Estimation of Parameters and Reliability Function of Exponentiated Exponential Distribution: Bayesian approach Under General Entropy Loss Function

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Abstract

In this paper we propose Bayes estimators of the parameters of exponentiated exponential distribution and reliability functions under general entropy loss function for Type II censored sample. The proposed estimators have been compared with the corresponding Bayes estimators obtained under squared error loss function and maximum likelihood estimators for their simulated risks (average loss over sample space).

Keywords: Exponentiated Exponential distribution, General entropy loss function, Type II censoring, Simulated risk.

1. Introduction

Two parameter gamma and Weibull distributions are the most popular distributions for analyzing life time data. These distributions possess both scale and shape parameters and hence, found to be quite flexible to analyze any positive real data. These have increasing as well as decreasing failure rates depending on their shape parameters. On the other hand, a major disadvantage of the gamma distribution is that its distribution function and survival function can not be expressed in a nice closed form if the shape parameter is not an integer. Perhaps, this is the reason for gamma distribution to be less popular than Weibull distribution. Several applications of Weibull distribution can be found inJohnson (1968) and for short comings, readers are referred toGorski (1968). One of the disadvantages that can be pointed out here is that the asymptotic convergence of the distribution of mles to normality is very slow Bain (1976). As an alternative to gamma and Weibull distribution, the exponentiated Exponential (EE) distribution can take place and this distribution has been studied by Gupta & Kundu (1999), Gupta & Kundu (2001a), Gupta & Kundu (2001b), Gupta & Kundu (2002), Jaheen (2004), Raquab & Ahsanullah (2001), Raquab & Madi (2005), Sarhan (2007) and Zheng (2002) etc. Gupta & Kundu (2001a) and Gupta & Kundu (2003), noted that the two-parameter EE distribution provides a better fit than the two-parameter Weibull distribution for some specific data. In fact EE distribution is a special case of a distribution that was used by Gompertz (1825).

The probability density function (pdf) of EE distribution is given below,

$$f(x;\alpha,\lambda)=\alpha\lambda(1-e^{-\lambda x})^{\alpha-1}e^{-\lambda x} \quad ; \quad x>0,\alpha>0,\lambda>0 \tag{1}$$

where α is the shape parameter and λ is the scale parameter of considered distribution. Its cumulative distribution function and reliability (i.e., survival) function are given by,

$$F(x)=[1-e^{-\lambda x}]^{\alpha} \tag{2}$$

and

$$R(t)=1-[1-e^{-\lambda t}]^{\alpha} \tag{3}$$

respectively.

Gupta & Kundu (2001b) studied different methods of parameter estimation for EE parameters which include maximum likelihood estimation, moment method of estimation and probability plot method of estimation based on complete random sample. An extensive survey of some recent developments for the two-parameter EE distribution based on complete random sample can be had from Gupta & Kundu (2007). It may be noted that either no or a very little attention has been paid to censored sample, although censoring is quite common in life testing and reliability.

Censoring arises in a life test when exact lifetimes are known for only a portion of the test units and the remaining lifetimes are known only to exceed or exceeded by certain value. There are various types of censoring scheme. One of the most common censoring scheme is Type II censoring in which, a total of n units is placed on test, but instead of continuing until all n units, the test is terminated at the time of the r^{th} $(1\le r\le n)$ unit failure. For example, in early childhood learning centers, interest often focuses upon testing children to determine when a child learns to accomplish certain specified tasks. The age at which a child learns the task would be considered the time to event. Some children undergoing testing, may be very slow learner out of the total n children and, therefore, the experimenter can not wait till all the children learn to accomplish the task and decides to stop the data collection as soon as observation on r^{th} (predetermined) child complete the learning to accomplish the task. For the remaining $n-r$ children, it is only known that their learning time is more than the learning time for r^{th} child. For inferences related to type II censored data, seeLawless (1982).

In this paper, we will consider type II censored data and try to develop estimators for the shape parameter when scale parameter is known and scale parameter when shape parameter is known. In both the cases, estimator for the reliability function will also be obtained.

In parameter estimation problem, the most popular loss function is squared error loss function (SELF) which can easily be justified on the grounds of minimum variance unbiased estimation. However, the greatest weakness of this loss function is that it is symmetric and gives equal weightage to over estimation and under estimation of same magnitude. A number of asymmetric loss functions are available in statistical literature and perhaps most widely used asymmetric loss function is LINEX loss function originally proposed by Varian (1975) **Error! Reference source not found.** and popularized by Zellner (1986). It has been pointed out by various authors that LINEX loss is not as appropriate for estimation of scale parameter as it is for location parameter (c.f.Basu et. al.(1991), Parsian & Sanjari (1993)). Keeping this point in mind Basu et. al.(1991) defined a modified LINEX loss function. A suitable alternative to the modified LINEX loss function is the General Entropy loss defined as,

$$L(\theta,\hat{\theta}) \propto \left(\frac{\hat{\theta}}{\theta}\right)^{c_1} - c_1 \ln\left(\frac{\hat{\theta}}{\theta}\right) - 1 \tag{4}$$

This loss is a simple generalization of the Entropy loss used by several authors where the shape parameter c_1 is taken equal to 1. The general version ((4)) allows different shapes for the loss function. It may be noted that when $c_1 > 0$, a positive error causes more serious consequences than a negative error. The Bayes estimator of θ under General Entropy loss is given as,

$$\hat{\theta}_G = [E_\theta(\theta^{-c_1})]^{(-1/c_1)} \tag{5}$$

provided that $E_\theta[.]$ exists and is finite. It can be noted that, when $c_1 = 1$, the Bayes estimate ((5)) coincides with the Bayes estimate under the weighted squared-error loss function. Similarly, when $c_1 = -1$, the Bayes estimate (5) coincides with the Bayes estimate under SELF. It is worthwhile to mention here that posterior distribution of shape and scale parameter for EE distribution involves integral expression in the denominator which can not be reduced in a nice closed form and hence the exact evaluation of posterior expectations for obtaining Bayes estimator for shape and scale parameters and Reliability function will not be possible. We have used numerical integration technique to obtain Bayes estimators under both the loss functions.

