

On Bayesian estimation of step stress accelerated life testing for exponentiated Lomax distribution based on censored samples

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Abstract

In reliability analysis and life-testing experiments, the researcher is often interested in the effects of changing stress factors such as “temperature”, “voltage” and “load” on the lifetimes of the units. Step-stress (SS) test, which is a special class from the well-known accelerated life-tests, allows the experimenter to increase the stress levels at some constant times to obtain information on the unknown parameters of the life models more speedily than under usual operating conditions. In this paper, a simple SS model from the exponentiated Lomax (ExpLx) distribution when there is time limitation on the duration of the experiment is considered. Bayesian estimates of the parameters assuming a cumulative exposure model with lifetimes being ExpLx distribution are resultant using Markov chain Monte Carlo (M.C.M.C) procedures. Also, the credible intervals and predicted values of the scale parameter, reliability and hazard are derived. Finally, the numerical study and real data are presented to illustrate the proposed study.

Key Words: Bayesian estimation, M.C.M.C method, credible interval, cumulative damage, exponentiated Lomax distribution.

1. Introduction and motivation

Generally, there are two well-known types of SS loadings which are concerned in accelerating life tests ALTs: The 1st is the linearly increasing stress and the 2nd is the stable stress. There are two types of real data which are found from ALT. The 1st type is the whole data set in which failure time (FT) of each unit is seen and the 2nd is the censored data in which FT of every sample unit mayn't be observed yet.

Miller and Nelsen (1983) studied finest test plans which is employed to minimize the asymptotic variance (Asy-Var.) of the MLE of the mean life at a plan (use) stress for the two-step ALT (2S- ALT) when every units are gone to death. Bai et al. (1989) further studied optimum 2S- ALT where a pre-determined censoring time is considered. Rene et al. (1996) developed a Bayesian estimation and studied the inferences of data from ALT. Li (2002) studied a Bayesian method for estimating the failure rate (FR) for exponential (Exp) distribution. For studies about the ALT see (DeGroot and Goel (1979), Nelson (1990), Dorp et al. (1996), Sha and Pan (2014), Dorp and Mazzuchi (2005), Lee and Pan (2010), Lee and Pan (2011), Chandra and Khan (2016), Ismail (2017), Jaheen (2017), Jaheen (2017)).

The major aim and motivation of this article is to provide a model for 2-SS ALTs based on the ExpLx model. We reflect the Bayesian estimation of the scale parameter, reliability, hazard rate of the distribution of FTs under regular operating conditions and the SS ALT model based on cumulative break that helps a log linear model. We find the probability density and cumulative distribution functions (P-D-F & C-D-F) for lifetimes from the ExpLx model. Finally, the performance of these methods is illustrated by a M.C.M.C simulation study and application under censoring type-I.

2. Basic assumption SS ALT model

The following basic assumptions are taken in the experiment of SS testing

For any stress $t_1 \leq t_2 \leq \dots \leq t_k$, the lifetime model is ExpLx (θ, β, λ) . The P-D-F is written as

$$f(t_{ij}, \theta, \beta, \lambda_j) = \frac{\theta \beta \lambda_j (1 + \lambda_j t_{ij})^{-(\beta+1)}}{(1 - (1 + \lambda_j t_{ij})^{-\beta})^{-\theta+1}} |_{(t_{ij}, \theta, \beta, \lambda > 0, j=1,2,\dots,k, i=1,\dots,r_j)} \cdot \tag{1}$$

θ and β are constants with respect to stress x , and the scale parameter λ is an affected by stress $x_j, j = 1, 2, \dots, k$ through the log linear model in the form

$$\lambda_j = \exp(a + bx_j),$$

where both a & b are unknown parameters. Suppose that for a certain pattern of stress x , units run at stress x_j starting at time $\tau_{(j-1)}$ and going to time

$$\tau_j |_{(j=1,2,\dots,k, \tau_0=0)}.$$

The conduct of these units is as follows:

At step 1, the population portion $F_1(t)$ of units failing by time τ under steady stress x_1 is

$$F_1(t) = [1 - (1 + \exp(a + bx_1) t)^{-\beta}]^\theta |_{(0 < t < \tau_1, a, b, \beta, \theta > 0)}.$$

Let $F(t)$ be the population C-D-F of units failing (dying) under SS, then in the 1st step let $F(t) = F_1(t), 0 < t < \tau_1$, where τ_1 is the time when the stress is grown from x_1 to be x_2 .

When the 2nd step starts, units which have same age u_1 created the alike fraction unsuccessful seen at the termination of the step.

