Multi-Objective Supply Chain Inventory Model with Demand Dependent Purchase Cost and Production Rate Dependent Production Cost

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Abstract

In this paper, we have developed an integrated production inventory model for a two echelon supply chain consisting of one manufacturer and one retailer. Purchase cost for the manufacturer is dependent on inventory lot size; production cost of the manufacturer is dependent on production rate. Also purchase cost of the retailer is dependent on demand rate of the customer. The proposed multi objective supply chain inventory model has been solved by various techniques like as Fuzzy programming technique with hyperbolic membership function (FPTHMF), Fuzzy non-linear programming technique (FNLP) and Fuzzy additive goal programming technique (FAGP), weighted Fuzzy non-linear programming technique (WFNLP) and weighted Fuzzy additive goal programming technique (WFAGP). A numerical example is provided to illustrate the proposed model. Finally sensitivity analysis and graphical representation have been shown.

Keywords: Inventory, Supply Chain, Multi-item, Fuzzy Techniques

1. Introduction

The supply chain inventory model deal with decision that minimum the total average cost or maximum the total average profit. In that way to construct a real life mathematical supply chain inventory model on base on various assumptions and notations and approximation. Supply Chain management has taken a very important and critical role for any company in the increase globalization and competition in the business. The success of any supply chain system in any business depends on its level of collaboration and integration. In this paper, it is important to mention that a supply chain composed of two stages is considered a complex system in which one is manufacturer and other one is retailer.

The supply chain may be coordinated in numerous approaches. Zhou and Li (2007) showed that the coordination among parties in the ordering method will increase the expected profit for the retailer as well as for the complete supply chain. A three layer multiple item production inventory model for multiple suppliers and retailers was formulated by Pal et. al (2012). Rau et al., (2003) studied on integrated inventory model for deteriorating items under a multi- echelon supply chain environment. Yang et al., (2000) discussed on economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. Pal et al., (2014) formulated on a multiple echelon production

inventory system with supply disruption. Pal et al. (2012) discussed three layer supply chain: a production inventory model for re-workable items. Goyal, (1976) discussed an integrated inventory model for a single supplier-single customer problem. Tzafestas and Kapsiotis (1994) coordinated control of manufacturing supply chain system using multi-level techniques. Cardenas-Barron and Sana (2014) studied on a production inventory model for a two echelon supply chain when demand is dependent on sales teams' initiatives.

In the real life situations, purchase cost or production cost depend on number of quantities, demand rate, selling price etc. Bhunia and Shaikh (2014) developed on a deterministic inventory model for deteriorating items with selling price dependent demand and three-parameter Weibull distributed deterioration. Sridevi et al (2010) studied on inventory model for deteriorating items with Weibull rate of replenishment and selling price dependent demand. Shah et al., (2009) discussed on lot size inventory model for the Weibull distributed deterioration rate with discounted selling price and stock-dependent demand. Alfares and Ghaithan (2016) considered an inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts multi item is also important in the inventory management system. Ghosh et al., (2015) presented a multi-item inventory model for deteriorating items in limited storage space with stock-dependent demand. Sadeghi (2015) discussed a multi-item integrated inventory model with different replenishment frequencies of retailers in a two-echelon supply chain management: a tuned-parameters hybrid meta-heuristic. Kotb and Fergancy (2011) developed multi-item EOQ model with both demand depended unit cost and varying lead time via geometric programming. Cárdenas-Barrón and Sana (2015) considered multi-item EOQ inventory model in a two-layer supply chain while demand varies with promotional effort.

The concept of fuzzy set theory was first introduced by Zadeh (1965). Afterward Zimmermann (1985) applied the fuzzy set theory concept with some useful membership functions to solve the linear programming problem with some objective functions. Roy and Maiti (1998) considered multi-objective inventory models of deteriorating items with some constraints in a fuzzy environment. Wee et al., (2009) studied on multi-objective joint replenishment inventory model of deteriorated items in a fuzzy environment. Mandal and Islam (2016) formulated a fuzzy inventory model for deteriorating items with time depended demand and shortages under fully backlogged condition.

In this article, an integrated production inventory model for a two echelon supply chain has been considered in which purchase cost for the manufacturer is dependent on inventory lot size, production cost of the manufacturer is dependent on production rate and Purchase cost of the retailer is dependent on demand rate of the customer. Idle time cost and multi-item have been considered in this supply chain inventory model. Average cost is calculated in manufacturer individual inventory level as well as retailer individual inventory level and these are adding in integrated inventory level. The proposed MOSCIM has been solved by various techniques which are

FPTHMF, FNLP, FAGP, WFNLP and WFAGP. A numerical example has been given to illustrate the paper. Sensitivity analysis and graphical representation have been shown. Finally conclusions have been drawn.

2. Notations

$I_{mri}(t)$: Manufacturer raw material inventory level of the ith item at time t.

$I_{mfi}(t)$: Manufacturer finished product level of the ith item at time t.

$I_{ri}(t)$: Retailer inventory level of the ith item at time t.

t_{i1} : End of production time of the ith item of the manufacturer.

P_i : Production rate of the manufacturer of the ith item.

Q_i : The order quantity in the duration of a cycle of length for i[th] item.

h_{mri} : Holding cost per unit per unit time of the raw material for i[th] item.

h_{mfi} : Holding cost per unit per unit time of the finished product for i[th] item.

h_{ri} : Holding cost per unit per unit time of the ith item for retailer.

A_{si} : Set-up cost per order of ith item for the manufacturer.

A_{ri} : Set-up cost per order of ith item for the retailer.

I_{si} : Cost per unit idle time of manufacturer.

i_{ri} : Cost per unit idle time of retailer.

T_i : The length of cycle time for i[th] item.

D_i : Customer demand rate per unit time for the ith item.