In the section 2 we have obtained the maximum likelihood estimators, Bayes estimators under GELF and SELF of λ and $R(t)$ when α is known for type II censored data. In section 3 we have also obtained the maximum likelihood estimators, Bayes estimators under GELF and SELF for α and $R(t)$ when λ is known. The estimators, thus obtained, i.e., mle and Bayes estimators under SELF and GELF have been compared in term of their risks (**average loss over sample space**). The comparison is based on Monte-Carlo study of 5000 simulated samples from EE distribution.

2. Maximum Likelihood and Bayes Estimators of λ and $R(t)$ when α is known

Let us consider that n identical items whose life time follow the p.d.f. (1), are put on test. The test is terminated as soon as we observe r ordered failure times, say $x_1 < x_2 < \cdots < x_r$. Naturally, $x_1, x_2, \cdots, x_r$ constitute type II censored sample. Therefore, the likelihood function is given as,

$$l(x_1, x_2, \cdots, x_r \mid \lambda) = \frac{n!}{(n-r)!} \alpha^r \lambda^r e^{-\lambda \sum_{i=1}^{r} x_i} \left(\prod_{i=1}^{r} (1 - e^{-\lambda x_i}) \right)^{\alpha - 1} \left(1 - (1 - e^{-\lambda x_r})^{\alpha} \right)^{n-r} \tag{6}$$

2.1 Maximum Likelihood Estimator of λ and $R(t)$

Differentiating the log of likelihood given in (6) with respect to λ and equating it to zero, we get the following normal equation,

$$\frac{r}{\lambda} + \sum_{i=1}^{r} \frac{x_i (\alpha e^{-\lambda x_i} - 1)}{1 - e^{-\lambda x_i}} - \frac{(n-r)\alpha x_r e^{-\lambda x_r} (1 - e^{-\lambda x_r})^{\alpha - 1}}{1 - (1 - e^{-\lambda x_r})^{\alpha}} = 0 \tag{7}$$

Note that this is an implicit equation in λ and can not be solved in nice closed form. Therefore numerical iteration method is to be used for solving it. We propose the use of Newton- Raphson method and denote the solution (i.e., mle) by $\hat{\lambda}_M$. Using the invariance property of mle, $\hat{R}_{M\lambda}$, the mle of $R(t)$ may be obtained by replacing λ by $\hat{\lambda}_M$ in ((3)) i.e.,

$$\hat{R}_{M\lambda} = 1 - (1 - e^{-t\hat{\lambda}_M})^{\alpha} \tag{8}$$

2.2 Bayes Estimators of λ and $R(t)$ under General Entropy and Squared Error Loss Functions

2.2.1 Bayes Estimator of λ

Consider gamma prior for λ having p.d.f.,

$$g_1(\lambda) = \frac{m^c}{\Gamma(c)} e^{-m\lambda} \lambda^{c-1} \qquad ; \qquad \lambda > 0, m > 0, c > 0 \tag{9}$$

For prior ((9)), the posterior pdf of λ given $x_1, x_2, \ldots, x_r$ is obtained as,

$$h_{11}(\lambda \mid x_1, x_2, \cdots, x_r) = \frac{e^{-(m + \sum_{i=1}^{r} x_i)\lambda} \lambda^{c+r-1} (\prod_{i=1}^{r} (1 - e^{-\lambda x_i}))^{\alpha - 1} (1 - (1 - e^{-\lambda x_r})^{\alpha})^{n-r}}{\int_0^{\infty} e^{-(m + \sum_{i=1}^{r} x_i)\lambda} \lambda^{c+r-1} (\prod_{i=1}^{r} (1 - e^{-\lambda x_i}))^{\alpha - 1} (1 - (1 - e^{-\lambda x_r})^{\alpha})^{n-r} d\lambda} \tag{10}$$

The Bayes estimator of λ under GELF (4) is, therefore, obtained as,

$$\hat{\lambda}_G = \left[\frac{I_1(c - c_1)}{I_1(c)} \right]^{-1/c_1} \tag{11}$$

where,

$$I_1(c)=\int_0^{\infty} e^{-(m+\sum_{i=1}^{r} x_i)\lambda} \lambda^{c+r-1}\left(\prod_{i=1}^{r}(1-e^{-\lambda x_i})\right)^{\alpha-1}\left(1-(1-e^{-\lambda x_r})^{\alpha}\right)^{n-r} d\lambda$$

and the Bayes estimator of λ denoted by $\hat{\lambda}_S$ under SELF for the posterior ((10)) can be obtained as,

$$\hat{\lambda}_S=\frac{I_1(c+1)}{I_1(c)} \tag{12}$$

The integrals $I_1(c)$, $I_1(c+1)$ and $I_1(c-c_1)$ are not reducible to nice closed form. Therefore, we propose to use Gauss - Laguerre quadrature formula for their evaluation.

2.2.2 Bayes Estimator of $R(t)$

For given t, from (3), we get,

$$\lambda=-\frac{1}{t}\ln(1-(1-R)^{1/\alpha})$$

Using this transformation, we find the posterior pdf of R from (10) as given below,

$$h_{12}(R\mid x_1,x_2,\cdots,x_r)=\frac{N_1}{\int_0^1 N_1 dR} \tag{13}$$

where,

$$N_1=\left(1-(1-R)^{1/\alpha}\right)^{\frac{1}{t}(m+\sum_{i=1}^{r}x_i)}\left(\ln(1-(1-R)^{1/\alpha})\right)^{c+r-1}$$
$$\left(\prod_{i=1}^{r}(1-(1-(1-R)^{1/\alpha})^{x_i/t})\right)^{\alpha-1}$$
$$\left(1-(1-(1-(1-R)^{1/\alpha})^{x_r/t})^{\alpha}\right)^{n-r}\frac{(1-R)^{\frac{1}{\alpha}-1}}{1-(1-R)^{1/\alpha}}$$

and the Bayes estimator of $R(t)$ under GELF is obtained as,

$$\hat{R}_{G\lambda}=\left[\frac{\int_0^1 N_1 R^{-c_1} dR}{\int_0^1 N_1 dR}\right]^{-1/c_1} \tag{14}$$

Similarly, the Bayes estimator $\hat{R}_{S\lambda}$ of $R(t)$ under SELF for the posterior (13) can be obtained as,

$$\hat{R}_{S\lambda}=\frac{\int_0^1 N_1 R dR}{\int_0^1 N_1 dR} \tag{15}$$

It may be mentioned here that the integrals involved in the above expressions are also not reducible in nice closed form. However, these can be evaluated using Gauss quadrature formula.