In other meaning, the stayers at the time τ_1 will be converted to a certain stress x_2 stating at u_2 , which can be controlled as the result of

$$F_2(u_1) = F_2(u_1)F_1(\tau_1),$$

$$F_2(u_1) = [1 - (1 + \exp(a + bx_j)u_{j-1})^{-\beta}]^\theta$$

$$F_2(u_1) = [1 - (1 + \exp(a + bx_j)(\Delta_{j-2} + u_{j-2}))^{-\beta}]^\theta,$$

where $\Delta_0 = \tau_1 - \tau_0, u_0 = 0$ and $\Delta_{j-2} = \tau_{j-1} - \tau_{j-2}, j=2,3,\dots,k$, by taking the root θ and taking the root $-\beta$ to two sides, the cumulative exposure model for j steps is written as follows

$$F(t) = [1 - [1 + \exp(a + bx_j)(t - \tau_{j-1} + u_{j-1})]^{-\beta}]^\theta,$$

where

$$u_{j-1} = -\exp(b(x_{j-1} - x_j))(\Delta_{j-2} + u_{j-2}).$$

It seen that $F(t)$, for a SS pattern next $F(t)$, can be printed in the form:

$$F(t) = \begin{cases} 0, & t \leq \tau_0 \\ F_1(u_1), & \tau_0 \leq t \leq \tau_1 \\ F_j(t - \tau_{j-1} + u_{j-1}), & \tau_{j-1} \leq t \leq \tau_j, j = 2, \dots, k - 1 \\ F_k(t - \tau_{k-1} + u_{k-1}), & \tau_{k-1} \leq t \leq \infty \end{cases} \quad (2)$$

and the associated P-D-F $f(t)$, is presented as the following form:

$$f(t) = \begin{cases} f_1(t), & \tau_0 \leq t \leq \tau_1 \\ f_j(t - \tau_{j-1} + u_{j-1}), & \tau_{j-1} \leq t \leq \tau_j, j = 2, \dots, k - 1 \\ F_k(t - \tau_{k-1} + u_{j-1}), & \tau_{k-1} \leq t \leq \infty \\ 0 & elsewhere \end{cases} \quad (3)$$

In this work, the ML process is consumed for the SS model. The P-D-F for each test is shown in (3) and it is the time derivative of the C-D-F given in (2). The sample likelihood is the product of such P-D-F's evaluated at FTs if the uncensored data is used or the seen P-D-F's of such survival function (SF) evaluated at censoring time when censoring is put on. It is indicated that $F(t)$, differs for units with unlike SS forms.

The MLE on Type I when there are 2 SS as special case. A special circumstance, we let $k=2$, τ_1 is the time at which the stress will change from x_1 to be x_2 and L is the time at which the experiment is finished (censoring time). In time SS, the stress x_{j-1} is grown to x_j at τ_{j-1} , $j = 2, \dots, k$. It is implicit that the test is stayed till all components die or until time L . The likelihood function has the identical form as

$$L(\beta, \theta, \lambda_j; \underline{t}) = \frac{\left[\prod_{i=1}^{n_1} \left(\frac{\theta \beta \exp(a+bx_1)(1+\exp(a+bx_1)t_{i1})^{-\beta-1}}{(1-(1+\exp(a+bx_1)t_{i1})^{-\beta})^{\theta-1}} \right)^{\delta_{i1}} \right]}{\left[\prod_{i=n_1+1}^{n_2} \left(\frac{\theta \beta \exp(a+bx_2)(1+\exp(a+bx_2)(t_{i2}-\tau_1+u_1))^{-\beta-1}}{(1-(1+\exp(a+bx_2)(t_{i2}-\tau_1+u_1))^{-\beta})^{\theta-1}} \right)^{\delta_{i2}} \right]^{-1}}, \quad (4)$$

The log likelihood function ℓ can be presented as

$$\ell = \sum_{i=1}^{n_1} \left[\begin{array}{c} \delta_{i1}(a + bx_1) \\ + \log(\theta + \beta) \\ + \delta_{i1}(-\beta - 1) \log(1 + \exp(a + bx_1)t_{i1}) \\ + \delta_{i1}(\theta - 1) \log(1 - (1 + \exp(a + bx_1)t_{i1})^{-\beta}) \end{array} \right] \\ + \sum_{i=n_1+1}^{n_2} \left[\begin{array}{c} \delta_{i2}(a + bx_2) \\ + \log(\theta + \beta) \\ + \delta_{i2}(-\beta - 1) \log(1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1)) \\ + \delta_{i2}(\theta - 1) \log(1 - (1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1))^{-\beta}) \end{array} \right] \quad (5)$$

3. Bayesian analysis

In this section, we introduce Bayesian estimation of the unknown parameters, reliability and hazard rate functions (H-R-Fs) of the SS ExpLx model based squared error loss (SEL) function and censoring from the type I.

Due to Sinha (1998), the Jeffrey’s rule for picking the non-informative prior P-D-F for the unrelated random variables (RVs) a, b, θ, β are studied as well-known uniform distribution.

