

## *ln*-Type Variance Estimators in Simple Random Sampling

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### Abstract

Until now, various types of estimators have been used for estimating the population variance in simple random sampling studies, including ratio, product, regression and exponential-type estimators. In this article, we propose a family of *ln*-type estimators for the first time in the simple random sampling and show that they are more efficient than the other types of estimators under certain conditions obtained theoretically. Numerical illustrations and a simulation study support our findings in theory. In addition, it has been shown how to determine the optimal points in order to reach the minimum MSE values with the properties of the *ln*-type estimators in the different data sets.

**Key Words:** Simple Random Sampling; Variance Estimator; *ln*-type Estimator; Efficiency.

**Mathematical Subject Classification:** 62D05

### 1. Introduction

Different types of estimators have been proposed in the literature for estimating the population variance in simple random sampling, including ratio-type (Singh and Solanki 2013, Yadav et al. 2015, Solanki et al. 2015 and Kadilar and Cekim 2017), regression-type (Diana and Tommasi 2004, Kadilar and Cingi 2007 and Asghar et al. 2017) and exponential-type (Yadav et al. 2015, Yadav and Kadilar 2013) estimators. Apart from these types of estimators, *ln* type estimators have used by Cekim and Kadilar (2020) in stratified random sampling methods and Hassan et al. (2020) in two phase sampling methods. Singh and Solanki (2013), Yadav et al. (2013, 2015) and Khan (2015) benefited from transformations of an auxiliary variable; Shabbir and Gupta (2014) and Adichwal et al. (2016) used the information of two auxiliary variables; Singh et al. (2016), Singh and Solanki (2014) and Subramani and Kumarapandiyam (2015) improved estimators as a family for the population variance.

The type of estimator used is decided by looking at the relationship between the study variable  $y$  and the auxiliary variable  $x$ . If the relationship is a straight line passing through the neighborhood of the origin, then ratio and product estimators are equal to regression estimators. Exponential estimators are proposed to make ratio and product estimators more effective than regression estimators when this condition is not satisfied. In this article, we achieve better estimators using the *ln*-function when the condition is not satisfied. We propose an *ln*-type estimator of the population variance in simple random sampling for the first time in the literature on Sampling Theory.

The usual variance estimator is given by

$$s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2, \quad (1)$$

where  $n$  is the sample size. Since this estimator is unbiased, its variance expression is given by

$$V(s_y^2) = \lambda S_y^4 \theta_{40}^*, \quad (2)$$

where  $\lambda = \frac{1}{n} - \frac{1}{N}$ ,  $S_y^2$  is the population variance and  $\theta_{jk} = \frac{\mu_{jk}}{\mu_{20}^{j/2} \mu_{02}^{k/2}}$ ,  $\theta_{jk}^* = \theta_{jk} - 1$ . Here,  $\mu_{jk} = \frac{1}{N-1} \sum_{i=1}^N (y - \bar{Y})^j (x - \bar{X})^k$ ,

$N$  is the size of the population,  $(y_i, x_i)$  are the values of the study and auxiliary variables, respectively, and  $(\bar{Y}, \bar{X})$  are the population means.

Isaki (1983) defined the classical ratio estimator to estimate the population variance of the study variable as follows:

$$t_{r1} = s_y^2 \frac{S_x^2}{s_x^2}, \tag{3}$$

where  $s_x^2$  and  $s_y^2$  are the variances of the samples of the auxiliary and the study variables, respectively,  $S_x^2$  is the population variance of the auxiliary variable. The bias and the mean square error (MSE) of the estimator in (3) are given respectively up to the first degree of approximation by

$$B(t_{r1}) = \lambda S_y^2 [\theta_{04}^* - \theta_{22}^*] \tag{4}$$

and

$$MSE(t_{r1}) = \lambda S_y^4 [\theta_{40}^* - 2\theta_{22}^* + \theta_{04}^*]. \tag{5}$$

