Pak.j.stat.oper.res. Vol.17 No. 1 2021 pp 113-122 DOI: http://dx.doi.org/10.18187/pjsor.v17i1.2704

Pakistan Journal of Statistics and Operation Research

Estimating E-Bayesian of Parameters of Inverse Weibull Distribution Using a Unified Hybrid Censoring Scheme

[1]Shahram Yaghoobzadeh Shahrastani [1], Iman Makhdoom[2*]

* Corresponding Author

1. Department of Statistics, Payame Noor University (PNU), P.O. Box, 19395-4697 Tehran, Iran, yagoubzade@gmail.com
2. Department of Statistics, Payame Noor University (PNU), P.O. Box, 19395-4697 Tehran, Iran makhdoom@pnu.ac.ir

Abstract

The combination of generalization Type-I hybrid censoring and generalization Type-II hybrid censoring schemes, scheme creates a new censoring called a Unified hybrid censoring scheme. Therefore, in this study, the E-Bayesian estimation of parameters of the inverse Weibull (IW) distribution is obtained under the unified hybrid censoring scheme, and the efficiency of the proposed method was compared with the Bayesian estimator using Monte Carlo simulation as well as, we use a real data set for practical purposes. Finally, we showed that in all schemes the E-Bayesian estimation parameters are better than their Bayesian estimations.

Key Words: E-Bayesian Estimation; Unified Hybrid Censoring Scheme; Inverse Weibull Distribution.

Mathematical Subject Classification: 62F15, 62N02

1. Introduction

Consider a lifetime test with n units. Suppose that the units have independent and identically lifetime with the probability density function $f(x;\,\theta)$ and the cumulative distribution function$F(x;\,\theta)$, and $Y_{1:n} < \cdots < Y_{n:n}$ are the lifetime of the units until their failure. For the first time, Epstein (1954) investigated a scheme in a survival experiment in which the experiment ended at time $T^* = \min(Y_{r:n}\,, T)$ and the values of T and r were pre-determined. Childs et al. (2003) called this Type-I hybrid censoring. In this scheme, there may be very few failures up to time T. Childs et al. (2003) investigated a scheme in which the experiment ended at time$T^* = \max(Y_{r:n}\,, T)$. This scheme was called the Type-II hybrid censoring scheme. Obviously, this scheme does not have a problem with the previous scheme. Even before time T, all units can fail, but the time to test is not predictable. Chandrasekar et al. (2004) introduced two Types of generalization hybrid censoring of Type I and II, so that the problem has somewhat improved the previous two schemes (not having the minimum failure in the Type-I hybrid censoring scheme and prolonging the test time in the Type-II hybrid censoring scheme).

In generalization Type-I hybrid censoring scheme, suppose $T \in (0, \infty)$ and the values of k and r such that $k < r$ are predetermined. If the k^{th} failure occurs before time T, the experiment at min $(Y_{r:n}\,, T)$ and if, after time T, the experiment ends at $Y_{k:n}$. Therefore, this scheme guarantees at least k failures.

In general Type-II hybrid censoring scheme, assume that r and $T_1, T_2 \in (0, \infty)$, so that $T_1 < T_2$, are constant and predetermined values. If the r^{th} failure occurs before time T_1, the experiment at time T_1, if between T_1 and T_2, occurs at time $Y_{r:n}$, and if after T_2, the experiment ends at T_2. Therefore, this scheme guarantees that the experiment ends up at time T_2.

The combination of the above scheme creates a new censoring called a **Unified hybrid censoring scheme**. This scheme was first introduced by Balakrishnan et al. (2008). In this scheme, the values T_1, T_2, r, and k, so that $T_1 < T_2$ and $k < r$, are predetermined before the experiment begins. If the k^{th} failure occurs before time T_1, the experiment at time min $(\max(Y_{r:n}, T_1), T_2)$, if between T_1 and T_2, occurs at time min $(Y_{r:n}, T_2)$, and if after T_2, the experiment ends at $Y_{k:n}$. In this censoring, one of the following six occurrences occurs. Suppose that for j = 1,2, d_j the number of failures is up to T_j. In this case, we have six types of observations.

