Pakistan Journal of Statistics and Operation Research

Bayesian Estimates Based On Record Values Under Weighted LINEX Loss Function

Fuad S. Al-Duais[1,2], Mohammed Alhagyan[3*]

* Corresponding Author

1. Mathematics Department, College of Humanities and Science in Al Aflaj, Prince Sattam Bin Abdulaziz University, Al Aflaj, Saudia Arabia, F.alduais@psau.edu.sa
2. Administration Department, Administrative Science College, Thamar University, Thamar, Yemen.
3. Mathematics Department, College of Humanities and Science in Al Aflaj, Prince Sattam Bin Abdulaziz University, Al Aflaj, Saudia Arabia, m.alhagyan@psau.edu.sa

Abstract

In this paper, we developed linear exponential (LINEX) loss function by merging weights to produce weighted linear exponential (WLINEX) loss function. Then we utilized WLINEX to derive scale parameter and reliability function of the Weibull distribution based on record values when the shape parameter is known. After, we estimated scale parameter and reliability function of Weibull distribution by using maximum likelihood (ML) estimation and by several Bayes estimations. The Bayes estimates were obtained with respect to symmetric loss function (squared error loss (SEL)), asymmetric loss function (LINEX) and asymmetric loss function (WLINEX). The ML and the different Bayes estimates were compared via a Monte Carlo simulation study. The result of simulation mentioned that the proposed WLINEX loss function is promising and can be used in real environment especially at the case of underestimate where it revealed better performance than LINEX loss function for estimating scale parameter.

Key Words: Bayesian estimate, Recorded values, Weighted Linex, Reliability

Mathematical Subject Classification: 62N02, 62N05, 62F15

1. Introduction

Record values arise naturally in many real life applications involving data relating to sport, weather and life testing studies. Many authors have studied record values and associated statistics, for example ; Nagaraja (1988), Ahsanullah (1995), Arnold, et.al. (1998), Ahsanullah (1980), Balakrishnan and Chan (1993), Shojaee et al (2012), Soliman et al (2006), Sultan (2008), Sultan, et al (2001), Asgharzadeh and Fallah (2010), Shawky and Badr (2012). These scholars investigated some inferential methods based on record values for the distributions of Rayleigh , exponential, Gumbel, Weibull, logistic distributions, inverse Weibull, exponentiated family, modified Weibull, Lomax and inverse Rayleigh.

The Weibull distribution (WD) is a very popular statistical model in reliability engineering and failure analysis, radar systems, manufacturing and delivery times in industrial engineering problems, weather forecasting and analysis of systems involving a weakest link.

In this paper, we will derive Bayes estimator under weighted linear exponential (WLINEX) loss function to estimate scale parameter and reliability function of the Weibull distribution based on record values. After, we will compare the proposed model with others.

Let $X_1, X_2, X_3, \cdots\cdots$ a sequence of independent and identically distributed (iid) random variables with *(cdf) F(x)* and *(pdf) f (x)*. Set $Y_m = \max\left(X_1, X_2, X_3, \cdots\cdots X_m\right)$, $m \geq 1$, we say that X_j is an upper record and denoted by $X_{U(i)}$ if $Y_j > Y_{j-1}, \quad j > 1$.

Assuming that $X_{U(1)}, X_{U(2)}, X_{U(3)}, \cdots\cdots X_{U(m)}$ are the first m upper record values arising from a sequence { X_i } of (iid) Weibull variables with density function *(pdf)*

$$f\left(x;\alpha,\beta\right) = \begin{cases} \alpha\,\beta\, x^{\alpha-1} \exp[-\beta\, x^{\alpha}]\,, & x \geq 0\,,\, \alpha,\beta > 0\,, \\ 0 & o.w. \end{cases} \tag{1}$$

and cumulative distribution function *(cdf)*

$$F\left(x;\alpha,\beta\right) = 1 - \exp[-\beta\, x^{\alpha}]\,, \qquad x \geq 0,\, \alpha,\beta > 0 \tag{2}$$

Where β and α are scale and shape parameters respectively. Also, The reliability function *R(t)*, and the hazard (instantaneous failure rate) function *H(t)* at mission time t for the Weibull distribution are given by

$$\begin{aligned} R\left(t;\alpha,\beta\right) &= 1 - F_T\left(t;\alpha,\beta\right) \\ &= \exp[-\beta\, x^{\alpha}]\,, \qquad t \geq 0 \end{aligned} \tag{3}$$

and

$$H(t) = \alpha\,\beta\, t^{\alpha-1}, \qquad t \geq 0 \tag{4}$$

2. Maximum Likelihood Estimates (MLE)

In this section, we discuss the maximum likelihood estimates of the parameters of WD given in (1) when the available data are record values. We consider the shape parameter α is known and the scale parameter β is unknown.

