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Paradox in The d-Dimensional Fixed Charge Transportation Problem and Algorithm for Finding The Paradox

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Abstract

The d-dimensional fixed charge transportation problem is a generalization of fixed charge transportation. This problem has d-type of constraints so that it can be applied to more complex problem. In the transportation problem, sometimes there are some cases when increasing the product in shipping, the number of costs incurred is less than before increasing the product. This problem is called the transportation paradox. In this research, it will be explained about the model of d-dimensional fixed charge transportation problem and sufficient condition for the occurrence of the paradox. Furthermore an algorithm is given in finding the paradox in the d-dimensional fixed charge transportation problem with an example to support the theory presented.

Key Words: d-Dimensional; Fixed Charge; Linear Programming; Transportation Paradox; Transportation Problem.

Mathematical Subject Classification: 90B06, 90C08.

1. Introduction

Linear programming is a branch of mathematics. Many researches have been carried out related to this topic such as: integer programming model for optimizing bus timetable using genetic algorithm (Wihartiko et al., 2017), frequency determination of bus rapid transit (Mayyani et al., 2017), analyze combination method for solving nonlinear equations for optimization (Silalahi et al., 2017), star catalog generation for satellite attitude navigation (Saifudin et al., 2015, comparison of sensitivity analysis on linear optimization (Silalahi and Dewi, 2014), analysis of upper bound for the iteration complexity of an interior-point method (Silalahi, 2014).

The classical transportation problem is a special case of linear programming (George et al., 2014). This problem was first formulated by Hitchcock in 1941 (Chvatal, 1983) and usually relates to the shipping of logistics. In the transportation problem, there are several cases that must provide fixed costs (Schrenk et al., 2011). This problem is called fixed charge transportation problem which was first formulated by Dantzig and Hirsch in 1954 (Kumar et al., 2010). The fixed costs can be in the form of vehicle rental costs, landing fees at the airport, set up costs for machines in manufacturing environment, etc. In transportation problem, sometimes there are cases when increasing the product in shipping, the number of costs incurred is less than before increasing the product (Joshi and Gupta, 2012). This problem is called the transportation paradox (Das et al., 2015).

Many researchers have developed the paradox of transportation. Some of these are paradox in fixed charge transportation problem (Acharya et al., 2015) and paradox in the d-dimensional transportation problem (Kautsar et al., 2018). Sufficient conditions for the occurrence of the paradoxes were analyzed in these researches. Model of fixed charge transportation problem is a transportation problem with 2 types of constraints. That transportation problem can be generalized into the model of d-dimensional fixed charge transportation problem. Therefore, in this research we will discuss sufficient condition for the occurrence of the paradox.

The remainder of the paper is organized as follows. In section 2, the model of d-dimensional fixed charge transportation problem is presented. In section 3, we discuss the dual of d-dimensional fixed charge transportation problem. The optimality of d-dimensional fixed charge transportation problem is presented in section 4. In section 5, the sufficient condition for the occurrence of the paradox is discussed. In section 6, we present the algorithm paradox in the d-dimensional fixed charge transportation problem. In section 7, numerical example is presented to support the theory in the previous section. The last section contains conclusions and summaries of this paper.

2. The d-Dimensional Fixed Charge Transportation Problem

The model of the d-dimensional fixed charge transportation problem is a generalization of the fixed charge transportation problem. The fixed charge transportation problem focuses on 2 types of constraints, namely supply and demand of the product. Then the fixed costs correspond to the supplying of products (Arora and Ahuja, 2000). Furthermore, on the d-dimensional fixed charge transportation problem, the type of constraints is generalized as many as d-type of constraints. Each type of constraint is seen as a vector type constraint and fixed costs correspond to vector type constraints. Consider the model of d-dimensional fixed charge transportation problem as follows.

$$\min \sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} x_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} F_{i_2,i_3,\ldots,i_d},$$

subject to

$$\begin{gathered}\sum_{\substack{i_1,i_2,\ldots,i_d\\ i_1=I_1}} x_{i_1,i_2,\ldots,i_d} = a_{I_1}(1), \quad I_1 = 1,2,\ldots,n_1,\\ \sum_{\substack{i_1,i_2,\ldots,i_d\\ i_k=I_k}} x_{i_1,i_2,\ldots,i_d} \le a_{I_k}(k), \quad k = 2,3,\ldots,d; \quad I_k = 1,2,\ldots,n_k,\\ x_{i_1,i_2,\ldots,i_d} \ge 0,\end{gathered} \tag{1}$$

where
$\sum_{I_2=1}^{n_2} a_{I_2}(2) = \sum_{I_3=1}^{n_3} a_{I_3}(3) = \cdots = \sum_{I_d=1}^{n_d} a_{I_d}(d) \ge \sum_{I_1=1}^{n_1+1} a_{I_1}(1)$,

