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An Improved Class of Estimators Of Population Mean of Sensitive Variable Using Optional Randomized Response Technique

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Abstract

In this paper we have suggested a class of estimators of population mean of sensitive variable under optional randomized response technique as reported in Gupta et al (2014). We have obtained the mean squared error (MSE) of the suggested class of estimators up to the first order of approximation. The optimum conditions are obtained at which the (MSE) of the proposed class of estimators is minimum. An empirical study is carried out to show the performance of the suggested class of estimators over existing estimators .It is found that the performance of proposed class of estimators is better than the existing estimators including Grover and Kaur (2019)

Key Words: Auxiliary variable, Bias, Efficiency, mean square error, Randomized response technique, Optional randomized response technique, Simple random sampling without replacement, Sensitive study variable, Percent relative efficiency.

Mathematical Subject Classification: 62D05

1. INTRODUCTION

When dealing with the sensitive issues such as gambling, illegal income, alcoholism, sexual abuse, drug addiction, abortion, tax evasion and many others, people often do not respond truthfully or even refuse to answer. This can cause substantial bias in the estimation of population parameters. To eliminate or reduce the bias we have a technique known as randomized response (RRT) given by Warner (1965). There are also so much work has been done with situation when the response to a sensitive question is quantitative variable. For making the technique more efficient various methods have been suggested by many authors. These include Multiplicative Scrambling models by Eichorn and Hayre (1983). Gupta et al (2002) introduced an optional randomized response technique where the choice to provide a scrambled response or truthful response totally depends on the respondent weather respondent considered the question is sensitive or not. Gupta et al (2010), Haung (2010), Gupta et al (2013), Gupta et al (2014), Tarray and Singh (2017) have studied optional RRT with improved additive models.

In sampling practice direct techniques for collecting information about non sensitive characters make massive use of auxiliary variables to improve sampling design and to achieve higher precision in population parameters estimates furthermore development on RRM have been focused on the use of auxiliary variable. Yan (2005–2006) used the auxiliary information directly at the estimation stage in simple random sampling to improve Warner (1965) estimator through the ratio method. Further many authors (Sousa et al. (2010) and Gupta et al. (2012)) have estimated the mean of the sensitive variable using auxiliary information. Diana and Perri (2009, 2011) also give best estimators using auxiliary information. When the study variable is sensitive many authors have done there tremendous work using auxiliary information, such as Bahl and Tuteja (1991), Grover and Kaur (2011), Singh and Solanki (2012), Singh and Vishwakarma (2007) and many more. Whereas when the study variable is sensitive, Sousa et al (2010), Gupta et al (2012), Koyuncu et al (2014), Kalucha et al (2015) and many more used auxiliary information in RRT under traditional additive model. But in both the condition auxiliary variable is non-sensitive. Koyuncu et al (2014) have studied exponential-type-estimator to get more efficient estimators. Grover and Kaur (2019) improved the efficiency of Koyuncu et al (2014)'s estimator of population mean of sensitive variable by replacing traditional RRT with optional RRT.

In this study, we have proposed a class of ratio cum exponential- type estimator of the mean of the sensitive variable using non sensitive auxiliary information. We also derived the Bias and MSE of the proposed class of estimators correct up to first order of approximation and compared it empirically with the Grover and Kaur (2019)'s estimator .

2. NOTATIONS AND EXISTING ESTIMATORS

Let a random sample of size n be drawn without replacement from a finite population $\Omega = (\Omega_1, \Omega_2, ..., \Omega_N)$. Let Y be the study variable, a sensitive variable which cannot be observed directly. Let X be a non-sensitive auxiliary variable that have a positive correlation with Y. Let W be the sensitivity level of the asked sensitive question. In the Optional RRT model respondent is asked to report a scrambled response for Y given by $Ƶ = Y + ST$ but is asked to provide a true response for X. Where T is the Bernoulli random variable with parameter W, so that $0 \le W \le 1$. And S, be a scrambling variable, whose mean is supposed to be zero i.e. $\bar{S} = E(S) = 0$ and its variance σ_s^2 is supposed to be known quantity. It is assumed that the variables S and T are two mutually independent variables which are further independent of variables Y and X .

If we take $W = 1$ in the above model then it boils down to the additive RRT model, and scrambled response is then written as $Z = Y + S$.

