Pak.j.stat.oper.res. Vol.16 No. 1 2020 pp119-130 DOI: http://dx.doi.org/10.18187/pjsor.v16i1.2691

Pakistan Journal of Statistics and Operation Research

Power Comparisons of Five Most Commonly Used Autocorrelation Tests

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Abstract

In regression analysis, autocorrelation of the error terms violates the ordinary least squares assumption that the error terms are uncorrelated. The consequence is that the estimates of coefficients and their standard errors will be wrong if the autocorrelation is ignored. There are many tests for autocorrelation, we want to know which test is more powerful. We use Monte Carlo methods to compare the power of five most commonly used tests for autocorrelation, namely Durbin-Watson, Breusch-Godfrey, Box–Pierce, Ljung Box, and Runs tests in two different linear regression models. The results indicate the Durbin-Watson test performs better in the regression model without lagged dependent variable, although the advantage over the other tests reduce with increasing autocorrelation and sample sizes. For the model with lagged dependent variable, the Breusch-Godfrey test is generally superior to the other tests.

Key Words: Correlated error terms; Ordinary least squares assumption; Residuals; Regression diagnostic; Lagged dependent variable.

Mathematical Subject Classification: 62J05; 62J20.

1. Introduction

An important assumption of the classical linear regression model states that there is no autocorrelation in the error terms. Autocorrelation of the error terms violates the ordinary least squares assumption that the error terms are uncorrelated, meaning that the Gauss Markov theorem (Plackett, 1949, 1950; Greene, 2018) does not apply, and that ordinary least squares estimators are no longer the best linear unbiased estimators.

A search on autocorrelation tests available in statistical software such as EViews, IBM SPSS, MATLAB, Minitab, R, SAS, SHAZAM, S-Plus, and Stata revealed that the commonly available autocorrelation tests in these software are Durbin-Watson, Breusch-Godfrey, Box–Pierce, Ljung Box, and Runs tests. Table 1 lists the autocorrelation tests available for these statistical software packages.

Comparison of the autocorrelation tests has received attention in the literature. L'Esperance and Taylor (1975), by Monte Carlo simulation, presents comparisons of four autocorrelation tests (Durbin-Watson bounds test, a test based on Theil's best linear unbiased scalar estimator, a test devised by Abrahamse, Koerts and Louter, and an exact test devised by Durbin). Smith (1976) compared different autocorrelation tests (the Durbin-Watson test, Durbin cumulated periodogram test, Durbin test, Geary sign test and Schmidt tests).

Table 1: Autocorrelation tests available in statistical software packages

Statistical software	Tests for autocorrelation				
	Durbin-Watson	Breusch-Godfrey	Box–Pierce	Ljung-Box	Runs test
EViews	√	√	-	√	√
IBM SPSS	√	-	-	-	√
MATLAB	√	√	-	√	√
Minitab	√	-	-	√	√
R	√	√	√	√	√
SAS	√	√	-	√	√
SHAZAM	√	√	-	√	√
SPlus	√	√	√	√	√
Stata	√	√	√	√	√

In this paper, we use Monte Carlo methods to compare the power of the five most commonly used statistical tests for autocorrelation, namely the Durbin-Watson (Durbin and Watson, 1950, 1951, 1971; Greene, 2018), Breusch-Godfrey (Breusch, 1978; Godfrey, 1978; Asteriou and Hall, 2017), Box– Pierce (Box and Pierce, 1970; Greene, 2018), Ljung-Box (Ljung and Box, 1978; Verbeek, 2017), and Runs test (Wald and Wolfowitz, 1943; Gujarati and Porter, 2009), in two different linear regression models: without lagged dependent variable and with lagged dependent variable.

2. Tests for Autocorrelation

We give a short description of the five methods of testing for Autocorrelation.

Durbin-Watson test

The most frequently used statistical test for detecting autocorrelation is the Durbin-Watson test (Durbin and Watson, 1950, 1951, 1971; Greene, 2018) which is defined based on the ordinary least squares residuals.

If u_t is the residual associated with the observation at time t, then the Durbin-Watson d-statistic is

$$d = \frac{\sum_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n}\hat{u}^2{}_t} \quad (1)$$

where n is the number of observations.

The value of Durbin Watson d-statistic in Equation (1) always lies between 0 and 4. Harvey (1990) notes that, for large sample sizes, the Durbin-Watson test statistic d is approximately normally distributed with mean = 2 and variance = $4/n$. Durbin-Watson d-statistic is not applicable when lagged dependent variables are included in the explanatory variables.