$x_{i_1,i_2,\ldots,i_d}$ = the number of product that corresponds to $(i_1, i_2, \ldots, i_d)$,
$c_{i_1,i_2,\ldots,i_d}$ = the shipping costs of product that correspond to $(i_1, i_2, \ldots, i_d)$,
$F_{i_2,i_3,\ldots,i_d}$ = the fixed cost that corresponds to $(i_2, i_3, \ldots, i_d)$,
$a_{I_k}(k)$ = the I_k-th element of k-th vector type constraints.

Suppose $F_{i_2,i_3,\ldots,i_d}$ have l-step, so that

$$F_{i_2,i_3,\ldots,i_d} = \sum_{j=1}^{k} \delta_{i_2,i_3,\ldots,i_d,j} F_{i_2,i_3,\ldots,i_d,j}$$

where

$$\delta_{i_2,i_3,\ldots,i_d,j} = \begin{cases} 1; & \sum_{i_1=1}^{n_1} x_{i_1,i_2,\ldots,i_d} > A_{i_2,i_3,\ldots,i_d,j} \\ 0; & \text{others} \end{cases}$$

$A_{i_2,i_3,\ldots,i_d,j}$ are constants that satisfies $0 = A_{i_2,i_3,\ldots,i_d,1} < A_{i_2,i_3,\ldots,i_d,2} < \cdots < A_{i_2,i_3,\ldots,i_d,l}$ and $F_{i_2,i_3,\ldots,i_d}$ are the fixed costs. Furthermore the constraint in Equation (1) can be added with slack variable which the shipping cost of product is zero. So that, the d-dimensional fixed charge transportation problem can be written as follows.

$$\min \sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} x_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} F_{i_2,i_3,\ldots,i_d}, \tag{2}$$

subject to

$$\sum_{\substack{i_1,i_2,\ldots,i_d \\ i_1=I_1}} x_{i_1,i_2,\ldots,i_d} = a_{I_1}(1), \quad I_1 = 1,2,\ldots,n_1,n_1+1,$$

$$\sum_{\substack{i_1,i_2,\ldots,i_d \\ i_k=I_k}} x_{i_1,i_2,\ldots,i_d} = a_{I_k}(k), \quad k = 2,3,\ldots,d; \quad I_k = 1,2,\ldots,n_k,$$

$$x_{i_1,i_2,\ldots,i_d} \geq 0,$$

where

$$\sum_{I_2=1}^{n_2} a_{I_2}(2) = \sum_{I_3=1}^{n_3} a_{I_3}(3) = \cdots = \sum_{I_d=1}^{n_d} a_{I_d}(d) = \sum_{I_1=1}^{n_1+1} a_{I_1}(1).$$

3. The Dual of d-Dimensional Fixed Charge Transportation Problem

The dual of d-dimensional fixed charge transportation problem in Equation (2) can be obtained by multiplying the primal constraints with dual variable, so that:

$$\sum_{i_1,i_2,\ldots,i_d} \left(\sum_{k=1}^{d} u_{i_k}(k) \right) x_{i_1,i_2,\ldots,i_k} = \sum_{I_1=1}^{n_1+1} a_{I_1}(1) u_{I_1}(1) + \sum_{I_k=1}^{n_k} \left(\sum_{k=2}^{d} a_{I_k}(k) u_{I_k}(k) \right)$$

Based on Weak Duality Theorem in Vanderbei (2014), the value of primal objective function in the minimization problem is greater than the value of dual objective function. As a result, we obtain an inequality as follows.