1. If $0 < Y_{k:n} < Y_{r:n} < T_1 < T_2$, the experiment ends at time T_1 with D failures.
2. If $0 < Y_{k:n} < T_1 < Y_{r:n} < T_2$, the experiment ends with the failure of r^{th}.
3. If $0 < Y_{k:n} < T_1 < T_2 < Y_{r:n}$, the experiment ends at time T_2 with d_2 failures.
4. If $0 < T_1 < Y_{k:n} < Y_{r:n} < T_2$, the experiment ends at time $Y_{r:n}$.
5. If $0 < T_1 < Y_{k:n} < T_2 < Y_{r:n}$, experiment ends at time T_2 with d_2 failures.
6. If $0 < T_1 < T_2 < Y_{k:n} < Y_{r:n}$, The experiment ends with the failure of k^{th}.

Note that in the first case, $d_1 = d_2 = D$, $T_1 < Y_{(D+1):n}$ and $r \leq D$, so that the experiment $(D+1)^{\text{th}}$ does not occur before T_1, and in the third and fifth cases, $T_2 < Y_{(d_2+1):n}$ and $k \leq d_2$ are such that That the $(d_1+1)^{\text{th}}$ experiment does not occur before T_2. If c is the stopping point and d is the number of failures until time c, then, the likelihood function of this hybrid censored sample is as follows:

$$L(\theta|\boldsymbol{y}) = \frac{n!}{(n-d)!}\prod_{i=1}^{d} f(y_{i:n};\ \theta)\,[1-F(c)]^{n-d} \tag{1}$$

where $\boldsymbol{y} = (y_{1:n}, \dots, y_{d:n})$, $d\epsilon\{D, d_1, d_2, k, r\}$, and $c\epsilon\{T_1, T_2, Y_{r:n}, Y_{k:n}\}$.

If the random variable Y has a Weibull distribution with the pdf
$f(y;\alpha,\lambda) = \alpha\lambda y^{\alpha-1}e^{-\lambda y^{\alpha}},\ \ y > 0,$
then the random variable $X = \frac{1}{Y}$ has an IW distribution with the pdf

$$f(x;\alpha,\lambda) = \alpha\lambda x^{-(\alpha+1)}e^{-\lambda x^{-\alpha}},\ \ x > 0. \tag{2}$$

The quantities $\alpha > 0$ and $\lambda > 0$ are the shape and scale parameters respectively. From now on it will be denoted by IW(α,λ). If X follow IW(α,λ), then the distribution function of X is given by

$$F(x;\alpha,\lambda) = e^{-\lambda x^{-\alpha}}, \qquad x > 0. \tag{3}$$

The IW model has been derived as a suitable model for describing the degradation phenomena of mechanical components, such as the dynamic components of diesel engines, see for example Murthy et al. (2004).The physical failure process given by Erto and Rapone (1984) also leads to the IW model. Erto and Rapone (1984) showed that the IW model provides a good fit to survival data such as the times to breakdown of an insulating fluid subject to the action of constant tension, see also Nelson (1982). Interpretation of IW distribution in the context of load strength relationship for a component was provided by Calabria and Pulcini (1994). In reliability engineering research, IW distribution is often used in statistical analysis of life time and response time data. Khan et al. (2008) in their theoretical analysis of IW distribution mention that numerous failure characteristics such aswear out periods and infant mortality can be modeled through IW distribution. They mention about the wide range of areas in reliability analysis where IW distribution model can be used successfully. Shafiei et al. (2016) mention that IW distribution is an appropriate model for situations where hazard function is unimodal. They further mention the distribution as one of the popular distributions in complementary risk problems.

The hierarchical Bayesian prior distribution was primarily introduced by Lindley and Smith (1972).Then it was examined by Han (1997), and E- Bayesian and hierarchical Bayesian methods were introduced. Recently, E-Bayesian and hierarchical Bayesian methods have been used by Han (2009, 2011) to estimate exponential distribution parameter and estimation of the reliability of the binomial distribution, by Jaheen and Okasha (2011) to estimate of the reliability of the Type 12 distribution based onType II progressive censoring samples, by Wang et al. (2012) and Yousefzadeh (2017) to estimate Pascal distribution parameters, by Yaghoobzadeh (2018) to estimate of scale parameter of gompertz distribution under type II censoring schemes based on fuzzy data. Also, Han (2017) gives the property of E-Bayesian estimation and hierarchical Bayesian estimation of the system reliability parameter. In this study, E-Bayesian of α and λ parameters of IW distribution Based On an unified hybrid censored sample using square error loss function (

$L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2$) are described in Section 2. A numerical example and a Monte Carlo simulation are presented in Section 3 for illustrative purposes. Section 4 is the conclusions.