Breusch–Godfrey test

Breusch (1978) and Godfrey (1978) (see also Asteriou and Hall, 2017) have developed a test of autocorrelation that is more flexible, covering autocorrelation of higher orders and applicable whether or not the regressors include lags of the dependent variable. Using simple regression model,

$$Y_t = \beta_1 + \beta_2 X_t + u_t. \quad (2)$$

The simple regression model Equation (2) is first fitted by ordinary least squares to obtain a set of sample residuals $\hat{u}_t$.

Breusch and Godfrey proved that, if the following auxiliary regression model is fitted

$$\hat{u}_t = \alpha_1 + \alpha_2 X_t + \hat{\rho}_1\hat{u}_{t-1} + \hat{\rho}_2\hat{u}_{t-2} + \cdots + \hat{\rho}_p\hat{u}_{t-p} + \varepsilon_t \quad (3)$$

and obtain R^2 from this auxiliary regression Equation (3), then, asymptotically, it can be shown that

$$(n-p)R^2 \sim \chi_p^2$$

Box–Pierce test

Box and Pierce (1970) (see also Greene, 2018) introduced the portmanteau statistic

$$Q = n\sum_{k=1}^{h} r_k^2 \quad (4)$$

where n is the sample size, r_k is the sample autocorrelation at lag k, and h is the number of lags being tested. Under $H_0$of no autocorrelations the statistic Q in Equation (4) follows a$\chi^2_{(h)}$. Box–Pierce test Q is not valid when lagged dependent variables exist in the regressor.

Ljung–Box test

Another test for assessing autocorrelation is the Ljung and Box (1978) (see also Verbeek, 2017). Ljung and Box (1978) modified Box and Pierce (1970) test statistic by

$$Q = n(n+2)\sum_{k=1}^{h} \frac{r_k^2}{n-k} \quad (5)$$

where n is the sample size, r_k is the sample autocorrelation at lag k, and h is the number of lags being tested. UnderH_0 of no autocorrelations the statistic Q follows a $\chi^2_{(h)}$. Hyndman and Athanasopoulos (2013) suggest using $h = min(10,\ n/5)$for Equation (5). Ljung-Box test Q is not valid when lagged dependent variables exist in the regressor.

Runs test

Runs test of randomness (Wald and Wolfowitz, 1943; see also Gujarati and Porter, 2009) is a statistical test that is used to know the randomness in data. Runs test of randomness is an alternative test to test autocorrelation in the residuals (Geary, 1970). If there is no autocorrelation, then the residuals are distributed randomly. A run is defined as a series of consecutive positive (or negative) values. Let n = total number of observations, n_1= number of positive residuals, n_2 = number of negative residuals, $n = n_1 + n_2$, and R = number of runs. Asuming that n_1 > 10 and n_2 > 10, then under the null hypothesis that the successive residuals are random, the number of runs is asymptotically normally distributed.

The test statistic is

$$z = \frac{R - E(R)}{s_R}$$

with mean

$$E(R) = \frac{2n_1n_2}{n} + 1$$

and variance

$$s_R^2 = \frac{2n_1n_2(2n_1n_2 - n)}{n^2(n+1)}.$$

3. Monte Carlo method

Power comparisons of tests for autocorrelation are made by using Monte Carlo method for simulation. We use *R* for Windows (*R* Core Team, 2019) for doing the simulation. We carried out simulations for six different sample sizes n = 15, 30, 60, 100, 150, 200. The number of simulation is 10 000 and the level of significance α = 0.05.

The model equations used in the simulation are model without lagged dependent variable with regressor $x_i = i$:

$$y_i = \beta_1 + \beta_2 i + u_i$$

and model with lagged dependent variable with regressor $x_i = y_{i-1}$:

$$y_i = \beta_1 + \beta_2 y_{i-1} + u_i$$

with the regression coefficients in both models $\beta_1 = 0.4, \beta_2 = -0.7$, and $u_i$is a stationary autoregressive AR(1) series, derived from standard normal innovations for autocorrelation ρ = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99.

We compute the power of the test as the proportion of times we correctly reject the null hypothesis.

The following are the simulation steps:

1. Generate the sample data for the model without lagged dependent variable and the model with a lagged dependent variable.
2. Perform the five autocorrelation tests and calculate their p-values.
3. Repeat steps 1 – 2, 10 000 times.
4. Calculate the power of the test as the proportion of times we correctly reject the null hypothesis.
5. Repeat steps 1 – 4 for $\rho = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99$
6. Repeat steps 1 – 5 for $n = 15, 30, 60, 100, 150, 200$.