$C_{P_m}^i$: The unit purchase cost of the manufacturer of the ith item.

$C_{P_m'}^i$: The unit production cost of the manufacturer of the ith item.

$C_{P_r}^i$: The unit purchase cost of the retailer of the ith item.

AC_m : Average cost of the manufacturer of the ith item.

AC_r : Average cost of the manufacturer of the ith item.

$TAC_i(Q, P, D)$: Joint total average cost of the ith item.

3. Assumptions

1. The multi item inventory system is developed.
2. The replenishment occurs instantaneously at infinite rate.
3. The lead time is neglected.
4. Shortages are not allowed.
5. Demand rate is constant.
6. The cost of idle times at manufacturer and retailer level are considered.

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7. The unit purchase cost $C^i_{P_m}$ of the ith item of the manufacturer is inversely related to the lot size of the raw materials Q_i. So we take the following form $C^i_{P_m}(Q_i) = \delta_i Q_i^{-a_i}$ where $\delta_i > 0$ and $a_i > 0$ are constant real numbers.

8. The unit production cost $C^i_{P_m}$ of the ith item of the manufacturer is considered as

$$C^i_{P_m}(P_i) = \frac{L_i}{P_i} + \sigma_i P_i \text{ where } L_i \text{ is the labor and energy costs of the i}^{th} \text{ item which is}$$

equally distributed over the production rate. This cost is reduced by increases the production lot size. The second term $\sigma_i P_i$ represents tool/die costs which are proportional to the production rate.

9. The unit purchase cost $C^i_{P_r}$ of the retailer is inversely related to the demand rate D_i of the customer. So we take the following form $C^i_{P_r}(D_i) = \mu_i D_i^{-b_i}$ where $\mu_i > 0$ and $b_i > 1$ are constant real numbers.

10. Holding cost of the raw materials, finished products of the supply chain is different.

11. Ratio of the raw material and finished good is $1:1$

4. Model formation in crisp for ith item

4.1 The manufacturer individual inventory model

At this manufacturer inventory level (figure 1), the production starts with raw materials order lot size is Q_i. In this level, two situations occur: the first one is when the raw material decreases with production rate P_i and it reaches to zero at time t_{i1}. The second one is when the finished products, after production, pile up with rate P_i up to time t_{i1}, and it reaches to Q_i at the time t_{i1}. The governing differential equations are as following

$$\frac{dI_{mri}(t)}{dt} = -P_i, 0 \le t \le t_{i1} \tag{1}$$

With the boundary conditions, $I_{mri}(0) = Q_i$, $I_{mri}(t_{i1}) = 0$

And for finished products

$$\frac{dI_{mfi}(t)}{dt} = P_i, 0 \le t \le t_{i1} \tag{2}$$

With the boundary conditions, $I_{mfi}(0) = 0$, $I_{mfi}(t_{i1}) = Q_i$

Solving the above differential equations (1) and (2) we get

$$I_{mri}(t) = Q_i - P_i t, \quad 0 \le t \le t_{i1} \tag{3}$$

and $I_{mfi}(t) = Q_i - P_i(t_{i1} - t), \quad 0 \le t \le t_{i1}$ (4)

where $\quad t_{i1} = \dfrac{Q_i}{P_i}$ (5)
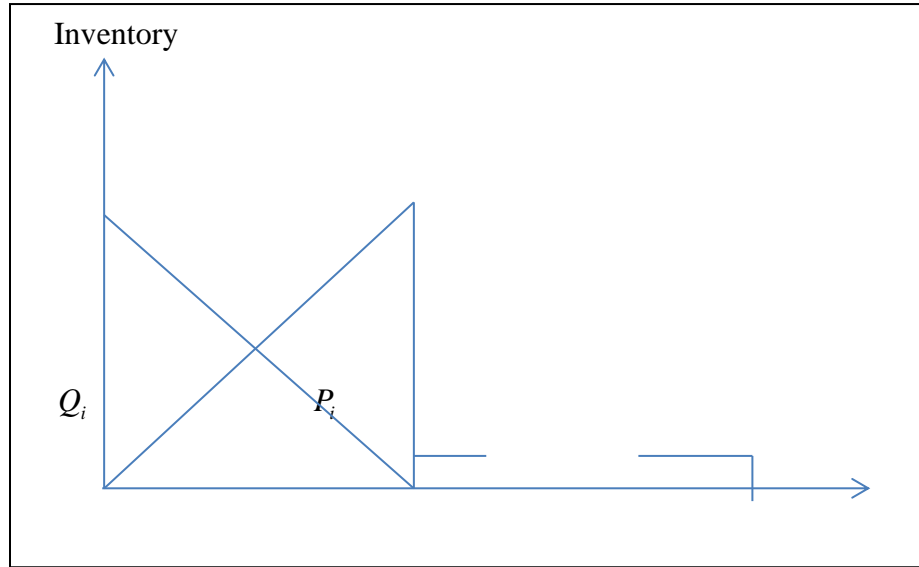

Figure 1: Inventory level for i[th] item of the manufacture

Now we are calculating the various cost as following

i) Average production cost $= \dfrac{Q_i \left(\dfrac{L_i}{P_i} + \sigma_i P_i \right)}{T_i}$

$\qquad\qquad\qquad\qquad = \dfrac{Q_i}{T_i} \left(\dfrac{L_i}{P_i} + \sigma_i P_i \right)$

ii) Average purchased cost $= \dfrac{\delta_i Q_i^{1-a_i}}{T_i}$

iii) Average holding cost of raw materials $= \dfrac{h_{mri}}{T_i} \displaystyle\int_0^{t_{i1}} I_{mri}(t)\,dt$

