we can denote $Z = \langle z_M, z_W \rangle$, where $z_M = (\frac{z_R+z_L}{2})$ is the mid-point and $z_W = (\frac{z_R-z_L}{2})$ is the width of interval Z.
According to Ishibuchi and Tanaka(1990), the mid-point and width of an interval can be regarded as the expected value and uncertainty of interval respectively. Since the objective function (1) of Problem-I is the cost function which is to be minimized, so our interest is to obtain minimum cost with minimum uncertainty.
Using (2.3), the left limit z_L in Problem-I can be expressed in terms of expected cost and uncertainty as follows:

$$z_L = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{m_{ij}} x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij},\ \text{when } x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{10}$$

where $c_{m_{ij}}$ is the mid-point and $c_{w_{ij}}$ is the width of the cost co-efficient of Z.
Minimizing (10) is equivalent to minimize the expected cost and maximize the uncertainty simultaneously.
Also our objective is to minimize the uncertainty of interval along with minimizing expected value of interval, which can be achieved by simultaneously minimizing the left-limit function z_L and uncertainty function z_W.
where,

$$z_W = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij} \tag{11}$$

$c_{w_{ij}} = \left(\frac{c_{R_{ij}} - c_{L_{ij}}}{2}\right)$ is the width of the cost coefficient of Z in Problem-I.

6. Equivalent crisp Problem of ITP (Problem-I)

The equivalent crisp problem of ITP (Problem-I) can be obtained using (10)-(11) and (6)-(9) as follows:

Problem-II:

$$\text{Minimize } z_L = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{m_{ij}} x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij} \tag{12}$$

$$\text{Minimize } z_W = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij} \tag{13}$$

Subject to;

$$\sum_{j=1}^{n} x_{ij} \leq a_{R_i},\ \sum_{j=1}^{n} x_{ij} \geq a_{L_i},\ i = 1, 2, \ldots, m \tag{14}$$

$$\sum_{i=1}^{m} x_{ij} \leq b_{R_j},\ \sum_{i=1}^{m} x_{ij} \geq b_{L_j},\ j = 1, 2, \ldots, n \tag{15}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{16}$$

with

$$\sum_{i=1}^{m} a_{L_i} = \sum_{j=1}^{n} b_{L_j} \text{ and } \sum_{i=1}^{m} a_{R_i} = \sum_{j=1}^{n} b_{R_j} \tag{17}$$

7. Procedure for obtaining ideal solution of ITP (Problem-I)

This section discusses the stepwise procedure to obtain the ideal expected value of overall transportation cost and ideal uncertainty of the interval in which the overall transportation cost lies. The stepwise procedure for obtained ideal solution of a generalised ITP is given below:

Step 1: Represent the objective function (1) in the form of center and width using definition (2.2),

$$Z = \langle z_M, z_W \rangle = \sum_{i=1}^{m}\sum_{j=1}^{n} \langle c_{m_{ij}}, c_{w_{ij}} \rangle x_{ij} \tag{18}$$

Step 2: Split the function (18) obtained in Step 1 into two separate functions with the help of definition (2.3),

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{m_{ij}} x_{ij} \tag{19}$$

and

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij} \tag{20}$$

Step 3: Using (19) and (20), construct two linear programming problems (say Problem-III and Problem-IV) as follows:

Problem-III:

$$\text{Minimize } z_M = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{m_{ij}} x_{ij}$$

Subject to; $(14 - 17)$

Problem-IV:

$$\text{Minimize } z_W = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{w_{ij}} x_{ij}$$
$$\text{Subject to; } (14-17)$$

where $c_{m_{ij}} = (\frac{c_{R_{ij}}+c_{L_{ij}}}{2})$ is the mid-point and $c_{w_{ij}} = (\frac{c_{R_{ij}}-c_{L_{ij}}}{2})$ is the width of the cost coefficient of Z in Problem-I.

Step 4: Solve Problem-III and Problem-IV independently and obtain their global minimums. Let z_M^* and z_W^* be the global minimums of Problem-III and Problem-IV respectively. So, $Z^*=\langle z_M^*, z_W^*\rangle$ is the ideal solution of the Problem-I.

Remark. *Let us suppose Z' and Z^o be the two solutions for Problem-I then according to Definition (3.1), if $d_{Z^*Z^o} \leq d_{Z^*Z'}$ then Z^o is the preferred solution otherwise Z'.*

8. Numerical illustration

Let us consider the following ITP

$$\text{Minimize } Z = \sum_{i=1}^{3}\sum_{j=1}^{4}[c_{L_{ij}}, c_{R_{ij}}]x_{ij}$$