The joint non-informative prior density function of the random parameters is found as

$$\Phi(a, b, \theta, \beta) \propto \phi_1 \phi_2 \phi_3 \phi_4, \mu_1 \leq a \leq \mu_2, \mu_3 \leq b \leq \mu_4, \mu_5 \leq \beta \leq \mu_6, \mu_7 \leq \theta \leq \mu_8, \quad (6)$$

The informative prior P-D-F of a, b, θ, β have a lognormal (LogN) distribution which are totally uncorrelated with the location and scale parameters $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$, respectively. It has the following P-D-F

$$\pi_1(a) = \frac{1}{av_1\sqrt{2\pi}} \exp\left[-\frac{(\ln(a)-v_1)^2}{2v_1^2}\right] |_{(a > 0)}, \quad (7)$$

$$\pi_2(b) = \frac{1}{bv_2\sqrt{2\pi}} \exp\left[-\frac{(\ln(b)-v_2)^2}{2v_2^2}\right] |_{(b > 0)}, \quad (8)$$

$$\pi_3(\beta) = \frac{1}{\beta v_3\sqrt{2\pi}} \exp\left[-\frac{(\ln(\beta)-v_3)^2}{2v_3^2}\right] |_{(\beta > 0)}, \quad (9)$$

$$\pi_4(\theta) = \frac{1}{\theta v_4\sqrt{2\pi}} \exp\left[-\frac{(\ln(\theta)-v_4)^2}{2v_4^2}\right] |_{(\theta > 0)}, \quad (10)$$

The joint informative prior P-D-F of the random parameters is given as

$$\Pi(a, b, \theta, \beta) = \frac{\exp\left[-\frac{(\ln(a)-v_1)^2}{2v_1^2} - \frac{(\ln(b)-v_2)^2}{2v_2^2} - \frac{(\ln(\beta)-v_3)^2}{2v_3^2} - \frac{(\ln(\theta)-v_4)^2}{2v_4^2}\right]}{av_1bv_2\beta v_3\theta v_4(2\pi)^2} |_{(a,b,\theta,\beta > 0)}, \quad (11)$$

and hence the posterior P-D-F with non-informative prior is provided as follows

$$\Phi_1(a, b, \theta, \beta) \propto \Phi(a, b, \theta, \beta) L(\beta, \theta, a, b; \underline{t}),$$

then

$$\begin{aligned} &\Phi_1(a, b, \theta, \beta) \propto \phi_1 \phi_2 \phi_3 \phi_4 \\ &\times \left[\prod_{i=1}^{n_1} \left(\theta \beta \exp(a + bx_1) (1 + \exp(a + bx_1)t_{i1})^{-\beta-1} \right)^{\delta_{i1}} \right. \\ &\quad \left. \times (1 - (1 + \exp(a + bx_1)t_{i1})^{-\beta})^{\theta-1} \right] \\ &\times \left[\prod_{i=n_1+1}^{n_2} \left(\theta \beta \exp(a + bx_2) \right. \right. \\ &\quad \left. \left. \times (1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1))^{-\beta-1} \right)^{\delta_{i2}} \right. \\ &\quad \left. \times (1 - (1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1))^{-\beta})^{\theta-1} \right], \quad (12) \end{aligned}$$

and hence the posterior P-D-F with informative prior is presented as follows

$$\Pi_1(a, b, \theta, \beta) \propto \Pi(a, b, \theta, \beta) L(\beta, \theta, a, b; \underline{t}), a, b, \theta, \beta > 0$$

then

$$\begin{aligned} \Pi_1(a, b, \theta, \beta) \propto & \frac{\exp \left[-\frac{(\ln(a) - v_1)^2}{2v_2^2} - \frac{(\ln(b) - v_3)^2}{2v_4^2} \right.}{av_2bv_4\beta v_6\theta v_8(2\pi)^2} \\ & \left. -\frac{(\ln(\beta) - v_5)^2}{2v_6^2} - \frac{(\ln(\theta) - v_7)^2}{2v_8^2} \right] \\ & \times \left[\prod_{i=1}^{n_1} \left(\theta\beta \exp(a + bx_1) (1 + \exp(a + bx_1)t_{i1})^{-\beta-1} \right)^{\delta_{i1}} \right. \\ & \left. \times (1 - (1 + \exp(a + bx_1)t_{i1})^{-\beta})^{\theta-1} \right] \\ & \times \left[\prod_{i=n_1+1}^{n_2} \left(\theta\beta \exp(a + bx_2) (1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1))^{-\beta-1} \right)^{\delta_{i2}} \right. \\ & \left. \times (1 - (1 + \exp(a + bx_2)(t_{i2} - \tau_1 + u_1))^{-\beta})^{\theta-1} \right], \end{aligned} \quad (13)$$

It is obvious that from the posterior P-D-F the usual Bayesian methods of parameter estimation with integration is hard so one of the M.C.M.C methods are spent to simulate or make direct draws from some the complex model. Then, we shall study the SEL function to get the a consistent estimators from the marginal posterior P-D-Fs.