3. Maximum Likelihood and Bayes Estimators of α and $R(t)$ when λ is known

3.1 Maximum Likelihood Estimator of α and $R(t)$

Differentiating the log of likelihood given in (6) with respect to α and equating it to zero (λ is known), we get the following normal equation,

$$\frac{r}{\alpha}+\sum_{i=1}^{r}\ln(1-e^{-\lambda x_i})-\frac{(n-r)(1-e^{-\lambda x_r})^{\alpha}\ln((1-e^{-\lambda x_r}))}{1-(1-e^{-\lambda x_r})^{\alpha}}=0 \tag{16}$$

This is an implicit equation in α. Therefore, as mentioned earlier, it can be solved by using Newton - Raphson method and denote the solution (i.e., mle) by $\hat{\alpha}_M$. Hence $\hat{R}_{M\alpha}$ (mle of R(t)) can be obtained as,

$$\hat{R}_{M\alpha}=1-(1-e^{-t\lambda})^{\hat{\alpha}_M} \tag{17}$$

3.2 Bayes Estimators of α and R under General Entropy and Squared Error Loss Functions

3.2.1 Bayes Estimator of α

We choose improper prior for α as Jeffery's prior with pdf,

$$g_2(\alpha)=\frac{1}{\alpha} \qquad ; \qquad \alpha>0 \tag{18}$$

Now, corresponding to this prior of α, the posterior pdf of α given $x_1, x_2, \cdots, x_r$ is obtained as,

$$h_{21}(\alpha\,|\,x_1,x_2,\cdots,x_r)=\frac{\alpha^{r-1}(\prod_{i=1}^{r}(1-e^{-\lambda x_i}))^{\alpha-1}(1-(1-e^{-\lambda x_r})^{\alpha})^{n-r}}{I_3(r)} \tag{19}$$

where,

$$I_2(r)=\int_0^{\infty}\alpha^{r-1}\left(\prod_{i=1}^{r}(1-e^{-\lambda x_i})\right)^{\alpha-1}\left(1-(1-e^{-\lambda x_r})^{\alpha}\right)^{n-r}d\alpha$$

If $\hat{\alpha}_G$ denote the Bayes estimator of α under GE loss function, then it obtained as,

$$\hat{\alpha}_G=\left[\frac{I_2(r-c_1)}{I_2(r)}\right]^{-1/c_1} \tag{20}$$

Furthermore, if $\hat{\alpha}_S$ be the Bayes estimator of α under SELF, then

$$\hat{\alpha}_S = \frac{I_2(r+1)}{I_2(r)} \tag{21}$$

The integrals $I_2(r)$, $I_2(r+1)$ and $I_2(r-c_1)$ are not reducible to nice closed form. Therefore we propose to use Gauss - Laguerre quadrature formula for its evaluation.

3.2.2 Bayes Estimators of $R(t)$

For given t, from ((3)), we get,

$$\alpha = \frac{\ln(1-R)}{\ln(1-e^{-\lambda t})}$$

By use of this transformation, we find the posterior pdf of R given $x_1, x_2, \cdots, x_r$ from posterior (19) of α as,

$$h_{22}(R \mid x) = \frac{N_2}{\int_0^1 N_2 dR} \tag{22}$$

where

$$N_2 = \frac{(\ln(1-R))^{r-1}}{1-R}\left(\prod_{i=1}^{r}(1-e^{-\lambda x_i})\right)^{\frac{\ln(1-R)}{\ln(1-e^{-\lambda t})}-1}\left(1-(1-e^{-\lambda x_r})^{\frac{\ln(1-R)}{\ln(1-e^{-\lambda t})}}\right)^{n-r}$$

Therefore, the Bayes estimator $\hat{R}_{G\alpha}$ of $R(t)$ under GELF is obtained as,

$$\hat{R}_{G\alpha} = \left[\frac{\int_0^1 N_2 R^{-c_1} dR}{\int_0^1 N_2 dR}\right]^{-1/c_1} \tag{23}$$

and hence the Bayes estimator $\hat{R}_{S\alpha}$ of $R(t)$ under SELF is obtained as,

$$\hat{R}_{S\alpha} = \frac{\int_0^1 N_2 R dR}{\int_0^1 N_2 dR} \tag{24}$$

As mentioned earlier the integrals involved in the above expressions can be evaluated using Gauss quadrature formula.

4. Simulation Studies

The estimators $\hat{\lambda}_M$ and $\hat{R}_{M\lambda}$ are the mle's of the parameter λ and reliability $R(t)$ for a specified t respectively and $\hat{\lambda}_G$, $\hat{R}_{G\lambda}$ and $\hat{\lambda}_S$, $\hat{R}_{S\lambda}$ are the corresponding Bayes estimators under GELF and SELF respectively, when α is known. Similarly, the estimators $\hat{\alpha}_M$ and $\hat{R}_{M\alpha}$ are the mle's of the parameter α and reliability $R(t)$ for a specified t respectively and $\hat{\alpha}_G$, $\hat{R}_{G\alpha}$ and $\hat{\alpha}_S$, $\hat{R}_{S\alpha}$ are the corresponding Bayes estimators under GELF and SELF respectively when λ is known.