Now the population mean of variable $Ƶ$ is given by $\bar{Ƶ} = E(\bar{Ƶ}) = E(Y + ST) = E(Y) = \bar{Y} = \mu_{Y\bar{Ƶ}}$ (say) as $E(S) = 0$, where $\bar{Y}$ is the population mean of variable. The population variance of variable $Ƶ$ is given by $S_Ƶ^2 = V(Y + ST)$ $= S_y^2 + WS_s^2$. Let $C_Ƶ$ be the coefficient of variation of variable $Ƶ$. So $C_Ƶ^2 = C_y^2 + W\dfrac{S_s^2}{\bar{Y}^2}$, where C_y is the coefficient of variation of variable Y. Let $\rho_{Ƶx}$ be the coefficient of correlation between variables $Ƶ$ and X .So $\rho_{Ƶx} = \dfrac{\rho_{yx}}{\sqrt{1 + W\dfrac{S_s^2}{S_y^2}}}$, where ρyx is the coefficient of correlation between variables Y and X,

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2,\ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2 \text{ and } S_s^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(s_i - \bar{S}\right)^2.$$

where $\bar{X}$ is the population mean of auxiliary variable X . Let C_X be the coefficient of variation of variable X.

The estimate of sensitivity level W in the above Optional RRT model may be obtained by using the same approach of Gupta et al (2014). According to them, the estimated value of W is

$$\hat{W}=\frac{\frac{1}{n}\sum_{i=1}^{n}Ƶ_i^2-\left\{\hat{V}(y)+\left(\frac{1}{n}\sum_{i=1}^{n}Ƶ_i\right)^2\right\}}{E\left(s^2\right)}$$, where $\hat{V}(y)$ is the estimate of variance of y .

They further found that,

$$\hat{W}=\frac{\frac{1}{n}\sum_{i=1}^{n}Ƶ_i^2-\left\{\frac{1}{n}\sum_{i=1}^{n}Ƶ_i+\left(\frac{1}{n}\sum_{i=1}^{n}Ƶ_i\right)^2\right\}}{E\left(s^2\right)}$$, when Y is assumed to follow Poisson distribution and

$$\hat{W}=\frac{\hat{S}_{Ƶ}^2-\left\{C_x\left(\frac{1}{n}\sum_{i=1}^{n}Ƶ_i\right)^2\right\}}{E\left(s^2\right)}$$, when it is assumed that $C_x=C_y$.

3. PROPOSED CLASS OF ESTIMATORS

We suggest the following class of estimators for population mean $\bar{Y}$ (with scrambling variable $Ƶ=Y+ST$) as

$$t_{(\alpha,\delta)}=\left\{m_1\bar{z}+m_2\left(\bar{X}-\bar{x}\right)\right\}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha}\exp\left\{\frac{\delta\left(\bar{X}-\bar{x}\right)}{\bar{X}+\bar{x}}\right\} \tag{1}$$

where (m_1, m_2) are suitable chosen constants and (α,δ) are real constants which generates different estimators. To obtain the Bias of $t_{(\alpha,\delta)}$ up to the first order of approximation, we write

$\bar{ƶ}=\bar{Ƶ}(1+e_0)$ and $\bar{x}=\bar{X}(1+e_1)$,

such that $E(e_0)=E(e_1)=0$

and $E\left(e_0^2\right)=\frac{(1-f)}{n}C_z^2=\lambda C_z^2=\lambda\left\{C_y^2+W\frac{S_s^2}{\bar{Y}^2}\right\}$,

$E\left(e_1^2\right)=\frac{(1-f)}{n}C_x^2=\lambda C_x^2$, $E(e_0e_1)=\frac{(1-f)}{n}C_{ƶx}=\lambda C_{ƶx}$,

where $C_{ƶx}=\rho_{ƶx}C_{ƶ}C_x$ and $\lambda=\frac{1}{n}-\frac{1}{N}$.

Expressing $t_{(\alpha,\delta)}$ at (1) in terms of e_0 and e_1 we have

$$t_{(\alpha,\delta)}=\left\{m_1\bar{Ƶ}(1+e_0)-m_2\bar{X}e_1\right\}(1+e_1)^{-\alpha}\exp\left\{\frac{-\delta e_1}{2+e_1}\right\},$$

$$=\bar{Z}\left\{m_1(1+e_0)-m_2\left(\frac{1}{R}\right)e_1\right\}(1+e_1)^{-\alpha}\exp\left\{\frac{-\delta e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right\} \tag{2}$$

where $R=\left(\frac{\bar{Z}}{\bar{X}}\right)=\left(\frac{\bar{Y}}{\bar{X}}\right)$, $\bar{Z}=\bar{Y}$.