4. Results and Conclusion

Tables 2 and 3 respectively report the powers of the five autocorrelation tests for model without lagged dependent variabel and model with lagged dependent variable.

The difference of the power of the tests becomes more apparent when the comparison is carried out graphically. Figures 1 and 2 respectively present the simulated power curves for Durbin-Watson, Breusch-Godfrey, Box–Pierce, Ljung-Box, and Runs tests for autocorrelation with $\rho = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99$ and six different sample sizes $n = 15, 30, 60, 100, 150, 200$. based on the results of Tables 2 and 3. The vertical axis of the figures measures the power of the five tests, and the horizontal axis represents the value of the autocorrelationρ.

Examining the results in Figure 1 and Table 2 reveal that the Durbin-Watson test performs better in the model without lagged dependent variable, although the advantage over the Breusch-Godfrey, Box-Pierce, Ljung Box, and Runs tests reduce with increasing autocorrelation ρ and sample size n.

For the model with lagged dependent variable, Figure 2 and Table 3 reveal that the Breusch-Godfrey test is generally superior to the other tests. The Durbin-Watson test, Box-Pierce, Ljung Box, and Runs tests have very low power except for very high correlations. Note that the Durbin-Watson, Box–Pierce, Ljung Box tests are not valid when lagged dependent variables exist in the regressor.

In conclusion, the Durbin-Watson test performs better in the regression model without lagged dependent variable, although the advantage over the other tests reduce with increasing autocorrelation and sample sizes. For the model with lagged dependent variable, the Breusch-Godfrey test is generally superior to the other tests.

Table 2: The powers of Durbin-Watson, Breusch-Godfrey, Box–Pierce, Ljung-Box, and Runs-test tests for autocorrelation with ρ = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99 and sample sizes n = 15, 30, 60, 100, 150, 200 (without lagged dependent variable)

		Autocorrelation						
	Samples(n)	0	0.2	0.4	0.6	0.8	0.9	0.99
Durbin-Watson test	15	0.054	0.095	0.224	0.398	0.564	0.628	0.655
	30	0.050	0.163	0.485	0.812	0.951	0.972	0.983
	60	0.052	0.313	0.834	0.989	1.000	1.000	1.000
	100	0.049	0.495	0.968	1.000	1.000	1.000	1.000
	150	0.049	0.659	0.997	1.000	1.000	1.000	1.000
	200	0.049	0.788	1.000	1.000	1.000	1.000	1.000
Breusch-Godfrey test	15	0.062	0.032	0.062	0.151	0.287	0.339	0.364
	30	0.052	0.078	0.322	0.685	0.895	0.939	0.958
	60	0.055	0.212	0.752	0.980	1.000	1.000	1.000
	100	0.050	0.406	0.950	1.000	1.000	1.000	1.000
	150	0.049	0.596	0.996	1.000	1.000	1.000	1.000
	200	0.051	0.745	1.000	1.000	1.000	1.000	1.000
Box-Pierce test	15	0.050	0.023	0.048	0.117	0.238	0.281	0.302
	30	0.048	0.072	0.307	0.668	0.885	0.931	0.953
	60	0.052	0.206	0.748	0.979	1.000	1.000	1.000
	100	0.049	0.402	0.950	1.000	1.000	1.000	1.000
	150	0.048	0.595	0.996	1.000	1.000	1.000	1.000
	200	0.051	0.744	1.000	1.000	1.000	1.000	1.000
Ljung-Box test	15	0.083	0.043	0.081	0.177	0.316	0.363	0.386
	30	0.063	0.088	0.344	0.700	0.901	0.941	0.960
	60	0.059	0.223	0.762	0.980	1.000	1.000	1.000
	100	0.052	0.414	0.953	1.000	1.000	1.000	1.000
	150	0.051	0.602	0.996	1.000	1.000	1.000	1.000
	200	0.053	0.748	1.000	1.000	1.000	1.000	1.000
Runs test	15	0.022	0.031	0.066	0.127	0.213	0.249	0.265
	30	0.036	0.063	0.179	0.392	0.635	0.732	0.775
	60	0.053	0.135	0.447	0.799	0.968	0.989	0.996
	100	0.052	0.234	0.703	0.968	1.000	1.000	1.000
	150	0.060	0.330	0.881	0.997	1.000	1.000	1.000
	200	0.053	0.420	0.953	1.000	1.000	1.000	1.000

Table 3: The powers of Durbin-Watson , Breusch-Godfrey, Box–Pierce, Ljung-Box, and Runs tests for autocorrelation with ρ = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99 and sample sizes n = 15, 30, 60, 100, 150, 200 (with lagged dependent variable).