$\qquad\qquad\qquad\qquad\qquad\qquad = \dfrac{h_{mri} Q_i^2}{2 T_i P_i}$

iv) Average holding cost of finished products $= \dfrac{h_{mfi}}{T_i} \displaystyle\int_0^{t_{i1}} I_{mfi}(t)\,dt$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \dfrac{h_{mfi} Q_i^2}{2 T_i P_i}$

v) Average set-up-cost $= \dfrac{A_{si}}{T_i}$

vi) Average idle time cost $= \dfrac{i_{si}\left(T_i - t_{i1}\right)}{T_i}$

Therefore average cost of the manufacturer is

$AC_m =<$ Average production cost $>+<$ Average purchasing cost $>+<$ Average holding cost of raw materials $>+<$ Average holding cost of finished products $>+<$ Average set-up-cost $>+<$ Average idle time cost $>$

$$AC_m = \frac{\delta_i Q_i^{1-a_i}}{T_i} + \frac{Q_i}{T_i}\left(\frac{L_i}{P_i} + \sigma_i P_i\right) + \frac{h_{mri} Q_i^2}{2T_i P_i} + \frac{h_{mfi} Q_i^2}{2T_i P_i} + \frac{A_{si}}{T_i} + \frac{i_{si}\left(T_i - t_{i1}\right)}{T_i} \tag{6}$$

Hence

$$AC_m\left(Q_i, P_i, D_i\right) = \frac{\delta_i Q_i^{-a_i} P_i D_i}{\left(P_i + D_i\right)} + \frac{P_i D_i}{\left(P_i + D_i\right)}\left(\frac{L_i}{P_i} + \sigma_i P_i\right) + \frac{h_{mri} Q_i D_i}{2\left(P_i + D_i\right)} + \frac{h_{mfi} Q_i D_i}{2\left(P_i + D_i\right)} +$$

$$\frac{A_{si} P_i D_i}{Q_i\left(P_i + D_i\right)} + \frac{i_{si} P_i}{\left(P_i + D_i\right)} \tag{7}$$

for $i = 1,2,3,......,n$

4.2 The retailer individual inventory model

At this inventory level (figure 2),the corresponding differential equation is

$$\frac{dI_{ri}(t)}{dt} = -D_i, \quad t_{i1} \leq t \leq T_i \tag{8}$$

With boundary conditions, $I_{ri}\left(t_{i1}\right) = Q_i$, $I_{ri}\left(T_i\right) = 0$

Solving the above differential equation (8) we get,

$$I_{ri}(t) = Q_i - D_i\left(t - t_{i1}\right), \quad t_{i1} \leq t \leq T_i \tag{9}$$
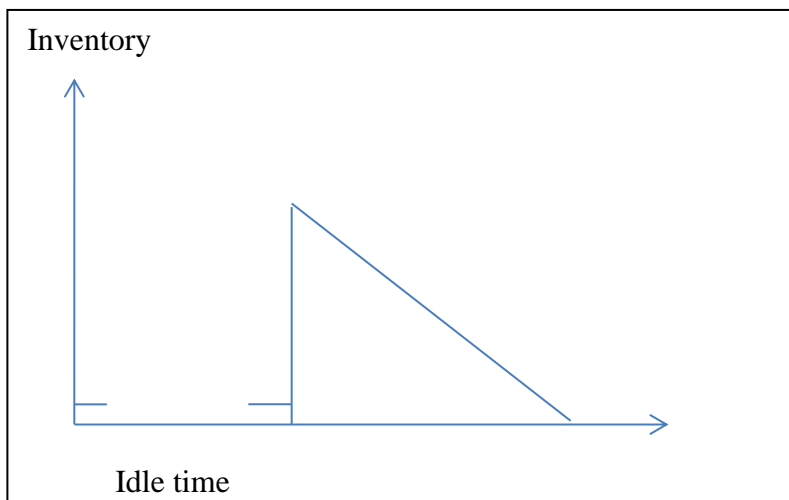
and $T_i = \dfrac{Q_i}{D_i} + \dfrac{Q_i}{P_i} \tag{10}$

Figure 2: Inventory situation for i^{th} item of the retailer

Now we calculate the various costs as following

i) Average purchased cost $= \dfrac{\mu_i D_i^{-b_i} Q_i}{T_i}$

ii) Average holding cost $= \dfrac{h_{ri}}{T_i} \displaystyle\int_{t_{i1}}^{T_i} I_{ri}(t)dt$

$$= \dfrac{h_{ri} Q_i^2}{2T_i D_i}$$

iii) Average set-up-cost $= \dfrac{A_{ri}}{T_i}$

iv) Average Idle time cost $= \dfrac{i_{ri} t_{i1}}{T_i}$

Therefore the average cost for the retailer individual inventory level is

$AC_r =<$ Average purchased cost $>+<$ Average holding cost $>+<$ Average set-up-cost $>+<$ Average Idle time cost $>$

$$AC_r = \dfrac{\mu_i D_i^{-b_i} Q_i}{T_i} + \dfrac{h_{ri} Q_i^2}{2T_i D_i} + \dfrac{A_{ri}}{T_i} + \dfrac{i_{ri} t_{i1}}{T_i}$$

$$AC_r(Q_i, P_i, D_i) = \dfrac{\mu_i D_i^{1-b_i} P_i}{(P_i + D_i)} + \dfrac{h_{ri} Q_i P_i}{2(P_i + D_i)} + \dfrac{A_{ri} P_i D_i}{(P_i + D_i)Q_i} + \dfrac{i_{ri} D_i}{(P_i + D_i)} \tag{11}$$

