An Alternative Procedure for Estimating the Population Mean in Simple Random Sampling

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Abstract

This paper deals with the problem of estimating the finite population mean using auxiliary information in simple random sampling. Firstly we have suggested a correction to the mean squared error of the estimator proposed by Gupta and Shabbir (2008). Later we have proposed a ratio type estimator and its properties are studied in simple random sampling. Numerically we have shown that the proposed class of estimators is more efficient than different known estimators including Gupta and Shabbir (2008) estimator.

Keywords: Study variate, Auxiliary variate, Finite population mean, Bias, Mean squared error.

1.Introduction

Consider a finite population $U=(U_1,U_2,...,U_N)$ consisting of N units. Let y and x be the auxiliary variables with population means $\bar{Y}$ and $\bar{X}$ respectively. Let a sample of size n be drawn from the population U using simple random sampling without replacement (SRSWOR) scheme. Let $\bar{y}$ and $\bar{x}$ be the sample means of y and x respectively. For estimating the population mean $\bar{Y}$, the usual unbiased estimator, classical ratio and product estimators are respectively defined by

$$t_0=\bar{y}, \tag{1}$$

$$t_R=\bar{y}\frac{\bar{X}}{\bar{x}}, \tag{2}$$

$$t_P=\bar{y}\frac{\bar{x}}{\bar{X}}. \tag{3}$$

Here it is assumed that the population mean $\bar{X}$ of the auxiliary variable x is known. The classical ratio and product estimators are considered to be practicable in many situations, but they have the limitations of having at the most the same efficiency as that of linear regression estimator. Regression estimator, in spite of its lesser practicability, seems to be holding a unique position due to its sound theoretical basis. Some authors including, Jhajj et al. (2006), Kadilar and Cingi (2004, 2006a, b, c), Singh and Espejo (2003, 2007), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh and Agnihotri (2008), Singh (1986) and the references cited therein, have attempted to formulate the modified estimators in order to provide better alternatives.

Using the transformation

$$z_i = \eta x_i + \lambda, \text{ i = 1, 2, …, N.} \tag{4}$$

on the auxiliary variable x, Gupta and Shabbir (2008) suggested the following ratio type estimator

$$t_1 = [w_1\bar{y} + w_2(\bar{X} - \bar{x})](\frac{\eta\bar{X} + \lambda}{\eta\bar{x} + \lambda}) \tag{5}$$

for the population mean $\bar{Y}$, where w_1 and w_2 are weights whose values are to be determined such that mean squared error of the estimator t_1 is minimum, and $\eta(\neq 0)$ and λ are either constants or functions of the known parameters such as standard deviation S_x, variance S_x^2, moment ratios $\beta_1(x)$, $\beta_2(x)$, coefficient of variation C_x and correlation coefficient ρ_{yx} between y and x etc.

The variance/MSE of $\bar{y}$ under SRSWOR is given by

$$MSE(t_0 = \bar{y}) = \frac{(1-f)}{n}\bar{Y}^2C_y^2, \tag{6}$$

where $f = n/N$, $C_y = S_y/\bar{Y}$ and $S_y^2 = \sum_{i=1}^{N}(y_i - \bar{Y})^2/(N-1)$.

To the first degree of approximation, the MSEs of classical ratio t_R and product t_P are respectively given by

$$MSE(t_R) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + C_x^2(1-2k)], \tag{7}$$

$$MSE(t_P) = \frac{(1-f)}{n}\bar{Y}^2[C_y^2 + C_x^2(1+2k)], \tag{8}$$

where

$C_x = S_x/\bar{X}, k = \rho_{yx}(C_y/C_x), \rho_{yx} = (S_{yx})/(S_yS_x), S_x^2 = \sum_{i=1}^{N}(x_i - \bar{X})^2/(N-1)$ and $S_{yx} = \sum_{i=1}^{N}(x_i - \bar{X})(y_i - \bar{Y})/(N-1)$.

To the first degree of approximation, the MSE of the estimator t_1 is obtained by Gupta and Shabbir (2008) as

$$MSE(t_1) \cong (w_1 - 1)^2 \bar{Y}^2 + \frac{(1-f)}{n}[w_1^2 \bar{Y}^2 \{C_y^2 + \tau C_x^2(\tau - 2k)\}$$
$$+ w_2^2 \bar{X}^2 C_x^2 - 2w_1 w_2 R \bar{X}^2 C_x^2 (k - \tau)], \quad (9)$$

where $R = \bar{Y}/\bar{X}$ and $\tau = \eta\bar{X}/(\eta\bar{X} + \lambda)$.

It is to be noted that the MSE expression obtained by Gupta and Shabbir (2008) is not correct and thus the entire study carried out in the paper by Gupta and Shabbir (2008) are erroneous except concerning the bias. Keeping this in view we have first obtained the correct MSE expression of the estimator t_1. Later we propose a general class of estimators for population mean $\bar{Y}$ along with its properties. An empirical study is carried out to show the performance of the suggested estimator over others.

2. MSE expression of Gupta and Shabbir (2008) estimator t_1

To obtain the MSE of t_1 we write

$$\bar{y} = \bar{Y}(1 + e_0), \ \bar{x} = \bar{X}(1 + e_1),$$

such that

$$E(e_0) = E(e_1) = 0$$

and

$$\left.\begin{aligned} E(e_0^2) &= \frac{(1-f)}{n} C_y^2, \\ E(e_1^2) &= \frac{(1-f)}{n} C_x^2, \\ E(e_0 e_1) &= \frac{(1-f)}{n} \rho_{yx} C_y C_x = \frac{(1-f)}{n} k C_x^2. \end{aligned}\right\} \quad (10)$$

Expressing t_1 in terms of e's we have

$$t_1 = [w_1 \bar{Y}(1 + e_0) - w_2 \bar{X} e_1](1 + \tau e_1)^{-1}. \quad (11)$$