		Autocorrelation						
	Samples(n)	0	0.2	0.4	0.6	0.8	0.9	0.99
Durbin-Watson test	15	0.012	0.004	0.004	0.004	0.003	0.003	0.011
	30	0.008	0.004	0.005	0.005	0.007	0.022	0.177
	60	0.007	0.009	0.019	0.014	0.030	0.269	0.719
	100	0.006	0.019	0.051	0.027	0.101	0.630	0.960
	150	0.006	0.033	0.122	0.049	0.195	0.858	0.997
	200	0.005	0.063	0.225	0.077	0.292	0.950	1.000
Breusch-Godfrey test	15	0.061	0.040	0.045	0.078	0.115	0.154	0.247
	30	0.054	0.055	0.146	0.307	0.485	0.586	0.719
	60	0.054	0.113	0.414	0.733	0.878	0.944	0.978
	100	0.050	0.204	0.693	0.928	0.967	0.993	0.999
	150	0.050	0.311	0.892	0.977	0.986	0.999	1.000
	200	0.051	0.424	0.967	0.990	0.991	1.000	1.000
Box-Pierce test	15	0.004	0.001	0.000	0.000	0.000	0.001	0.018
	30	0.005	0.001	0.001	0.002	0.005	0.034	0.230
	60	0.005	0.005	0.012	0.007	0.038	0.312	0.756
	100	0.006	0.014	0.038	0.020	0.120	0.660	0.966
	150	0.005	0.027	0.104	0.040	0.218	0.872	0.998
	200	0.005	0.053	0.200	0.064	0.314	0.956	1.000
Ljung-Box test	15	0.010	0.003	0.001	0.001	0.001	0.003	0.033
	30	0.007	0.003	0.002	0.003	0.007	0.044	0.256
	60	0.006	0.007	0.013	0.009	0.043	0.328	0.767
	100	0.006	0.015	0.043	0.022	0.127	0.673	0.967
	150	0.005	0.029	0.110	0.042	0.224	0.876	0.998
	200	0.005	0.055	0.206	0.067	0.319	0.957	1.000
Runs test	15	0.017	0.025	0.035	0.036	0.037	0.032	0.031
	30	0.016	0.024	0.033	0.037	0.041	0.055	0.128
	60	0.029	0.039	0.057	0.057	0.068	0.171	0.457
	100	0.030	0.057	0.093	0.080	0.113	0.380	0.803
	150	0.030	0.074	0.138	0.097	0.159	0.578	0.952
	200	0.029	0.086	0.162	0.108	0.194	0.707	0.992