4.3 The integrated inventory model

Therefore the total average cost for i^{th} item in integrated inventory model is

$$TAC_i(Q_i, P_i, D_i) = AC_m(Q_i, P_i, D_i) + AC_r(Q_i, P_i, D_i)$$

$$TAC_i(Q_i, P_i, D_i) = \dfrac{\delta_i Q_i^{-a_i} P_i D_i}{(P_i + D_i)} + \dfrac{P_i D_i}{(P_i + D_i)}\left(\dfrac{L_i}{P_i} + \sigma_i P_i\right) + \dfrac{h_{mri} Q_i D_i}{2(P_i + D_i)} + \dfrac{h_{mfi} Q_i D_i}{2(P_i + D_i)} +$$

$$\dfrac{\mu_i D_i^{1-b_i} P_i}{(P_i + D_i)} + \dfrac{h_{ri} Q_i P_i}{2(P_i + D_i)} + \dfrac{A_{ri} P_i D_i}{(P_i + D_i)Q_i} + \dfrac{i_{ri} D_i}{(P_i + D_i)} + \dfrac{A_{si} P_i D_i}{Q_i(P_i + D_i)} + \dfrac{i_{si} P_i}{(P_i + D_i)}$$

$D_i > 0, P_i > 0, Q_i > 0$ for $i = 1,2,3,......,n$ $\tag{12}$

Here total average cost per unit time for each item can be considered as one objective function. So our proposed multi objective supply chain inventory model (MOSCIM) can be written as

Min $\{TAC_1, TAC_2, TAC_3, ..., TAC_n\}$

Subject to, $D_i > 0, P_i > 0, Q_i > 0$

Where

$$TAC_i(Q_i, P_i, D_i) = \frac{\delta_i Q_i^{-a_i} P_i D_i}{(P_i + D_i)} + \frac{P_i D_i}{(P_i + D_i)}\left(\frac{L_i}{P_i} + \sigma_i P_i\right) + \frac{h_{mri} Q_i D_i}{2(P_i + D_i)} + \frac{h_{mfi} Q_i D_i}{2(P_i + D_i)} +$$

$$\frac{\mu_i D_i^{1-b_i} P_i}{(P_i + D_i)} + \frac{h_{ri} Q_i P_i}{2(P_i + D_i)} + \frac{A_{ri} P_i D_i}{(P_i + D_i)Q_i} + \frac{i_{ri} D_i}{(P_i + D_i)} + \frac{A_{si} P_i D_i}{Q_i(P_i + D_i)} + \frac{i_{si} P_i}{(P_i + D_i)}$$

for $i = 1,2,3,......,n$ (13)

5. Fuzzy programming technique with hyperbolic membership function (FPTHMF) for solving MOSCIM.

Solve the MOSCIM (13) as a single objective NLP using only one objective at a time and we are ignoring the all other objectives. Repeat the process n times for n different objective functions. So we get the ideal solutions. Pay-off matrix is prepared by using the ideal solutions as follows:

$$\begin{array}{cccc} & TAC_1(Q_1, P_1, D_1) & TAC_2(Q_2, P_2, D_2) & .... & TAC_n(Q_n, P_n, D_n) \\ (Q_1^1, P_1^1, D_1^1) & \begin{bmatrix} TAC_1^*(Q_1^1, P_1^1, D_1^1) & TAC_2(Q_1^1, P_1^1, D_1^1) & .... & TAC_n(Q_1^1, P_1^1, D_1^1) \\ (Q_2^2, P_2^2, D_2^2) & TAC_2(Q_2^2, P_2^2, D_2^2) & TAC_2^*(Q_2^2, P_2^2, D_2^2) & .... & TAC_n(Q_2^2, P_2^2, D_2^2) \\ .... & .... & .... & .... & .... \\ (Q_n^n, P_n^n, D_n^n) & TAC_1(Q_n^n, P_n^n, D_n^n) & TAC_2(Q_n^n, P_n^n, D_n^n) & .... & TAC_n^*(Q_n^n, P_n^n, D_n^n) \end{bmatrix} \end{array}$$

(14)

Let $U^k = \text{Max}\left\{TAC_k(Q_i^i, P_i^i, D_i^i), i = 1,2,.....,n\right\}$ for $k = 1,2,3,....,n$ (15)

and $L^k = TAC_k^*(Q_k^k, P_k^k, D_k^k)$ for $k = 1,2,3,....,n$ (16)

Therefore $L^k \le TAC_k(Q_i^i, P_i^i, D_i^i) \le U^k$ for $i = 1,2,3,....,n$ and $k = 1,2,3,....,n$

The non-linear hyperbolic membership function $\mu_{TAC_i}^H(TAC_i(Q_i, P_i, D_i))$ for the ith objective function $TAC_i(Q_i, P_i, D_i)$ is defined as follows:

$$\mu_{TAC_i}^H(TAC_i(Q_i, P_i, D_i)) = \frac{1}{2}\tanh\left(\left(\frac{U^i + L^i}{2} - TAC_i(Q_i, P_i, D_i)\right)\rho_i\right) + \frac{1}{2}$$ (17)

where ρ_i is a parameter, $\rho_i = \dfrac{3}{(U^i - L^i)\big/2} = \dfrac{6}{U^i - L^i}$ for $i = 1,2,3,....,n$

Using the above membership function, fuzzy non-linear programming problem is formulated as follows:

Max λ

Subject to, $\dfrac{1}{2}\tanh\left(\left(\dfrac{U^i + L^i}{2} - TAC_i(Q_i, P_i, D_i)\right)\rho_i\right) + \dfrac{1}{2} \ge \lambda, \lambda \ge 0$ (18)

After simplification the above non-linear programming problem (18) can be written as

Max y

Subject to, $y + \rho_i \left(TAC_i \left(Q_i, P_i, D_i \right) \right) \leq \dfrac{U^i + L^i}{2} \rho_i, \; y \geq 0, \; D_i > 0, P_i > 0, Q_i > 0$

for $i = 1,2,3,....,n$ (19)

The programming problem (19) can be solved by suitable mathematical programming algorithm and we shall get the solution of the MOSCIM (13).