We assume that $|\tau e_1| < 1$ so that the term $(1 + \tau e_1)^{-1}$ is expandable. Expanding the right hand side of (11) we have

$$t_1 = [w_1 \bar{Y}(1 + e_0) - w_2 \bar{X} e_1][1 - \tau e_1 + \tau^2 e_1^2 - ...]$$
$$= [w_1 \bar{Y}(1 + e_0) - w_2 \bar{X} e_1 - w_1 \tau \bar{Y}(e_1 + e_0 e_1) + w_2 \tau \bar{X} e_1^2$$
$$+ w_1 \bar{Y}(\tau^2 e_1^2 + \tau^2 e_0 e_1^2) - ...].$$

Neglecting terms of e's having power greater than two we have

$$t_1 \cong [w_1\overline{Y}(1+e_0) - w_2\overline{X}e_1 - w_1\tau\overline{Y}(e_1 + e_0e_1) + w_2\tau\overline{X}e_1^2 + w_1\overline{Y}\tau^2e_1^2].$$

Subtracting $\overline{Y}$ from both sides of the above expression, we have

$$(t_1 - \overline{Y}) \cong [w_1\overline{Y}\{1+e_0 - \tau(e_1 + e_0e_1 - \tau e_1^2)\} - w_2\overline{X}(e_1 - \tau e_1^2) - \overline{Y}]. \qquad (12)$$

Squaring both sides of (12) and neglecting terms of e's having power greater than two we have

$$(t_1 - \overline{Y})^2 \cong [\overline{Y}^2 + w_1^2\overline{Y}^2\{1+e_0^2 + \tau^2e_1^2 + 2e_0 - 2\tau(e_1 + e_0e_1 - \tau e_1^2) - 2\tau e_0e_1\}$$
$$+ w_2^2\overline{X}^2e_1^2 - 2w_1\overline{Y}^2\{1+e_0 - \tau(e_1 + e_0e_1 - \tau e_1^2)\}$$
$$+ 2w_2\overline{Y}\,\overline{X}(e_1 - \tau e_1^2) - 2w_1w_2\overline{Y}\,\overline{X}\{(e_1 + e_0e_1 - \tau e_1^2) - \tau e_1^2\}]$$

or

$$(t_1 - \overline{Y})^2 = [\overline{Y}^2 + w_1^2\overline{Y}^2(1+2e_0 - 2\tau e_1 + e_0^2 + 3\tau^2e_1^2 - 4\tau e_0e_1) + w_2^2\overline{X}^2e_1^2$$
$$- 2w_1w_2\overline{Y}\,\overline{X}(e_1 + e_0e_1 - 2\tau e_1^2) - 2w_1\overline{Y}^2\{1+e_0$$
$$- \tau(e_1 + e_0e_1 - \tau e_1^2)\} + 2w_2\overline{Y}\,\overline{X}(e_1 - \tau e_1^2)]. \qquad (13)$$

Taking expectation of both sides of (13) we get the MSE of t_1 to the first degree of approximation as

$$MSE(t_1) = [\overline{Y}^2 + w_1^2\overline{Y}^2\alpha_1 + w_2^2\overline{X}^2\alpha_2 - 2w_1w_2\overline{Y}\,\overline{X}\alpha_3 - 2w_1\overline{Y}^2\alpha_4 - 2w_2\overline{Y}\,\overline{X}\alpha_5], \quad (14)$$

where

$$\alpha_1 = [1 + \frac{(1-f)}{n}\{C_y^2 + \tau C_x^2(3\tau - 4k)\}],$$

$$\alpha_2 = \frac{(1-f)}{n}C_x^2,$$

$$\alpha_3 = \frac{(1-f)}{n}C_x^2(k - 2\tau),$$

$$\alpha_4 = [1 - \frac{(1-f)}{n}\tau C_x^2(k-\tau)],$$

$$\alpha_5 = \frac{(1-f)}{n}\tau C_x^2.$$

Now setting $\frac{\partial MSE(t_1)}{\partial w_i} = 0$, (i =1, 2), we have

$$\begin{bmatrix} \overline{Y}^2\alpha_1 & -\alpha_3\overline{Y}\,\overline{X} \\ -\alpha_3\overline{Y}\,\overline{X} & \alpha_2\overline{X}^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} \overline{Y}^2\alpha_4 \\ \overline{X}\,\overline{Y}\alpha_5 \end{bmatrix}. \qquad (15)$$

Solving (15) we get the optimum values of w_1 and w_2 as

$$\left.\begin{aligned} w_1 &= \frac{(\alpha_2\alpha_4 + \alpha_3\alpha_5)}{(\alpha_1\alpha_2 - \alpha_3^2)} = w_1^* \\ w_2 &= \frac{R(\alpha_1\alpha_5 + \alpha_3\alpha_4)}{(\alpha_1\alpha_2 - \alpha_3^2)} = w_2^* \end{aligned}\right\}. \quad (16)$$

It is assumed that the unknown parameters involved in α_i's (i = 1, 2, 3, 4, 5) and R are easily estimable from the preliminary data as in Singh and Singh (1984), Tracy and Singh (1997), Tracy et al. (1998), Upadhyaya and Singh (1999), Singh and Vishwakarma (2006), Singh and Espejo (2007), Kadilar and Cingi (2006a) and Singh et al. (2008).

Substitution of (16) in (14) yields the minimum MSE of t_1 as

$$MSE_{\min}(t_1) = \bar{Y}^2[1 - \frac{(\alpha_2\alpha_4^2 + 2\alpha_3\alpha_4\alpha_5 + \alpha_1\alpha_5^2)}{(\alpha_1\alpha_2 - \alpha_3^2)}]. \quad (17)$$

Thus we state the following theorem.

Theorem 1: To the first degree of approximation,

$$MSE(t_1) \geq \bar{Y}^2[1 - \frac{(\alpha_2\alpha_4^2 + 2\alpha_3\alpha_4\alpha_5 + \alpha_1\alpha_5^2)}{(\alpha_1\alpha_2 - \alpha_3^2)}]$$

with equality holding if

$w_1 = w_1^*$ and $w_2 = w_2^*$.