$$\sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} x_{i_1,i_2,\ldots,i_d} \geq \sum_{i_1,i_2,\ldots,i_d} \left(\sum_{k=1}^{d} u_{i_k}(k) \right) x_{i_1,i_2,\ldots,i_k}$$

$$\iff c_{i_1,i_2,\ldots,i_d} \geq \sum_{k=1}^{d} u_{i_k}(k)$$

So that the dual of d-dimensional transportation problem in Equation (2) is

$$\max \sum_{I_1=1}^{n_1+1} a_{I_1}(1) u_{I_1}(1) + \sum_{I_2=1}^{n_2} a_{I_2}(2) u_{I_2}(2) + \sum_{I_3=1}^{n_3} a_{I_3}(3) u_{I_3}(3) + \cdots + \sum_{I_d=1}^{n_d} a_{I_d}(d) u_{I_d}(d),$$

subject to

$$u_{I_1}(1) + u_{I_2}(2) + \cdots + u_{I_d}(d) \leq c_{i_1,i_2,\ldots,i_d},$$

$$\begin{gathered} I_1 = 1,2,\ldots,n_1,n_1+1, \\ I_2 = 1,2,\ldots,n_2, \\ I_3 = 1,2,\ldots,n_3, \\ \vdots \\ I_d = 1,2,\ldots,n_d. \end{gathered}$$

4. The Optimality of d-Dimensional Transportation Problem

Based on Strong Duality Theorem in Vanderbei (2014), the value of primal and dual objective function have the same value. As a result, we obtain an equation as follows.

$$\sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} x_{i_1,i_2,\ldots,i_d} = \sum_{i_1,i_2,\ldots,i_d} \left(\sum_{k=1}^{d} u_{i_k}(k) \right) x_{i_1,i_2,\ldots,i_k}$$

$$\iff c_{i_1,i_2,\ldots,i_d} = \sum_{k=1}^{d} u_{i_k}(k).$$

Based on the basic solution definition in Eiselt and Sandblom (2018), the values of basic variable are $x_{i_1,i_2,\ldots,i_d} > 0$ and the values of the non basic variable are $x_{i_1,i_2,\ldots,i_d} = 0$. To obtain the optimal solution, Equation (3) must meet this condition

$$\begin{aligned} u_{I_1}(1) + u_{I_2}(2) + \cdots + u_{I_d}(d) &= c_{I_1,I_2,\ldots,I_d}, \quad \forall (I_1, I_2, \ldots, I_d) \in B \\ u_{I_1}(1) + u_{I_2}(2) + \cdots + u_{I_d}(d) &\le c_{I_1,I_2,\ldots,I_d}, \quad \forall (I_1, I_2, \ldots, I_d) \notin B, \end{aligned}$$

where B is index set of basic solution.

5. The Sufficient Condition for The Occurrence of The Paradox

The following is given a theorem about the sufficient condition for the occurrence of the paradox.

Theorem 5.1. *Suppose ΔF is a change in fixed costs, θ is the number of increase product, and $v_1, v_2, \ldots, v_d$ is an index with $1 \le v_1 \le n_1 + 1$, $1 \le v_k \le n_k$, $\forall k = 2, 3, \ldots, d$. The sufficient condition for occurrence of the paradox in Equation (2) if there is at least an index $(v_1, v_2, \ldots, v_d) \in B$ where $a_{v_1}(1), a_{v_2}(2), \ldots, a_{v_d}(d)$ is replaced with $a_{v_1}(1) + \theta, a_{v_2}(2) + \theta, \ldots, a_{v_d}(d) + \theta$ and $\theta(a_{v_1}(1) + a_{v_2}(2) + \cdots + a_{v_d}(d)) + \Delta F < 0$.*

Proof. Suppose Z^0 is the value of objective function, $\sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d}$ is the fixed costs, $x^0_{i_1,i_2,\ldots,i_d}$ are the optimal solution, and $u_{i_1}(1), u_{i_2}(2), \ldots, u_{i_d}(d)$ are dual variable with $u_{i_1}(1) + u_{i_2}(2) + \cdots + u_{i_d}(d) = c_{i_1,i_2,\ldots,i_d}$, then

$$\begin{aligned} Z^0 &= \sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} x^0_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d} \\ &= \sum_{i_1,i_2,\ldots,i_d} \big(u_{i_1}(1) + u_{i_2}(2) + \cdots + u_{i_d}(d)\big) x^0_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d} \\ &= \sum_{i_1,i_2,\ldots,i_d} x^0_{i_1,i_2,\ldots,i_d} u_{i_1}(1) + \sum_{i_1,i_2,\ldots,i_d} x^0_{i_1,i_2,\ldots,i_d} u_{i_2}(2) + \cdots + \sum_{i_1,i_2,\ldots,i_d} x^0_{i_1,i_2,\ldots,i_d} u_{i_d}(d) \\ &\quad + \sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d} \\ &= \sum_{i_1=1}^{n_1+1} a_{I_1}(1) u_{I_1}(1) + \sum_{I_2=1}^{n_2} a_{I_2}(2) u_{I_2}(2) + \cdots + \sum_{I_d=1}^{n_d} a_{I_d}(d) u_{I_d}(d) + \sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d}. \end{aligned}$$