Table 1: Ratio estimators for different values of  $\tau$  and  $\kappa$

Constant values				
$i$	$\tau$	$\kappa$	Ratio estimators	Authors
2	1	$-\beta_2(x)$	$t_{r2} = s_y^2 \frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}$	Kadilar and Cingi (2006)
3	1	$-C_x$	$t_{r3} = s_y^2 \frac{S_x^2 - C_x}{s_x^2 - C_x}$	Kadilar and Cingi (2006)
4	$\beta_2(x)$	$-C_x$	$t_{r4} = s_y^2 \frac{\beta_2(x)S_x^2 - C_x}{\beta_2(x)s_x^2 - C_x}$	Kadilar and Cingi (2006)
5	$C_x$	$-\beta_2(x)$	$t_{r5} = s_y^2 \frac{C_x S_x^2 - \beta_2(x)}{C_x s_x^2 - \beta_2(x)}$	Kadilar and Cingi (2006)
6	1	$\beta_2(x)$	$t_{r6} = s_y^2 \frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}$	Upadhyaya and Singh (1999)

The estimator in (3) is motivated for different values of  $\tau$  and  $\kappa$  by Upadhyaya and Singh (1999) and Kadilar and Cingi (2006), as presented in Table 1, when the classical ratio estimator is shown as a class by

$$t_{ri} = s_y^2 \frac{\tau S_x^2 + \kappa}{\tau s_x^2 + \kappa}, i = 2, 3, \dots, 6,$$

where  $\tau$  and  $\kappa$  are constant values or a function of known population parameters of the auxiliary variable  $x$  such as the population mean ( $\bar{X}$ ), the population coefficient of kurtosis ( $\beta_2(x)$ ), the population coefficient of variation ( $C_x$ ), the population coefficient of skewness ( $\beta_1(x)$ ), the population coefficient of correlation ( $\rho_{xy}$ ), etc.

The bias and the MSE of the estimators provided in Table 1 are given respectively up to the first degree of approximation by

$$B(t_{ri}) = \lambda S_y^2 [\phi^2 \theta_{04}^* - \phi \theta_{22}^*] \tag{6}$$

and

$$MSE(t_{ri}) = \lambda S_y^4 [\theta_{40}^* - 2\phi \theta_{22}^* + \phi^2 \theta_{04}^*], i = 2, 3, \dots, 6, \tag{7}$$

where  $\phi = \frac{\tau S_x^2}{\tau S_x^2 + \kappa}$ . Similarly, Isaki (1983) defined the classical regression estimator for  $S_y^2$  as

$$t_{reg} = s_y^2 + b_{(s_y^2, s_x^2)} (S_x^2 - s_x^2), \tag{8}$$

where  $b_{(s_y^2, s_x^2)} = \frac{s_y^2 \theta_{22}^*}{s_x^2 \theta_{04}^*}$ .

Since the estimator in (8) is unbiased, its variance is given by

$$V(t_{reg}) = \lambda S_y^4 \theta_{40}^* (1 - \rho^{*2}), \tag{9}$$

where  $\rho^* = \frac{\theta_{22}^*}{\sqrt{\theta_{40}^* \theta_{04}^*}}$ .

Bahl and Tuteja [24] developed the exponential ratio estimator as follows:

$$t_{exp} = s_y^2 \exp\left(\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right). \tag{10}$$

The bias and the MSE expressions of the estimator given in (10), up to the first degree of approximation, are

$$B(t_{exp}) = \frac{\lambda S_y^2}{2} \left[ \frac{3}{4} \theta_{04}^* - \theta_{22}^* \right] \tag{11}$$

and

$$MSE(t_{exp}) = \lambda S_y^4 \left[ \theta_{40}^* - \theta_{22}^{*2} + \frac{\theta_{04}^*}{4} \right], \tag{12}$$

respectively.