Subject to;

$$\sum_{j=1}^{4} x_{1j} = [7,9],\ \sum_{j=1}^{4} x_{2j} = [17,21],\ \sum_{j=1}^{4} x_{3j} = [16,18],$$
$$\sum_{i=1}^{3} x_{i1} = [10,12],\ \sum_{i=1}^{3} x_{i2} = [2,4],\ \sum_{i=1}^{3} x_{i3} = [13,15],$$
$$\sum_{i=1}^{3} x_{i4} = [15,17],\ x_{ij} \geq 0,\ i=1,2,3,\ j=1,2,3,4$$

where,

$$C = \begin{bmatrix} [7,9] & [8,14] & [3,4] & [6,7] \\ [3,10] & [5,8] & [7,12] & [9,10] \\ [6,12] & [4,15] & [7,8] & [12,13] \end{bmatrix}$$

Using Eqs.(12)-(13), we write the left-limit z_L and width z_W of the interval objective function as:

$$\text{Minimize } z_L = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{L_{ij}} x_{ij},\ \text{Minimize } z_W = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{w_{ij}} x_{ij}$$

where,

$$c_{L_{ij}} = \begin{bmatrix} 7 & 8 & 3 & 6 \\ 3 & 5 & 7 & 9 \\ 6 & 4 & 7 & 12 \end{bmatrix},\ c_{w_{ij}} = \begin{bmatrix} 1 & 3 & 0.5 & 0.5 \\ 3.5 & 1.5 & 2.5 & 0.5 \\ 3 & 5.5 & 0.5 & 0.5 \end{bmatrix}$$

Using eqs.(14) and (15) we write the crisp constraints as follows:

$$\sum_{j=1}^{4} x_{1j} \leq 9,\ \sum_{j=1}^{4} x_{1j} \geq 7,\ \sum_{j=1}^{4} x_{2j} \leq 21,\ \sum_{j=1}^{4} x_{2j} \geq 17, \tag{21}$$

$$\sum_{j=1}^{4} x_{3j} \leq 18,\ \sum_{j=1}^{4} x_{3j} \geq 16,\ \sum_{i=1}^{3} x_{i1} \leq 12,\ \sum_{i=1}^{3} x_{i1} \geq 10, \tag{22}$$

$$\sum_{i=1}^{3} x_{i2} \leq 4,\ \sum_{i=1}^{3} x_{i2} \geq 2,\ \sum_{i=1}^{3} x_{i3} \leq 15,\ \sum_{i=1}^{3} x_{i3} \geq 13, \tag{23}$$

$$\sum_{i=1}^{3} x_{i4} \leq 17,\ \sum_{i=1}^{3} x_{i4} \geq 15,\ x_{ij} \geq 0,\ i = 1,2,3,\ j = 1,2,3,4 \tag{24}$$

Using fuzzy programming techniques (Bit et al., 1992) , the Pareto optimal solution of the problem is obtained as follows, $x_{11} = 2.71$, $x_{14} = 4.28$, $x_{21} = 4.28$, $x_{22} = 2.0$, $x_{24} = 10.71$, $x_{31} = 3.0$, $x_{33} = 13$,
$Z = [272.88, 360.25]$=$\langle z_M, z_W\rangle$=$\langle 316.5, 43.6\rangle$.
To obtain the ideal solution of given problem we form the following two single objective problems as follows:

Problem-V:

$$\text{Minimize } z_M = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{m_{ij}} x_{ij}$$
$$\text{Subject to constraints; } (21-24)$$

Problem-VI:

$$\text{Minimize } z_W = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{w_{ij}} x_{ij}$$
$$\text{Subject to constraints; } (21-24)$$

where,

$$c_{m_{ij}} = \begin{bmatrix} 8 & 11 & 3.5 & 6.5 \\ 6.5 & 6.5 & 9.5 & 9.5 \\ 9 & 9.5 & 7.5 & 12.5 \end{bmatrix},\ c_{w_{ij}} = \begin{bmatrix} 1 & 3 & 0.5 & 0.5 \\ 3.5 & 1.5 & 2.5 & 0.5 \\ 3 & 5.5 & 0.5 & 0.5 \end{bmatrix}$$

The ideal solutions of the (Problem-V and Problem-VI) are $x_{14} = 7, x_{21} = 7, x_{22} = 2, x_{24} = 8, x_{31} = 3, x_{33} = 13$ and $x_{11} = 9$,$x_{22} = 2$,$x_{24} = 15$,$x_{31} = 1$,$x_{33} = 15$ respectively with the ideal value of the objective function Z^*=$\langle z^*_M, z^*_W\rangle$=$\langle 304.5, 30\rangle$.
Using Definition (3.1), the distance from Z^*=$\langle z^*_M, z^*_W\rangle$=$\langle 304.5, 30\rangle$ to Z=$\langle z_M, z_W\rangle$=$\langle 316.5, 43.6\rangle$ is 18.13.

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The following table shows the comparison of the proposed method with the existing method given by Das et al.(1999) for a set of twenty simulated problems.