Thus the statement "the specific values of η and λ used in defining various transformations of the auxiliary variable x play no role" given by Gupta and Shabbir (2008, p. 563) is not correct. Expression (17) clearly indicates that there is role of the specific values of η and λ as minimum MSE of $\mathbf{t_1}$ depends on (η, λ). Thus we provide list of some estimators in Table 1 which are members of the class of estimators $\mathbf{t_1}$.

2.1 Efficiency comparisons

Kadilar and Cingi (2004) proposed the following class of estimators for population mean $\bar{Y}$ as

$$t_{KC_i} = [\bar{y} + b(\bar{X} - \bar{x})]a_i, \text{ i =1, 2, 3, 4, 5.} \quad (18)$$

where b is the sample regression coefficient of y on x and a_i's are defined as

$$a_1 = \frac{\bar{X}}{\bar{x}},\ a_2 = \frac{\bar{X} + C_x}{\bar{x} + C_x},\ a_3 = \frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)},\ a_4 = \frac{\bar{X}\beta_2(x) + C_x}{\bar{x}\beta_2(x) + C_x},\ a_5 = \frac{\bar{X}C_x + \beta_2(x)}{\bar{x}C_x + \beta_2(x)}.$$

To the first degree of approximation, the MSE of t_{KC_i}, (i =1, 2, 3, 4, 5) are given by

$$MSE(t_{KC_i}) = \frac{(1-f)}{n}\overline{Y}^2[a_i^* C_x^2 + C_y^2(1-\rho_{yx}^2)], \tag{19}$$

where

$$a_1^* = 1,\ a_2^* = \overline{X}/(\overline{X}+C_x),\ a_3^* = \overline{X}/(\overline{X}+\beta_2(x)),\ a_4^* = \overline{X}\beta_2(x)/(\overline{X}\beta_2(x)+C_x)$$
$$a_5^* = \overline{X}C_x/(\overline{X}C_x+\beta_2(x)).$$

Kadilar and Cingi (2006c) have suggested another class of estimators for $\overline{Y}$ as

$$t_{KC_i}^* = [\overline{y}+b(\overline{X}-\overline{x})]b_i,\ i=1, 2, 3, 4, 5. \tag{20}$$

where

$$b_1 = \frac{\overline{X}+\rho_{yx}}{\overline{x}+\rho_{yx}},\ b_2 = \frac{\overline{X}C_x+\rho_{yx}}{\overline{x}C_x+\rho_{yx}},\ b_3 = \frac{\overline{X}\rho_{yx}+C_x}{\overline{x}\rho_{yx}+C_x},\ b_4 = \frac{\overline{X}\beta_2(x)+\rho_{yx}}{\overline{x}\beta_2(x)+\rho_{yx}},$$
$$b_5 = \frac{\overline{X}\rho_{yx}+\beta_2(x)}{\overline{x}\rho_{yx}+\beta_2(x)}.$$

To the first degree of approximation, the MSE of $t_{KC_i}^*$ (i = 1, 2, 3, 4, 5) are given by

$$MSE(t_{KC_i}^*) = \frac{(1-f)}{n}\overline{Y}^2[b_i^* C_x^2 + C_y^2(1-\rho_{yx}^2)],\ i=1, 2, 3, 4, 5. \tag{21}$$

where

$$b_1^* = \frac{\overline{X}}{\overline{X}+\rho_{yx}},\ b_2^* = \frac{\overline{X}C_x}{\overline{X}C_x+\rho_{yx}},\ b_3^* = \frac{\overline{X}\rho_{yx}}{\overline{X}\rho_{yx}+C_x},\ b_4^* = \frac{\overline{X}\beta_2(x)}{\overline{x}\beta_2(x)+\rho_{yx}},$$
$$b_5^* = \frac{\overline{X}\rho_{yx}}{\overline{X}\rho_{yx}+\beta_2(x)}.$$

Kadilar and Cingi (2006a) have further considered the following class of estimators for population mean $\overline{Y}$ as

$$t_{KC_j}^{**} = k_1[\overline{y}+b(\overline{X}-\overline{x})]a_1 + k_2[\overline{y}+b(\overline{X}-\overline{x})]a_j,\ j=2,3,4,5. \tag{22}$$

where k_1 and k_2 are weight such that $k_1+k_2=1$.

To the first degree of approximation the common minimum MSE of $t_{KC_j}^{**}$, (j =1,2,3,4,5) is given by

$$MSE_{min}(t_{KC_j}^{**}) = \frac{(1-f)}{n}\overline{Y}^2 C_y^2(1-\rho_{yx}^2),\ (j=2,3,4,5) \tag{23}$$

$$= \overline{Y}^2[1+\alpha_1 - 2\alpha_4 - \frac{(\alpha_3+\alpha_5)^2}{\alpha_2}] \quad (24)$$

$$= MSE(t_{Reg}),$$

where $t_{Reg} = \overline{y} + b(\overline{X} - \overline{x})$ is the usual linear regression estimator of the population mean $\overline{Y}$.

From (6), (17), (19), (21), (23) and (24) we have

$$MSE(\overline{y}) - \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} = \frac{(1-f)}{n}\overline{Y}^2 C_y^2 \rho_{yx}^2 > 0,$$
$$(j = 2,3,4,5) \quad (25)$$

$$MSE(t_{KC_i}) - \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} = \frac{(1-f)}{n}\overline{Y}^2 C_x^2 a_i^* > 0,$$
$$(i = 1,2,3,4,5,\ j = 2,3,4,5) \quad (26)$$

$$MSE(t^*_{KC_i}) - \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} = \frac{(1-f)}{n}\overline{Y}^2 C_x^2 b_i^* > 0,$$
$$(i = 1,2,3,4,5\ ,\ j = 2,3,4,5) \quad (27)$$

$$\{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} - MSE_{min}(t_1)$$
$$= \frac{\overline{Y}^2[\alpha_2(\alpha_1 - \alpha_4) - \alpha_3(\alpha_3+\alpha_5)]^2}{\alpha_2(\alpha_1\alpha_2 - \alpha_3^2)} > 0,\ (j = 2,3,4,5) \quad (28)$$