In this section, we shall compare the estimators obtained under GELF with corresponding Bayes estimators under SELF and their mle's. The comparisons are based on the **simulated risks(average loss over sample space)** under GELF and SELF both. The exact expressions for the risks can not be obtained, therefore the risks of the estimators are estimated on the basis of **Monte-Carlo simulation study** of 5000 samples. It may be noted that the risks of the estimators will be the function of n, r, m, c, α, λ, t and c_1. In order to consider wide variation of these values, we have obtained the simulated risks for $n = 15[3]33$ and $r = 6[2]18$. The various values of the hyper parameters are taken as $m = 1[1]7$ and $c = 1[1]7$. For over estimation case, the variation in c_1 considered here are c_1=0.5[0.5]3 and for under estimation case $c_1 = -3.0[0.5]-0.5$. The model parameters are considered as $\alpha = 1[1]7$ and $\lambda = 1[1]7$ and t is arbitrarily taken as 0.5. From the results thus obtained effect of variation of various parameters / hyper parameters on the risk of estimators has been noted for the study of the behavior of the estimators and are summarized below. It may be mentioned here that graphs of all results are not shown here due to space restriction.

4.1 For estimation of λ and R when α is known

We observed that under GELF, the proposed estimator $\hat{\lambda}_G$ performed well (**in sense of having smaller risk**) than $\hat{\lambda}_M$ and $\hat{\lambda}_S$ for all choices of c_1. The risk of the estimators increases with increase in the value of $|c_1|$ for all estimators, but more increase noted for $\hat{\lambda}_M$ as compared to $\hat{\lambda}_G$ and $\hat{\lambda}_S$. The least increase in the risk is observed for $\hat{\lambda}_G$. For $c_1 = 0$, all the three estimators have more or less equal risks (see, figure (1)). Keeping this point in mind, we have included the graphs for value of $|c_1| = 1.5$ (a moderate value).

On varying r, we saw that our proposed estimator $\hat{\lambda}_G$ again perform well for all choices of r keeping other parameters fixed and risks of all the estimators decreases with increase in the value of r (see, figure (2)). It further noted that as the value of n increases, the risk decreases almost all for all estimators provided that the values of other parameters are kept constant, although the decrease is very small (see, fig.(3)). Therefore, for showing the behavior of risk of estimators, we have fixed (n,r) at $(15,12)$ in rest of the graphs.

In over estimation situation (i.e., for fixed $c_1 = +1.5$), we observed that $\hat{\lambda}_G$ perform better than other estimators for small values of hyper shape parameter $m \le 2$ (see, figure (4)). It may be noted here that as n increases the prior distribution becomes more and more peaked. Further for large m, the estimator $\hat{\lambda}_G$ performs worst than other estimators. On the other hand as c increases the prior variance increases. In this situation we noted that proposed estimator perform better than other estimators (see, figure (5)). Thus we may infer that the choice of the hyper parameters c and m should be such that the prior variance is not very small and actual value of λ is close to the prior mean. A similar trend can be noted for under estimation case (when c_1 is negative), see figures (8) and (9).

While studying the effect of variation of λ on the risk of estimators, it was noted that as λ increases in general risk increases (see, figures (7) and (11)). It may further be seen that the proposed estimator $\hat{\lambda}_G$ has smaller risk for small values of λ which are expected value of parameter under the assumed hyper parameters. It was also observed that as α increases, the risk decreases (see, figures (6) and (10)).

The performance of the proposed estimator $\hat{\lambda}_G$ under SELF is such that it performs better than mle in most of the cases. In over estimation situation, the proposed estimator $\hat{\lambda}_G$ performs better than $\hat{\lambda}_S$ for lower value of $m \le 3$, all c, all choices of α and lower value of $\lambda = 1$. While in under estimation situation it perform better for all α and $\lambda \le 2$ when we choose smaller values of m and c.

In case of estimators of reliability, in over estimation case, under GELF, it is observed that our proposed estimator $\hat{R}_{G\lambda}$ perform well when $3 \le \lambda \le 5$ with lower values of α, m and c (see, figure (12)). While in under estimation situation it perform well for almost all values of m and c, lower values of $\alpha = 1$ and $\lambda \le 3$. Further, in some situations, not presented here under SELF, $\hat{R}_{G\lambda}$ also perform better than $\hat{R}_{S\lambda}$ and $\hat{R}_{M\lambda}$.

4.2 For estimation of α and R when λ is known

As in case of estimators of λ, we observe under GELF that the risks of estimators of α also increases with increase in the value of $|c_1|$ where all the rest parameters were fixed at some values and for all choices of $|c_1|$, risk of our proposed estimator $\hat{\alpha}_G$ is smaller than those of other estimators $\hat{\alpha}_S$ and $\hat{\alpha}_M$. Also, we see that difference in the risk of estimators increases with increase in the value of $|c_1|$. Thus, for further study we have again fixed $|c_1|=1.5$ (a moderate value).

On varying r in over estimation situation (i.e., for $c_1=1.5$), we saw that the proposed estimator $\hat{\alpha}_G$ again perform better for all choices of r keeping other parameters fixed and as r increase, the risk of the estimators decreases but very slowly.

It is further noted that, $\hat{\alpha}_G$ perform better than all other estimators for all considered values of n keeping other parameter fixed and risks of the estimators decreases with increase in the value of n. Therefore, for further study of behavior of risk of estimators, we have fixed (n,r) at $(15,12)$ as in previous subsection. In both over and under estimation situations (i.e., for fixed $|c_1|=1.5$) under GELF, we again get $\hat{\alpha}_G$ perform better for all model parameters α and λ.

Next, under SELF, in over estimation situation $\hat{\alpha}_G$ perform well again for all α and λ, while in under estimation situation, for some choices of α and λ, the proposed estimator $\hat{\alpha}_G$ perform better than $\hat{\alpha}_S$ and $\hat{\alpha}_M$.

Finally, in some of the situations, the proposed estimator $\hat{R}_{G\alpha}$ of reliability perform better than $\hat{R}_{S\alpha}$ and $\hat{R}_{M\alpha}$ under both the losses either in over estimation situation or in under estimation situation.