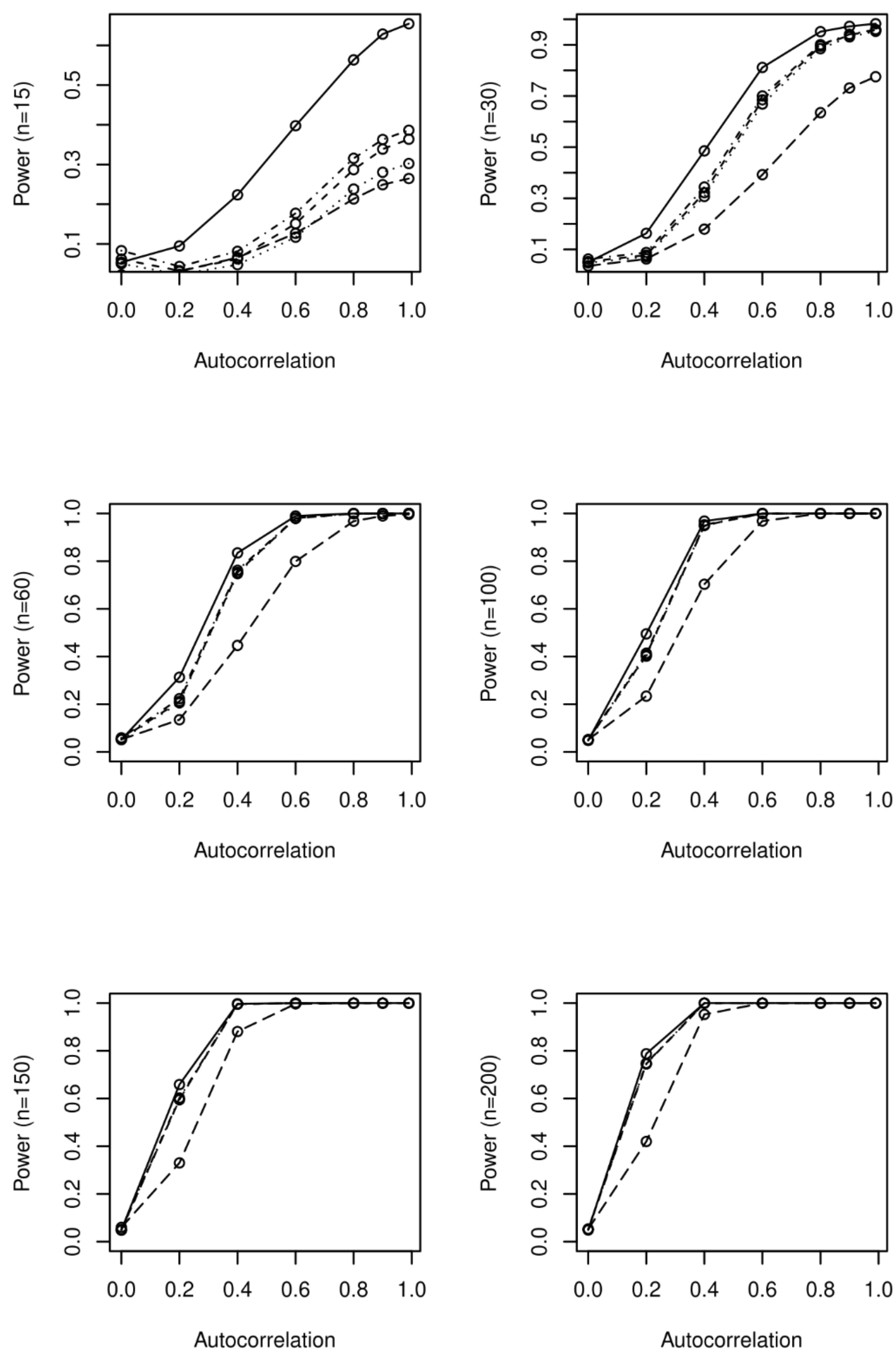

Figure 1: Simulated power curves for Durbin-Watson (solid), Breusch-Godfrey (dashed), Box–Pierce (dotted), Ljung-Box (dotdash), and Runs (longdash) tests for autocorrelation with $\rho = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99$ and sample sizes $n = 15, 30, 60, 100, 150, 200$ (without lagged dependent variable).