2. Estimating the E-Bayesian of α and λ Parameters

Suppose $y_{1:n}$, ..., $y_{n:n}$ is a random sample based on unified hybrid censored scheme, and are identical to the probability density function (2). Also, assume that A and B are independent, and each has a prior gamma distribution as follows

$$\pi_1(\alpha|a_1,b_1)\propto \alpha^{a_1-1}e^{-b_1\alpha},\qquad \alpha>0,$$

$$\pi_2(\lambda|a_2,b_2)\propto \lambda^{a_2-1}e^{-b_2\lambda},\qquad \lambda>0$$

Wherea_1, b_1, a_2 , and b_2 are positive and known values. The derivative of $\pi(\alpha|a_1,b_1)$ with respect to α is

$$\frac{d\pi(\alpha|a_1,b_1)}{d\alpha}=\frac{b_1^{a_1}\alpha^{a_1-2}e^{-b_1\alpha}}{\Gamma(a_1)}\big((a_1-1)-b_1\alpha\big)$$

According to Han (1997), a_1 and b_1 should be chosen to guarantee that $\pi(\alpha|a_1,b_1)$ is a decreasing function of α. Thus, $b_1>0$ and $0<a_1<1$. Given $a_1=1$, and the larger the value of b_1, the thinner the tail of the density function is. Berger (1985) showed that the thinner tailed prior distribution often reduces the robustness of the Bayesian estimation. Consequently, the hyperparameter b_1 should be chosen under the restriction $0<b_1<c_1$, where c_1 is a given upper bound (c_1 is a positive constant). In this study, we only consider the case when $a_1=1$. In this case, the density function $\pi(\alpha|a_1,b_1)$ becomes

$$\pi(\alpha|b_1)=b_1e^{-b_1\alpha},\qquad \alpha>0, \tag{4}$$

Also, we consider the prior distribution b_1 as $\pi(b_1)=\frac{1}{c_1}$,$0<b_1<c_1$.

As the same way, $\pi(\lambda|a_2,b_2)$ becomes

$$\pi(\lambda|b_2)=b_2e^{-b_2\lambda},\qquad \lambda>0, \tag{5}$$

and $\pi(b_2)=\frac{1}{c_2}$,$0<b_2<c_2$. Therefore, the Bayesian estimation of each function of α and λ as $h(\alpha,\lambda)$, under square error loss function is as follows.

$$\hat{h}=E_{(\alpha,\lambda|\boldsymbol{y})}\big(h(\alpha,\lambda)\big)=\left(\int_0^\infty\int_0^\infty h(\alpha,\lambda)\pi(\alpha,\lambda|\boldsymbol{y})d\alpha d\lambda\right)\Big/\left(\int_0^\infty\int_0^\infty \pi(\alpha,\lambda|\boldsymbol{y})d\alpha d\lambda\right) \tag{6}$$

where

$$\pi(\alpha,\lambda|\boldsymbol{y})\propto\alpha^d\left(\prod_{i=1}^d y_{i:n}\right)^{-(\alpha+1)}e^{-b_1\alpha}\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}f_{\lambda|\alpha}(a_1^*,b_1^*,j)$$

where $f_{\lambda|\alpha}(a_1^*,b_1^*,j)$ is the gamma distribution with the parameter $a_1^*=d+1$ of shape and with the scale parameter as follows.

$$b_1^*=b_2+\sum_{i=1}^d y_{1:n}^{-\alpha}+jc^{-\alpha}$$

By considering $h(\alpha,\lambda)=\alpha$ and $h(\alpha,\lambda)=\lambda$ in relation (6), the Bayesian estimations for the α and λ representing the symbols $\hat{\alpha}_B(b_1,b_2)$ and $\hat{\lambda}_B(b_1,b_2)$, respectively, are as follows.