Suppose we observed the first *m* upper record values each of which has the WD whose *pdf* and *cdf* are respectively, given by (1) and (2). Based on those upper record values, we have the joint density function of the first *m* upper record values $\underline{x} \equiv x_{U(1)}, x_{U(2)}, x_{U(3)}, \cdots\cdots x_{U(m)}$ is given by Soliman (2006) as

$$\begin{aligned} f_{1,2,3\cdots\cdots m}\left(x_{U(1)}, x_{U(2)}, x_{U(3)}, \cdots\cdots x_{U(m)}\right) = f\left(x_{U(m)}\right) \prod_{i=1}^{m-1} \frac{f\,(x_{U(i)})}{1 - F\,(x_{U(i)})}\,, \\ 0 \leq x_{U(1)} < x_{U(2)} < x_{U(3)}, \cdots\cdots < x_{U(m)} < \infty, \end{aligned} \tag{5}$$

where $f(.)$ *and* $F(.)$ are given respectively, by (1) and (2) after replacing *x* by $x_{U(i)}$. The likelihood function based on the m upper record values *x* is given by

$$L\left(\alpha,\beta \,\middle|\, \underline{x}\right) = \left(\alpha\,\beta\right)^m u\, \exp\left[-\beta\, T_m\right], \qquad u = \prod_{i=1}^{m} x_{U(i)}^{\alpha-1} \quad , \quad T_m = x_{U(m)}^{\alpha}, \tag{6}$$

From (6), the natural logarithm of the likelihood function is given by

$$L\left(\alpha,\beta \,\middle|\, \underline{x}\right) \equiv Ln(\ell) = m\, Ln\left(\alpha\,\beta\right) - \beta\, T_m + \left(\alpha - 1\right)\sum_{i=1}^{m} Ln\,\left(x_{U(i)}\right), \tag{7}$$

where T_m is given by equation (6).

When the shape parameter α is known , the maximum likelihood estimate of (MLE) $\hat{\beta}_{ML}$ can be obtained from (7) as

$$\hat{\beta}_{ML} = \frac{m}{T_m} \quad , \tag{8}$$

By the invariance property of the maximum likelihood estimator of reliability function $\hat{R}(t)_{ML}$ of $R(t)$, can be obtained after replacing β by $\hat{\beta}_{ML}$ in Equation (3)

$$\hat{R}(t)_{ML} = \exp[-\hat{\beta}_{ML} t^{\alpha}], \qquad t \geq 0 \tag{9}$$

3. Loss Function

We consider three different types of loss functions as follows:

3.1 Squared-Error Loss function :

This type is classified as a symmetric function and considers equal importance to the losses for overestimation and underestimation of equal magnitude. Squared error loss function is given by

$$L(\hat{\theta},\theta) = (\hat{\theta}-\theta)^2 \tag{10}$$

3.2 LINEX Loss function

In 1975, Varian introduced LINEX loss function, which is asymmetric. This loss functions was widely used by several authors, among them, Zellener (1986), Basu and Ebrahim (1991), and Pandey and Rai (1992). The LINEX loss function can be expressed as

$$L(\Delta) \propto [\exp[-a\Delta] - a\Delta - 1] \qquad ; a \neq 0 \tag{11}$$

where $\Delta = (\hat{\theta}-\theta)$ .The sign and magnitude of a reflects the direction and degree of asymmetry, respectively. The Bayes estimator relative to LINEX loss function, denoted by $\hat{\theta}_{BL}$, is given by

$$\hat{\theta}_{BL} = -\frac{1}{a}\ln[\mathrm{E}_{\theta}(\exp\{-a\theta\})] \qquad ; a \neq 0 \tag{12}$$

provided that $E_{\theta} = (e^{-a\theta})$ exists and finite, where E_{θ} denotes the expected value.