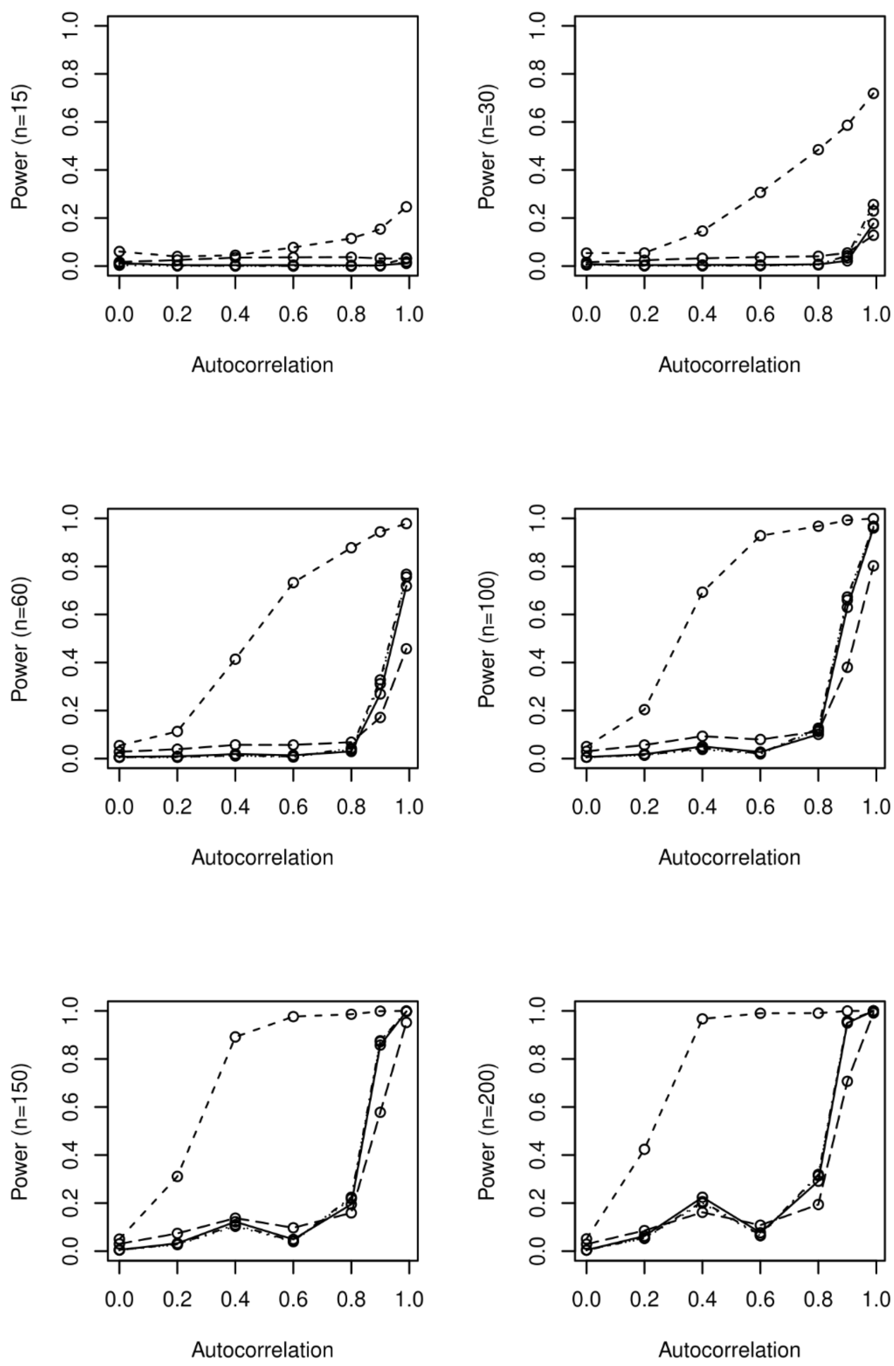

Figure 2: Simulated power curves for Durbin-Watson (solid), Breusch-Godfrey (dashed), Box–Pierce (dotted), Ljung-Box (dotdash), and Runs (longdash) tests for autocorrelation with ρ = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99 and six different sample sizes n = 15, 30, 60, 100, 150, 200 (with lagged dependent variable).

Acknowledgements

The author would like to thank the reviewers for their valuable comments and suggestions which led to improvements of this paper.

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Appendix 1. R code for Comparison of Five Autocorrelation Tests