5. Conclusion

The performance of proposed estimator $\hat{\lambda}_G$ in comparison to $\hat{\lambda}_S$ and $\hat{\lambda}_M$ have been discussed in the previous section. On the basis of this discussion, we may conclude that the proposed estimator $\hat{\lambda}_G$ performs better (in sense of having smaller risk) than $\hat{\lambda}_S$ and $\hat{\lambda}_M$ under both General entropy and Squared error loss functions, for certain choices of loss parameter and hyper parameters as discussed in the previous section. Similarly, from previous section we may conclude that our proposed estimator $\hat{R}_{G\lambda}$, $\hat{\alpha}_G$ and $\hat{R}_{G\alpha}$ perform well in comparison to the corresponding estimators $\hat{R}_{S\lambda}$, $\hat{R}_{M\lambda}$; $\hat{\alpha}_S$, $\hat{\alpha}_M$ and $\hat{R}_{S\alpha}$, $\hat{R}_{M\alpha}$ respectively even in both the losses for certain choices of loss parameter and hyper parameters. Thus, the uses of proposed estimators $\hat{\lambda}_G$, $\hat{R}_{G\lambda}$, $\hat{\alpha}_G$ and $\hat{R}_{G\alpha}$ are recommended, even if quadratic loss function seems to be justified loss function.

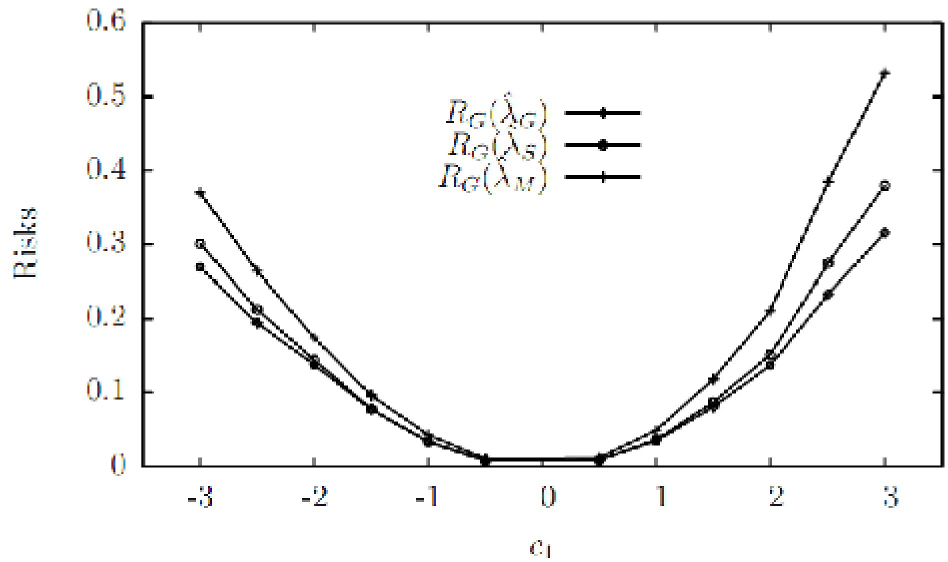

Figure 1: Risks of estimators of λ under GE loss function for fixed $n = 15, r = 12, m = 1, c = 1, \alpha = 1, \lambda = 1$

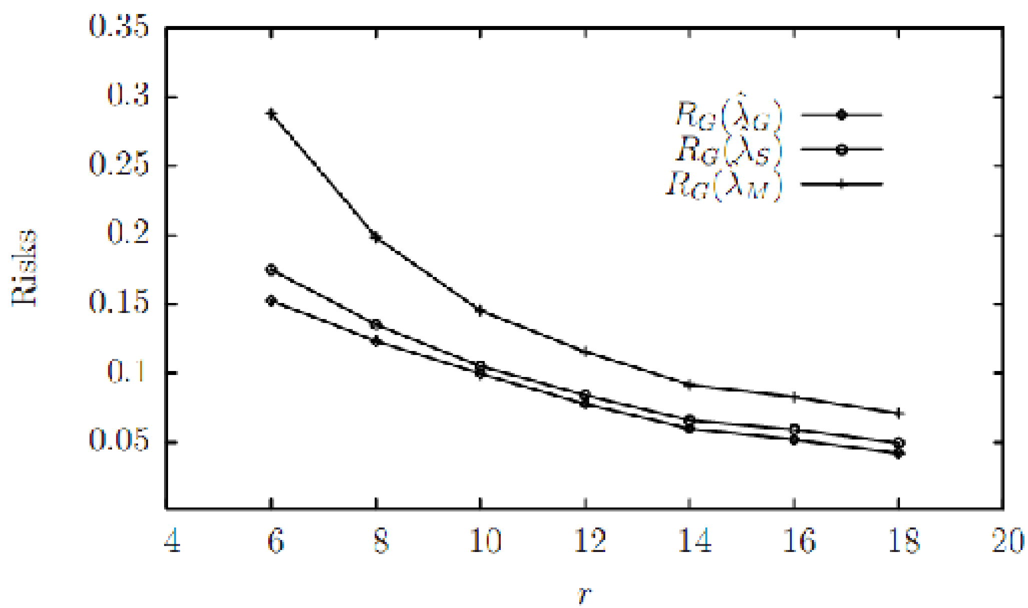

Figure 2: Risks of estimators of λ under GE loss function for fixed $n = 20, m = 1, c = 1, \alpha = 1, \lambda = 1, c_1 = +1.5$