3.3 Weighted LINEX loss functions

This function was proposed by the scholars depending on weighted loss function (LINEX) as following:

$$L(\hat{\theta}-\theta) = w(\theta)[\exp[-a\Delta] - a\Delta - 1] \qquad ; a \neq 0 \tag{13}$$

where $\hat{\theta}$ represents estimated parameter that makes the expectation of loss function (Equation (13)) as smallest as possible. While, $w(\theta)$ represents the proposed weighted function which equal to

$$w(\theta) = \exp[-z\theta] \tag{14}$$

Depending on the posterior distribution of the parameter θ, and by using the proposed weighted function as in Equation (14) , we can get the estimated weighted Bayes parameter θ as the following :

$$\mathrm{E}\, L_w(\hat{\theta},\theta) = \int_{\forall\theta} L_w(\hat{\theta},\theta)\, f(\theta|\underline{x})\, d\theta$$

$$= \int_{\forall\theta} w(\theta)[\exp[a(\hat{\theta}-\theta)] - a(\hat{\theta}-\theta) - 1]\; f(\theta|\underline{x})\, d\theta$$

$$= \int_{\forall\theta} \exp[-z\theta]\exp[a(\hat{\theta}-\theta)]\; f(\theta|\underline{x})\, d\theta - \exp[-z\theta]\{a(\hat{\theta}-\theta)\}\, f(\theta|\underline{x})\; d\theta - \exp[-z\theta]\; f(\theta|\underline{x})\, d\theta$$

$$= \exp[a\hat{\theta}]\int_{\forall\theta} \exp[-\theta(a+z)]\; f(\theta|\underline{x})\, d\theta - a\hat{\theta}\int_{\forall\theta}\exp[-z\theta] + a\int_{\forall\theta}\theta\exp[-z\theta]\; f(\theta|\underline{x})d\theta - \int_{\forall\theta}\exp[-z\theta]\, f(\theta|\underline{x})d\theta$$

$$= \exp[a\hat{\theta}]\, E(\exp[-\theta(z+a)]|\underline{x}) - a\hat{\theta}\, E(\exp[-z\theta]|\underline{x}) + a\, E(\theta\exp[-z\theta]|\underline{x}) - E(\exp[-z\theta]|\underline{x})$$

$$\frac{\partial L_w(\hat{\theta},\theta)}{\partial\hat{\theta}} = a\exp[a\hat{\theta}]E(\exp[-\theta(z+a)]|\underline{x}) - a\; E(\exp[-z\theta]|\underline{x}) = 0$$

Consequently, the estimated weighted Bayes parameter using the proposed loss function will be

 DOI: http://dx.doi.org/10.18187/pjsor.v16i1.2854

$$\hat{\theta}_{WBL} = \frac{1}{a} Ln\left[\frac{E\ \left(\exp[-z\theta] \mid \underline{x}\right)}{E\ \left(\exp\left[-\theta\left(z+a\right)\ \right] \mid \underline{x}\right)}\right] \qquad ; a \neq 0 \tag{15}$$

Note that when $z = 0$ in Equation (14), then we get LINEX loss function, i.e. the LINEX loss function is just a special case of the proposed WLINEX loss function.

4. Bayes Estimates

In this section, Bayes estimates of the parameter β and the reliability $R(t)$ of the Weibull distribution are derived. We use three different loss functions including the squared error loss function, the LINEX loss functions, and the weighted LINEX loss functions.

4.1 Estimates Based on Squared Error Loss Function

Let gamma (δ, b) be a conjugate prior for (β) as

$$g(\beta) = \frac{b^s}{\Gamma(\delta)} \beta^{\delta-1} \exp[-b\beta], \qquad \beta > 0, \quad b, \delta > 0 \tag{16}$$