4. Bayesian estimators under the squared loss function

It is well known that the Bayesian estimate based on SEL function, is the posterior mean. The SEL function is a symmetric loss function and reads as

$$SEL(a, \hat{a}) = c(\hat{a} - a)^2,$$

where c denotes a constant and \hat{a} is an estimator. The Bayesian estimators of a, b, θ, β of SS ExpLx(a, b, θ, β) under SEL can be got when the parameters are unknown, respectively as follows

$$\begin{aligned} \hat{a}_{SEL} = E_1(a|\underline{t}) &= \int_0^\infty \int_0^\infty \int_0^\infty c(\hat{a} - a)^2 \Phi_1(a, b, \theta, \beta) dbd\theta d\beta, \\ \hat{b}_{SEL} = E_1(b|\underline{t}) &= \int_0^\infty \int_0^\infty \int_0^\infty c(\hat{b} - b)^2 \Phi_1(a, b, \theta, \beta) dad\theta d\beta, \\ \hat{\theta}_{SEL} = E_1(\theta|\underline{t}) &= \int_0^\infty \int_0^\infty \int_0^\infty c(\hat{\theta} - \theta)^2 \Phi_1(a, b, \theta, \beta) dadb d\beta, \\ \hat{\beta}_{SEL} = E_1(\beta|\underline{t}) &= \int_0^\infty \int_0^\infty \int_0^\infty c(\hat{\beta} - \beta)^2 \Phi_1(a, b, \theta, \beta) dbd\theta da, \end{aligned} \quad (14)$$

5. Credible intervals

The confidence intervals (CIs) estimation in Bayesian technique is straighter than the non-Bayesian method which build on CIs. Once the marginal P-D-F of φ has been derived, a symmetric $100(1 - \omega)\%$ two-sided Bayesian probability interval estimate of φ , denoted by $[\mathbf{L}_\varphi, \mathbf{U}_\varphi]$, can be built easily. The Bayesian equivalent to the CIs is called a credibility interval. Generally, we have

$$p(\mathbf{L}(\underline{t}) \leq \varphi \leq \mathbf{U}(\underline{t})) = \int_{\mathbf{L}(\underline{t})}^{\mathbf{U}(\underline{t})} \Pi_1(a, b, \theta, \beta | \underline{t}) d\varphi = 1 - \omega, \quad (15)$$

for the limits \mathbf{L}_φ and \mathbf{U}_φ . Another time, these equations are not in closed forms, so we concern proper numerical techniques to find a solution of these non-linear equations.

6. Numerical illustration

Under Bayesian estimation, we shall assume three M.C.M.C chains with dissimilar initial values for many levels of integration necessary to find the standardizing constant and the marginal posterior P-D-Fs.

M.C.M.C simulation is the simplest way to get consistent simulation results exclusive of evaluating the complex integrals (see Gelman et al. (2003)). M.C.M.C algorithm is valuable in upper dimensional problems is the alternating conditional sampling called Gibbs sampling.

WinBUGS Spiegel halter et al. (2003), a specialized software package for implementing M.C.M.C simulation and Gibbs sampling is applied in this section.

7. The simulation Algorithm

Accelerated life data from ExpLx model are generated using R language at different samples of size (n=40, n=60), $\lambda = 2, \theta = 2, \beta = 3$. Two accelerated stress stages

$x_1 = 1$ and $x_2 = 1.5$ are measured and the normal levels is determined as $x_u = 0.5$.

Since, the three chains with different initials run at the same time of simulation. Each chain stays for N=50000 iterations. The values of starting solution for a, b, θ, β are shown where the 1st chain is with initial values a = 0.5, b = 0.5, $\theta = 0.9$ and $\beta = 1$. The 2nd is with initial values a = 0.4, b = 0.6, $\theta = 0.5$ and $\beta = 0.9$, and the 3rd is with initial values a = 0.3, b = 0.7, $\theta = 0.5$ and $\beta = 0.5$. The reliability (r_1, r_2) and H-R-F (h_1, h_2) are predicted at natural conditions as $x_u = 0.5, t_0 = 0.5, 0.7$.

If the experiment is ended once wholly the objects fail or when a fixed censoring time t is reached (Type I censoring). When $n_1 = n_2 = 20, t_{c_j} = 0.90, 0.80$ and $n_1 = n_2 = 30, t_{c_j} = 0.90, 0.80$, respectively. The analogous likelihood function is found in (4).