From (25), (26), (27) and (28) we have the following inequalities,

$$\{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j}) < MSE(\overline{y}),\ (j = 2,3,4,5) \quad (29)$$

$$\{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j}) < MSE(t_{KC_i}),\ (i = 1,2,3,4,5\ ,\ j = 2,3,4,5) \quad (30)$$

$$\{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j}) < MSE(t^*_{KC_i}),\ (i = 1,2,3,4,5\ ,\ j = 2,3,4,5) \quad (31)$$

$$MSE_{min}(t_1) < \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\},\ (j = 2,3,4,5) \quad (32)$$

From (29), (30), (31) and (32) we have

$$MSE_{min}(t_1) < \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} < MSE(\overline{y}),\ (j = 2,3,4,5) \quad (33)$$

$$MSE_{min}(t_1) < \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} < MSE(t_{KC_i}),$$
$$(i = 1,2,3,4,5\ ,\ j = 2,3,4,5) \quad (34)$$

$$MSE_{min}(t_1) < \{MSE(t_{Reg}) = MSE_{min}(t^{**}_{KC_j})\} < MSE(t^*_{KC_i}),$$
$$(i = 1,2,3,4,5\ ,\ j = 2.3.4,5) \quad (35)$$

It follows from (33), (34) and (35) that the class of estimators t_1 due to Gupta and Shabbir (2008) is better than usual unbiased estimator $\bar{y}$, usual linear regression estimator t_{Reg} [and hence the usual ratio (t_R) and product (t_P) estimators] and the estimators due to Kadilar and Cingi (2004, 2006a, c).

3.A general class of estimators

We define a class of estimators for population mean $\bar{Y}$ as

$$t_2 = \psi_1 \bar{y}\left(\frac{\eta\bar{X}+\lambda}{\eta\bar{x}+\lambda}\right) + \psi_2(\bar{X}-\bar{x})\left(\frac{\eta\bar{X}+\lambda}{\eta\bar{x}+\lambda}\right)^2, \tag{36}$$

where ψ_1 and ψ_2 are suitably chosen constants such that MSE of t_2 is minimum and (η,λ) are same as defined earlier. A large number of estimators can be generated from the suggested estimator t_2 for suitable values of $(\psi_1,\psi_2,\eta,\lambda)$.

Expressing t_2 in terms of e's we have

$$t_2 = \psi_1\bar{Y}(1+e_0)(1+\tau e_1)^{-1} - \psi_2\bar{X}e_1(1+\tau e_1)^{-2}. \tag{37}$$

We assume that $\left|\tau e_1\right| < 1$, so that $(1+\tau e_1)^{-1}$ and $(1+\tau e_1)^{-2}$ are expandable. Expanding the right hand side of (37), multiplying out and neglecting terms of e's having power greater than two we have

$$t_2 \cong \psi_1\bar{Y}[1+e_0-\tau e_1-\tau e_0 e_1+\tau^2 e_1^2] - \psi_2\bar{X}[e_1-2\tau e_1^2]$$

or

$$(t_2-\bar{Y}) \cong \bar{Y}[\psi_1\{1+e_0-\tau e_1-\tau e_0 e_1+\tau^2 e_1^2\} - \psi_2(\frac{1}{R})(e_1-2\tau e_1^2)-1] \tag{38}$$

Taking expectation of both sides of (38) we get the bias of t_2 to the first degree of approximation as

$$B(t_2) = \bar{Y}[\psi_1\{1+\frac{(1-f)}{n}\tau(\tau-k)C_x^2\} + 2\psi_2(\frac{1}{R})\frac{(1-f)}{n}\tau C_x^2 - 1]$$

$$= \bar{Y}(\psi_1-1) + \frac{(1-f)}{n}\tau C_x^2[\psi_1\{\tau-k\} + 2\psi_2(\frac{1}{R})]. \tag{39}$$

Squaring both sides of (38) and neglecting terms of e's having power greater than two we have

$$(t_2-\bar{Y})^2 = \bar{Y}^2[\psi_1^2\{1+2e_0-2\tau e_1+e_0^2-4\tau e_0 e_1+3\tau^2 e_1^2\} + \psi_2^2(\frac{1}{R^2})e_1^2$$

$$-2\psi_1\psi_2(\frac{1}{R})\{e_1+e_0e_1-3\tau e_1^2\} - 2\psi_1\{1+e_0-\tau e_1-\tau e_0 e_1$$

$$+\tau^2 e_1^2\} + 2\psi_2(\frac{1}{R})(e_1 - 2\tau e_1^2) + 1]. \quad (40)$$

Taking expectation of both sides of (40) we get the MSE of t_2 to the first degree of approximation as

$$MSE(t_2) = \overline{Y}^2[1 + \psi_1^2 A + \psi_2^2(\frac{1}{R^2})B + 2\psi_1\psi_2(\frac{1}{R})C - 2\psi_1 D - 2\psi_2(\frac{1}{R})E], \quad (41)$$

where

$$A = [1 + \frac{(1-f)}{n}\{C_y^2 + \tau C_x^2(3\tau - 4k)\}],$$

$$B = \frac{(1-f)}{n}C_x^2,$$

$$C = \frac{(1-f)}{n}C_x^2(3\tau - k),$$

$$D = [1 + \frac{(1-f)}{n}\tau C_x^2(\tau - k)],$$

$$E = \frac{2(1-f)}{n}\tau C_x^2.$$

Differentiating (41) with respect to ψ_1 and ψ_2 partially and equating to zero we get

$$\begin{bmatrix} A & (\frac{1}{R})C \\ (\frac{1}{R})C & (\frac{1}{R^2})B \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} D \\ (\frac{1}{R})E \end{bmatrix}. \quad (42)$$

Solving (42) we get the optimum values of ψ_1 and ψ_2 as

$$\left.\begin{aligned} \psi_1 &= \frac{(BD - CE)}{(AB - C^2)} = \psi_1^*(say) \\ \psi_2 &= \frac{(AE - CD)}{(AB - C^2)} = \psi_2^*(say) \end{aligned}\right\}. \quad (43)$$

Thus the resulting minimum MSE of t_2 is given by

$$MSE_{min}(t_2) = \overline{Y}^2[1 - \frac{(BD^2 - 2CDE + AE^2)}{(AB - C^2)}]. \quad (44)$$

Now we established the following theorem.