## 2. Proposed Family of *ln*-Type Estimators and its Properties

We propose the family of *ln*-type estimators for the population variance as

$$t_{ln} = s_y^2 \ln\left(\frac{\tau s_x^2 + \kappa}{\tau S_x^2 + \kappa} + \frac{\tau s_x^2 + \kappa}{\tau S_x^2 + \kappa}\right). \tag{13}$$

Table 2. Examples of the proposed estimators

Constant values			Proposed estimators
<i>j</i>	$\tau$	$\kappa$	
1	1	1	$t_{ln1} = s_y^2 \ln\left(\frac{s_x^2}{S_x^2} + \frac{s_x^2}{s_x^2}\right)$
2	1	$-\beta_2(x)$	$t_{ln2} = s_y^2 \ln\left(\frac{s_x^2 - \beta_2(x)}{S_x^2 - \beta_2(x)} + \frac{s_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right)$
3	1	$-C_x$	$t_{ln3} = s_y^2 \ln\left(\frac{s_x^2 - C_x}{S_x^2 - C_x} + \frac{s_x^2 - C_x}{s_x^2 - C_x}\right)$
4	$\beta_2(x)$	$-C_x$	$t_{ln4} = s_y^2 \ln\left(\frac{\beta_2(x)s_x^2 - C_x}{\beta_2(x)S_x^2 - C_x} + \frac{\beta_2(x)s_x^2 - C_x}{\beta_2(x)s_x^2 - C_x}\right)$
5	$C_x$	$-\beta_2(x)$	$t_{ln5} = s_y^2 \ln\left(\frac{C_x s_x^2 - \beta_2(x)}{C_x S_x^2 - \beta_2(x)} + \frac{C_x s_x^2 - \beta_2(x)}{C_x s_x^2 - \beta_2(x)}\right)$
6	1	$\beta_2(x)$	$t_{ln6} = s_y^2 \ln\left(\frac{s_x^2 + \beta_2(x)}{S_x^2 + \beta_2(x)} + \frac{s_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right)$

Here, we suppose that  $\frac{\tau S_x^2 + \kappa}{\tau S_x^2 + \kappa} + \frac{\tau S_x^2 + \kappa}{\tau S_x^2 + \kappa} > 1$ . Note that the variances are always positive.

Some examples of the proposed estimators are demonstrated in Table 2 for different values of  $\tau$  and  $\kappa$ . We consider the following error terms to obtain the equations of the bias and the MSE of the proposed estimators:

$$\mathcal{G}_0 = \frac{S_y^2}{S_y^2} - 1, \mathcal{G}_1 = \frac{S_x^2}{S_x^2} - 1, E(\mathcal{G}_0) = E(\mathcal{G}_1) = 0,$$

$$E(\mathcal{G}_0^2) = \lambda \theta_{40}^*, E(\mathcal{G}_1^2) = \lambda \theta_{04}^*, E(\mathcal{G}_0 \mathcal{G}_1) = \lambda \theta_{22}^*.$$

By expanding the estimator proposed in (13) and using the notation of  $\mathcal{G}$ 's up to the first order approximation, for  $|\phi \mathcal{G}_j| < 1$ , we have

$$t_{lnj} = S_y^2 (1 + \mathcal{G}_0) \ln \left[ (1 + \phi \mathcal{G}_1)^{-1} + (1 + \phi \mathcal{G}_1) \right], j = 1, 2, \dots, 6, \\ \cong S_y^2 (1 + \mathcal{G}_0) \ln(2). \tag{14}$$

After subtracting the population variance from both sides of (14), we obtain

$$t_{lnj} - S_y^2 \cong S_y^2 \{ (\ln(2) - 1) + \ln(2) \mathcal{G}_0 \}, j = 1, 2, \dots, 6. \tag{15}$$