$$\hat{\alpha}_B(b_1,b_2)=\frac{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty\left(\frac{\alpha}{b_1^*}\right)^{d+1}\left(\prod_{i=1}^d y_{i:n}\right)^{-(\alpha+1)}e^{-b_1\alpha}d\alpha}{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty\frac{\alpha^d}{b_1^{*\,d+1}}\left(\prod_{i=1}^d y_{i:n}\right)^{-(\alpha+1)}e^{-b_1\alpha}d\alpha} \tag{7}$$

$$\hat{\lambda}_B(b_1,b_2)=\frac{(d+1)\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \frac{\alpha^d}{b_1^{*\,d+2}}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha}{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \frac{\alpha^d}{b_1^{*\,d+1}}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha} \quad (8)$$

The definition for E-Bayesian estimation was originally proposed by Han (2009), relate as follows.
Definition 1: With $\hat{\theta}_B(b_1,b_2)$ being continuous,

$$\hat{\theta}_{EB}=\int\int_D \hat{\theta}_B(b_1,b_2)\pi(b_1,b_2)\,db_1db_2 \quad (9)$$

is called the E-Bayesian estimation of θ (briefly E-Bayesian estimation, the full name should be expected Bayesian estimation), which is assumed to be finite, where D is the domain of b_1 and b_2, $\hat{\theta}_B(b_1,b_2)$ is the Bayesian estimation of θ with hyper parameters b_1 and b_2, and $\pi(b_1,b_2)$ is the density function of b_1 and b_2 over D. Definition 1 indicates that the E-Bayesian estimation of θ is just the expectation of the Bayesian estimation of θ for all the hyperparameters. Therefore, with respect to (8) and (9) and definition (1), the E-Bayesian estimations α ($\hat{\alpha}_{EB}$) and λ ($\hat{\lambda}_{EB}$) are as follows.

$$\hat{\alpha}_{EB}=\frac{1}{c_1c_2}\int_0^{c_1}\int_0^{c_2}\left(\frac{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \left(\frac{\alpha}{b_1^*}\right)^{d+1}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha}{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \frac{\alpha^d}{b_1^{*\,d+1}}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha}\right)db_1db_2$$

$$\hat{\lambda}_{EB}=\frac{1}{c_1c_2}\int_0^{c_1}\int_0^{c_2}\left(\frac{(d+1)\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \frac{\alpha^d}{b_1^{*\,d+2}}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha}{\sum_{j=0}^{n-d}\frac{(-1)^j}{j!\,(n-d-j)!}\int_0^\infty \frac{\alpha^d}{b_1^{*\,d+1}}\left(\prod_{i=1}^{d} y_{i:n}\right)^{-(\alpha+1)} e^{-b_1\alpha}d\alpha}\right)db_1db_2$$

3. Numerical Experiments

In this section, a numerical example and a Monte Carlo simulation are presented to illustrate all the estimation methods described in the section 2.

3.1. Simulation Study

In this section, the simulation results are presented for comparing different unified hybrid censored schemes and the performance of estimation of Bayesian and E-Bayesian parameters are based on the mean square error (MSE) criterion. For this purpose, we generate a random sample of 50 of the IW distribution with $\alpha = 2.5$ and $\lambda = 0.05$. Then, the Bayesian, and E-Bayesian estimations of α and λ were estimated. The performance of all estimates have been compared numerically of the MSE value. This process, have been irritated 1000 times, and the average all estimates and their MSEs were estimated and are listed in Tables 1 to 4. The simulation is conducted by R software.

Drawn upon the simulation results, we found out that:

1. According to Tables 1 and 2, in both the cases (b_1,b_2), for fixed r, k, and T_2 , when T_1 is increased, the performance of the E-Bayesian estimation of the parameters α and λ is more than their Bayesian estimations. Also, the MSE of all estimators decreases with increasing T_1, and the numerical value of the estimators approaches the real values of the parameters by increasing T_1.
2. According to Tables 3 and 4, in both the cases (b_1,b_2), for fixed r, k, and T_1 , when T_2 is increased, the performance of the Bayesian estimation of the parameters α and λ is more than their E-Bayesian estimations. Also, the MSE of all estimators decreases with increasing T_2, and the numerical value of the estimators approaches the real values of the parameters by increasing T_2.

