Stratified Two-Phase Ranked Set Sampling

Raghunath Arnab
Department of Statistics, University of Botswana and Department of Statistics, University of KwaZulu-Natal, South Africa
Email: arnabr@mopipi.ub.bw

George Anderson
Department of Computer Science, University of Botswana, Botswana
Email: andersong@mopipi.ub.bw

John. O. Olaomi
Department of Statistics, University of South Africa, South Africa
Email: olaomjo@unisa.ac.za

B. C. Rodríguez
Department of Statistics and Operations Research, Universidad de Granada, Spain
Email: beacr@ugr.es

Abstract

We propose an alternative two-phase stratified ranked set sampling. A comparison of the performances of the proposed estimators made by simulation studies using both real and simulated data sets. It is found that the proposed two-phase stratified regression estimator beats its competitors in literature.

AMS Subject Classification: 62D05

Keywords: Order statistics, ranked set sampling, relative efficiency, sampling with replacement, two-phase sampling.

1. Introduction

Ranked set sampling (RSS) introduced by McIntyre (1952), was used to estimate the mean pasture and forage yield. The RSS is employed when precise measurement of the variable of interest is difficult or expensive, but one can easily rank the variable without measuring the variable by an inexpensive method such as visual perception, judgment and auxiliary information. For example, in the problem of estimating the mean height of trees in a forest, one can rank the heights of a small sample of two or three trees standing nearby easily by visual inspection without measuring them. In estimating the number of bacterial cells per unit volume, we can rearrange two or three test tubes easily in order of concentration using optical instruments without measuring exact values. In the RSS, instead of selecting a single sample of size m, we select m- sets of samples each of size m. In each set, we rank all the elements but we only measure one of them. Finally, the average of the m- measured units is taken as an estimate of the population mean.

Takahashi and Wakimoto (1968) and Takahasi (1970) provided the theoretical justification for using the ranked set sampling. They proved that when the ranking is perfect, the sample mean of the RSS is an unbiased estimator of the population mean and the variance of the RSS mean is smaller than that of the sample mean of the simple random sampling with replacement (SRSWR) of the same size. Dell and Clutter (1972) proved that the sample mean based on the RSS is unbiased for the population mean regardless of the ranking error and it is at least as precise as the SRSWR sample mean of the same size. Stokes (1977) considered the performance of the Dell and Clutter estimator when the regression of the study variable (y) and the ranking variable (x) is linear, and y and x follow certain model. Yu and Lam (1997) proposed regression estimator when x and y follow a bivariate normal distribution and found on the basis of simulation studies that their proposed regression estimator performs better than the naive estimator, unless the correlation between x and y is low ($|\rho|< 0.4$). Kadilar et al., (2006) and Arnab and Olaomi (2015) proposed an improved estimator of mean μ_y, the population mean of the study variable y using the ranking variable as an auxiliary variable x when the population mean μ_x of x is unknown. Zamanzade and Al-Omari (2016) developed a new ranked set sampling for estimating the population mean and variance, called neoteric ranked set sampling (NRSS) under perfect and imperfect ranking conditions while Mahdizadeh and Zamanzade (2018) introduced stratified pair ranked set sampling (SPRSS) and utilized it in estimating the population mean, with some theoretical results.

In this paper, we propose two alternative estimators for two-phase sampling where in the first phase; information only on the ranking variable x is collected. Based on the observed x - values, the population is divided into a number of homogeneous strata. From each of the stratum so formed, one selects ranked set samples independently using proportional allocation. The performances of the proposed estimators are compared by simulation studies using both real tree data collected by Platt et al. (1988) and generated bivariate normal data. We found that the proposed two-phase stratified regression estimator performs better in respect of relative bias (RB) and mean-square error (MSE) than those of naïve and Yu and Lam (1997) estimators for the tree data and it behaves better in most situations for the simulated bivariate data.

1.1. Rank set sampling by SRSWR method

First, we choose a small number m (set size) such that one can easily rank the m elements of the population with sufficient accuracy. Then the selection of RSS is as follows: Select a sample of m^2 units from a population U by SRSWR method. Allocate these m^2 units at random into m sets each of size m. Rank all the units in a set with respect to the values of the variable of interest y from 1 (minimum) to m (maximum) by a very inexpensive method such as eye inspection. At this stage, no actual measurement is done. After the ranking has been completed, the unit holding rank i ($i = 1,..,m$) in the i th set is actually measured. This completes a cycle of the sampling. One repeats the process for r cycles to obtain the desired sample of size $n = mr$. Thus, in a RSS, a total of $m^2 r$ units are drawn from the population but only mr of them are measured and the rest $mr(m-1)$ are discarded. We call these measured mr observations "ranked set sample". Since the ordering of a large number of observations is difficult, increase of sample size $n = mr$ is done by increasing the number of cycles r. It is well known that $\hat{\mu}_{rss}(m,r)$, the sample mean the RSS of size $n = mr$ is unbiased for the population mean μ_y.