Theorem 2: To the first degree of approximation,

$$MSE(t_2) \geq \bar{Y}^2[1 - \frac{(BD^2 - 2CDE + AE^2)}{(AB - C^2)}]$$

with equality holding if

$\psi_1 = \psi_1^*$ and $\psi_2 = \psi_2^*$.

3.1 Particular case $(\psi_1 = 1)$

Putting $\psi_1 = 1$ in (36), we get an estimator for $\bar{Y}$ as

$$t_2^* = \bar{y}(\frac{\eta\bar{X} + \lambda}{\eta\bar{x} + \lambda}) + \psi_2(\bar{X} - \bar{x})(\frac{\eta\bar{X} + \lambda}{\eta\bar{x} + \lambda})^2 . \quad (45)$$

To the first degree of approximation, the bias and MSE of t_2^* are respectively obtained by putting $\psi_1 = 1$ in (39) and (41) as

$$B(t_2^*) = \frac{(1-f)}{n}\tau C_x^2[(\tau - k) + 2\psi_2(\frac{1}{R})], \quad (46)$$

$$MSE(t_2^*) = \bar{Y}^2[1 + A - 2D + \psi_2^2(\frac{1}{R^2})B + 2\psi_2(\frac{1}{R})(C - E)] . \quad (47)$$

The $MSE(t_2^*)$ at (47) is minimized for

$$\psi_2 = \frac{(C - E)R}{B} = \psi_{20} \text{ (say)}. \quad (48)$$

Thus the resulting minimum MSE of t_2^* is given by

$$MSE_{min}(t_2^*) = \bar{Y}^2[1 + A - 2D - \frac{(C - E)^2}{B}] = \frac{(1-f)}{n}S_y^2(1 - \rho^2), \quad (49)$$

which equals to the approximate variance of the usual regression estimator $t_{Reg} = \bar{y} + b(\bar{X} - \bar{x})$.

Thus we established the following theorem.

Theorem 3: To the first degree of approximation,

$$MSE(t_2^*) \geq \bar{Y}^2[(1 + A - 2D) - \frac{(C - E)^2}{B}]$$

with equality holding if

$$\psi_2 = \psi_{20} .$$

From (44) and (49) we have

$$MSE_{min}(t_2^*) - MSE_{min}(t_2) = \frac{\bar{Y}^2[B(A-D)-C(C-E)]^2}{(AB-C^2)} > 0, \qquad (50)$$

which clearly shows that the proposed class of estimators t_2 is better than t_2^* (or usual linear regression estimator t_{Reg}) and hence the usual unbiased estimator $\bar{y}$, ratio estimator t_R, product estimator t_P and the estimators due to Kadilar and Cingi (2004, 2006 a, c).

Remark 3.1: It is observed from the expressions in (43) and (48) that the proposed classes of estimators t_1 and t_2^* will attained their minimum mean squared errors respectively in (44) and (49) only when the unknown population parameters C_y, C_x, $\beta_1(x)$, $\beta_2(x)$, k, ρ_{yx} and S_x are known. To use such estimators in practice one has to use some guessed values of C_y, C_x, $\beta_1(x)$, $\beta_2(x)$, k, ρ_{yx} and S_x, either through past experience or through a pilot sample survey [see Srivastava and Jhajj (1980, p. 92)]. Das and Tripathi (1978, sec. 3) have illustrated that even if the values of the constants used in the estimators are not exactly equal to their optimum values as given by (43) and (48) but are close enough, the resulting estimators will be better than the usual unbiased estimator $\bar{y}$. For more detailed discussion on this issue, the reader is referred to Reddy (1973, 1974), Sahai and Ray (1980), Ray and Sahai (1980), Prasad (1989), Lui (1990), Prasad and Singh (1990, 1992) and Ahmed et al. (2000, 2003). On the otherhand, following Srivastava and Jhajj (1983) it can be also shown that the estimator t_2 and t_2^* with estimated optimum values obtained by their consistent estimators, attain the same minimum mean squared errors of estimators t_2 and t_2^* based on optimum values, up to the first order of approximation [see, Jhajj et al. (2005, p. 28)].

Now we provide list of some ratio-type estimators in Table 2, which are members of the proposed estimator t_2. A list of some known estimators are also given in Table 3, which are members of the estimators t_1 and t_2.

Table 1: Some ratio-type estimators which are members of t_1.

Estimators	Values of constants (η,λ)	
	η	λ
$t_{1(1)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{C_x\bar{X}+1}{C_x\bar{x}+1})$	C_x	1
$t_{1(2)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}\{\frac{\beta_2(x)\bar{X}+1}{\beta_2(x)\bar{x}+1}\}$	$\beta_2(x)$	1
$t_{1(3)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}\{\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\}$	$\beta_2(x)$	C_x
$t_{1(4)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}})$	1	ρ_{yx}
$t_{1(5)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{C_x\bar{X}+\rho_{yx}}{C_x\bar{x}+\rho_{yx}})$	C_x	ρ_{yx}
$t_{1(6)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}\{\frac{\beta_2(x)\bar{X}+\rho_{yx}}{\beta_2(x)\bar{x}+\rho_{yx}}\}$	$\beta_2(x)$	ρ_{yx}
$t_{1(7)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{S_x\bar{X}+1}{S_x\bar{x}+1})$	S_x	1
$t_{1(8)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}\{\frac{\beta_2(x)\bar{X}+S_x}{\beta_2(x)\bar{x}+S_x}\}$	$\beta_2(x)$	S_x
$t_{1(9)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{S_x\bar{X}+\rho_{yx}}{S_x\bar{x}+\rho_{yx}})$	S_x	ρ_{yx}
$t_{1(10)}^{r}=\{w_1\bar{y}+w_2(\bar{X}-\bar{x})\}(\frac{\bar{X}}{\bar{x}})$	1	0

Table 2: Some ratio-type estimators which are members of t_2.