DOI: http://dx.doi.org/10.18187/pjsor.v17i1.2704

Table 1. Estimate, the mean square error for α and $T_2 = 100$

(b_1, b_2)	(k, r)	T_1	$\hat{\alpha}_B$	$\hat{\alpha}_{EB}$	MSE($\hat{\alpha}_B$)	MSE($\hat{\alpha}_{EB}$)
(1.5, 2)	(11, 20)	80	1.0084356	1.3116907	0.9722919	0.9582604
		85	1.6295644	1.8872927	0.9291750	0.8110088
		95	2.2896661	2.5735351	0.8350252	0.7265117
	(15, 20)	80	0.9002296	1.4889474	1.2943128	1.0106295
		85	2.0657321	2.4462040	1.0263026	0.9604438
		95	2.3750732	2.6011219	0.9364447	0.8430386
	(18, 20)	80	0.9603170	1.2968166	0.7655433	0.7446662
		85	1.0417214	2.1999589	0.6260000	0.5859107
		95	1.8491109	2.5619906	0.5438144	0.4673512
	(12, 25)	80	3.968900	2.3963982	1.7479096	1.0443045
		85	2.897733	2.4516401	0.9601863	0.8931725
		95	2.753317	2.5567550	0.8093399	0.7713700
	(12, 35)	80	3.118313	2.2918441	1.0693892	0.9788446
		85	2.965890	2.3658907	1.0433348	0.8797319
		95	2.770518	2.5194235	0.8389121	0.6179893
(2.5, 3)	(11, 20)	80	1.264413	1.6612946	1.2980096	1.2388655
		85	1.536037	2.2058489	1.1631997	1.1217848
		95	2.234662	2.5497267	0.9900068	0.9096081
	(18, 20)	80	1.860379	2.1271919	1.6419446	1.5753926
		85	2.0782481	2.2608274	0.9715295	0.8473973
		95	2.9904854	2.5826947	0.7014477	0.6473973
	(12, 25)	80	1.7583538	2.1329156	1.4209078	1.0292515
		85	2.0836629	2.2943991	1.0031588	0.9421133
		95	2.9015001	2.5290020	0.9064419	0.8547526
	(12, 35)	80	1.0192650	2.0194721	1.1086039	1.0041249
		85	1.8166442	2.2617467	0.9388403	0.8259847
		95	2.1086032	2.5030348	0.8166442	0.7741249

Table 2. Estimate, the mean square error for λ and $T_2 = 100$

(b_1, b_2)	(k, r)	T_1	$\hat{\lambda}_B$	$\hat{\lambda}_{EB}$	MSE($\hat{\lambda}_B$)	MSE($\hat{\lambda}_{EB}$)
(1.5, 2)	(11, 20)	80	0.01002324	0.02018169	0.36583143	0.15168012
		85	0.01388574	0.02082876	0.24540068	0.11009306
		95	0.07551424	0.05698260	0.13283998	0.09900334
	(15, 20)	80	0.00854059	0.00881930	0.15417093	0.11921365
		85	0.00985218	0.02058568	0.08650327	0.07451532
		95	0.07288751	0.05165167	0.07900718	0.06650365
	(18, 20)	80	0.01298031	0.02154525	0.18194735	0.11264996
		85	0.02697772	0.03872098	0.11207455	0.09469526
		95	0.03939734	0.05098933	0.10112370	0.08978639
	(12, 25)	80	0.09020828	0.02195403	0.09984382	0.08134841
		85	0.08257069	0.03250745	0.07433193	0.06635288
		95	0.03631063	0.06295896	0.06405985	0.05471148
	(12, 35)	80	0.01250740	0.02827069	1.01471148	0.99984382
		85	0.01295896	0.03920828	0.88134841	0.78433193
		95	0.02195403	0.05631063	0.76635288	0.69405985
(2.5, 3)	(11, 20)	80	0.01217904	0.02987496	0.72154542	0.66436972
		85	0.02047071	0.03594608	0.52135136	0.40379023
		95	0.03962807	0.05881317	0.30398702	0.28200912
	(18, 20)	80	0.01714689	0.02243666	0.68180587	0.51885518
		85	0.02714689	0.03948523	0.42243629	0.37489675
		95	0.03172106	0.04950494	0.27156115	0.11788791
	(12, 25)	80	0.01152291	0.02391643	0.63932464	0.51376332
		85	0.02841107	0.03120920	0.43404868	0.39150965
		95	0.03992588	0.05114683	0.27755420	0.14960132
	(12, 35)	80	0.01637617	0.03174601	0.21078954	0.10921629
		85	0.03275874	0.04146268	0.16296289	0.08314580
		95	0.04064067	0.05030860	0.12041944	0.07720966