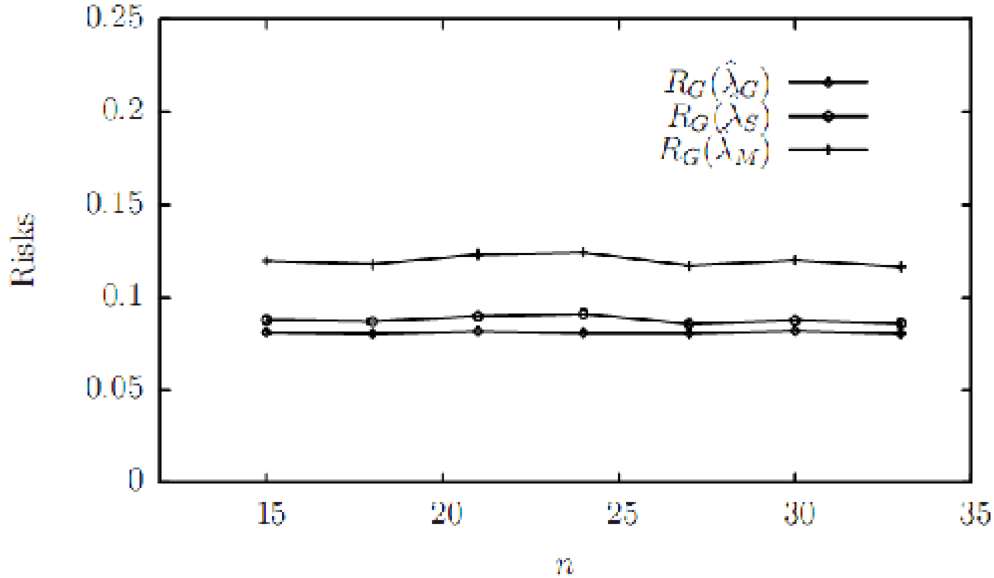

Figure 3: Risks of estimators of λ under GE loss function for fixed $r=12, m=1, c=1, \alpha=1, \lambda=1, c_1=1.5$

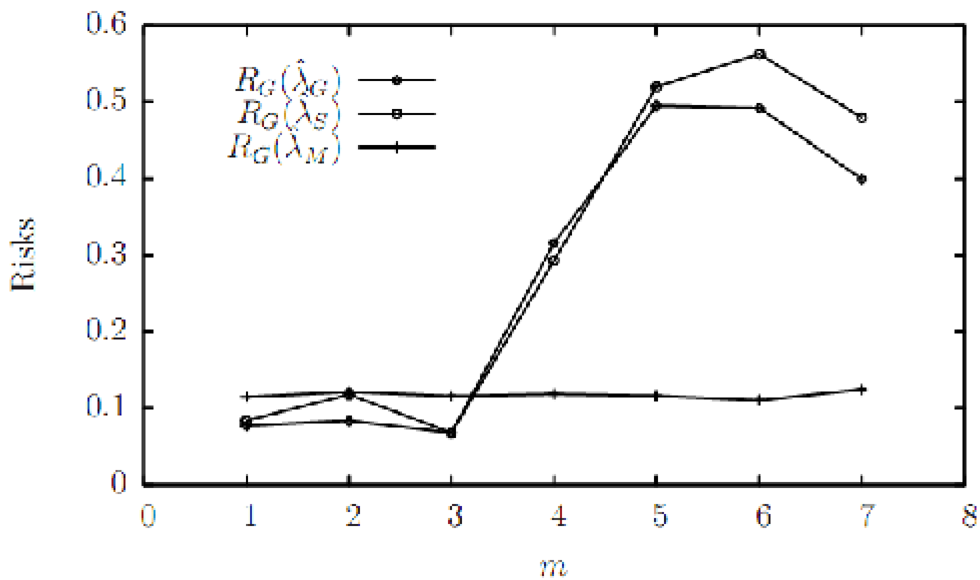

Figure 4: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, c=1, \alpha=1, \lambda=1, c_1=1.5$

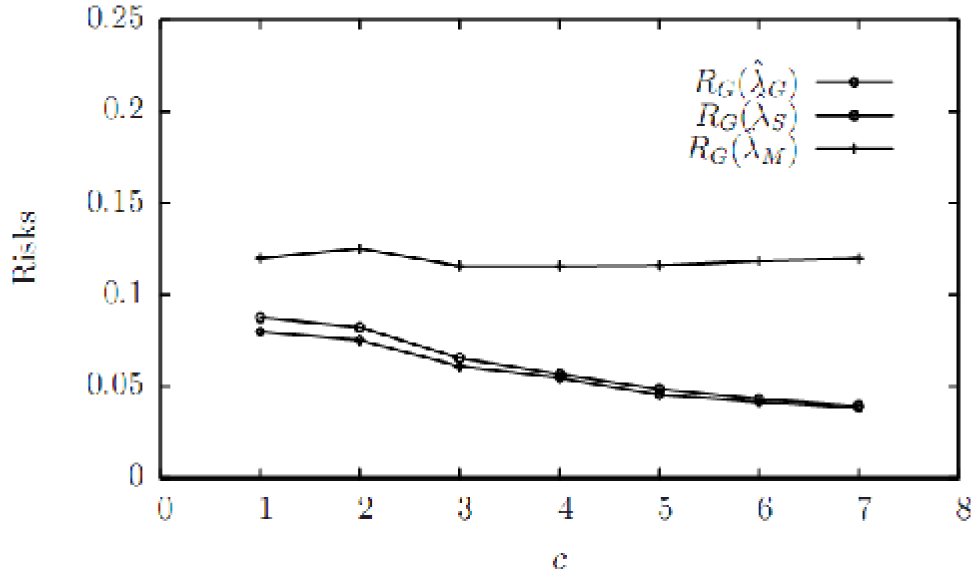

Figure 5: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, \alpha=1, \lambda=1, c_1=1.5$