Now suppose at least an index $(v_1, v_2, \ldots, v_d) \notin B$ where $\hat{a}_{v_1}(1) = a_{v_1}(1) + \theta, \hat{a}_{v_2}(2) = a_{v_2}(2) + \theta, \ldots, \hat{a}_{v_d}(d) = a_{v_d}(d) + \theta$, $\hat{Z}$ is value of objective function after the increase of θ, and $\hat{x}_{i_1,i_2,\ldots,i_d}$ is optimal solution after the increase of θ, then

$$\begin{aligned} \hat{Z} &= \sum_{i_1,i_2,\ldots,i_d} c_{i_1,i_2,\ldots,i_d} \hat{x}_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} \hat{F}_{i_2,i_3,\ldots,i_d} \\ &= \sum_{i_1,i_2,\ldots,i_d} \big(u_{i_1}(1) + u_{i_2}(2) + \cdots + u_{i_d}(d)\big) \hat{x}_{i_1,i_2,\ldots,i_d} + \sum_{i_2,i_3,\ldots,i_d} \hat{F}_{i_2,i_3,\ldots,i_d} \\ &= \sum_{i_1,i_2,\ldots,i_d} \hat{x}_{i_1,i_2,\ldots,i_d} u_{i_1}(1) + \sum_{i_1,i_2,\ldots,i_d} \hat{x}_{i_1,i_2,\ldots,i_d} u_{i_2}(2) + \cdots + \sum_{i_1,i_2,\ldots,i_d} \hat{x}_{i_1,i_2,\ldots,i_d} u_{i_d}(d) \\ &\quad + \sum_{i_2,i_3,\ldots,i_d} \hat{F}_{i_2,i_3,\ldots,i_d} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i_1=1}^{n_1+1} a_{I_1}(1)u_{I_1}(1) + \sum_{I_2=1}^{n_2} a_{I_2}(2)u_{I_2}(2) + \cdots + \sum_{I_d=1}^{n_d} a_{I_d}(d)u_{I_d}(d) + \sum_{i_2,i_3,\ldots,i_d} \hat{F}_{i_2,i_3,\ldots,i_d} \\
&= \sum_{\substack{I_1=1\\ I_1\neq v_1}}^{n_1+1} a_{I_1}(1)u_{I_1}(1) + \hat{a}_{v_1}(1)u_{v_1}(1) + \sum_{\substack{I_2=1\\ I_2\neq v_2}}^{n_2} a_{I_2}(2)u_{I_2}(2) + \hat{a}_{v_2}(2)u_{v_2}(2) + \cdots + \\
&\quad \sum_{\substack{I_d=1\\ I_d\neq v_d}}^{n_d} a_{I_d}(d)u_{I_d}(d) + \hat{a}_{v_d}(d)u_{v_d}(d) + \sum_{i_2,i_3,\ldots,i_d} \hat{F}_{i_2,i_3,\ldots,i_d} \\
&= \sum_{i_1=1}^{n_1+1} a_{I_1}(1)u_{I_1}(1) + \theta u_{v_1}(1) + \sum_{I_2=1}^{n_2} a_{I_2}(2)u_{I_2}(2) + \theta u_{v_2}(2) + \cdots + \sum_{I_d=1}^{n_d} a_{I_d}(d)u_{I_d}(d) \\
&\quad +\theta u_{v_d}(d) + \sum_{i_2,i_3,\ldots,i_d} F^0_{i_2,i_3,\ldots,i_d} + \Delta F \\
&= Z^0 + \theta\big(a_{v_1}(1) + a_{v_2}(2) + \cdots + a_{v_d}(d)\big) + \Delta F.
\end{aligned}
$$