6. Fuzzy programming technique (based on max-min and max-additive operators)

In this technique for solving MOSCIM (13), first we have to make pay-off matrix which has been shown in the above (14) and then to find U^i and L^i, shown in equation no. (15) and (16). In this technique, the membership function $\mu_{TAC_i} \left(TAC_i \left(Q_i, P_i, D_i \right) \right)$ for the ith objective function $TAC_i \left(Q_i, P_i, D_i \right)$ is defined as follows:

$$\mu_{TAC_i} \left(TAC_i \left(Q_i, P_i, D_i \right) \right) = \begin{cases} 1 & for \; TAC_i(Q_i, P_i, D_i) < L^i \\ \dfrac{U^i - TAC_i(Q_i, P_i, D_i)}{U^i - L^i} & for \; L^i \leq TAC_i(Q_i, P_i, D_i) \leq U^i \\ 0 & for \; TAC_i(Q_i, P_i, D_i) > U^i \end{cases}$$

for $i = 1,2,3,....,n$.   (20)

6.1 Fuzzy non-linear programming technique (FNLP) based on max-min operator

Using the above membership function (20), fuzzy non-linear programming problem based on max-min operator is formulated as follows:

Max $\alpha^{'}$

Subject to, $TAC_i \left(Q_i, P_i, D_i \right) + \alpha^{'} \left(U^i - L^i \right) \leq U^i, 0 \leq \alpha^{'} \leq 1, D_i > 0, P_i > 0, Q_i > 0$

for $i = 1,2,3,....,n$ (21)

The programming problem (21) can be solved by suitable mathematical programming algorithm and we shall get the solution of the MOSCIM (13).

6.2 Fuzzy additive goal programming technique (FAGP) based on max-additive operator

In this technique, using membership function (20), the fuzzy additive goal programming problem based on max-additive operator is formulated as follows:

Max $\displaystyle\sum_{i=1}^{n} \dfrac{U^i - TAC_i \left(Q_i, P_i, D_i \right)}{U^i - L^i}$

Subject to, $L^i - TAC_i \left(Q_i, P_i, D_i \right) \leq 0, \; D_i > 0, P_i > 0, Q_i > 0$ for $i = 1,2,3,....,n$ (22)

The programming problem (22) can be solved by suitable mathematical programming algorithm and we shall get the solution of the MOSCIM (13).

7. Weighted fuzzy programming technique (based on max-min and max-additive operators)

7.1 Weighted fuzzy non-linear programming technique (WFNLP) based on max-min operator

For this technique we consider positive weight ω_i for each objective $TAC_i(Q_i, P_i, D_i)$. Using the above membership function (20), the WFNLP based on max-min operator is stated as follows:

Max α

Subject to, $\omega_i \mu_{TAC_i}(TAC_i(Q_i, P_i, D_i)) \geq \alpha,\ 0 \leq \alpha \leq 1,\ D_i > 0, P_i > 0, Q_i > 0$

and $\sum_{i=1}^{n} \omega_i = 1$, for $i = 1, 2, 3, ..., n$ (23)

The programming problem (23) can be solved by suitable mathematical programming algorithm and we shall get the solution of the MOSCIM (13).

7.2 Weighted fuzzy additive goal programming technique (WFAGP) based on max-additive operator (WFAGP)

Again using the above membership function (20), the WFAGP based on max-additive operator is formulated as follows

Max $\sum_{i=1}^{n} \omega_i \mu_{TAC_i}(TAC_i(Q_i, P_i, D_i))$

Subject to, $0 \leq \mu_{TAC_i}(TAC_i(Q_i, P_i, D_i)) \leq 1,\ D_i > 0, P_i > 0, Q_i > 0$

and $\sum_{i=1}^{n} \omega_i = 1$, for $i = 1, 2, 3, ..., n$ (24)

The programming problem (24) can be solved by suitable mathematical programming algorithm and we shall get the solution of the MOSCIM (13).

8. Numerical Example

Let us consider two items with parameters value in proper unit.

Table 1: Input imprecise data for shape parameters

Item	Parameters												
	δ	a	L	σ	μ	b	h_{mr}	h_{mf}	h_r	A_s	i_s	A_r	i_r
I	100	5	500	0.5	200	10	1.5	3	5	200	4	300	6
II	200	7	800	0.6	300	11	1.0	2	4	300	3	400	5

Table 2: Optimal solutions of MOSCIM using different methods

Methods	$P_1{}^*$	$D_1{}^*$	$Q_1{}^*$	$TAC_1{}^*$	$P_2{}^*$	$D_2{}^*$	$Q_2{}^*$	$TAC_2{}^*$
FPTHMF	28.51	1.41	16.44	137.23	33.57	1.44	22.07	162.08
FNLP	28.51	1.41	16.33	137.23	33.57	1.44	22.07	162.08
FAGP	28.39	1.41	16.34	137.23	33.57	1.44	22.07	162.08

Table 3: Optimal solution of MOSCIM using different weights by WFNLP method

weights	$P_1{}^*$	$D_1{}^*$	$Q_1{}^*$	$TAC_1{}^*$	$P_2{}^*$	$D_2{}^*$	$Q_2{}^*$	$TAC_2{}^*$
$w_1 = 0.4 , w_2 = 0.6$	28.49	1.41	16.42	137.23	29.81	1.43	17.83	164.39
$w_1 = 0.5 , w_2 = 0.5$	28.34	1.41	16.31	137.23	33.57	1.44	22.07	162.08
$w_1 = 0.6 , w_2 = 0.4$	31.91	1.41	16.77	137.49	33.57	1.44	22.07	162.08