Estimators	Values of constants (η, λ)	
	η	λ
$t^r_{2(1)} = \psi_1 \bar{y}(\frac{C_x \bar{X} + 1}{C_x \bar{x} + 1}) + \psi_2 (\bar{X} - \bar{x})(\frac{C_x \bar{X} + 1}{C_x \bar{x} + 1})^2$	C_x	1
$t^r_{2(2)} = \psi_1 \bar{y}\{\frac{\beta_2(x)\bar{X} + 1}{\beta_2(x)\bar{x} + 1}\} + \psi_2 (\bar{X} - \bar{x})\{\frac{\beta_2(x)\bar{X} + 1}{\beta_2(x)\bar{x} + 1}\}^2$	$\beta_2(x)1$	1
$t^r_{2(3)} = \psi_1 \bar{y}\{\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x}\} + \psi_2 (\bar{X} - \bar{x})\{\frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x}\}^2$	$\beta_2(x)$	C_x
$t^r_{2(4)} = \psi_1 \bar{y}(\frac{\bar{X} + \rho_{yx}}{\bar{x} + \rho_{yx}}) + \psi_2 (\bar{X} - \bar{x})(\frac{\bar{X} + \rho_{yx}}{\bar{x} + \rho_{yx}})^2$	1	ρ_{yx}
$t^r_{2(5)} = \psi_1 \bar{y}(\frac{C_x \bar{X} + \rho_{yx}}{C_x \bar{x} + \rho_{yx}}) + \psi_2 (\bar{X} - \bar{x})(\frac{C_x \bar{X} + \rho_{yx}}{C_x \bar{x} + \rho_{yx}})^2$	C_x	ρ_{yx}
$t^r_{2(6)} = \psi_1 \bar{y}\{\frac{\beta_2(x)\bar{X} + \rho_{yx}}{\beta_2(x)\bar{x} + \rho_{yx}}\} + \psi_2 (\bar{X} - \bar{x})\{\frac{\beta_2(x)\bar{X} + \rho_{yx}}{\beta_2(x)\bar{x} + \rho_{yx}}\}^2$	$\beta_2(x)$	ρ_{yx}
$t^r_{2(7)} = \psi_1 \bar{y}(\frac{S_x \bar{X} + 1}{S_x \bar{x} + 1}) + \psi_2 (\bar{X} - \bar{x})(\frac{S_x \bar{X} + 1}{S_x \bar{x} + 1})^2$	S_x	1
$t^r_{2(8)} = \psi_1 \bar{y}\{\frac{\beta_2(x)\bar{X} + S_x}{\beta_2(x)\bar{x} + S_x}\} + \psi_2 (\bar{X} - \bar{x})\{\frac{\beta_2(x)\bar{X} + S_x}{\beta_2(x)\bar{x} + S_x}\}^2$	$\beta_2(x)$	S_x
$t^r_{2(9)} = \psi_1 \bar{y}(\frac{S_x \bar{X} + \rho_{yx}}{S_x \bar{x} + \rho_{yx}}) + \psi_2 (\bar{X} - \bar{x})(\frac{S_x \bar{X} + \rho_{yx}}{S_x \bar{x} + \rho_{yx}})^2$	S_x	ρ_{yx}
$t^r_{2(10)} = \psi_1 \bar{y}(\frac{\bar{X}}{\bar{x}}) + \psi_2 (\bar{X} - \bar{x})(\frac{\bar{X}}{\bar{x}})^2$	1	0

Table 3: Some known estimators which are members of the estimators t_1 and t_2

Estimators	Values of constants			
	η	λ	ψ_1	ψ_2
$t_{SD} = \bar{y}(\frac{\bar{X}+C_x}{\bar{x}+C_x})$ Sisodia and Dwivedi (1981)	1	C_x	1	0
$t_{US1} = \bar{y}\{\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\}$	$\beta_2(x)$	C_x	1	0
$t_{US2} = \bar{y}\{\frac{C_x\bar{X}+\beta_2(x)}{C_x\bar{x}+\beta_2(x)}\}$ Upadhyaya and Singh (1999)	C_x	$\beta_2(x)$	1	0
$t_{ST} = \bar{y}(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}})$ Singh and Tailor (2003)	1	ρ_{yx}	1	0
$t_{S1} = \bar{y}(\frac{\bar{X}+S_x}{\bar{x}+S_x})$	1	S_x	1	0
$t_{S2} = \bar{y}\{\frac{\beta_2(x)+S_x}{\beta_2(x)+S_x}\}$ Singh (2003)	$\beta_2(x)$	S_x	1	0
$t_{STK} = \bar{y}\{\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\}$ Singh et al. (2004)	1	$\beta_2(x)$	1	0
$t_{K1} = \bar{y}(\frac{C_x\bar{X}+\rho_{yx}}{C_x\bar{x}+\rho_{yx}})$	C_x	ρ_{yx}	1	0
$t_{K2} = \bar{y}(\frac{\rho_{yx}\bar{X}+C_x}{\rho_{yx}\bar{x}+C_x})$	ρ_{yx}	C_x	1	0
$t_{K3} = \bar{y}\{\frac{\beta_2(x)\bar{X}+\rho_{yx}}{\beta_2(x)\bar{x}+\rho_{yx}}\}$	$\beta_2(x)$	ρ_{yx}	1	0
$t_{K4} = \bar{y}\{\frac{\rho_{yx}\bar{X}+\beta_2(x)}{\rho_{yx}\bar{x}+\beta_2(x)}\}$ Kadilar and Cingi (2006b)	ρ_{yx}	$\beta_2(x)$	1	0

4. Empirical study

In this section, we evaluate the performances of various estimators using following data sets which arepreviously used in the literature.

Population 1: [Source: Kadilar and Cingi (2006c)]

N = 200, n = 50, $\bar{Y}$ = 500, $\bar{X}$ = 25, C_y = 15, C_x = 2, ρ_{yx} = 0.90, $\beta_2(x)$ = 50.