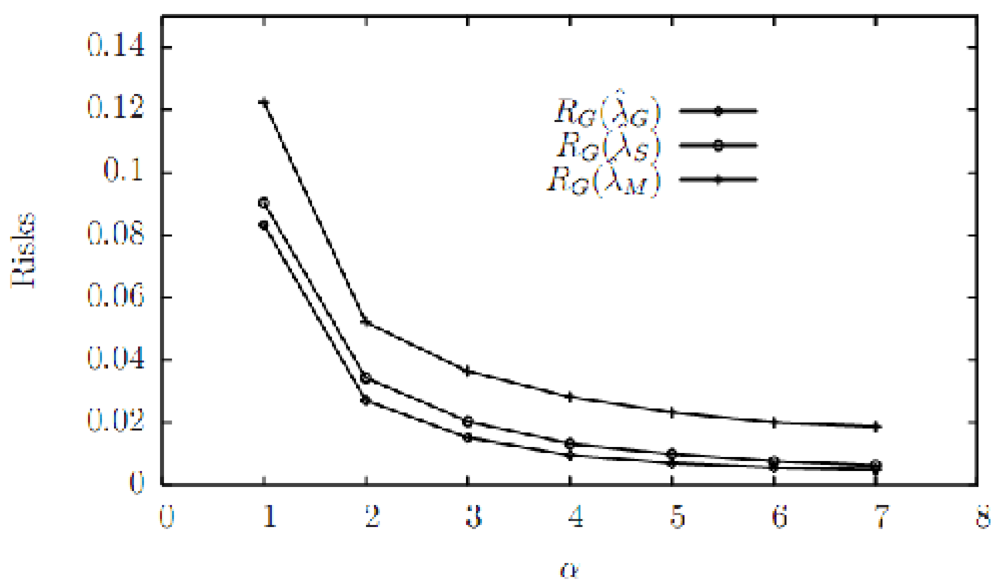

Figure 6: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, c=1, \lambda=1, c_1=1.5$

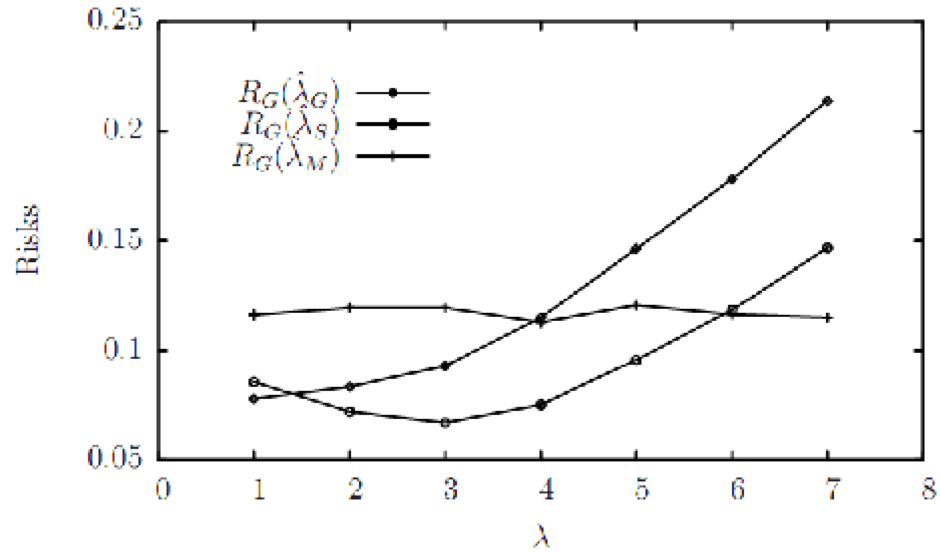

Figure 7: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, c=1, \alpha=1, c_1=1.5$

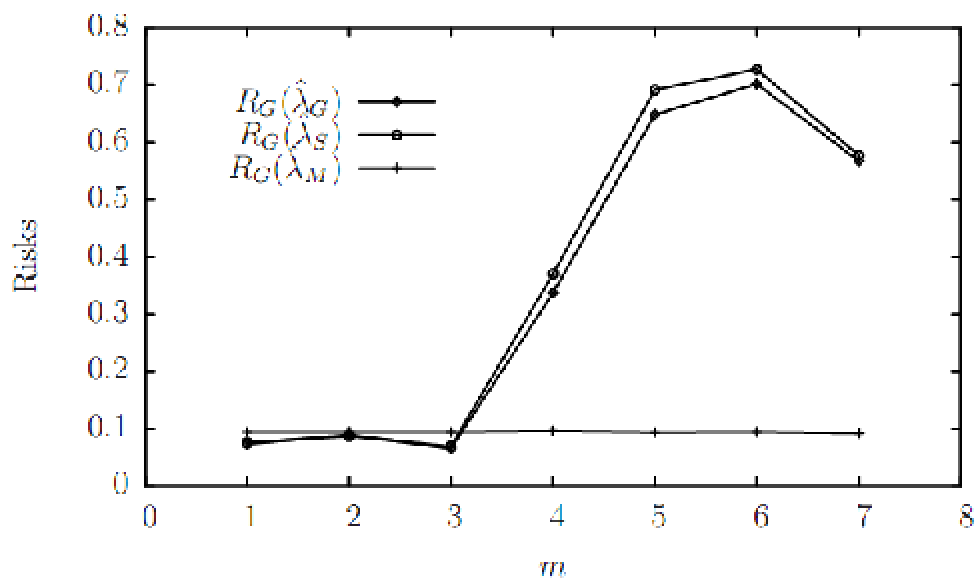

Figure 8: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, c=1, \alpha=1, \lambda=1, c_1=-1.5$

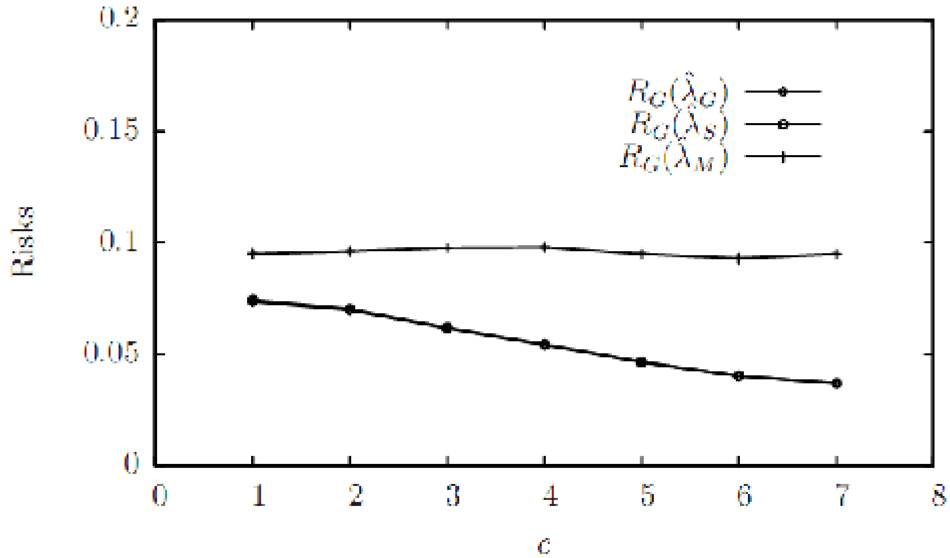

Figure 9: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, \alpha=1, \lambda=1, c_1=-1.5$