Table 1: Priors distributions for Bayesian approach

Types	a	b	θ	β
P-I	U(0,6)	U(0,3)	U(0,7)	U(0,5)
P-II	Inor(0.0001,1000)	Inor(0.0001,1000)	Inor(0.0001,1000)	Inor(0.0001,1000)

where P-I=the non-informative prior, U= uniform distribution and P-II= informative prior, Inor =LogN. The posterior mean and variance of t_p , given t can be designed, respectively, as follows

$$E(t_p(x_u)|\underline{t}) = \int_0^\infty t_p f(t_p(x_u)|\underline{t}) dt_p(x_u), \tag{16}$$

and

$$V(t_p(x_u)|\underline{t}) = \int_0^\infty [t_p(x_u) - E(t_p(x_u))]^2 f(t_p(x_u)|\underline{t}) dt_p(x_u), \tag{17}$$

where

$$t_p = \frac{1}{\varphi} \left[1 - \left(1 - p^{\frac{1}{\theta}} \right)^{\frac{-1}{\beta}} \right],$$

and p is the pth percentile.

7. An application

A real data set of FTs of “84 aircrafts windshield” is used to compare the fits of the Exp-Lx model. We consider the data on FTs for a specified model windshield given in Murthy et al. (2004). These data were analyzed by Ramos et al. (2013) and El-Bassiouny et al. (2015). The FTs of “84 Aircraft Windshield” are given in the Appendix. The initial values of a; b, β and θ in 3 chains and priors distribution used in this real data set are the similar in the simulation work. The results of the application are listed in Tables 4.

8. Concluding remarks

In this paper, a simple SS model under the ExpLx model when there is time limitation on the duration of the experiment is considered. Bayesian estimates of the parameters assuming a cumulative exposure model with lifetimes being ExpLx are resultant using Markov chain Monte Carlo (M.C.M.C) procedures. Credible intervals and predicted values of the scale parameter, reliability and H-R-F are derived. Finally, the numerical study and real data are presented to illustrate the proposed study.

Table 2 Posterior statistics of model parameters when sample size, n=40.

Non-informative												
$t_{c1} = 0.80$							$t_{c1} = 0.90$					
E	mean	Sd	MC error	2.5%	Median	97.5 %	mean	Sd	MC error	2.5 %	Median	97.5 %
\hat{a}	5.994	0.0057	0.0800	5.979	5.996	6.0	5.994	0.0059	0.803E-5	5.979	5.996	6.0
\hat{b}	2.994	0.0063	2.0E-5	2.977	2.996	3.0	2.994	0.0063	2.092E-5	2.977	2.996	3.0
$\hat{\theta}$	6.969	0.0309	5.483E-5	6.886	6.978	6.999	6.969	0.0309	9.483E-5	6.886	6.978	6.999
$\hat{\beta}$	0.8101	0.0528	9.973E-5	0.7102	0.8089	0.9172	0.8101	0.0528	9.973E-5	0.710	0.8089	0.917
$\hat{\lambda}_u$	1792.0	11.76	0.03746	1762.0	1795	1806.0	1792.0	11.76	0.03746	1762	1795	1806
$\hat{\lambda}_l$	8005.0	68.26	0.2218	7834.0	8021	8091.0	8005.0	68.26	0.2218	7834	8022	8091
$\hat{\lambda}_h$	35770.0	394.5	1.294	34760	3586	36250	35770	394.5	1.294	4760	35860	36250
$\hat{r}_{0.5}$	0.0296	0.0106	0.016E-5	0.0135	0.028	0.0544	0.0296	0.0105	2.016E-5	0.013	0.0281	0.054
$\hat{r}_{0.7}$	0.0173	0.0068	1.311E-5	0.0072	0.017	0.0335	0.0172	0.0068	1.311E-5	0.007	0.0161	0.033
$\hat{h}_{0.5}$	1.598	0.1115	2.101E-4	1.385	1.596	1.822	1.598	0.1115	2.101E-4	1.385	1.596	1.822
$\hat{h}_{0.7}$	0.8038	0.0547	1.032E-4	0.6994	0.8028	0.9139	0.8038	0.0547	1.032E-4	0.699	0.8028	0.913
$\hat{t}_p(x_u)$	0.04E-5	9.9E-7	0.004598	0.0025	0.0034	9.9E-7	0.0034	9.9E-7	0.003372	0.002	5.2E-4	9.E-7
$\hat{t}_p(x_l)$	7.67E-4	1.2E-4	2.217E-7	0.0010	0.7E-4	7.6E-4	7.6E-4	1.2E-4	2.217E-7	7E-4	5.7E-4	7E-4
$\hat{t}_p(x_h)$	1.71E-4	0.6E-5	4.967E-8	2.3E-4	1.6E-4	5.E-5	1.7E-4	2.6E-5	4.967E-8	2E-4	1.6E-4	1E-4
informative												
$t_{c1} = 0.90$						$t_{c1} = 0.80$						
\hat{a}	1.233	0.0436	8.117E-5	1.15	1.232	1.321	1.231	0.0434	7.672E-5	1.148	1.23	1.318
\hat{b}	1.207	0.0419	7.358E-5	1.127	1.206	1.291	1.202	0.0418	7.782E-5	1.123	1.202	1.286
$\hat{\theta}$	1.248	0.0394	7.182E-5	1.173	1.248	1.327	1.249	0.0394	7.342E-5	1.174	1.248	1.328
$\hat{\beta}$	1.168	0.0357	6.634E-5	1.099	1.167	1.239	1.153	0.035	6.57E-5	1.086	1.152	1.223
$\hat{\lambda}_u$	6.283	0.3033	5.569E-4	5.723	6.271	6.911	6.252	0.3012	5.306E-4	5.697	6.239	6.874
$\hat{\lambda}_l$	11.49	0.6924	0.00125	10.22	11.46	12.94	11.41	0.6878	0.001226	10.16	11.38	12.85
$\hat{\lambda}_h$	21.03	1.606	0.002871	18.13	20.95	24.41	20.84	1.593	0.002877	17.96	20.76	24.2
$\hat{r}_{0.5}$	0.2322	0.0157	2.739E-5	0.202	0.232	0.2638	0.2382	0.0159	2.858E-5	0.208	0.2379	0.27
$\hat{r}_{0.7}$	0.1218	0.0106	1.845E-5	0.1019	0.1216	0.1434	0.1261	0.0107	1.942E-5	0.106	0.1258	0.148
$\hat{h}_{0.5}$	1.723	0.0589	1.049E-4	1.61	1.722	1.841	1.698	0.0578	1.057E-4	1.587	1.697	1.813
$\hat{h}_{0.7}$	0.994	0.0318	5.765E-5	0.933	0.9935	1.058	0.9801	0.0312	5.775E-5	0.920	0.9796	1.043