We assume that $e_1 << 1$ so that $(1+e_1)^{-\alpha}$ is expandable. Now expanding the right hand side of (2), multiplying out and neglecting terms of $e's$ having power greater than two, we have

$$t_{(\alpha,\delta)}\cong\bar{Y}\left[m_1\left\{1+e_0-\theta e_1-\theta e_0e_1+\frac{\theta(\theta+1)}{2}e_1^2\right\}+m_2(1/R)\left(\theta e_1^2-e_1\right)\right]$$

or

$$\left(t_{(\alpha,\delta)}-\bar{Y}\right)\cong\bar{Y}\left[m_1\left\{1+e_0-\theta e_1-\theta e_0e_1+\frac{\theta(\theta+1)}{2}e_1^2\right\}+m_2(1/R)\left(\theta e_1^2-e_1\right)-1\right] \tag{3}$$

where $\theta=\frac{2\alpha+\delta}{2}$.

Taking expectation on both side of (3) we get the Bias of $t_{(\alpha,\delta)}$ to the first degree of approximation as

$$B\left(t_{(\alpha,\delta)}\right)=\bar{Y}\left[m_1\left\{1+\lambda\left(\frac{\theta(\theta+1)}{2}C_x^2-\theta\rho_{zx}C_zC_x\right)\right\}+\frac{m_2\lambda\theta}{R}C_x^2-1\right] \tag{4}$$

Squaring both side of (3) and neglecting terms of $e's$ having power greater than two we have

$$\begin{aligned}\left(t_{(\alpha,\delta)}-\bar{Y}\right)^2\cong\bar{Y}^2\Big[&1+m_1^2\left\{1+2e_0-2\theta e_1+e_0^2-4\theta e_0e_1+\theta(2\theta+1)e_1^2\right\}+m_2^2\left(1/R^2\right)e_1^2\\&+2m_1m_2(1/R)\left\{2\theta e_1^2-e_1-e_0e_1\right\}\\&-2m_1\left\{1+e_0-\theta e_1-\theta e_0e_1+\frac{\theta(\theta+1)}{2}e_1^2\right\}-2m_2(1/R)\left(\theta e_1^2-e_1\right)\Big]\end{aligned} \tag{5}$$

Taking expectation of both sides of (5) we get the MSE of $t_{(\alpha,\delta)}$ up to the first order of approximation as

$$MSE\left(t_{(\alpha,\delta)}\right)=\bar{Y}^2\left[1+m_1^2A_1+m_2^2A_2+2m_1m_2A_3-2m_1A_4-2m_2A_5\right] \tag{6}$$

where

$$A_1=\left[1+\lambda\left\{C_z^2-4\theta\rho_{zx}C_zC_x+\theta(2\theta+1)C_x^2\right\}\right],$$

$$A_2=\frac{\lambda}{R^2}C_x^2,$$

$$A_3=\frac{\lambda}{R}\left\{2\theta C_x^2-\rho_{zx}C_zC_x\right\},$$

$$A_4=\left[1+\lambda\left\{\frac{\theta(\theta+1)}{2}C_x^2-\theta\rho_{zx}C_zC_x\right\}\right],$$

$$A_5=\frac{\lambda\theta}{R}C_x^2.$$

The $MSE\left(t_{(\alpha,\delta)}\right)$ at (6) is minimized for

$$m_{10} = \frac{(A_2 A_4 - A_3 A_5)}{\left(A_1 A_2 - A_3^2\right)} \tag{7}$$

$$m_{20} = \frac{(A_1 A_5 - A_3 A_4)}{\left(A_1 A_2 - A_3^2\right)} \tag{8}$$

Putting (7) and (8) in eq (6) we get the resulting minimum MSE of $t_{(\alpha,\delta)}$ as

$$MSE_{\min}\left(t_{(\alpha,\delta)}\right) = \bar{Y}^2\left[1 - \frac{\left(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2\right)}{\left(A_1 A_2 - A_3^2\right)}\right] \tag{9}$$

Thus we arrived at the following theorem.

Theorem-1 – Up to the terms of order n^{-1},

$$MSE_{\min}\left(t_{(\alpha,\delta)}\right) \geq \bar{Y}^2\left[1 - \frac{\left(A_2 A_4^2 - 2A_3 A_4 A_5 + A_1 A_5^2\right)}{\left(A_1 A_2 - A_3^2\right)}\right]$$

with equality holding if

$m_1 = m_{10}$,
$m_2 = m_{20}.$

where m_{10} and m_{20} are respectively defined in (7) and (8).

A large number of estimators can be generated from $t_{(\alpha,\delta)}$ just by putting suitable values of constants $(m_1, m_2, \alpha, \delta)$. Some known members of $t_{(\alpha,\delta)}$ are shown in Table 1.