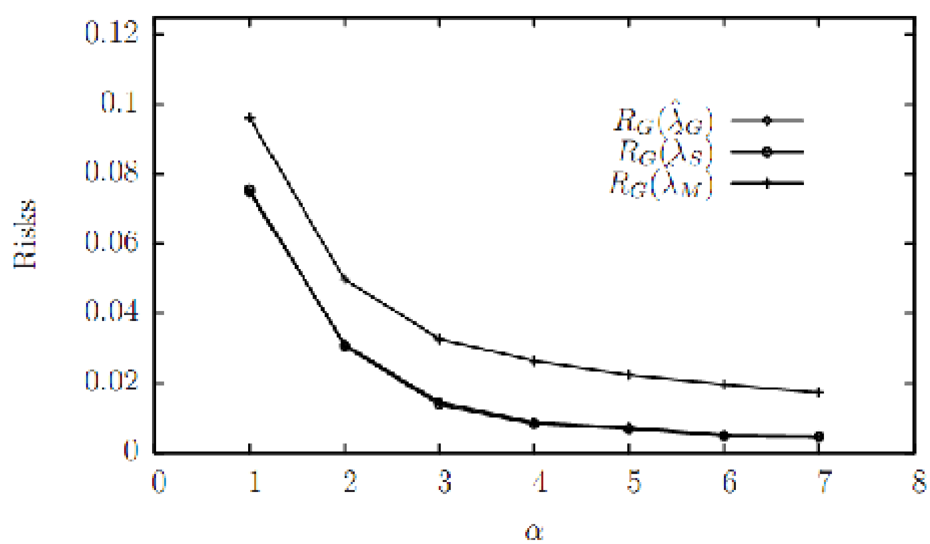

Figure 10: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, c=1, \lambda=1, c_1=-1.5$

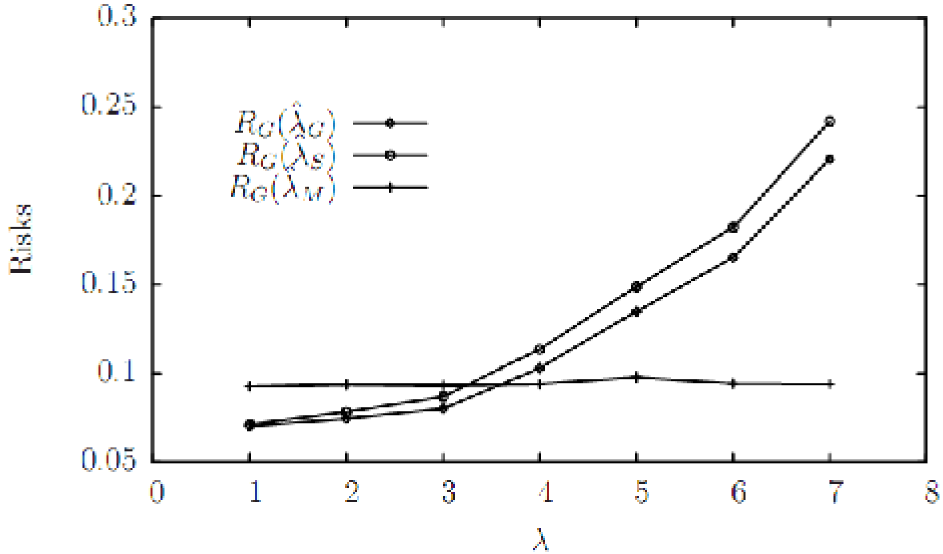

Figure 11: Risks of estimators of λ under GE loss function for fixed $n=15, r=12, m=1, c=1, \alpha=1, c_1=-1.5$

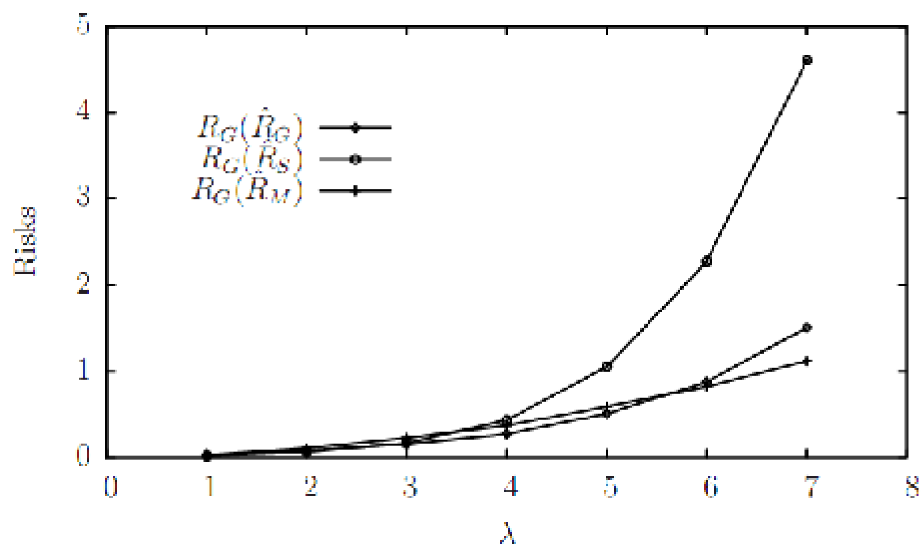

Figure 12: Risks of estimators of $R(t)$ under GE loss function for fixed $n=15, r=12, m=1, c=1, \alpha=1, c_1=1.5, t=0.5$

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