Table 4: Optimal solution of MOSCIM using different weights by WFAGP method

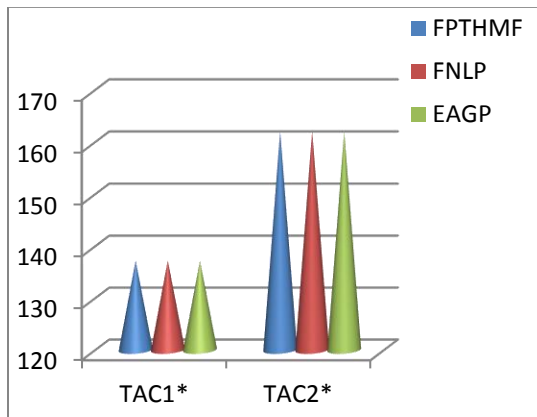| weights	$P_1{}^*$	$D_1{}^*$	$Q_1{}^*$	$TAC_1{}^*$	$P_2{}^*$	$D_2{}^*$	$Q_2{}^*$	$TAC_2{}^*$
$w_1 = 0.4 , w_2 = 0.6$	28.46	1.40	16.44	137.24	3.57	1.41	22.07	162.27
$w_1 = 0.5 , w_2 = 0.5$	28.48	1.41	16.34	137.23	33.57	1.44	22.07	162.08
$w_1 = 0.6 , w_2 = 0.4$	28.24	1.41	16.41	137.23	33.57	1.44	22.07	162.08

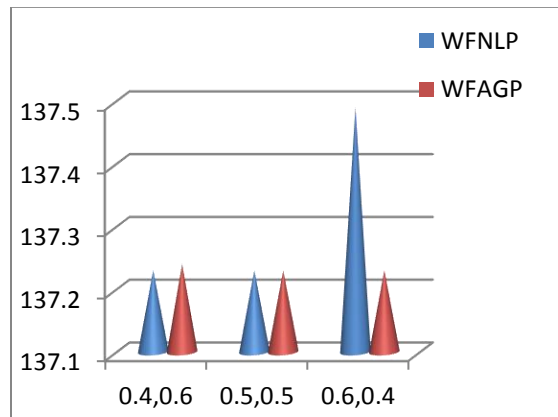Figure 3: minimizing cost of 1st and 2nd
items using different methods.

Figure 4: minimizing cost of 1st item using
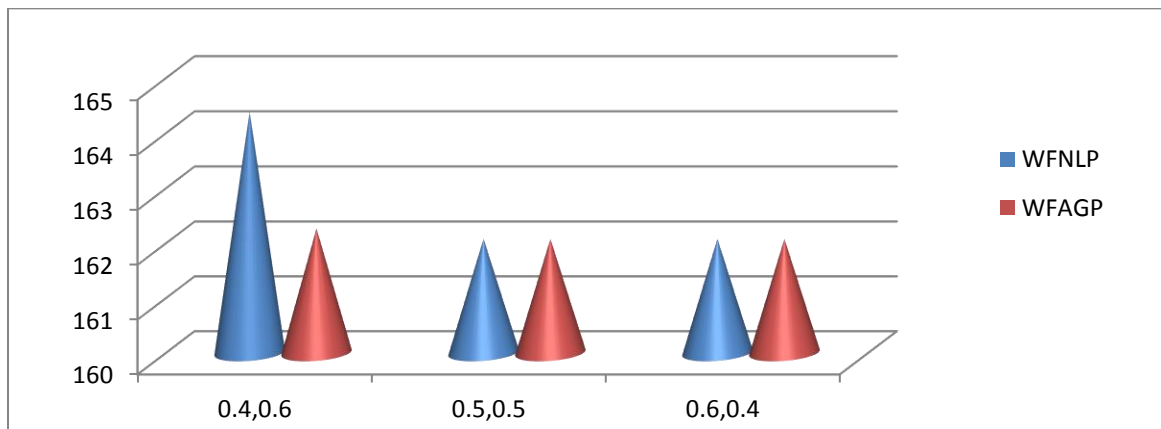different methods and different Weights.

Figure 5: minimizing cost of 2nd item using different methods and different weights.

From the above figure 3 shows that minimum costs of both items are same for all different methods. Figure 4 shows that minimum costs for 1st item are almost same for different weights in WFNLP, WFAGP methods. Figure 5 shows that minimum costs for 2nd item are almost same for different weights in WFNLP, WFAGP methods.

9. Sensitivity Analysis

Table 5: Optimal solution of MOSCIM by FPTHMF, FNLP and FAGP methods for different values of L_1, L_2.