$\hat{t}_p(x_u)$	0.0507	0.0040	7.089E-6	0.0590	0.0506	0.0432	0.0518	0.0041	7.493E-6	0.06	0.0516	0.044
$\hat{t}_p(x_l)$	0.0277	0.0024	4.253E-6	0.0327	0.0276	0.0233	0.0284	0.0024	4.471E-6	0.033	0.0283	0.023
$\hat{t}_p(x_h)$	0.0152	0.0014	2.637E-6	0.0183	0.0151	0.0124	0.0156	0.0015	2.771E-6	0.018	0.0155	0.012

Table 3 Posterior statistics of model parameters when sample size, n=60.

Non-informative												
E	$t_{c1} = 0.80$						$t_{c1} = 0.90$					
	mean	Sd	MC error	2.5%	Median	97.5%	mean	Sd	MC error	2.5%	Median	97.5%
\hat{a}	5.994	0.0059	1.856E-5	5.978	5.996	6.0	5.994	0.0058	1.8E-5	2.978	3.996	4.0
\hat{b}	2.994	0.0063	1.919E-5	2.977	2.996	3.0	2.994	0.0064	2.0E-5	2.976	2.996	3.0
$\hat{\theta}$	6.969	0.0305	9.583E-5	6.887	6.979	6.999	6.969	0.0308	9.9E-5	6.886	6.978	6.999
$\hat{\beta}$	0.8538	0.0560	1.012E-4	0.747	0.8524	0.9674	0.8516	0.0559	9.9E-5	0.7456	0.850	0.9649
$\hat{\lambda}_{u.}$	1792.0	11.92	0.03747	1761	1795.0	1806.0	1792.0	11.91	0.037	1761.0	1795	1806.0
$\hat{\lambda}_l$	8004.0	68.84	0.2141	7831.	8020.0	8091.0	8004.0	69.1	0.216	7831.0	8020.	8091.0
$\hat{\lambda}_h$	35760	396.7	1.223	34760	35850.0	36250	35760.0	399.2	1.251	34750.0	35850	36250
$\hat{r}_{0.5}$	0.0223	0.0086	1.554E-5	0.009	0.02101	0.0425	0.02262	0.0085	1.5E-5	0.00982	0.021	0.0429
$\hat{r}_{0.7}$	0.0125	0.0053	9.816E-6	0.005	0.01169	0.0255	0.01279	0.0053	1.0E-5	0.00504	0.012	0.0258
$\hat{h}_{0.5}$	0.0223	0.008	1.554E-5	0.009	0.02101	0.0425	1.685	0.1168	2.08E-4	1.462	1.683	1.919
$\hat{h}_{0.7}$	0.0125	0.0053	9.816E-6	0.004	0.01169	0.0255	0.8466	0.0575	1.02E-4	0.7369	0.845	0.9622
$\hat{t}_p(x_u)$	0.003	4.5E-4	8.194E-7	0.0040	0.003	0.0023	0.003062	4.54E-4	8.18E-7	0.00408	0.003	0.0023
$\hat{t}_p(x_l)$	6.81E-4	1.00E-4	1.836E-7	9.08E-4	6.715E-4	5.13E-4	6.853E-4	1.01E-4	1.83E-7	9.134E-4	6.7E-4	5.16E-4
$\hat{t}_p(x_h)$	1.52E-4	2.25E-5	4.121E-8	2.03E-4	1.503E-4	1.15E-4	1.534E-4	2.27E-5	4.10E-8	2.045E-4	1.5E-4	1.15E-4
informative												
E	$t_{c1} = 0.90$						$t_{c1} = 0.80$					
	mean	Sd	MC error	2.5%	Median	97.5%	mean	Sd	MC error	2.5%	Median	97.5%
\hat{a}	1.235	0.0436	8.065E-5	1.152	1.234	1.322	1.234	0.0437	7.79E-5	1.151	1.234	1.322
\hat{b}	1.207	0.0420	7.626E-5	1.127	1.206	1.292	1.207	0.0419	7.72E-5	1.127	1.206	1.291
$\hat{\theta}$	1.248	0.0393	6.839E-5	1.172	1.247	1.326	1.248	0.0394	7.17E-5	1.172	1.247	1.326