Table 1: Comparison of proposed method with the method given by Das et al.(1999)

		Solution		Distance of the solution from ideal solution	
Problem	Ideal Solution $\langle z_M^*, z_W^* \rangle$	Proposed Method	Method proposed by Das et al.(1999)	Proposed Method	Method proposed by Das et al.(1999)
1	$\langle$304.5,30$\rangle$	[272.88,360.25] $\langle$316.5,43.6$\rangle$	[254,355] $\langle$304.5,50.5$\rangle$	**18.13**	20.5
2	$\langle$452,88$\rangle$	[364.72,542] $\langle$453.36,88.64$\rangle$	[362,542] $\langle$452,90$\rangle$	**1.50**	2
3	$\langle$552.5,40.5$\rangle$	[553.8,648.87] $\langle$601.33,47.53$\rangle$	[495,610] $\langle$552.5,57.5$\rangle$	49	**17**
4	$\langle$913,117$\rangle$	[890.53,1202.82] $\langle$1046.67,156.14$\rangle$	[692,1134] $\langle$913,221$\rangle$	139.28	**104**
5	$\langle$851.5,138.5$\rangle$	[700.95,1090.25] $\langle$895.6,194.65$\rangle$	[621,1082] $\langle$851.5,230.5$\rangle$	**71.39**	92
6	$\langle$301.5,51$\rangle$	[296.45,417.9] $\langle$357.17,60.7$\rangle$	[228,375] $\langle$301.5,73.5$\rangle$	56.5	**22.5**
7	$\langle$469.5,95$\rangle$	[415.5,642.41] $\langle$528.95,113.45$\rangle$	[335,606] $\langle$470.5,135.5$\rangle$	62.24	**40.51**
8	$\langle$944.5,183.5$\rangle$	[769.18,1157.28] $\langle$963.23,194.05$\rangle$	[739,1150] $\langle$944.5,205.5$\rangle$	**21.49**	22
9	$\langle$751,127$\rangle$	[625.48,879.87] $\langle$752.67,127.19$\rangle$	[626,878] $\langle$752,126$\rangle$	1.680	**1.414**
10	$\langle$580,111$\rangle$	[477.36,771.68] $\langle$624.52,147.16$\rangle$	[425.2,776.34] $\langle$600.75,175.57$\rangle$	**57.35**	67.82
11	$\langle$772,190$\rangle$	[594.84,1053.99] $\langle$824.4,229.57$\rangle$	[528,1016] $\langle$772,244$\rangle$	65.66	**54**
12	$\langle$600.5,121$\rangle$	[470.7,761.64] $\langle$616.17,145$\rangle$	[425.5,747] $\langle$586.25,160.75$\rangle$	**28.66**	42.22
13	$\langle$697.5,118.5$\rangle$	[563.56,852.54] $\langle$708.05,144.49$\rangle$	[535,868] $\langle$701.5,166.5$\rangle$	**28.04**	48.16
14	$\langle$589,182$\rangle$	[353.59,832.54] $\langle$593.06,239.47$\rangle$	[391.5,822.5] $\langle$607,215.5$\rangle$	57.61	**38.02**
15	$\langle$897,194$\rangle$	[718.55,1213.22] $\langle$965.88,247.33$\rangle$	[673,1217] $\langle$945,272$\rangle$	**87.11**	91.58
16	$\langle$892,116.5$\rangle$	[955.3,1382.33] $\langle$1168.8,213.51$\rangle$	[564.96,1223.23] $\langle$894.0,329.13$\rangle$	293.3	**212.6**
17	$\langle$1326.5,257$\rangle$	[1049.66,1824.55] $\langle$1437.10,387.44$\rangle$	[874.72,1840.4] $\langle$1357.56,482.84$\rangle$	**171**	227.9
18	$\langle$1495.5,136.5$\rangle$	[1376.65,1666.6] $\langle$1521.63,144.98$\rangle$	[1331,1660] $\langle$1495.5,164.5$\rangle$	**27.4**	28
19	$\langle$772,190$\rangle$	[594.84,1053.99] $\langle$824.4,229.57$\rangle$	[528,1016] $\langle$772,244$\rangle$	65.66	**54**
20	$\langle$452,130$\rangle$	[315,632.05] $\langle$473.5,158.25$\rangle$	[279,627] $\langle$453,174$\rangle$	**35.5**	44.01

9. Comparison and Conclusion

The present paper proposes an alternative solution approach for solving ITP where the cost coefficient of the objective function and source and destination parameters have been considered as an interval. Firstly, the single objective interval transportation problem is converted into a bi-objective crisp transportation problem where the objectives are to minimize the left-limit z_L of the interval (i.e. best case) simultaneously by minimizing the width z_W (i.e. uncertainty) of the interval. After that, the fuzzy programming technique is used to obtain the Pareto optimal solution of the transformed bi-objective transportation problem. Using definition (3.1) the results of the proposed method have been compared with that of the method developed by Das et al.(1999) . The comparison Table 1 shows that in eleven out of twenty problems the proposed method provides a better solution than the existing method. So, the proposed approach can be considered as an alternative approach for solving ITP if decision maker is interested in finding the minimum cost with minimum uncertainty.

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