Table 1 – Some known members of $t_{(\alpha,\delta)}$.

S.no.	Traditional RRT model $Z = Y + S$	Optional RRT Models $Ƶ = Y + ST$	Choice of constants			
			α	δ	m_1	m_2
1.	Suggested by Sousa et al (2010) $t_0 = \frac{1}{n}\sum_{i=1}^{n} z_i = \bar{z}$ $MSE(t_0) = \lambda\left(S_y^2 + S_s^2\right)$,	Suggested by Gupta et al (2014) $t_4 = \frac{1}{n}\sum_{i=1}^{n} ƶ_i = \bar{ƶ}$ $MSE(t_4) = \lambda\left(S_y^2 + WS_s^2\right)$,	0	0	1	0
2.	Suggested by Sousa et al (2010) $t_1 = \bar{z}\frac{\bar{X}}{\bar{x}}$	Suggested by Gupta et al (2014) $t_5 = \bar{ƶ}\frac{\bar{X}}{\bar{x}}$	1	0	1	0

	$MSE(t_1) = \lambda \bar{Y}^2 \left(C_y^2 + \frac{S_s^2}{\bar{Y}^2} + C_x^2 - 2\rho_{yx} C_y C_x \right)$	$MSE(t_5) = \lambda \bar{Y}^2 \left(C_y^2 + W \frac{S_s^2}{\bar{Y}^2} + C_x^2 - 2\rho_{yx} C_y C_x \right)$				
3.	Suggested by Gupta et al (2012) $t_2 = \{k_1 \bar{z} + k_2 (\bar{X} - \bar{x})\} \frac{\bar{X}}{\bar{x}}$ $MSE(t_2) = \frac{M_1 (1 - \lambda C_x^2)}{\frac{M_1}{\bar{Y}^2} (1 - \lambda C_x^2)}$	Suggested by Grover and Kaur (2019) $t_6 = \{k_1 \bar{z} + k_2 (\bar{X} - \bar{x})\} \frac{\bar{X}}{\bar{x}}$ $MSE(t_6) = \frac{M_2 (1 - \lambda C_x^2)}{\frac{M_2}{\bar{Y}^2} (1 - \lambda C_x^2)}$	1	0	k_1	k_2
4.	Suggested by Koyuncu et al (2014) $t_3 = \{w_1 \bar{z} + w_2 (\bar{X} - \bar{x})\} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ $MSE(t_3) = M_1 - \varphi_1 - \varphi_2$	Suggested by Grover and Kaur (2019) $t_7 = \{w_1 \bar{z} + w_2 (\bar{X} - \bar{x})\} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ $MSE(t_6) = M_2 - \varphi_3 - \varphi_4$	0	1	w_1	w_2

where $\varphi_1 = \frac{\frac{M_1^2}{\bar{Y}^2}}{1 + \frac{M_1}{\bar{Y}^2}} > 0$, $\varphi_2 = \frac{\lambda C_x^2 \left\{ M_1 + \lambda \frac{1}{16} C_x^2 \bar{Y}^2 \right\}}{4 \left\{ 1 + \frac{M_1}{\bar{Y}^2} \right\}} > 0$

$\varphi_3 = \frac{\frac{M_2^2}{\bar{Y}^2}}{1 + \frac{M_2}{\bar{Y}^2}} > 0$, $\varphi_4 = \frac{\lambda C_x^2 \left\{ M_2 + \lambda \frac{1}{16} C_x^2 \bar{Y}^2 \right\}}{4 \left\{ 1 + \frac{M_2}{\bar{Y}^2} \right\}} > 0$

$M_1 = \lambda S_y^2 \left\{ \left(1 + \frac{S_s^2}{S_y^2} \right) - \rho_{yx}^2 \right\}$ and $M_2 = \lambda S_y^2 \left\{ \left(1 + W \frac{S_s^2}{S_y^2} \right) - \rho_{yx}^2 \right\}$

Gupta et al (2012) suggested a regression estimator for $\bar{Y}$ under traditional RRT model $Z = Y + S$ as

$$t_{lr} = \bar{z} + \hat{\beta}_{zx} (\bar{X} - \bar{x}) , \tag{10}$$

where $\hat{\beta}_{zx}$ is the estimates of regression coefficients β_{zx}

The MSE of t_{lr} up to the first order of approximation, is given by

$$MSE(t_{lr}) = \lambda S_y^2 \left\{ \left(1 + \frac{S_s^2}{S_y^2} \right) - \rho_{yx}^2 \right\} = M_1 \tag{11}$$

Further Gupta et al (2014) envisaged the regression estimator for population mean $\bar{Y}$ under optional RRT model $Z = Y + ST$ as

$$t_{lr(0)} = \bar{z} + \hat{\beta}_{zx} \left(\bar{X} - \bar{x} \right) , \tag{12}$$

where $\hat{\beta}_{zx}$ are estimates of regression coefficients of β_{zx}.