Taking the expectation on both sides of (15), we get the bias of the proposed  $ln$ -type estimator to the first order of approximation as

$$B(t_{lnj}) \cong S_y^2 (\ln(2) - 1), j = 1, 2, \dots, 6. \tag{16}$$

By squaring and then taking the expectation on both sides of (15), we derive the MSE of  $t_{ln}$  to the first order of approximation as

$$MSE(t_{lnj}) \cong S_y^4 \{ (\ln(2) - 1)^2 + (\ln(2))^2 \lambda \theta_{40}^* \}, j = 1, 2, \dots, 6. \tag{17}$$

The MSE equation obtained in (17) does not depend on the values of  $\tau$  and  $\kappa$ . The MSE values depend only on the values of  $S_y^4$  and  $\theta_{40}^*$  when the population size is big. In numerical illustration section, we will investigate the effect of the variations of the  $S_y^4$  and  $\theta_{40}^*$  values on the MSE values of the proposed estimator.

### 3. Comparisons

In this section, we compare the MSEs of the proposed estimators  $t_{lnj}, j = 1, 2, \dots, 6$  with the MSEs of the traditional estimators  $t_{ri}, i = 1, 2, \dots, 6$ ,  $t_{reg}$ , and  $t_{exp}$ , respectively. We obtain the following conditions:

$$MSE(t_{lnj}) < MSE(t_{ri}), i, j = 1, 2, \dots, 6, \\ \text{if} \\ \lambda \left[ 0.52 \theta_{40}^* - 2 \phi \theta_{22}^* + \phi^2 \theta_{04}^* \right] - 0.094 > 0. \tag{18}$$

$$MSE(t_{lnj}) < V(t_{reg}), i, j = 1, 2, \dots, 6 \\ \text{if} \\ \lambda \theta_{40}^* (0.52 - \rho^{*2}) - 0.094 > 0 \text{ and then} \\ \rho^{*2} < (0.52) - \frac{0.094}{\lambda \theta_{40}^*}. \tag{19}$$

$$MSE(t_{lnj}) < MSE(t_{exp}), i, j = 1, 2, \dots, 6 \\ \text{if} \\ \lambda \left[ (0.52) \theta_{40}^* - \theta_{22}^* + \frac{\theta_{04}^*}{4} \right] - 0.094 > 0. \tag{20}$$

When the conditions (18)-(20) are satisfied, the proposed estimator is more efficient than the ratio, regression and exponential estimators. Note that  $0.094 \cong (\ln(2) - 1)^2$  and  $0.52 \cong 1 - (\ln(2))^2$ .

### 4. Numerical Illustration

We use the percent relative efficiency (PRE) to evaluate the performance of the considered estimators based on the classical ratio estimator:

$$PRE = \frac{MSE(t_{r1})}{MSE(t_j)} \times 100,$$

where  $j = r_i, reg, exp, ln_j, i, j = 1, 2, \dots, 6$ .

Table 3. Descriptive statistics of the populations in (Kadilar and Cingi, 2006) .

Parameters							
Populations	$N$	$n$	$\theta_{40}^*$	$\theta_{04}^*$	$\theta_{22}^*$	$S_y$	$\rho^*$
1	106	20	78.34	27.46	32.30	6425.09	0.71
2	204	20	54.05	28.78	20.09	2389.77	0.51
3	9	5	3.77	1.58	0.14	383.03	0.06

For the efficiency comparisons of the mentioned estimators, we apply the data set of Kadilar and Cingi (2006), whose population parameters are given in Table 3. The study variable ( $Y$ ) is the amount of apple production and the auxiliary variable ( $X$ ) is the number of apple trees in all populations. Population 1 is the Marmara region, Population 2 is the Black Sea region and Population 3 is the Edirne city in Turkey.

Table 4. PRE values of the estimators with respect to  $S_y^2$ .