We know Z^0 is the value of objective function and the value for $\theta\big(a_{v_1}(1) + a_{v_2}(2) + \cdots + a_{v_d}(d)\big) + \Delta F$ is negative. As a result the value of $\hat{Z}$ is lower than the value of Z^0, so paradox occur. $\square$

6. Algorithm

1. Determining paradoxical pair (Z^0, F^0), where Z^0 is the value of objective function and F^0 is the number of product that sent for the optimal solution X^0.
2. Determining $i = 1$.
3. Finding index $(v_1, v_2, \ldots, v_d) \notin B$ that satisfy $\theta(a_{v_1}(1) + a_{v_2}(2) + \cdots + a_{v_d}(d)) + \Delta F < 0$, otherwise go to step 8.
4. Increasing the number of product by 1 unit to vector type constraints that correspond to the dual variable, then determining new optimal solution X^i.
5. Determining new (Z^i, F^i).
6. Determining $i = i + 1$.
7. Go to step 3
8. Write paradoxical pair $(Z^*, F^*) = (Z^i, F^i)$ for optimal solution $X^* = X^i$.

7. Numerical Example

Given the d-dimensional fixed charge transportation problem as follows.

$$
a_{I_1}(1) = \begin{bmatrix} 10 \\ 7 \\ 8 \end{bmatrix}, \quad a_{I_2}(2) = \begin{bmatrix} 20 \\ 15 \end{bmatrix}, \quad a_{I_3}(3) = \begin{bmatrix} 13 \\ 13 \\ 9 \end{bmatrix}, \quad a_{I_4}(4) = \begin{bmatrix} 17 \\ 18 \end{bmatrix}
$$

$$
c_{I_1,I_2,1,1} = \begin{bmatrix} 18 & 7 \\ 17 & 20 \\ 17 & 15 \end{bmatrix}, \quad c_{I_1,I_2,1,2} = \begin{bmatrix} 18 & 8 \\ 7 & 20 \\ 17 & 15 \end{bmatrix}
$$

$$
c_{I_1,I_2,2,1} = \begin{bmatrix} 17 & 11 \\ 8 & 20 \\ 17 & 4 \end{bmatrix}, \quad c_{I_1,I_2,2,2} = \begin{bmatrix} 17 & 11 \\ 7 & 20 \\ 17 & 14 \end{bmatrix}
$$

$$
c_{I_1,I_2,3,1} = \begin{bmatrix} 18 & 19 \\ 8 & 19 \\ 17 & 14 \end{bmatrix}, \quad c_{I_1,I_2,3,2} = \begin{bmatrix} 18 & 19 \\ 8 & 19 \\ 17 & 14 \end{bmatrix}
$$

$$F_{1,1,I_4,j}=\begin{bmatrix}15&7&5\\12&4&2\end{bmatrix},\quad F_{1,2,I_4,j}=\begin{bmatrix}14&6&4\\10&5&3\end{bmatrix},\quad F_{1,3,I_4,j}=\begin{bmatrix}10&7&5\\14&6&3\end{bmatrix}$$

$$F_{2,1,I_4,j}=\begin{bmatrix}10&5&2\\11&7&3\end{bmatrix},\quad F_{2,2,I_4,j}=\begin{bmatrix}8&3&2\\15&8&4\end{bmatrix},\quad F_{2,3,I_4,j}=\begin{bmatrix}10&7&5\\8&5&3\end{bmatrix}$$

where

$$\delta_{I_2,I_3,I_4,1}=\begin{cases}1; & \sum_{I_1=1}^{3}x_{I_1,I_2,I_3,I_4}>0\\0; & \text{others}\end{cases}$$

$$\delta_{I_2,I_3,I_4,2}=\begin{cases}1; & \sum_{I_1=1}^{3}x_{I_1,I_2,I_3,I_4}>4\\0; & \text{others}\end{cases}$$

$$\delta_{I_2,I_3,I_4,3}=\begin{cases}1; & \sum_{I_1=1}^{3}x_{I_1,I_2,I_3,I_4}>7\\0; & \text{others.}\end{cases}$$

Based on Equation (2), vector type constraints become as follows.

$$a_{I_1}(1)=\begin{bmatrix}10\\7\\8\\10\end{bmatrix},\quad a_{I_2}(2)=\begin{bmatrix}20\\15\end{bmatrix},\quad a_{I_3}(3)=\begin{bmatrix}13\\13\\9\end{bmatrix},\quad a_{I_4}(4)=\begin{bmatrix}17\\18\end{bmatrix}.$$