The MSE of $t_{lr(0)}$ up to the first order of approximation, is given by $MSE(t_{lr(0)}) = \lambda S_y^2 \left\{ \left(1 + W \frac{S_s^2}{S_y^2} \right) - \rho_{yx}^2 \right\} = M_2$,

(13)

We note that the estimators listed in Table 1 are members of the proposed class of estimators $t_{(\alpha,\delta)}$. So the proposed class of estimators $t_{(\alpha,\delta)}$ will definitely have smaller (or equal to) minimum MSE than those estimators listed in Table 1 for some selected values of (α, δ).

To illustrate this we have carried out an empirical study in Section 4.

4. EMPIRICAL STUDY

To judge the merits of the suggested class of estimators over existing estimators, we have taken the same populations as considered by Grover and Kaur (2019). We have computed the percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to $t_0 = \frac{1}{n}\sum_{i=1}^{n} z_i = \bar{z}$ by using the formula

$$PRE\left(t_{(\alpha,\delta)}, t_0\right) = \frac{MSE(t_0)}{MSE_{\min}\left(t_{(\alpha,\delta)}\right)} * 100 \tag{14}$$

POPULATION I :{ Source: Koyuncu et al. (2014)}
$N = 5336$, $\rho_{xy} = 0.9632$, $\bar{X} = 22.99$, $\bar{Y} = 30.19$, $S_x = 172.09$, $S_y = 138.65$ and $n = 500$

POPULATION II :{ Source: Sousa et al. (2010)}
$N = 1000$, $\rho_{xy} = 0.8783$, $\bar{X} = 2$, $\bar{Y} = 2$, $S_x = 2.4495$, $S_y = 1.4142$ and $n = 50$.

We have completed the percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for population I and II (for different values of (α, δ)) and findings are compiled in Tables 2 – 4 and 5 - 7.

TABLE- 2: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.1$ for different values of (α,δ) for population I.

PRE of ($t_{(\alpha,\delta)}$ ***)***											
α	δ	**W**									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**0.25**	68071.78	31615.8	20590.17	15266.8	12130.9	10064	8599.05	7506.52	6660.42	5985.82
	-0.1	3996.54	3748.05	3528.69	3333.6	3158.97	3001.75	2859.46	2730.06	2611.88	2503.52
	0	2693.88	2580.02	2475.42	2378.98	2289.78	2207.04	2130.08	2058.32	1991.24	1928.4
0	**3**	3734.27	3514.55	3319.28	3144.6	2987.4	2845.19	2715.93	2597.93	2489.77	2390.27
	3.1	10095.84	8613.07	7510.24	6657.9	5979.42	5426.52	4967.29	4579.78	4248.41	3961.81
	2.6	1580.28	1541.72	1504.99	1469.99	1436.58	1404.66	1374.13	1344.9	1316.9	1290.04
0.5	**2.1**	10095.84	8613.07	7510.24	6657.91	5979.42	5426.52	4967.29	4579.78	4248.41	3961.81
	2	3734.27	3514.55	3319.27	3144.59	2987.4	2845.19	2715.93	2597.93	2489.77	2390.27
	1.6	1580.29	1541.72	1504.99	1469.99	1436.58	1404.66	1374.13	1344.9	1316.9	1290.04
1	**-2.6**	1541.33	1504.89	1470.12	1436.93	1405.21	1374.86	1345.8	1317.95	1291.23	1265.57
	-3	2693.88	2580.03	2475.42	2378.98	2289.78	2207.04	2130.08	2058.32	1991.24	1928.4
	-3.2	9884.06	8475.29	7418.13	6595.55	5937.27	5398.53	4949.49	4569.45	4243.66	3961.27
1.5	**-0.4**	1580.29	1541.72	1504.99	1469.99	1436.58	1404.66	1374.13	1344.9	1316.9	1290.04
	0	3734.27	3514.55	3319.28	3144.59	2987.4	2845.19	2715.93	2597.93	2489.77	2390.27
	0.1	10095.84	8613.07	7510.24	6657.91	5979.42	5426.52	4967.29	4579.78	4248.41	3961.81
-1	**0.8**	9884.06	8475.29	7418.14	6595.55	5937.27	5398.53	4949.49	4569.45	4243.66	3961.27
	1	2693.88	2580.03	2475.42	2378.98	2289.78	2207.04	2130.08	2058.32	1991.24	1928.4
	1.4	1541.33	1504.89	1470.13	1436.93	1405.21	1374.86	1345.8	1317.95	1291.23	1265.57

TABLE- 3: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.2$ for different values of (α,δ) for population I.