| Method	L_1	L_2	$Q_1{}^*$	$P_1{}^*$	$D_1{}^*$	$TAC_1{}^*$	$Q_2{}^*$	$P_2{}^*$	$D_2{}^*$	$TAC_2{}^*$
	500	800	16.44	28.51	1.41	137.23	22.07	33.57	1.44	162.08
	600	900	16.41	31.66	1.40	141.69	22.02	35.87	1.43	166.05
FPTHMF	700	1000	16.35	34.97	1.39	145.74	22.04	38.04	1.43	169.77
	800	1100	16.36	37.26	1.39	149.47	22.02	40.10	1.42	173.29
	500	800	16.34	28.51	1.41	137.23	22.07	33.57	1.44	162.08
	600	900	16.41	31.66	1.40	141.69	22.05	35.89	1.43	166.05
FNLP	700	1000	16.49	34.54	1.40	145.74	22.04	38.04	1.43	169.77
	800	1100	16.41	37.64	1.39	149.47	22.10	39.83	1.42	173.29
	500	800	16.34	28.39	1.41	137.23	22.07	33.57	1.44	162.08
	600	900	16.42	31.92	1.40	141.69	21.94	36.02	1.43	166.05
FAGP	700	1000	16.54	34.47	1.39	145.74	22.04	38.04	1.43	169.77
	800	1100	16.28	37.35	1.39	149.47	21.74	40.20	1.42	173.29
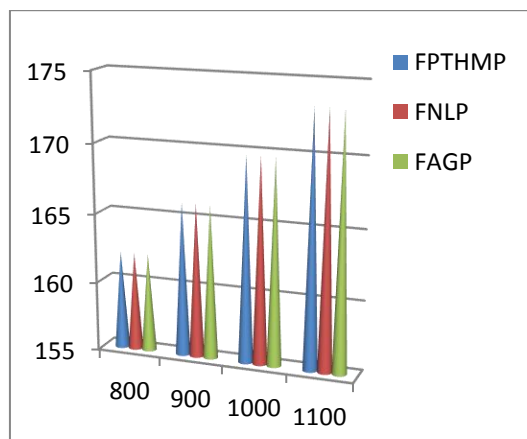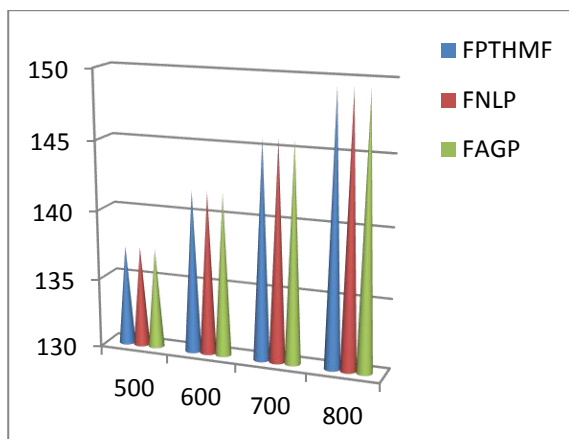
Figure 6: minimizing cost of 1st item using different Methods for different values of L_1

Figure 7: minimizing cost of 2nd item using different Methods for different values of L_2

From the above figures 6 & 7 shows that minimum costs of the both items are increased when values of L_1, L_2 are increased and it is same for all methods.

Table 6: Optimal solution of MOSCIM by FPTHMF, FNLP and FAGP methods

for different values of σ_1, σ_2.

| Method	σ_1	σ_2	Q_1^*	P_1^*	D_1^*	TAC_1^*	Q_2^*	P_2^*	D_2^*	TAC_2^*
FPTHMF	0.5	0.6	16.44	28.51	1.41	137.23	22.07	33.57	1.44	162.08
	0.6	0.7	16.40	26.17	1.40	140.85	23.00	30.97	1.43	166.59
	0.7	0.8	16.29	23.86	1.40	144.14	21.91	28.77	1.42	170.58
	0.8	0.9	16.23	22.23	1.39	147.16	21.81	27.38	1.42	174.37
FNLP	0.5	0.6	16.33	28.51	1.41	137.23	22.07	33.57	1.44	162.08
	0.6	0.7	16.38	25.89	1.40	140.85	22.00	30.97	1.43	166.50
	0.7	0.8	16.29	23.86	1.40	144.14	21.86	28.91	1.43	170.58
	0.8	0.9	16.22	22.20	1.39	147.16	21.81	27.38	1.42	174.37
FAGP	0.5	0.6	16.34	28.39	1.41	137.23	22.07	33.57	1.44	162.08
	0.6	0.7	16.35	25.88	1.40	140.85	22.00	30.97	1.43	166.50
	0.7	0.8	16.28	23.86	1.40	144.14	21.85	28.91	1.43	170.58
	0.8	0.9	16.23	22.33	1.39	147.16	21.81	27.38	1.42	174.37
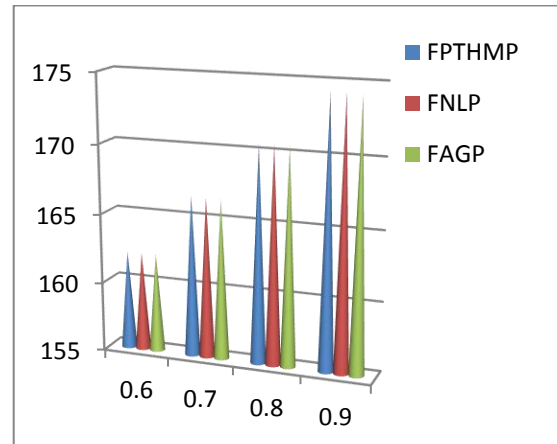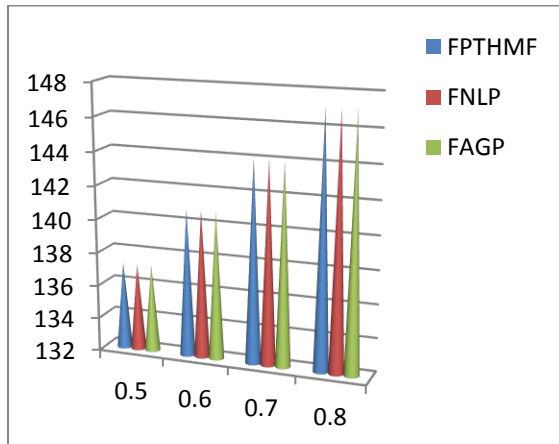
Figure 8: minimizing cost of 1st item using different Methods for different values of σ_1

Figure 9: minimizing cost of 2nd item using different Methods for different values of σ_2

From the above figures 8 & 9 shows that minimum costs of the both items are increased when values of σ_1, σ_2 are increased and it is same for all methods.

Table 7: Optimal solution of MOSCIM by FPTHMF, FNLP and FAGP methods for different values of a_1, a_2.