Estimators	Population 1	Population 2	Population 3
$t_{r1}$	100.0000000000	100.0000000000	100.0000000000
$t_{r2}$	100.0000003299	99.9999994118	99.9999976556
$t_{r3}$	100.0000000243	99.9999999661	99.9999924533
$t_{r4}$	100.0000000088	99.9999999989	99.9999970703
$t_{r5}$	100.0000016352	99.9999996574	99.9999971729
$t_{r6}$	99.9999997003	100.0000000588	100.0000023443
$t_{reg}$	103.3184418766	106.5467078126	134.7111699567
$t_{exp}$	76.3393775256	103.6187580449	125.8387270697
$t_{ln1}$	144.8869487298	143.3929844530	196.8975481467
$t_{ln2}$	144.8869487298	143.3929844530	196.8975481467
$t_{ln3}$	144.8869487298	143.3929844530	196.8975481467
$t_{ln4}$	144.8869487298	143.3929844530	196.8975481467
$t_{ln5}$	144.8869487298	143.3929844530	196.8975481467
$t_{ln6}$	144.8869487298	143.3929844530	196.8975481467

The results in Tables 4 support the theoretical finding that the proposed estimator is the most efficient; the PRE values of the proposed estimator are higher than those of the other estimators for all three populations. Therefore, we recommend using estimators based on the  $ln$ -function rather than the exponential function for estimating the population variance. For detailed analysis of the MSE values of the proposed estimators computed using (17), we examine the MSE values for different values of  $S_y^4$  and  $\theta_{40}^*$  as shown in Figures 1-3. We observe the effect of the MSE values when the  $S_y^4$  or  $\theta_{40}^*$  values increase for each population. Note that when  $S_y^4$  increases,  $\theta_{40}^*$  decreases

and vice-versa, since  $\theta_{40}^* = \frac{\mu_{40}}{S_y^4} - 1$  where  $\mu_{40} = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^4$ .

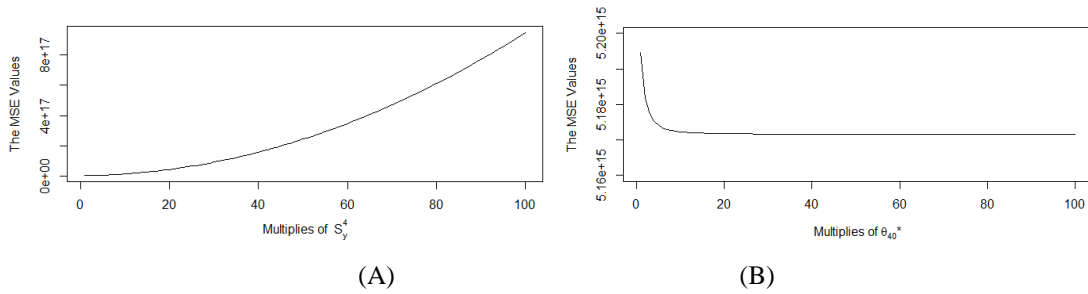


Figure 1. Graphs of the MSE values versus the increasing values of  $S_y^2$  or  $\theta_{40}^*$  in Population 1.

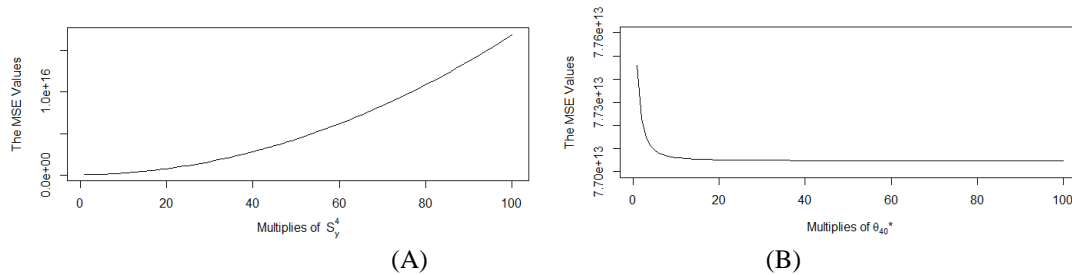


Figure 2. Graphs of the MSE values versus the increasing values of  $S_y^2$  or  $\theta_{40}^*$  in Population 2.