PRE of **(** $t_{(\alpha,\delta)}$ **)**											
α	δ	**W**									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**-0.28**	108399	13461.8	7178.35	4894.88	3714.06	2992.55	2505.99	2155.69	1891.44	1685
	-0.1	3485.33	2854.32	2416.97	2095.98	1850.37	1656.38	1499.28	1369.46	1260.39	1167.5
	0.3	1604.29	1462.05	1343.03	1241.97	1155.1	1079.63	1013.44	954.925	902.824	856.14
0	**3.1**	6960.94	4788.23	3649.82	2949.14	2474.44	2131.58	1872.33	1669.43	1506.3	1372.3
	3	3287.72	2716.17	2314.15	2015.96	1785.99	1603.22	1454.48	1331.07	1227.03	1138.1
	2.7	1673.68	1518.1	1389.05	1280.28	1187.37	1107.07	1036.99	975.282	920.541	871.65
0.5	**2.1**	6960.94	4788.23	3649.82	2949.14	2474.44	2131.58	1872.33	1669.43	1506.3	1372.3
	2	3287.72	2716.17	2314.15	2015.96	1785.99	1603.22	1454.48	1331.07	1227.03	1138.1
	1.6	1536.9	1406.12	1295.91	1201.77	1120.43	1049.44	986.947	931.509	881.996	837.51
1	**-3.3**	6895.75	4777.43	3655.19	2960.21	2487.5	2145.13	1885.73	1682.4	1518.72	1384.1
	-3	2487.26	2152	1896.5	1695.33	1532.82	1398.8	1286.39	1190.75	1108.38	1036.7
	-2.7	1604.29	1462.05	1343.03	1241.97	1155.1	1079.63	1013.44	954.925	902.824	856.14
1.5	**-0.1**	2330.28	2032.12	1801.74	1618.39	1468.99	1344.92	1240.24	1150.73	1073.32	1005.7
	0	3287.72	2716.17	2314.15	2015.96	1785.99	1603.22	1454.48	1331.07	1227.03	1138.1
	0.1	6960.94	4788.23	3649.82	2949.14	2474.44	2131.58	1872.33	1669.43	1506.3	1372.3
-1	**0.8**	6895.75	4777.43	3655.19	2960.21	2487.5	2145.13	1885.73	1682.4	1518.72	1384.1
	1	2487.26	2152	1896.5	1695.33	1532.82	1398.8	1286.39	1190.75	1108.38	1036.7
	1.3	1604.29	1462.05	1343.03	1241.97	1155.1	1079.63	1013.44	954.925	902.824	856.14

TABLE- 4: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.3$ for different values of (α,δ) for population I.

PRE of ($t_{(\alpha,\delta)}$)											
α	δ	**W**									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**-0.32**	72258.9	6621.67	3471.88	2353.68	1780.84	1432.6	1198.53	1030.38	903.75	804.95
	-0.1	2928.89	2108.13	1647.11	1351.83	1146.54	995.536	879.801	788.27	714.07	652.7
	0.2	1666.39	1372.75	1167.29	1015.48	898.73	806.15	730.941	668.632	616.166	571.38
0	**3.2**	47190.5	6227.13	3336.05	2279.48	1731.87	1396.85	1170.75	1007.87	884.956	788.9
	3	2791.96	2031.42	1597.04	1316.05	1119.4	974.071	862.29	773.643	701.623	641.95
	2.8	1779.67	1445.28	1216.95	1051.12	925.225	826.386	746.728	681.161	626.25	579.59
0.5	**2.2**	47190.5	6227.13	3336.05	2279.48	1731.87	1396.85	1170.75	1007.87	884.956	788.9
	2	2791.96	2031.42	1597.04	1316.05	1119.4	974.071	862.29	773.643	701.623	641.95
	1.7	1779.67	1445.28	1216.95	1051.12	925.225	826.386	746.728	681.161	626.25	579.59
1	**-3.3**	20078.1	5360.16	3094.56	2175.98	1678.39	1366.33	1152.34	996.475	877.879	784.61
	-3	2232.92	1726.79	1408.01	1188.8	1028.82	906.916	810.94	733.416	669.489	615.87
	-2.7	1534.13	1283.81	1103.89	968.33	862.531	777.66	708.066	649.965	600.727	558.47
1.5	**-0.2**	1779.67	1445.28	1216.95	1051.12	925.225	826.386	746.728	681.161	626.25	579.59
	0	2791.96	2031.42	1597.04	1316.05	1119.4	974.071	862.29	773.643	701.623	641.95
	0.2	47190.5	6227.13	3336.05	2279.48	1731.87	1396.85	1170.75	1007.87	884.956	788.9
-1	**0.7**	20078.1	5360.16	3094.56	2175.98	1678.39	1366.33	1152.34	996.475	877.879	784.61
	1	2232.92	1726.79	1408.01	1188.8	1028.82	906.916	810.94	733.416	669.489	615.87
	1.2	1666.39	1372.75	1167.29	1015.48	898.73	806.15	730.941	668.632	616.166	571.38