1.2. Judgment ranking

Sometimes, perfect ranking (no error in ranking) is not possible. In such cases, we use judgment ranking where each of the selected samples is ranked by an approximate method such as visual inspection, expert opinion or use of concomitant variable. It should be noted that some tests have been developed in the literature to assess the assumption of perfect ranking in RSS. Some of these include Frey, et al (2007), Zamanzade, et al (2012) and Zamanzade and Vock (2018).

Let $y_{i\langle j\rangle|k}$ be the smallest *j* th "judgment order statistic" corresponding to order statistic $x_{i(j)|k}$ of the concomitant variable x in the *i* th set of the cycle *k*. In case the judgment ranking is perfect $y_{i\langle j\rangle|k}$ becomes equal to the j th order statistic $y_{i(j)|k}$, otherwise if the judgment process is imperfect, we find $y_{i\langle j\rangle|k} \neq y_{i(j)|k}$.

Stokes (1977) derived the following results:

Theorem 1.

(i) $\hat{\mu}_{\langle rss\rangle} = \frac{1}{r}\sum_{k=1}^{r}\bar{y}_{\langle m\rangle|k}$ is an unbiased estimator for μ_y.

(ii) The variance of $\hat{\mu}_{\langle rss\rangle}$ is

$$V\left(\hat{\mu}_{\langle rss\rangle}\right) = \frac{1}{n}\left[\sigma_y^2 - \frac{1}{m}\sum_{j=1}^{m} \tau_{\langle j\rangle|m}^2\right]$$

(iii) An unbiased estimator of the variance of $V\left(\hat{\mu}_{\langle rss\rangle}\right)$ is

$$\hat{V}\left(\hat{\mu}_{\langle rss\rangle}\right) = \frac{1}{r(r-1)}\sum_{k=1}^{r}\left(\bar{y}_{\langle m\rangle|k} - \hat{\mu}_{\langle rss\rangle}\right)^2$$

1.3. Use of auxiliary variable with known mean μ_x

Stokes (1977) considered the linear regression of y on x as follows:

$$y = \mu_y + B\left(x - \mu_x\right) + \in \quad (1)$$

where x and $\in$ are independent random variables, $E(\in|\, x) = 0$, $V(\in|\, x) = \sigma_y^2(1-\rho^2)$, $B = \rho\frac{\sigma_y}{\sigma_x}$ and ρ is the correlation coefficient between x and y.

The equation (1) yields

$$y_{i\langle i\rangle|k} = \mu_y + B\left(x_{i(i)|k} - \mu_x\right) + \in_{i(i)|k} \quad (2)$$

Stokes (1977) further assumed that $\frac{y-\mu_y}{\sigma_y}$ and $\frac{x-\mu_x}{\sigma_x}$ have the same marginal distribution, which holds for bivariate normal and bivariate Pareto distributions. Stokes (1977) derived the following Theorem:

Theorem 2.

(i) $E\left(\hat{\mu}_{y\langle rss\rangle}\right) = \mu_y$

(ii) $V\left(\hat{\mu}_{y\langle rss\rangle}\right) = \frac{1}{mr}\left[\sigma_y^2 - \frac{\rho^2}{m}\sum_{j=1}^{m}\tau_{y\langle j\rangle|m}^2\right]$

where $\hat{\mu}_{y\langle rss\rangle} = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m} y_{j\langle j\rangle|k}$ and $\tau_{y\langle j\rangle|m} = E\left(y_{i\langle j\rangle|k}\right) - \mu_y$

Following Stokes (1977), Yu and Lam (1997) proposed the following estimator of the population mean μ_y of y as

$$\hat{\mu}_{yreg} = \hat{\mu}_{y\langle rss\rangle} - \hat{B}\left(\hat{\mu}_{x(rss)} - \mu_x\right) \tag{3}$$

where

$$\hat{\mu}_{x(rss)} = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m} x_{j(j)|r} \text{ and } \hat{B} = \frac{\sum_{k=1}^{r}\sum_{j=1}^{m}\left(y_{j\langle j\rangle|k} - \hat{\mu}_{y\langle rss\rangle}\right)\left(x_{j(j)|k} - \hat{\mu}_{x(rss)}\right)}{\sum_{k=1}^{r}\sum_{j=1}^{m}\left(x_{j(j)|k} - \hat{\mu}_{x(rss)}\right)^2} \tag{4}$$

Yu and Lam (1997) derived the following theorem:

Theorem 3.