The R code for comparison of the five autocorrelation tests is as follows

```
##################################################################
# R code  for Power Comparison of the Five Autocorrelation Tests  #
# using R-3.6.1 for Windows                                       #
##################################################################
library(lmtest)
library(car)
library(lawstat)

rm(list=ls(all=TRUE))
samples = c(15, 30, 60, 100, 150, 200)    # Sample sizes
autocorr = c(0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99)
coef =c(0.4, -0.7)
alpha <-0.05
sims <- 10000  #10000  #Number of simulations to conduct for each of the samples
  DW.powers <- matrix(nrow =length(samples), ncol = length(autocorr))
  BG.powers <- matrix(nrow =length(samples), ncol = length(autocorr))
  BP.powers <- matrix(nrow =length(samples), ncol = length(autocorr))
  LB.powers <- matrix(nrow =length(samples), ncol = length(autocorr))
  RT.powers <- matrix(nrow =length(samples), ncol = length(autocorr))

  DW.powers1 <- matrix(nrow =length(samples), ncol = length(autocorr))
  BG.powers1 <- matrix(nrow =length(samples), ncol = length(autocorr))
  BP.powers1 <- matrix(nrow =length(samples), ncol = length(autocorr))
  LB.powers1 <- matrix(nrow =length(samples), ncol = length(autocorr))
  RT.powers1 <- matrix(nrow =length(samples), ncol = length(autocorr))
rows <- 1
cols <- 1
set.seed(99999)
for(i in samples)
{
  for (j in autocorr)
   {
      pval <- matrix(nrow=sims, ncol=5)
      pval1 <- matrix(nrow=sims, ncol=5)

      #colnames(pval) <- c("DWtest", "BGtest", "Box-Pierce", "Ljung-Box", "Runs Test")

      for(l in 1:sims)
        {

          err <- as.vector(filter(rnorm(i), j,  method = "recursive"))

           # Model Non-Lagged DV
             x <- 1:i
             y <- coef[1] + coef[2] * x + err

           # Model Lagged DV
             y1 <- rep(NA, i)
             y1[1] <- coef[1] + err[1]
             for(k in 2:i )
                {
                   y1[k] <- coef[1] + coef[2] * y1[k-1] + err[k]
                }
            x1 <- c(0, y1[1:(i-1)])

          pval[l,1] <- dwtest(y ~ x,  alternative = "two.sided")$p.value
          pval[l,2] <- bgtest(y ~ x)$p.value
          pval[l,3] <- Box.test(lm(y ~ x)$residuals, lag= 1, type="Box-Pierce")$p.value
          pval[l,4] <- Box.test(lm(y ~ x)$residuals, lag= 1, type="Ljung-Box")$p.value
          pval[l,5] <- runs.test(lm(y ~ x)$residuals)$p.value

          pval1[l,1] <- dwtest(y1 ~ x1,  alternative = "two.sided")$p.value
          pval1[l,2] <- bgtest(y1 ~ x1)$p.value
          pval1[l,3] <- Box.test(lm(y1 ~ x1)$residuals, lag = 1, type="Box-Pierce")$p.value
          pval1[l,4] <- Box.test(lm(y1 ~ x1)$residuals, lag = 1, type="Ljung-Box")$p.value
          pval1[l,5] <- runs.test(lm(y1 ~ x1)$residuals)$p.value
```

```
        }
      DW.powers[rows,cols] <- mean(pval[,1] < alpha)
      BG.powers[rows,cols] <- mean(pval[,2] < alpha)
      BP.powers[rows,cols] <- mean(pval[,3] < alpha)
      LB.powers[rows,cols] <- mean(pval[,4] < alpha)
      RT.powers[rows,cols] <- mean(pval[,5] < alpha)

      DW.powers1[rows,cols]  <- mean(pval1[,1] < alpha)
      BG.powers1[rows,cols]  <- mean(pval1[,2] < alpha)
      BP.powers1[rows,cols]  <- mean(pval1[,3] < alpha)
      LB.powers1[rows,cols]  <- mean(pval1[,4] < alpha)
      RT.powers1[rows,cols]  <- mean(pval1[,5] < alpha)

      cols <- cols+1
   }
  rows <- rows + 1
  cols <- 1
}
coln <- c(15, 30, 60, 100, 150, 200, 15, 30, 60, 100, 150, 200,
15, 30, 60, 100, 150, 200, 15, 30, 60, 100, 150, 200, 15, 30, 60, 100, 150, 200)
powers <- rbind(DW.powers[1,], DW.powers[2,],DW.powers[3,], DW.powers[4,],
DW.powers[5,], DW.powers[6,], BG.powers[1,], BG.powers[2,], BG.powers[3,],
BG.powers[4,], BG.powers[5,], BG.powers[6,], BP.powers[1,], BP.powers[2,],
BP.powers[3,], BP.powers[4,], BP.powers[5,], BP.powers[6,], LB.powers[1,],
LB.powers[2,], LB.powers[3,],LB.powers[4,],LB.powers[5,],LB.powers[6,],
RT.powers[1,],RT.powers[2,],RT.powers[3,],RT.powers[4,],RT.powers[5,],
RT.powers[6,])
powers <- cbind(coln, powers)
colnames(powers) <- c('n', 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99)
rownames(powers) <- c("Durbin-Watson test"," "," "," "," "," ",
"Breusch-Godfrey test"," "," "," "," "," ", "Box-Pierce test",
" "," "," "," "," ", "Ljung-Box test", " "," "," "," "," ",
"Runs test", " "," "," "," "," ")

powers1 <- rbind(DW.powers1[1,], DW.powers1[2,],DW.powers1[3,],
DW.powers1[4,], DW.powers1[5,], DW.powers1[6,], BG.powers1[1,],