The optimal solutions for primal problem are as follows.

$$\begin{array}{llll}x_{1122}=3, & x_{1211}=7, & x_{2112}=5, & x_{2122}=2,\\ x_{3221}=8, & x_{4112}=1, & x_{4131}=2, & x_{4132}=7.\end{array}$$

The optimal solutions for dual problem are as follows.

$$\begin{array}{llllll}u_1(1)=7, & u_2(1)=-3, & u_3(1)=4, & u_4(1)=-10, & u_1(2)=10, & u_2(2)=0,\\ u_1(3)=0, & u_2(3)=0, & u_3(3)=0, & u_1(4)=0, & u_2(4)=0.\end{array}$$

The fixed cost occur if there are basic feasible solutions (Robers and Cooper, 1976), so the fixed costs is $F_{i_1,i_2,i_3,i_4}=59$ and paradoxical pair is $(Z^0,F^0)=(240,25)$. Applying step 2, 3, and 4 then selecting an index $(2,2,1,1)\notin B$. Increasing the product by 1 unit to corresponding vector type constraints so

$$a_{I_1}(1)=\begin{bmatrix}10\\7+1\\8\\10\end{bmatrix},\quad a_{I_2}(2)=\begin{bmatrix}20\\15+1\end{bmatrix},\quad a_{I_3}(3)=\begin{bmatrix}13+1\\13\\9\end{bmatrix},\quad a_{I_4}(4)=\begin{bmatrix}17+1\\18\end{bmatrix}.$$

The optimal solutions for primal problem are as follows.

$$\begin{array}{llll}x_{1122}=2, & x_{1211}=8, & x_{2112}=5, & x_{2122}=3,\\ x_{3221}=8, & x_{4112}=1, & x_{4131}=2, & x_{4132}=7.\end{array}$$

The optimal solutions for dual problem are as follows.

$$\begin{array}{llllll}u_1(1)=7, & u_2(1)=-3, & u_3(1)=4, & u_4(1)=-10, & u_1(2)=10, & u_2(2)=0,\\ u_1(3)=0, & u_2(3)=0, & u_3(3)=0, & u_1(4)=0, & u_2(4)=0.\end{array}$$

The fixed cost is $F_{i_1,i_2,i_3,i_4}=61$. The change of fixed cost is $\Delta F=61-59=2$. Because dual variable that selecting and ΔF with increasing the product by 1 unit meet Theorem 1

$$u_2(1)+u_2(2)+u_1(3)+u_1(4)+\Delta F=-1<0$$

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then paradox exists in this problem. The new paradoxical pair $(Z^1, F^1) = (239, 26)$. Going back to step 3 until $i = 3$ with the optimal solutions for primal problem are as follows.

$$\begin{array}{llll} x_{1211} = 8, & x_{2112} = 5, & x_{2122} = 5, & x_{3221} = 8, \\ x_{4112} = 1, & x_{4131} = 2, & x_{4132} = 7. & \end{array}$$

The optimal solutions for dual problem are as follows.

$$\begin{array}{llllll} u_1(1) = 7, & u_2(1) = -3, & u_3(1) = 4, & u_4(1) = 0, & u_1(2) = 0, & u_2(2) = 0, \\ u_1(3) = 0, & u_2(3) = 0, & u_3(3) = 0, & u_1(4) = 0, & u_2(4) = 0. & \end{array}$$

The paradoxical pair is $(Z^3, F^3) = (235, 28)$. Because there is no index that satisfies step 3, then best paradoxical pair is $(Z^*, F^*) = (Z^3, F^3) = (235, 28)$.

8. Conclusion

The model of the d-dimensional fixed charge transportation problem is a generalization of the fixed charge transportation problem. Each type of constraint is seen as a vector type constraint and the fixed costs are issued if shipping product occurs. The sufficient condition for the occurrence of the paradox is obtained with compare the value of the objective function before and after increasing of product. The dual variables are also involved in the determination of the value of the objective function to obtain optimal solution. The algorithm for finding paradox is done by increasing the product by as much as one item. Then evaluate the value of the objective function, it is done repeatedly until the paradox does not occur again. This algorithm is used to find lower costs than before, not to find the lowest cost that can be achieved from the d-dimensional fixed charge transportation problem.

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