Combining the likelihood function in (6) with the prior pdf of β in (16), we get the posterior of β as

$$\pi\left(\beta \mid \underline{x}\right) \frac{L\ (\beta; \underline{x})\, g\ (\beta)}{\int_0^\infty L\ (\beta; \underline{x})\, g\ (\beta)\ d\beta} = \frac{v^{m+\delta}}{\Gamma(m+\delta)} \beta^{m+\delta-1} \exp\ [-\beta v] \qquad \beta > 0, \quad b, \delta > 0 \tag{17}$$

where $\underline{x} = x_{U(1)}, x_{U(2)}, x_{U(3)} \cdots\cdots x_{U(m)}$

and

$$v = \left(b + T_m\right) \tag{18}$$

From Equation (17), the Bayes estimates of β and $R(t)$ based on the squared error loss function can be derived, respectively, as

$$\begin{aligned} \hat{\beta}_{BS} = E\left(\beta \mid \underline{x}\right) &= \int_0^\infty \beta\, \pi\left(\beta \mid \underline{x}\right) d\beta = \int_0^\infty \beta \frac{v^{m+\delta}}{\Gamma(m+\delta)} \beta^{m+\delta-1} \exp[-\beta v]\, d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)} \int_0^\infty \beta^{m+\delta} \exp[-\beta v]\, d\beta = \frac{m+s}{v} \end{aligned} \qquad \beta > 0 \tag{19}$$

and

$$\begin{aligned} \hat{R}_{BS}(t) &= E\ \left(\exp\left[-\beta t^\alpha \mid \underline{x}\right]\right) \\ &= \int_0^\infty \exp\ \left[-\beta t^\alpha\right] \pi\left(\beta \mid \underline{x}\right) d\beta = \int_0^\infty \frac{v^{m+\delta}}{\Gamma(m+\delta)} \beta^{m+\delta-1} \exp\ \left[-\beta (t^\alpha + v\right] d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)} \int_0^\infty \beta^{m+\delta-1} \exp\left[-\beta (t^\alpha + v\right]\, d\beta = \left(\frac{v}{v+t^\alpha}\right)^{m+s} \end{aligned} \tag{20}$$

4.2 Estimates Based on LINEX Loss Function

Under the LINEX loss function, and by using Equation (12), the Bayes estimator $\hat{\beta}_{BL}$ for β, is given by

$$\begin{aligned}\hat{\beta}_{BL}(t) &= -\frac{1}{a} Ln\left(E\left[\exp\left[-a\beta | \underline{x}\right]\right]\right) \\ &= -\frac{1}{a} Ln\int_0^\infty \frac{1}{\Gamma(m+\delta)} v^{m+\delta} \beta^{m+\delta=1} \exp\left[-\beta(v+a)\right] d\beta \\ &= -\frac{1}{a} Ln\left(1+\frac{a}{v}\right)^{-(m+\delta)}\end{aligned} \quad (21)$$

and the Bayes estimators for $R(t)$ is given by

$$\begin{aligned}\hat{R}_{BL}(t) &= -\frac{1}{a} Ln \left(\mathrm{E}\left[\exp\left[-a R(t)\right] \| \underline{x}\right]\right) \\ &= -\frac{1}{a} Ln\int_0^\infty \frac{1}{\Gamma(m+\delta)} v^{m+\delta} \beta^{m+\delta=1} \exp\left[-\beta v\right] \exp\left[-a \ \exp[-\beta t^{\alpha}]\right] d\beta \\ &= -\frac{1}{a} Ln \left[\sum_{i=0}^{\infty} \frac{(-a)^i}{i!}\left(1+\frac{i t^{\alpha}}{v}\right)^{-(m+\delta)}\right]\end{aligned} \quad (22)$$

4.3 Estimates Based on weighted LINEX Loss Function

Under the weighted LINEX loss function, and by using (15), the Bayes estimator $\hat{\beta}_{WBL}$ for β , is given by

$$\begin{aligned}\hat{\beta}_{WBL} &= \frac{1}{a} Ln\left[\frac{E\left(\exp\{-z\beta\}\right)}{E\left(\exp\{-\beta(z+a)\}\right)}\right] \\ &= \frac{1}{a} Ln\left[\frac{I_1}{I_2}\right]\end{aligned} \quad (23)$$