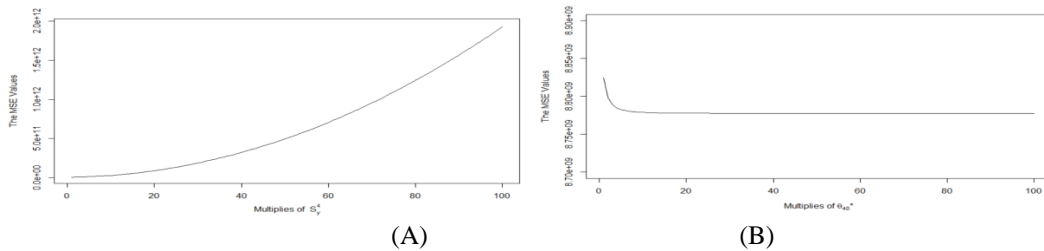


Figure 3. Graphs of the MSE values versus the increasing values of  $S_y^2$  or  $\theta_{40}^*$  in Population 3.

In Figures 1-3, for graphs A, we multiply  $S_y^2$  by 1 to 100, and for graphs B, we multiply  $\theta_{40}^*$  by 1 to 100, given in  $x$  - axis. The MSE values are shown in the  $y$  - axis. From Figures 1-3, we observe that the increasing values of  $S_y^2$  increase the MSE values, while the increasing values of  $\theta_{40}^*$  decrease the MSE values of the proposed estimators. From this result, an optimal point might be found to obtain the minimum MSE for the data sets.

### 5. Simulation Study

We perform a simulation study using Population 3 to support the theoretical results and the numerical illustrations. Moreover, the simulation provides some properties of the proposed  $ln$ -type estimators. The simulation steps are as follows:

- (i) Select a sample with size  $n$  from the population.
- (ii) Repeat (i)  $(R - 1)$  times, where  $R = \binom{N}{n} = \binom{9}{5} = 126$ .
- (iii) Compute the values of the considered estimators  $(s_i^2)$  in the article for each sample.
- (iv) Calculate the MSE values of the estimators using the formula:  $MSE(s_i^2) = \frac{1}{R} \sum_{i=1}^R (s_i^2 - S_y^2)^2$ .

Table 5. PRE values of the estimators for Population 3 in the simulation study

Estimators	R=126
$t_{r1}$	100.00000000
$t_{r2}$	99.9999843792
$t_{r3}$	99.9999972869
$t_{r4}$	99.9999431747
$t_{r5}$	99.9999811622
$t_{r6}$	100.0000156208
$t_{reg}$	221.6154839889
$t_{exp}$	176.0599703444
$t_{ln1}$	315.6211517558
$t_{ln2}$	315.6211458360
$t_{ln3}$	315.6211507276
$t_{ln4}$	315.6211446169
$t_{ln5}$	315.6211507395
$t_{ln6}$	117.31755457

The PRE values of the mentioned estimators over the classical ratio estimator obtained in the simulation study are provided in Table 5. It can be seen that similar to the results of the numerical illustration for Population 3, the proposed estimators have higher PRE values compared to the other estimators, showing that the proposed estimators are more efficient than the others.

### 6. Conclusions

The aim of the sampling studies in the literature is to obtain the minimum MSE using various types of estimators. In this article, a family of  $ln$ -type estimators was proposed as an alternative to the ratio, regression and exponential estimators. It is found that the proposed estimators achieve a better performance when estimating the population variance in simple random sampling compared to the ratio, regression and exponential estimators in both theory and practice. From this result, we can conclude that using the  $ln$ -function in variance estimators improves the efficiency of the estimators. In the future, the proposed estimator can be extended to a family of estimators, such as in the studies of Yadav et al. (2015) and Singh et al. (2017). Also, different estimators of other sampling methods can be developed using the  $ln$ -function.

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