Population 2: [Source: Kadilar and Cingi(2004), Kadilar and Cingi (2006a)]

$N = 106$, $n = 20$, $\overline{Y} = 2212.59$, $\overline{X} = 27421.70$, $C_y = 5.22$,

$C_x = 2.10$, $\rho_{yx} = 0.86$, $\beta_2(x) = 34.57$.

Population 3: [Source: Kadilar and Cingi (2006b)]

$N = 104$, $n = 20$, $\overline{Y} = 625.37$, $\overline{X} = 13.93$, $C_y = 1.866$, $C_x = 1.653$,

$\rho_{yx} = 0.865$, $\beta_2(x) = 17.516$.

Table 4: MSEs and PREs of different known estimators

Estimators	Population 1		Population 2		Population 3	
	MSE	PRE	MSE	PRE	MSE	PRE
$t_0 = \bar{y}$	843750.00	100.000	5411348.28	100.000	54993.75	100.000
t_R	656250.00	128.571	2542740.30	212.816	13869.96	396.495
t_{SD}	669110.08	126.100	2542892.90	212.803	14140.05	388.922
t_{US1}	656525.60	128.517	2542744.71	212.815	13858.25	396.831
t_{US2}	746250.00	113.065	2543936.23	212.716	21047.63	261.282
t_{ST}	662262.32	127.404	2542802.79	212.810	13898.70	395.676
t_{S1}	777916.67	108.463	4294609.80	126.003	29357.64	187.323
t_{S2}	662906.80	127.280	2659735.92	203.454	14015.67	392.373
t_{KC2}	173172.58	487.231	2284777.20	236.844	48331.50	113.784
t_{KC3}	161979.17	520.900	2282707.29	237.058	22314.56	246.448
t_{KC4}	175264.61	481.415	2284907.44	236.830	56422.67	97.467
t_{KC5}	164062.50	514.286	2283860.74	236.939	27766.95	198.055
t_{K1}	659304.79	127.976	2542770.06	212.813	13852.97	396.982
t_{K2}	670431.59	125.852	2542917.74	212.801	14252.91	385.842
t_{K3}	656374.12	128.547	2542742.10	212.815	13863.32	396.685
t_{K4}	782349.88	107.848	2545659.17	212.572	27813.65	197.722
t^*_{KC1}	174288.14	484.112	2284856.39	236.835	52102.80	105.549
t^*_{KC2}	174786.74	482.731	2284885.16	236.832	53933.14	101.967
t^*_{KC3}	172963.48	487.820	2284755.36	236.846	47217.40	116.469
t^*_{KC4}	175290.92	481.343	2284909.73	236.830	56697.15	96.996
t^*_{KC5}	161757.21	521.615	2282349.28	237.096	21014.10	261.699
t^{**}_{KCj} or t_{Reg}	160312.50	526.316	1409115.09	384.025	13846.05	397.180

* (j = 2,3,4,5)

Table 5: Corrected optimum values (w_1^*, w_2^*), MSEs and PREs of different estimators $(t_{1(i)}^r, i = 1 to 10)$ generated from Gupta and Shabbir (2008) estimator t_1.

Estimators	Population 1				Population 2				Population 3			
	w_1^*	w_2^*	MSE	PRE	w_1^*	w_2^*	MSE	PRE	w_1^*	w_2^*	MSE	PRE
$t_{1(1)}^r$	0.60	76.61	95396.05	884.471	0.74	0.09	1043368.08	518.642	0.96	2.41	13321.23	412.828
$t_{1(2)}^r$	0.59	76.48	95304.34	885.322	0.74	0.09	1043366.16	518.643 *	0.96	0.87	13316.65	412.970
$t_{1(3)}^r$	0.59	76.49	95308.27	885.285	0.74	0.09	1043366.29	518.643 *	0.96	0.98	13316.98	412.960
$t_{1(4)}^r$	0.60	76.71	95468.42	883.800	0.74	0.09	1043369.73	518.641	0.96	3.10	13323.21	412.766
$t_{1(5)}^r$	0.60	76.59	95386.75	884.557	0.74	0.09	1043367.79	518.642	0.96	2.19	13320.59	412.848
$t_{1(6)}^r$	0.59	76.48	95303.94	885.325	0.74	0.09	1043366.14	518.643 *	0.96	0.85	13316.58	412.972
$t_{1(7)}^r$	0.59	76.48	95304.34	885.322	0.74	0.09	1043366.03	518.643 *	0.96	0.83	13316.52	412.974
$t_{1(8)}^r$	0.60	76.73	95485.97	883.638	0.75	0.09	1049814.73	515.457	0.96	4.25	13326.36	412.669
$t_{1(9)}^r$	0.59	76.48	95303.94	885.325	0.74	0.09	1043366.03	518.643 *	0.96	0.81	13316.47	412.975
$t_{1(10)}^r$	0.59	76.47	95300.40	885.358 *	0.74	0.09	1043366.03	518.643 *	0.96	0.70	13316.13	412.986 *

* indicates the largest PRE

Table 6: Optimum values (ψ_1^*, ψ_2^*), MSEs and PREs of different estimators $(t_{2(i)}^r, i = 1 \ to \ 10)$ generated from proposed estimator t_2.

Estimators	Population 1				Population 2				Population 3			
	ψ_1^*	ψ_2^*	MSE	PRE	ψ_1^*	ψ_2^*	MSE	PRE	ψ_1^*	ψ_2^*	MSE	PRE
$t_{2(1)}^r$	0.53	3.98	45246.37	1864.791	0.50	1.57	202185.29	2676.430	0.94	0.13	12986.83	423.458
$t_{2(2)}^r$	0.53	3.98	44081.39	1914.073	0.50	1.57	202155.53	2676.824	0.94	0.09	13116.65	419.267
$t_{2(3)}^r$	0.53	3.98	44131.01	1911.921	0.50	1.57	202157.65	2676.796	0.94	0.10	13107.25	419.567
$t_{2(4)}^r$	0.53	3.98	46177.73	1827.179	0.50	1.57	202210.82	2676.092	0.94	0.15	12931.31	425.276
$t_{2(5)}^r$	0.53	3.98	45127.48	1869.703	0.50	1.57	202180.85	2676.489	0.94	0.12	13005.06	422.864
$t_{2(6)}^r$	0.53	3.98	44076.42	1914.289	0.50	1.57	202155.27	2676.828	0.94	0.09	13118.60	419.204
$t_{2(7)}^r$	0.53	3.98	44081.39	1914.073	0.50	1.57	202155.53	2676.824	0.94	0.09	13116.65	419.267
$t_{2(8)}^r$	0.53	3.98	46405.23	1818.222	0.54	1.52	297759.98	1817.352	0.94	0.17	12844.01	**428.167** *
$t_{2(9)}^r$	0.53	3.98	44076.42	1914.289	0.50	1.57	202153.61	**2676.850** *	0.94	0.09	13121.61	419.108
$t_{2(10)}^r$	0.53	3.98	44031.68	**1916.234** *	0.50	1.57	202153.61	**2976.850** *	0.94	0.09	13131.21	418.802