Where

$$\begin{aligned}I_1 = E\left(\exp\{-z\beta\}\right) &= \int_0^\infty \exp\{-z\beta\}\, \pi\left(\beta | \underline{x}\right) d\beta \\ &= \int_0^\infty \exp\{-z\beta\} \frac{v^{m+\delta}}{\Gamma(m+\delta)} \beta^{m+\delta-1} \exp\left[-\beta v\right] d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)} \int_0^\infty \beta^{m+\delta-1} \exp\left[-\beta\,(z+v)\right] d\beta \\ &= \left(1+\frac{z}{v}\right)^{-(m+\delta)}\end{aligned} \quad (24)$$

$$\begin{aligned}I_2 = E\left(\exp\{-\beta(z+a)\}\right) &= \int_0^\infty \exp\{-\beta\,(z+a)\}\pi\left(\beta | \underline{x}\right) d\beta \\ &= \int_0^\infty \exp\{-\beta(z+a)\} \frac{v^{m+\delta}}{\Gamma(m+\delta)} \beta^{m+\delta-1} \exp\left[-\beta v\right] d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)} \int_0^\infty \beta^{m+\delta-1} \exp\left[-\beta\,(z+a+v)\right] d\beta \\ &= \left(1+\frac{z}{a+v}\right)^{-(m+\delta)}\end{aligned} \quad (25)$$

and the Bayes estimator $\hat{R}(t)_{WBL}$ for $R(t)$, is given by

$$\begin{aligned}\hat{R}(t)_{WBL} &= \frac{1}{a} Ln\left[\frac{E\left(\exp\{-z\,R(t)\}\right)}{E\left(\exp\{-R(t)(z+a)\}\right)}\right] \\ &= \frac{1}{a} Ln\left[\frac{I_3}{I_4}\right]\end{aligned} \tag{26}$$

Where

$$\begin{aligned}I_3 = E\left(\exp\{-z\,R(t)\}\right) &= \int_0^\infty \exp\{-z\,R(t)\}\,\pi(\beta|\underline{x})\,d\beta \\ &= \int_0^\infty \exp\left\{-z\exp[\beta\, t^\alpha]\right\}\frac{v^{m+\delta}}{\Gamma(m+\delta)}\beta^{m+\delta-1}\exp[-\beta v]\,d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)}\int_0^\infty \beta^{m+\delta-1}\exp[-\beta v]\exp[-z\exp[-\beta t^\alpha]]\,d\beta \\ &= \sum_{i=0}^\infty \frac{(-z)^i}{i!}\left(1+\frac{i\,t^\alpha}{v}\right)^{-(m+\delta)}\end{aligned} \tag{27}$$

and

$$\begin{aligned}I_4 = E\left(\exp\{-(z+a)\,R(t)\}\right) &= \int_0^\infty \exp\{-(z+a)\,R(t)\}\,\pi(\beta|\underline{x})\,d\beta \\ &= \int_0^\infty \exp\left\{-(z+a)\exp[\beta\, t^\alpha]\right\}\frac{v^{m+\delta}}{\Gamma(m+\delta)}\beta^{m+\delta-1}\exp[-\beta v]\,d\beta \\ &= \frac{v^{m+\delta}}{\Gamma(m+\delta)}\int_0^\infty \beta^{m+\delta-1}\exp[-\beta v]\exp[-(z+a)\exp[-\beta t^\alpha]]\,d\beta \\ &= \sum_{i=0}^\infty \frac{(-(z+a))^i}{i!}\left(1+\frac{i\,t^\alpha}{v}\right)^{-(m+\delta)}\end{aligned} \tag{28}$$

5. Simulation Study and Comparisons

In this section, we conducted a simulation study by adopting the squared error loss function (SEL), LINEX loss function and WLINEX loss function in addition to MLE to estimate the scale parameter and reliability function of Weibull distribution when the shape parameter are known.

In order to compare the ML and Bayes estimates, we calculated the mean square error for each estimate according the following steps:

1. For given values $(\delta = 2, b = 1)$, we generate a random value $\beta = 1.383$ from the prior pdf as in Equation (16).
2. By using the value $\beta = 1.383$ from step 1, with selected values of $\alpha = 2$, $\alpha = 3$, $z = 0.5$ and $z = 3$ we generate m , ($m = 3, 5, 7$) upper record values from Weibull distribution whose pdf is given by Equation (1),
3. The different estimates of β, $R(t)$ at time t (chosen to be 0 .5) are computed.
4. Steps 1 to 3 are repeated 10,000 times and the mean square error (MSE) for each estimate (say $\hat{\theta}$) was calculated by using

$$MSE = \frac{1}{10000}\sum_{i=1}^{10000}\left(\hat{\theta}-\theta\right)^2$$

where $\hat{\theta}$ is the estimate at the i^{th} run.

 DOI: http://dx.doi.org/10.18187/pjsor.v16i1.2854

Tables 1- 8 show the mean squared error of the different estimates based on 10000 runs of Monte Carlo simulation and records up to 7.