$\hat{\beta}$	1.169	0.0358	6.882E-5	1.101	1.169	1.241	1.169	0.0357	6.29E-5	1.1	1.168	1.24
$\hat{\lambda}_{u.}$	6.293	0.3034	5.5E-4	5.733	6.281	6.923	6.291	0.3039	5.32E-4	5.731	6.279	6.921
$\hat{\lambda}_l$	11.52	0.6936	0.001241	10.24	11.48	12.96	11.51	0.6942	0.00121	10.24	11.48	12.96
$\hat{\lambda}_h$	21.08	1.61	0.002869	18.17	20.99	24.48	21.07	1.61	0.00284	18.15	20.99	24.46
$\hat{r}_{0.5}$	0.2313	0.0157	2.834E-5	0.2011	0.2311	0.2628	0.2316	0.0157	2.69E-5	0.2015	0.231	0.2631
$\hat{r}_{0.7}$	0.1212	0.0105	1.922E-5	0.1013	0.121	0.1427	0.1214	0.0105	1.80E-5	0.1016	0.121	0.143
$\hat{h}_{0.5}$	1.726	0.0591	1.101E-4	1.613	1.726	1.845	1.725	0.0589	1.00E-4	1.612	1.724	1.843
$\hat{h}_{0.7}$	0.9956	0.0319	6.036E-5	0.9343	0.9951	1.06	0.9949	0.0318	5.50E-5	0.9337	0.995	1.059
$\hat{t}_p(x_u)$	0.05057	0.0040	7.048E-6	0.0588	0.05043	0.0431	0.05061	0.0040	7.06E-6	0.05888	0.050	0.0432
$\hat{t}_p(x_l)$	0.02767	0.0023	4.227E-6	0.0326	0.02758	0.0232	0.0277	0.0023	4.21E-6	0.03265	0.027	0.0233
$\hat{t}_p(x_h)$	0.01514	0.0015	2.631E-6	0.0182	0.01508	0.0124	0.01516	0.0015	2.61E-6	0.01824	0.015	.0236

Table 4 Posterior statistics of model parameters when sample size, n=84.

Non-informative												
E	$t_{c1} = 0.90$						$t_{c1} = 0.80$					
	mean	Sd	MC error	2.5%	Median	97.5%	mean	Sd	MC error	2.5%	Median	97.5%
\hat{a}	5.999	6.64E-4	2.041E-6	5.998	6.0	6.0	5.999	6.64E-4	2.01E-6	5.998	6.0	6.0
\hat{b}	2.999	5.31E-4	1.707E-6	2.998	3.0	3.0	2.999	5.33E-4	1.59E-6	2.998	3.0	3.0
$\hat{\theta}$	6.996	0.0039	1.264E-5	6.985	6.997	7.0	6.996	0.0039	1.22E-5	6.985	6.997	7.0
$\hat{\beta}$	2.116	0.0502	9.364E-5	2.019	2.115	2.215	2.101	0.0501	8.65E-5	2.004	2.101	2.201
$\hat{\lambda}_{u.}$	1806.0	1.291	0.003957	1803.0	1807.0	1808.0	1806.0	1.291	0.0039	1803.0	1807.0	1808.0
$\hat{\lambda}_l$	8093.0	6.877	0.02124	8076.0	8095.0	8102.0	8093.0	6.879	0.0209	8076.0	8095.0	8102.0
$\hat{\lambda}_h$	36260	37.57	0.1171	36170	36270.0	36310.0	36260.	37.62	0.1138	36170.0	36270.0	36310.0
$\hat{r}_{0.5}$	4.12E-6	1.43E-6	2.72E-9	1.97E-6	3.905E-6	7.54E-6	4.54E-6	1.58E-6	2.83E-9	2.175E-6	4.308E-6	8.321E-6
$\hat{r}_{0.7}$	9.65E-7	3.72E-7	7.07E-10	9.02E-7	1.864E-6	9.65E-7	1.07E-6	4.15E-7	4.73E-7	1.006E-6	2.077E-6	1.074E-6
$\hat{h}_{0.5}$	4.227	0.1004	1.871E-4	4.033	4.226	4.426	4.198	0.1002	1.73E-4	4.004	4.197	4.397
$\hat{h}_{0.7}$	2.115	0.0502	9.359E-5	2.017	2.114	2.214	2.1	0.0501	8.65E-5	2.003	2.1	2.2