Table 3. Estimate, the mean square error for α and $T_1 = 45$

(b_1, b_2)	(k, r)	T_2	$\hat{\alpha}_B$	$\hat{\alpha}_{EB}$	MSE($\hat{\alpha}_B$)	MSE($\hat{\alpha}_{EB}$)
(1.5, 2)	(11, 20)	90	0.9957537	0.6339161	1.062178	1.214407
		110	2.3118858	1.1531517	0.830641	0.719226
		150	2.6339161	1.7957537	0.762178	0.614407
	(15, 20)	90	1.7065898	0.8937124	1.037677	1.954498
		110	2.0185215	1.0552049	0.908985	1.490925
		150	2.4611235	1.4480276	0.836010	0.959404
	(18, 20)	90	1.1308251	0.8127624	0.831108	0.927974
		110	1.9309987	1.0086287	0.774999	0.864279
		150	2.5122400	1.8806394	0.658258	0.749992
	(12, 25)	90	1.1822798	0.9285195	0.8584485	0.9079119
		110	1.9889285	1.0088465	0.7596543	0.8111743
		150	2.4374748	1.8500879	0.6094413	0.7384733
	(12, 35)	90	1.5718727	1.1097288	1.3895278	1.4073459
		110	1.9807946	1.6409883	1.0069845	1.2506338
		150	2.6149597	2.0560069	0.84163644	0.9255014
(2.5, 3)	(11, 20)	90	1.8285346	1.4242101	1.38407889	1.5504698
		110	2.1850752	1.8895607	1.11091642	1.4645827
		150	2.7077718	2.1627746	0.96143870	1.0019672
	(18, 20)	90	1.1078642	0.8345830	0.79081568	1.0080633
		110	2.1139134	1.1009293	0.73245607	0.8379379
		150	2.4554278	2.0875900	0.62469905	0.7161964
	(12, 25)	90	0.9926499	0.8910045	1.07836062	1.84205811
		110	1.1296560	0.7888355	0.79102405	0.82499218
		150	2.5287248	1.7491727	0.69914508	0.70318675
	(12, 35)	90	1.9232352	1.2534225	1.78851980	1.86673045
		110	2.2694444	1.7918619	1.06024561	1.73262390
		150	2.5128730	2.1681149	0.94087910	1.16673042

Table 4. Estimate, the mean square error for λ and $T_1 = 45$

(b_1, b_2)	(k, r)	T_2	$\hat{\lambda}_B$	$\hat{\lambda}_{EB}$	MSE($\hat{\lambda}_B$)	MSE($\hat{\lambda}_{EB}$)
(1.5, 2)	(11, 20)	90	0.03067451	0.01754875	0.089908104	0.356086732
		110	0.04394579	0.02913630	0.070822873	0.236063086
		150	0.05279717	0.03434380	0.051552638	0.164494967
	(15, 20)	90	0.02808769	0.02030007	0.161504991	0.209289078
		110	0.03180027	0.02436261	0.127928345	0.178411826
		150	0.04985280	0.03367812	0.102193247	0.154849690
	(18, 20)	90	0.02733432	0.01501363	0.115439526	0.137946388
		110	0.04519281	0.02207054	0.082897327	0.106536726
		150	0.05386042	0.03622065	0.068481334	0.085626029
	(12, 25)	90	0.03302440	0.02761198	0.087186751	0.105403070
		110	0.04360056	0.03335961	0.080150728	0.091038057
		150	0.05222345	0.04173082	0.072282442	0.081038058
	(12, 35)	90	0.02557485	0.01339570	0.139337354	0.197229546
		110	0.03144091	0.02524225	0.107611978	0.172822455
		150	0.04888983	0.03173394	0.081336791	0.151957044
(2.5, 3)	(11, 20)	90	0.02766431	0.01985449	0.142921258	0.184132366
		110	0.03932568	0.02065841	0.128337187	0.152294538
		150	0.05599842	0.04142739	0.088214903	0.124841527
	(15, 20)	90	0.01655514	0.01043330	0.221247390	0.269735181
		110	0.03960635	0.03177747	0.136787245	0.182609868
		150	0.04705481	0.03644419	0.091725247	0.099176133
	(18, 20)	90	0.02150275	0.01451502	0.097394016	0.124588687
		110	0.03133300	0.02014775	0.085431039	0.121805690
		150	0.04753325	0.03622597	0.062548587	0.107863095
	(12, 25)	90	0.01674199	0.01173110	0.090087877	0.153396965
		110	0.03900441	0.02676016	0.088175630	0.134686308
		150	0.51420139	0.03914584	0.060002891	0.115577656
	(12, 35)	90	0.01086794	0.01283105	0.111859032	0.245535310
		110	0.03178846	0.02982290	0.075802935	0.090254322
		150	0.04825255	0.03503170	0.061635373	0.078183760