Our observations about the results are stated in the following points:

5. All Tables 1, 3, 5 and 7 show that Bayes estimates under the proposed loss function (WLINEX loss function) presents the best estimation of β according to the smallest value of MSE in the case of underestimate when $z=3$. While, Bayes estimates under WLINEX loss function has the best estimation of β depending on the smallest MSE in the case of overestimate when $z=0.5$.
6. At underestimate case, Tables 1, 3,5 and 7 always show that the SEL function is better than LINEX loss function.
7. Tables 2,4,6 and 8, show that Bayes estimates under LINEX loss function reveals the best performance in estimate the reliability as it has smallest MSE at the case of underestimate. Whereas, Bayes estimates under SEL function come in second stage with trivial difference from WLINEX loss function.
8. ML estimation reveals the worst performance in estimate both of β and reliability function. Therefore, it is recommended to use Bayes estimate instead of ML estimates (see Tables 1-8).
9. From all Tables 1-8, one can observe that there is an inverse relationship between the value of MSE and the values of records.

Table 1. MSEs of the estimates of β $at\, \alpha=3$ and $z=3$

m	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$			$\hat{\beta}_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	4.4668	0.6711	2.8311	0.2605	0.1505	0.2437	0.3880	0.4521
5	1.0872	0.4848	1.2026	0.2499	0.1617	0.1706	0.2671	0.3155
7	0.5591	0.3558	0.6903	0.2049	0.1452	0.1337	0.1977	0.2328

Table 2. MSEs of the estimates of $R(t)$ $at\, \alpha=3$ and $z=3$

m	$\hat{R}(t)_{ML}$	$\hat{R}(t)_{BS}$	$\hat{R}(t)_{BL}$			$\hat{R}(t)_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	0.0196	0.0055	0.0051	0.0059	0.0064	0.0082	0.0094	0.0101
5	0.0084	0.0042	0.0040	0.0045	0.0048	0.0056	0.0063	0.0068
7	0.0049	0.0032	0.0031	0.0033	0.0036	0.0040	0.0043	0.0047

Table 3. MSEs of the estimates of β $at\, \alpha=3$ and $z=0.5$

m	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$			$\hat{\beta}_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	4.4668	0.6711	2.8106	0.2586	0.1533	0.7506	0.1340	0.1355
5	1.0872	0.4848	1.1820	0.2460	0.1564	0.5009	0.1458	0.1237
7	0.5591	0.3558	0.6886	0.2040	0.1383	0.3637	0.1335	0.1112

Table 4. MSEs of the estimates of $R(t)$ $at\,\alpha = 3\ and\ z = 0.5$

m	$\hat{R}(t)_{ML}$	$\hat{R}(t)_{BS}$	$\hat{R}(t)_{BL}$			$\hat{R}(t)_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	0.0196	0.0055	0.0051	0.0059	0.0065	0.0055	0.0064	0.0070
5	0.0084	0.0042	0.0039	0.0044	0.0047	0.0041	0.0047	0.0050
7	0.0049	0.0032	0.0030	0.0033	0.0034	0.0032	0.0034	0.0035

Table 5. MSEs of the estimates of β $at\,\alpha = 2\ and\ z = 3$

m	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$			$\hat{\beta}_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	4.7893	0.6676	2.8204	0.2723	0.1510	0.2460	0.3870	0.4515
5	1.1692	0.4967	1.2385	0.2457	0.1562	0.1717	0.2681	0.3158
7	0.5610	0.3556	0.6913	0.2164	0.1424	0.1333	0.1967	0.2347

Table 6. MSEs of the estimates of $R(t)$ $at\,\alpha = 2\ and\ z = 3$

m	$\hat{R}(t)_{ML}$	$\hat{R}(t)_{BS}$	$\hat{R}(t)_{BL}$			$\hat{R}(t)_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	0.0354	0.0121	0.0110	0.0138	0.0147	0.0196	0.0240	0.0253
5	0.0187	0.0098	0.0091	0.0103	0.0110	0.0140	0.0157	0.0168
7	0.0117	0.0075	0.0071	0.0082	0.0084	0.0102	0.0118	0.0120