* indicates the largest PRE

It is observed from Table 4 that the regression estimator t_{Reg} and Kadilar and Cingi's (2006 a) estimators t^{**}_{KCj} (j = 2,3,4,5) at the optimum condition, are more efficient than the usual unbiased estimator t_0, ratio estimator t_R, Sisodia and Dwivedi's (1981) estimator t_{SD}, Upadhyaya and Singh's (1999) estimators t_{US1}, t_{US2}; Singh and Tailor's (2003) estimator t_{ST}, Singh's (2003) estimators t_{S1}, t_{S2}; Singh et al.'s (2004) estimator t_{STK}, Kadilar and Cingi's (2004) estimators t_{KCi} (i = 1,2,3,4,5), Kadilar and Cingi's (2006 b) estimators t_{Ki} (i = 1,2,3,4) and Kadilar and Cingi's (2006 c) estimators t^{*}_{KCi} (i = 1,2,3,4,5) for all three populations.

Table 5 clearly shows that the minimum MSEs of the estimators $t^{r}_{1(j)}$ (j = 1 to 10) depend on the transformations used. The estimators $t^{r}_{1(6)}$ and $t^{r}_{1(9)}$ have smallest MSE (at optimum conditions) among all the estimators $t^{r}_{1(j)}$ (j = 1 to 9) and largest PRE for population I. However the estimators $t^{r}_{1(j)}$ (j = 1 to 10) except the estimator $t^{r}_{1(8)}$ are almost equally efficient for all three populations I, II and III. It is further observed from Table 4 and Table 5 that there is larger gain in efficiency by using the estimator $t^{r}_{1(j)}$ (j = 1 to 10), (which are members of Gupta and Shabbir (2008) estimator t_1) over regression estimator t_{Reg} and Kadilar and Cingi's (2006a) estimator t^{**}_{KCj} (j = 2,3,4,5). We also note that the estimator $t^{r}_{1(10)}$ (based only on the population mean $\overline{X}$) has smaller MSE (at optimum condition) among the estimators $t^{r}_{1(j)}$ (j = 1 to 10) for population I and III while for population II the MSEs (at optimum condition) of the estimators $t^{r}_{1(7)}, t^{r}_{1(9)}$ and $t^{r}_{1(10)}$ are same.

It is observed from Table 6 that the estimator $t^{r}_{2(10)}$ has largest PRE for population I. The estimators $t^{r}_{2(9)}$ and $t^{r}_{2(10)}$ are equally efficient but have largest efficiency among allthe estimator in population II. In population III the estimator $t^{r}_{2(8)}$ has the largest efficiency among all the estimators $t^{r}_{2(j)}$ (j = 1 to 10).

Finally we conclude that the estimators $t^{r}_{2(j)}$ (j = 1 to 10) (generated from the proposed class of estimators t_2) are more efficient than the estimator t_0, t_R, t_{SD}, t_{US1}, t_{US2}, t_{ST}, t_{S1}, t_{S2}, t_{STK}, t^{*}_{KCi} (i = 1 to 5), t_{Ki} (i = 1 to 4), t^{*}_{KCi} (i = 1 to 5), t^{**}_{KCj} (j = 2,3,4,5), the regression estimator t_{Reg} and the estimators $t^{r}_{1(j)}$ (j = 1 to 10) which are generated from Gupta and Shabbir (2008) class of estimators t_1.

There is no significant role of transformations used in Gupta and Shabbir (2008) estimator t_1 and the proposed class of estimators t_2 as the estimator $t^r_{1(10)}$ and $t^r_{2(10)}$ (which are only based on population mean $\bar{X}$ of the auxiliary variable x) appears to the best estimator in the sense of having largest efficiency for all three populations.

However this conclusion should not be extrapolated in general. There may be possibility of populations in practical situations where the transformations used in the estimators t_1 and t_2 may play significant role. Thus based on the above discussions we recommend the estimators $t^r_{2(j)}$(j = 1 to 10) generated from the proposed class of estimators t_2 for their use in practice.

5. Conclusion

A revisit to the Gupta and Shabbir (2008) estimator t_1 has been made in this paper. We have derived the correct MSE expression of Gupta and Shabbir (2008) estimator t_1. The correct MSE expression depend upon the transformation used, so different estimators $t^r_{1(j)}$ (j = 1 to 9) give the different minimum MSEs and hence PREs. Similar is the case with the estimators $t^r_{2(j)}$(j = 1 to 9) generated from the proposed class of estimators t_2. Theoretically and empirically it has been shown that the estimators t_1 and t_2 (at their optimum conditions) are better than usual regression estimator t_{Reg} and other competing estimators considered here. Finally with help of the three numerical data we have shown that the estimators $t^r_{2(j)}$ (j = 1 to 10) generated from proposed class of estimators t_2 are more efficient than the estimators $t^r_{1(j)}$ (j = 1 to 10) generated from Gupta and Shabbir (2008) estimator t_1, usual regression estimator t_{Reg} and other competing estimators considered here. Thus our recommendation goes in the favor of proposed class of estimators t_2.

Acknowledgement

The authors are deeply grateful to the Editor Dr. Shahid Kamal and to the referees for their helpful comments and valued suggestions that led to this improved version of the paper.

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