$\hat{t}_p(x_u)$	6.16E-4	2.08E-5	3.882E-8	6.58E-4	6.161E-4	5.77E-4	6.22E-4	2.11E-5	3.66E-8	-6.65E-4	-6.221E-4	-5.824E-4
$\hat{t}_p(x_l)$	1.37E-4	4.64E-6	8.669E-9	1.47E-4	1.375E-4	1.28E-4	1.38E-4	4.73E-6	8.18E-9	1.486E-4	1.388E-4	1.3E-4
$\hat{t}_p(x_h)$	3.07E-5	1.03E-6	1.936E-9	3.28E-5	3.069E-5	2.87E-5	3.10E-5	1.05E-6	1.82E-9	3.316E-5	3.099E-5	2.901E-5
informative												
$t_{c1} = 0.90$						$t_{c1} = 0.80$						
\hat{a}	4.824	0.0195	3.664E-5	4.785	4.824	4.862	4.825	0.0195	3.57E-5	4.787	4.825	4.863
\hat{b}	1.01	6.642E-	1.353E-6	1.009	1.01	1.011	1.01	6.59E-4	1.34E-6	1.009	1.01	1.011
$\hat{\theta}$	3.348	0.0251	4.59E-5	3.298	3.348	3.397	3.348	0.0251	4.56E-5	3.298	3.348	3.397
$\hat{\beta}$	3.178	0.0610	1.126E-4	3.057	3.178	3.297	3.148	0.0616	1.08E-4	3.026	3.148	3.268
$\hat{\lambda}_{uu}$	206.3	4.035	0.007553	198.4	206.3	214.2	206.5	4.034	0.0074	198.7	206.5	214.4
$\hat{\lambda}_l$	341.9	6.689	0.01252	328.8	341.9	355.0	342.2	6.688	0.0122	329.2	342.2	355.4
$\hat{\lambda}_h$	566.5	11.09	0.02076	544.9	566.5	588.3	567.0	11.09	0.0203	545.5	567.0	588.9
$\hat{r}_{0.5}$	1.35E-6	3.93E-7	7.27E-10	7.46E-7	1.296E-6	2.27E-6	1.54E-6	4.55E-7	108.E-7	1.484E-6	2.619E-6	1.549E-6
$\hat{r}_{0.7}$	1.53E-7	5.15E-8	9.60E-11	7.73E-8	1.453E-7	2.76E-7	1.79E-7	6.96E-8	1.0E-10	1.7E-7	3.258E-7	1.797E-7
$\hat{h}_{0.5}$	6.295	0.1208	2.226E-4	6.056	6.295	6.53	6.236	0.122	2.15E-4	5.995	6.236	6.472
$\hat{h}_{0.7}$	3.163	0.0607	1.119E-4	3.043	3.163	3.281	3.133	0.0613	1.08E-4	3.012	3.133	3.252
$\hat{t}_p(x_u)$	0.0017	4.98E-5	8.927E-8	0.0018	0.001716	0.00162	0.0017	5.09E-5	8.91E-8	0.00183	0.001734	0.00163
$\hat{t}_p(x_l)$	0.0010	5.38E-8	0.001036	0.0010	0.001036	9.79E-4	0.001	3.07E-5	5.38E-8	0.00111	0.001046	9.884E-4
$\hat{t}_p(x_h)$	6.25E-4	1.81E-5	0.2E-8	6.62E-4	6.251E-4	5.91E-4	6.31E-4	1.85E-5	3.25E-8	6.693E-4	6.314E-4	5.965E-4

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Appendix: "0.040, 1.8660, 2.385, 3.4430, 0.301, 1.8760, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779, 1.248, 2.010, 2.688, 3.924, 1.2810, 2.038, 2.4810, 3.467, 0.3090, 1.899, 2.610, 3.4780, 0.5570, 1.9110, 2.625, 3.5780, 3.117, 4.4850, 1.652, 2.2290, 3.166, 4.570, 1.652, 2.300, 3.344, 2.0890, 2.902, 4.1670, 1.4320, 4.6020, 1.7570, 2.324, 0.9430, 1.912, 2.6320, 3.5950, 1.070, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 2.82, 3.000, 4.0350, 1.281, 2.0850, 2.890, 4.1210, 1.3030, 3.000, 4.305, 1.5680, 2.194, 3.103, 4.3760, 1.615, 2.2230, 3.114, 4.4490, 1.619, 2.224, 3.3760, 4.6630".