Table 7. MSEs of the estimates of β $at\,\alpha = 2\ and\ z = 0.5$

m	$\hat{\beta}_{ML}$	$\hat{\beta}_{BS}$	$\hat{\beta}_{BL}$			$\hat{\beta}_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	4.7893	0.6676	2.6857	0.2595	0.1494	0.7213	0.1369	0.1322
5	1.1692	0.4967	1.1675	0.2415	0.1570	0.5015	0.1447	0.1246
7	0.5610	0.3556	0.7011	0.2010	0.1449	0.3710	0.1332	0.1152

Table 8. MSEs of the estimates of $R(t)$ $at\,\alpha = 2\ and\ z = 0.5$

m	$\hat{R}(t)_{ML}$	$\hat{R}(t)_{BS}$	$\hat{R}(t)_{BL}$			$\hat{R}(t)_{WBL}$		
			$a=-1$	$a=1$	$a=2$	$a=-1$	$a=1$	$a=2$
3	0.0354	0.0121	0.0107	0.0133	0.0148	0.0118	0.0146	0.0163
5	0.0187	0.0098	0.0088	0.0101	0.0111	0.0095	0.0109	0.0120
7	0.0117	0.0075	0.0072	0.0077	0.0086	0.0076	0.0082	0.0091

Acknowledgement

This Publication was supported by the Deanship of Scientific Research at Prince Sattam bin Abdulaziz University

References

1. Ahsanullah, M. (1980). Linear prediction of record values for the two parameter exponential distribution. *Annals of the Institute of Statistical Mathematics*, *32*(3), 363-368.
2. Ahsanullah, M. (1995). *Record statistics*. Nova Science Publishers.
3. Asgharzadeh, A., & Fallah, A. (2010). Estimation and prediction for exponentiated family of distributions based on records. *Communications in Statistics—Theory and Methods*, *40*(1), 68-83.
4. Arnold, B. C., Balakrishnan, N. and Nagaraja, H. N. (1998). Records. John Wiley & Sons, New York.Jafari Jozani, M., Marchand, E., Parsian, A. (2006). On estimation with weighted -balanced- type loss function. Statistics & Probability Letters, 76, 773-780.
5. Balakrishnan, N., Chan, P. S., & Ahsanullah, M. (1993). Recurrence relations for moments of record values from generalized extreme value distribution. *Communications in Statistics-Theory and Methods*, *22*(5), 1471-1482.
6. Basu, A. P., & Ebrahimi, N. (1991). Bayesian approach to life testing and reliability estimation using asymmetric loss function. *Journal of statistical planning and inference*, *29*(1-2), 21-31.
   Nagaraja, H. N. (1988). Record values and related statistics-a review. *Communications in Statistics-Theory and Methods*, *17*(7), 2223-2238.
7. Pandey, B. N., & Rai, O. (1992). Bayesian estimation of mean and square of mean of Normal distribution using LINEX loss function. *Communications in Statistics-Theory and Methods*, *21*(12), 3369-3391.
8. Shawky, A. I., & Badr, M. M. (2012). Estimations and prediction from the inverse Rayleigh model based on lower record statistics. *Life Science Journal*, *9*(2), 985-990.
9. Shojaee, O., Azimi, R., & Babanezhad, M. (2012). Empirical Bayes estimators of parameter and reliability function for compound Rayleigh distribution under record data. *American Journal of Theoretical and Applied Statistics*, *1*(1), 12-15.
10. Soliman, A. A., Ellah, A. A., & Sultan, K. S. (2006). Comparison of estimates using record statistics from Weibull model: Bayesian and non-Bayesian approaches. *Computational Statistics & Data Analysis*, *51*(3), 2065-2077.
11. Sultan, K. S. (2008). Bayesian estimates based on record values from the inverse Weibull lifetime model. *Quality Technology & Quantitative Management*, *5*(4), 363-374.
12. Sultan, K. S., Moshref, M. E., & Abd-El-Hakim, N. S. (2001). Estimation of parameters of Lomax distribution based on record values. *The Egyptian Statistical Journal ISSR*, *45*(2), 135-142.
13. Varian, H. R. (1975). A Bayesian approach to real estate assessment. *Studies in Bayesian econometric and statistics in Honor of Leonard J. Savage*, 195-208.
14. Zellner, A. (1986). On assessing prior distributions and Bayesian regression analysis with g-prior distributions. *Bayesian inference and decision techniques*.