BG.powers1[2,], BG.powers1[3,], BG.powers1[4,], BG.powers1[5,],
BG.powers1[6,], BP.powers1[1,], BP.powers1[2,], BP.powers1[3,],
BP.powers1[4,], BP.powers1[5,], BP.powers1[6,],  LB.powers1[1,],
LB.powers1[2,], LB.powers1[3,],LB.powers1[4,],LB.powers1[5,],
LB.powers1[6,], RT.powers1[1,],RT.powers1[2,],RT.powers1[3,],
RT.powers1[4,],RT.powers1[5,],RT.powers1[6,])
powers1 <- cbind(coln, powers1)
colnames(powers1) <- c('n', 0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.99)
rownames(powers1) <- c("Durbin-Watson test"," "," "," "," "," ",
"Breusch-Godfrey test"," "," "," "," "," ", "Box-Pierce test",
" "," "," "," "," ", "Ljung-Box test", " "," "," "," "," ",
"Runs test", " "," "," "," "," ")

powers
powers1

win.graph()
par(mfrow=c(3,2))
plot(autocorr, DW.powers[1,], yaxt="n", xlab="Autocorrelation",
ylab="Power (n=15)", type = "o")
axis(side=2,at=seq(0,1,0.1))
lines(autocorr,BG.powers[1,], lty=2, type = "o")
lines(autocorr,BP.powers[1,], lty=3, type = "o")
lines(autocorr,LB.powers[1,], lty=4, type = "o")
lines(autocorr,RT.powers[1,], lty=5, type = "o")

plot(autocorr, DW.powers[2,], yaxt="n", xlab="Autocorrelation",
ylab="Power (n=30)", type = "o")
axis(side=2,at=seq(0,1,0.1))
lines(autocorr,BG.powers[2,], lty=2, type = "o")
lines(autocorr,BP.powers[2,], lty=3, type = "o")
lines(autocorr,LB.powers[2,], lty=4, type = "o")
lines(autocorr,RT.powers[2,], lty=5, type = "o")

plot(autocorr, DW.powers[3,], ylim=c(0, 1), xlab="Autocorrelation",
```

```
ylab="Power (n=60)", type = "o")
lines(autocorr,BG.powers[3,], lty=2, type = "o")
lines(autocorr,BP.powers[3,], lty=3, type = "o")
lines(autocorr,LB.powers[3,], lty=4, type = "o")
lines(autocorr,RT.powers[3,], lty=5, type = "o")

plot(autocorr, DW.powers[4,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=100)", type = "o")
lines(autocorr,BG.powers[4,], lty=2, type = "o")
lines(autocorr,BP.powers[4,], lty=3, type = "o")
lines(autocorr,LB.powers[4,], lty=4, type = "o")
lines(autocorr,RT.powers[4,], lty=5, type = "o")

plot(autocorr, DW.powers[5,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=150)", type = "o")
lines(autocorr,BG.powers[5,], lty=2, type = "o")
lines(autocorr,BP.powers[5,], lty=3, type = "o")
lines(autocorr,LB.powers[5,], lty=4, type = "o")
lines(autocorr,RT.powers[5,], lty=5, type = "o")

plot(autocorr, DW.powers[6,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=200)", type = "o")
lines(autocorr,BG.powers[6,], lty=2, type = "o")
lines(autocorr,BG.powers[6,], lty=3, type = "o")
lines(autocorr,LB.powers[6,], lty=4, type = "o")
lines(autocorr,RT.powers[6,], lty=5, type = "o")

win.graph()
par(mfrow=c(3,2))
plot(autocorr, DW.powers1[1,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=15) - Lagged DV", type = "o")
lines(autocorr,BG.powers1[1,], lty=2, type = "o")
lines(autocorr,BP.powers1[1,], lty=3, type = "o")
lines(autocorr,LB.powers1[1,], lty=4, type = "o")
lines(autocorr,RT.powers1[1,], lty=5, type = "o")

plot(autocorr, DW.powers1[2,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=30) - Lagged DV",  type = "o")
lines(autocorr,BG.powers1[2,], lty=2, type = "o")
lines(autocorr,BP.powers1[2,], lty=3, type = "o")
lines(autocorr,LB.powers1[2,], lty=4, type = "o")
lines(autocorr,RT.powers1[2,], lty=5, type = "o")

plot(autocorr, DW.powers1[3,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=60) - Lagged DV", type = "o")
lines(autocorr,BG.powers1[3,], lty=2, type = "o")
lines(autocorr,BP.powers1[3,], lty=3, type = "o")
lines(autocorr,LB.powers1[3,], lty=4, type = "o")
lines(autocorr,RT.powers1[3,], lty=5, type = "o")

plot(autocorr, DW.powers1[4,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=100) - Lagged DV", type = "o")
lines(autocorr,BG.powers1[4,], lty=2, type = "o")
lines(autocorr,BP.powers1[4,], lty=3, type = "o")
lines(autocorr,LB.powers1[4,], lty=4, type = "o")
lines(autocorr,RT.powers1[4,], lty=5, type = "o")

plot(autocorr, DW.powers1[5,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=150) - Lagged DV", type = "o")
lines(autocorr,BG.powers1[5,], lty=2, type = "o")
lines(autocorr,BP.powers1[5,], lty=3, type = "o")
lines(autocorr,LB.powers1[5,], lty=4, type = "o")
lines(autocorr,RT.powers1[5,], lty=5, type = "o")

plot(autocorr, DW.powers1[6,], ylim=c(0, 1), xlab="Autocorrelation",
ylab="Power (n=200) - Lagged DV", type = "o")
lines(autocorr,BG.powers1[6,], lty=2, type = "o")
lines(autocorr,BP.powers1[6,], lty=3, type = "o")
lines(autocorr,LB.powers1[6,], lty=4, type = "o")
lines(autocorr,RT.powers1[6,], lty=5, type = "o")
```