TABLE- 5: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.1$ for different values of (α,δ) for population II.

PRE of ($t_{(\alpha,\delta)}$)											
α	δ	W									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**5.6**	14164.20	9720.10	7398.80	5972.63	5007.46	4310.88	3784.50	3372.65	3041.70	2769.89
	-0.1	466.39	445.67	440.02	434.51	429.14	423.91	418.80	413.81	408.94	404.18
	-1.7	20045.40	12195.00	8763.50	6839.09	5607.75	4752.21	4123.20	3641.26	3260.20	2951.39
0	**4.6**	14164.20	9720.10	7398.80	5972.63	5007.46	4310.88	3784.50	3372.65	3041.70	2769.89
	3.1	461.22	455.00	448.94	443.04	437.29	431.696	426.24	420.91	415.73	410.69
	2.9	456.06	449.99	444.08	438.325	432.71	427.25	421.92	416.72	411.65	406.702
0.5	**3.6**	14164.20	9720.10	7398.80	5972.63	5007.46	4310.88	3784.50	3372.65	3041.70	2769.89
	2	461.22	455.00	448.94	443.042	437.29	431.696	426.24	420.91	415.73	410.66
	1.9	456.06	449.99	444.08	438.32	432.71	427.25	421.92	416.72	411.65	406.70
1	**2.6**	14164.20	9720.10	7398.80	5972.63	5007.46	4310.88	3784.50	3372.65	3041.70	2769.89
	2.5	2753.92	2530.70	2340.90	2177.66	2035.69	1911.11	1800.90	1702.73	1614.70	1535.34
	0.9	456.06	449.99	444.08	438.32	432.71	427.25	421.92	416.72	411.65	406.70
1.5	**1.6**	14164.20	9720.10	7398.80	5972.63	5007.46	4310.88	3784.50	3372.65	3041.70	2769.89
	1	715.32	699.81	684.95	670.71	657.05	643.942	631.34	619.23	607.57	596.34
	-0.1	456.06	449.99	444.08	438.32	432.71	427.25	421.92	416.72	411.65	406.70
-1	**-0.8**	20045.40	12195.00	8763.50	6839.09	5607.75	4752.21	4123.20	3641.26	3260.20	2951.39
	0	676.91	663.08	649.80	637.04	624.78	612.97	601.62	590.66	580.11	569.92
	1	460.23	454.04	448.01	442.14	436.42	430.85	425.42	420.12	414.96	409.92

TABLE- 6: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.2$ for different values of (α,δ) for population II.

PRE of ($t_{(\alpha,\delta)}$***)***											
α	δ	**W**									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**0**	480.77	456.84	435.17	415.47	397.49	381	365.83	351.81	338.85	326.80
	-1.5	1616.90	1360.50	1174.3	1033.01	922.07	832.67	759.10	697.48	645.13	600.09
	-1.7	7436.70	3959.40	2698.2	2046.44	1648.39	1380.03	1186.90	1041.17	927.35	835.99
0	**-2.8**	7436.700	3959.40	2698.2	2046.44	1648.39	1380.03	1186.90	1041.17	927.35	835.99
	-2.5	1616.90	1360.50	1174.3	1033.01	922.07	832.67	759.10	697.48	645.13	600.09
	-1	480.77	456.84	435.17	415.47	397.49	381	365.83	351.81	338.85	326.80
0.5	**3.6**	6494.53	3667.40	2555.3	1960.90	1590.93	1338.47	1155.20	1016.12	906.95	818.99
	3.5	2367.95	1851.50	1520.1	1289.36	1119.48	989.19	886.10	802.48	733.31	675.12
	2	481.75	457.70	435.93	416.15	398.08	381.53	366.31	352.24	339.23	327.14
1	**-3**	480.77	456.84	435.17	415.47	397.49	381	365.83	351.81	338.85	326.80
	-4	692.71	642.28	598.71	560.67	527.19	497.49	470.97	447.14	425.61	406.05
	-4.7	7436.70	3959.40	2698.2	2046.44	1648.39	1380.03	1186.90	1041.17	927.35	835.99
1.5	**1**	729.31	673.34	625.35	583.76	547.36	515.25	486.70	461.16	438.17	417.36
	0.5	537.79	507.45	480.36	456.01	434.03	414.07	395.87	379.20	363.89	349.77
	0	481.75	457.70	435.93	416.15	398.08	381.53	366.31	352.24	339.23	327.14
-1	**0**	692.71	642.28	598.71	560.67	527.19	497.49	470.97	447.14	425.61	406.05
	0.5	537.79	507.45	480.36	456.01	434.03	414.07	395.87	379.20	363.89	349.77
	1	729.31	673.34	625.35	583.76	547.36	515.25	486.70	461.16	438.17	417.36