3.2 Application with real data set

In this subsection, a real data set is used to analyze α and λ estimation methods. The data set represent repair times (in h) for an airborne communication transceiver. They were first analyzed by Von Alven (1964). The data is presented in Table 5. Before analyzing the data, we fit the IW model to this data set. We used the Kolmogorov-Smirnov (K-S) distance between the fitted the empirical distribution functions, and corresponding p-values. It is observed that for this data, the K-S and corresponding p-value are 0.0815 and 0.9197, respectively. We observe the IW model fit quite well to this data set.

Table 5. Repair times (in h) for an airborne communication transceiver

0.2	0.3	0.5	0.5	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.7	0.8	1.0	1.0	1.0	1.0	1.1	1.3	1.5	1.5
1.5	1.5	2.0	2.0	2.2	2.5	2.7	3.0	3.0	3.3	3.3	4.0	4.0	4.5	4.7	5.0	5.4	5.4	7.0	7.5	8.8
9.0	10.3	7.5	8.8	9.0	10.3															

To compute the Bayesian and E-Bayesian estimations, since we do not have any prior information, we assumed that $b_1 = b_2 = 0.01$. Therefore, for $c_1 = c_2 = 1$, for these data, six unified hybrid censored schemes are considered under the following conditions.

Scheme 1: $K = 14$, $r = 30$, T_1 =3.5, $T_2 = 4.5$

Scheme 2: $K = 19$, $r = 25$, $T_1 = 1.5$, $T_2 = 3$

Scheme 3: $K = 19$, $r = 30$, $T_1 = 1.5$, $T_2 = 2$

Scheme 4: $K = 30$, $r = 32$, $T_1 = 2$, $T_2 = 4$

Scheme 5: $K = 30$, $r = 32$, $T_1 = 1$, $T_2 = 3$

Scheme 6: $K = 18$, $r = 20$, $T_1 = 1$, $T_2 = 3$

In all schemes, the Bayesian and E-Bayesian estimates of the parameters have been obtained. These results are presented in Table 6. Also for this data, in a complete uncensored sample, the maximum likelihood estimation for parameters α and λ are 1.011941 and 1.125229, respectively.

Table 6. Bayesian and E-Bayesian estimations of parameters α and λ

Scheme	$\hat{\alpha}_B$	$\hat{\alpha}_{EB}$	$\hat{\lambda}_B$	$\hat{\lambda}_{EB}$
1	0.075438204	0.094656141	0.827177291	1.144378477
2	0.940139616	1.009714642	1.022434427	1.108393285
3	0.983522234	1.013935819	1.772656778	1.031561386
4	1.263046540	1.030439965	1.795299730	1.176216987
5	0.925065487	1.013107987	2.348933240	1.025538408
6	2.006398994	1.094094911	2.5755073419	1.500216398

Table 6, shows that in all schemes, the E-Bayesian estimation of the parameters are closer to their estimated value in the complete sample. Therefore, estimating E-Bayesian parameters is better than their Bayesian estimations.