Method	a_1	a_2	TAC_1^*	TAC_2^*
FPTHMF	2	2	137.72	169.01
	3	3	137.26	158.47
	4	4	137.23	158.42
	5	5	137.22	158.39
FNLP	2	2	137.84	168.97
	3	3	137.76	158.67
	4	4	137.45	158.53

	5	5	137.23	158.42
	2	2	137.72	169.01
	3	3	137.61	158.89
FAGP	4	4	137.34	158.76
	5	5	137.23	158.39

Table 8: Optimal solution of MOSCIM by FPTHMF, FNLP and FAGP methods for different values of b_1, b_2.

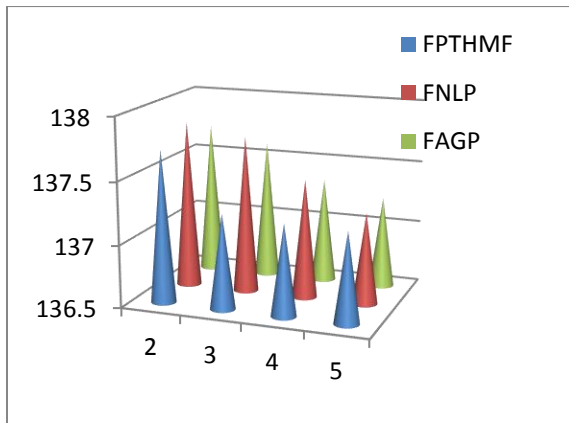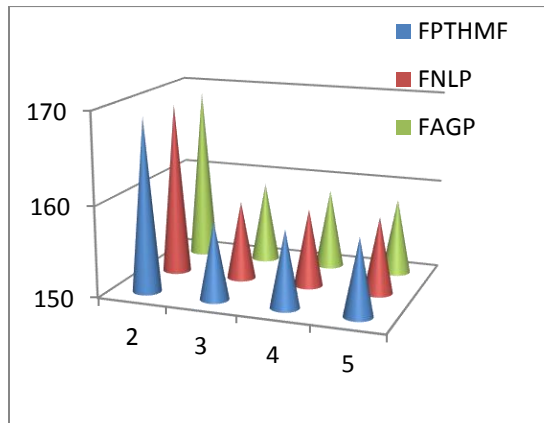| Method	b_1	b_2	$TAC_1{}^*$	$TAC_2{}^*$
	8	8	144.71	172.08
	9	9	140.56	166.52
FPTHMF	10	10	137.23	162.18
	11	11	134.49	162.08
	8	8	144.91	172.18
	9	9	140.65	166.59
FNLP	10	10	137.23	162.28
	11	11	134.43	162.08
	8	8	144.78	172.05
	9	9	140.66	166.52
FAGP	10	10	137.23	162.25
	11	11	134.49	162.08

Figure 10: minimizing cost of 1st item using different methods for different values of a_1

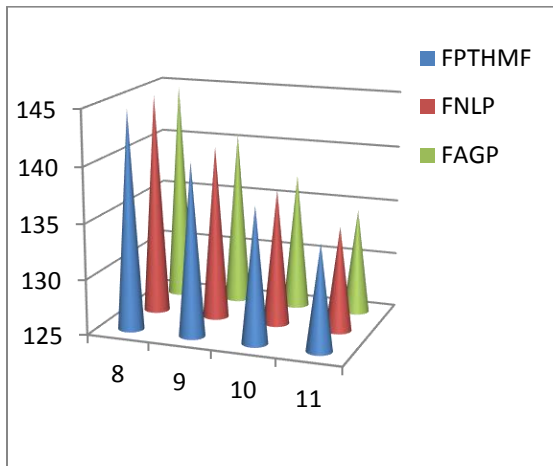Figure 11: minimizing cost of 2nd item using different methods for different values of a_2

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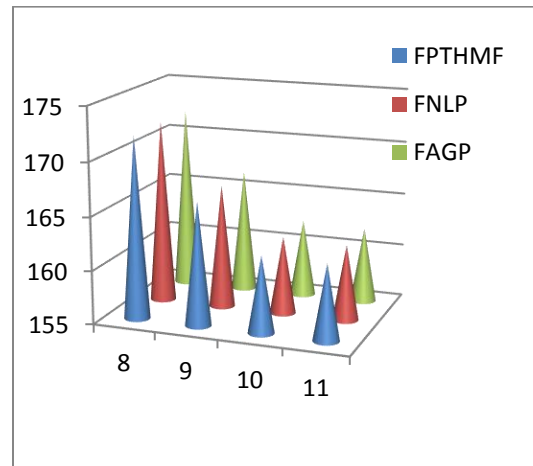Figure 12: minimizing cost of 1ˢᵗ item using different methods for different values of b_1

Figure 13: minimizing cost of 2ⁿᵈ item using different methods for different values of b_2

From the above figures 10 & 11 shows that minimum costs of the both items are decreased when values of a_1, a_2 are increased and it is same for all methods. Also from figures 12 & 13 shows that minimum costs of the both items are decreased when values of b_1, b_2 are increased and it is same for all methods.

10. Conclusions

In this paper, purchase cost for the manufacturer is dependent on inventory lot size and production cost of the manufacturer is dependent on production rate. Also purchase cost of the retailer is dependent on demand rate of the customer. Multi item is considered in this model. The proposed MOSCIM has been solved by FPTHMF, FNLP, FAGP, WFNLP and WFAGP methods. LINGO13 software has been used to find the solution of the numerical example.

Any limitation is not considered in this paper. So in the future study, various limitations can be used such as on total storage area, ordering cost etc. Due to uncertainty, various types of fuzzy number like as triangular fuzzy number, trapezoidal fuzzy number, generalized trapezoidal fuzzy number etc. can used to form the fuzzy model in fuzzy sense.

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