TABLE- 7: The percent relative efficiency (PRE) of the proposed class of estimators $t_{(\alpha,\delta)}$ with respect to t_0 for $\eta = 0.3$ for different values of (α,δ) for population II.

PRE of $(t_{(\alpha,\delta)})$											
α	δ	**W**									
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
-0.5	**-1.7**	4019.90	2070.50	1394.60	1051.56	844.05	705.01	605.35	530.42	472.02	425.24
	-1.5	1483.89	1104.90	880.20	731.53	625.87	546.93	485.70	436.82	396.91	363.70
	-0.3	534.81	478.91	433.61	396.15	364.66	337.83	314.69	294.52	276.80	261.09
0	**-2.7**	4019.90	2070.50	1394.60	1051.56	844.05	705.01	605.35	530.42	472.02	425.24
	-2	715.28	616.20	541.27	482.60	435.44	396.70	364.30	336.81	313.19	292.68
	-1.5	561.43	499.74	450.28	409.75	375.93	347.28	322.71	301.39	282.72	266.24
0.5	**3.7**	435559	4173.50	2097.50	1401.06	1052	842.28	702.37	602.37	527.35	468.98
	3.6	3750.45	1991.90	1356.30	1028.43	828.30	693.45	596.42	523.25	466.11	420.24
	2.5	567.41	504.32	453.88	412.63	378.27	349.21	324.32	302.74	283.87	267.22
1	**-4.7**	4019.90	2070.50	1394.60	1051.56	844.05	705.01	605.35	530.42	472.02	425.24
	-4.5	1483.89	1104.90	880.20	731.53	625.87	546.93	485.70	436.82	396.91	363.70
	-3.5	561.43	499.74	450.28	409.75	375.93	347.28	322.71	301.39	282.72	266.24
1.5	**0.3**	4019.90	2070.50	1394.60	1051.56	844.05	705.01	605.35	530.42	472.02	425.24
	1	715.28	616.20	541.27	482.60	435.44	396.70	364.30	336.81	313.19	292.68
	1.5	561.43	499.74	450.28	409.75	375.93	347.28	322.71	301.39	282.72	266.24
-1	**-0.7**	4019.90	2070.50	1394.60	1051.56	844.05	705.01	605.35	530.42	472.02	425.24
	0	715.28	616.20	541.27	482.60	435.44	396.70	364.30	336.81	313.19	292.68
	0.5	561.43	499.74	450.28	409.75	375.93	347.28	322.71	301.39	282.72	266.24

Comparing the results of the Table 2 to Table 7 (in which the PRE of proposed class of estimators $t_{(\alpha,\delta)}$ with respect to $t_0 = \hat{\mu}_{yz}$ are given) with the results of Table 5.4 to Table 5.6 given in Grover and Kaur (2019, pp. 55-58) (in which PRE's of estimator $t_7 = \hat{\mu}_{\exp z}$ due to Grover and Kaur (2019) with respect to $t_0 = \hat{\mu}_{yz}$ are given) we found that the proposed class of estimators $t_{(\alpha,\delta)}$ is better than Grover and Kaur (2019) estimator $t_7 = \hat{\mu}_{\exp z}$ with substantial gain in efficiency for selected values of (α,δ). So we conclude that there is enough scope of selecting the values of (α,δ) for obtaining better estimators than $t_0, t_1, t_2, t_3, t_4, t_5, t_6$ and Grover and Kaur (2019) estimator t_7. Thus we recommended our proposed class of estimators $t_{(\alpha,\delta)}$ for its use in practice.

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