4. Conclusion

In this study, the Bayesian and E-Bayesian estimations of the inverse Weibull distribution parameters were obtained under the unified hybrid censored scheme with squared error loss function. In this study, six unified hybrid censored schemes are considered, and, using a real data set, we showed that in all schemes the E-Bayesian estimation parameters are better than their Bayesian estimations. Also, using Monte Carlo simulation, the conditions of superiority of the estimator were obtained with respect to each other.

Acknowledgments

The authors would like to thank the Editor and the anonymous referees for their constrictive comments and suggestions that appreciably improved the quality of presentation of this manuscript.

References

1. Berger, J. O. (1985), *Statistical Decision Theory and Bayesian Analysis*, second ed., Springer-Verlag, New York.
2. Calabria, R. and G. Pulcini. (1994), Bayes 2-sample prediction for the inverse Weibull distribution, *Communications in Statistics-Theory and Methods*, 23 (6),1811–1824.
3. Childs, A., Chandrasekhar, B., Balakrishnan, N. and Kundu, D. (2003), Exact Liklihood Inference Based on Type-I and Type-II Hybrid Censored Sampales from the Exonential distribution, *Annals of the Institute of Statistical Mathematics*, 55, 319-330.
4. Chandrasekhar, B., Childs, A. and Balakrishnan, N. (2004), Exact Liklihood Inference for the Exonential distribution under Generalized Type-I and Type-II Hybrid Censoring, Naval Research Logistics, 51, 994-1004.
5. Epstein, B. (1954), Truncated Life-Tests in the Exponential Case, *Annals of Mathematical Statistics*, 25, 555-564.
6. Erto, P. and Rapone, M. (1984), Non-informative and practical Bayesian confidence bounds for reliable life in the Weibull model, *Reliability Engineering*, 7, 181-191.
7. Han, M. (1997), The structure of hierarchical prior distribution and its applications, *Chinese Operations Research and Management Science*, 63, 31–40.
8. Han, M. (2009), E-Bayesian estimation and hierarchical Bayesian estimation of failure rate, *Applied Mathematical Modelling*, 33(4), 1915-1922.
9. Han, M. (2011), E-Bayesian estimation of the reliability derived from Binomial distribution, *Applied Mathematical Modelling*, 35, 2419-2424.
10. Han, M. (2017), The E-Bayesian and hierarchical Bayesian estimations for the system reliability parameter, *Communications in Statistics-Theory and Methods*, 46(4), 1606-1620.
11. Jaheen, Z. F. and Okasha, H. M. (2011), E-Bayesian estimation for the Burr type XII model based on type-2 censoring, *Applied Mathematical Modelling*, 35, 4730-4737.
12. Khan, M. S., Pasha, G. R. and Pasha, A. H. (2008), Theoretical analysis of inverse Weibull distribution, *WSEAS Transactions on Mathematics*, 7(2), 30–38.
13. Lindley, D. V. and Smith, A. F. (1972), Bayes estimation for the linear model, *Journal of the Royal Statistical Society-Series B*, 34, 1–41.
14. Murthy, D.N.P. Xie, M. and Jiang, R. (2004), *Weibull Models*, Wiley, New York.
15. Nelson, W. (1982), *Applied Lifetime Data Analysis*, Wiley, New York.
16. Shafiei, S., Darijani, S. and Saboori, H. (2016), InverseWeibull power series distributions: properties and applications, *Journal of Statistical Computation and Simulation*, 86 (6), 1069–1094.
17. Von Alven, W.H. (1964), *Reliability engineering by ARINC*, Prentice-Hall, Englewood Cliffs.
18. Wang, J., Li, D. and Chen, D. (2012), E-Bayesian Estimation and Hierarchical Bayesian Estimation of the System Reliability Parameter, *Systems Engineering Procedia*, 3, 282-289.
19. Yousefzadeh, F. (2017), E-Bayesian and hierarchical Bayesian estimations for the system reliability parameter based on asymmetric loss function, *Communications in Statistics-Theory and Methods*, 46(1), 1-8.
20. Yaghoobzadeh, S. S. (2018), Estimating E-bayesian and hierarchical bayesian of scalar parameter of Gompertz distribution under type II censoring schemes based on fuzzy data, *Communications in Statistics - Theory and Methods*,doi. org/10.1080/036109262017.1417438.