(i) $E\left(\hat{\mu}_{yreg}\right) = \mu_y$

(ii) The optimum value of $\hat{B}$ that minimizes the variance of $\hat{\mu}_{yreg}$ is B.

$$(iii)\ Var\left(\hat{\mu}_{yreg}\right) = \frac{\sigma_y^2\left(1-\rho^2\right)}{mr}\left[1 + E\left(\frac{\bar{Z}_{(rss)}^2}{S_z^2}\right)\right] = \frac{\sigma_y^2\left(1-\rho^2\right)}{mr}\left[1 + E\left(\frac{\left(\bar{x}_{(rss)} - \mu_x\right)^2}{s_{x(rss)}^2}\right)\right]$$

where

$$\bar{Z}_{(rss)} = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m} z_{j(j)|k} = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m}\frac{x_{j(j)|k} - \mu_x}{\sigma_x} = \frac{\bar{x}_{(rss)} - \mu_x}{\sigma_x}$$

$$S_z^2 = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m}\left(z_{j(j)|k} - \bar{Z}_{(rss)}\right)^2 = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m}\left(\frac{x_{j(j)|k} - \bar{x}_{(rss)}}{\sigma_x}\right)^2 = s_{x(rss)}^2 / \sigma_x^2 \text{ and}$$

$$s_{x(rss)}^2 = \frac{1}{mr}\sum_{k=1}^{r}\sum_{j=1}^{m}\left(x_{j(j)|k} - \bar{x}_{(rss)}\right)^2 .$$

1.4. Population mean with unknown μ_x

Since μ_x is unknown, Yu and Lam (1997) considered a two-phase sampling procedure where in the first-phase, a relatively large sample $\tilde{s}$ of size $\tilde{n}$ is selected by the simple random sampling without replacement (SRSWOR) method from a population of size N and only information on the auxiliary variable x is collected. On the second-phase, a sub-sample s of size $n(=rm)$ is selected from $\tilde{s}$ using ranked set sampling with r cycles and information of study variable y is obtained using x as ranking variable. The proposed estimator for the population mean μ_y is

$$\hat{\mu}_{YM} = \hat{\mu}_{y\langle rss\rangle} - \hat{B}\left(\hat{\mu}_{x(rss)} - \bar{x}'\right) \tag{5}$$

where $\bar{x}' = \sum_{i\in\tilde{s}} x_i / \tilde{n}$ and $\hat{B}$ is defined in (4).

Yu and Lam (1997) derived the following results for large N and assuming the model (2) holds.

Theorem 4.

(i) $E\left(\hat{\mu}_{YM}\right) = \mu_y$

(ii) The optimum value of $\hat{B}$ that minimizes the variance of $\hat{\mu}_{YM}$ is B.

$$(iii)\ Var\left(\hat{\mu}_{YM}\right) = \frac{\sigma_y^2\left(1-\rho^2\right)}{mr}\left[1+E\left\{\frac{\left(\bar{Z}_{(rss)}-\bar{Z}\right)^2}{S_z^2}\right\}\right]+\frac{\rho^2\sigma_y^2}{\tilde{n}}$$

$$= \frac{\sigma_y^2\left(1-\rho^2\right)}{mr}\left[1+E\left\{\frac{\left(\bar{x}_{(rss)}-\bar{x}'\right)^2}{s_{x(rss)}^2}\right\}\right]+\frac{\rho^2\sigma_y^2}{\tilde{n}} \qquad (6)$$

where $\bar{Z}=(\bar{x}'-\mu_x)/\sigma_x$, $\bar{Z}_{(rss)}$, $\bar{x}_{(rss)}$, S_Z^2 and $s_{x(rss)}^2$ are defined as in Theorem 3.

2. Two-phase stratified ranked set sampling

Initially, a relatively large sample $\tilde{s}$ of size $\tilde{n}$ is selected from the entire population by SRSWOR method. From each of the selected units of $\tilde{s}$, information only on the concomitant variable x is obtained similar to Yu and Lam (1977) in two-phase sampling. Here, we assume that the condition of two-phase sampling is valid i.e. the cost of collecting data on x is much cheaper than that of the study variable y. Observing the values of x, the sampled units are classified into a number of strata H so that each of the stratum becomes homogeneous with respect to the variable under study y. The number of strata will certainly depend on the characteristics of the variable y and sample size $\tilde{n}$. For example, noting eye estimates of heights or date of plantation, one can classify the plants as small, medium or big. Similarly, noting the CD counts of HIV patients, we may classify the conditions of the HIV infected patients into bad, very bad and severe. Let $\tilde{s}_h$ be the set of units of size $\tilde{n}_h$ falling in the h th stratum. Here $\tilde{n}_h$ is a random variable taking values from 0 to min $(\tilde{n}_h, N_h)$ where N_h is the total number of the units in the h th stratum of the population. From the sampled $\tilde{n}_h$ units of the h th stratum, a sub-sample of $s_h(\subset \tilde{s}_h)$ of size $n_h=\gamma_h\tilde{n}_h=mr_h$ units is selected using a ranked-set sampling procedure with r_h cycles of set-size m each using the x as ranking variable where $\gamma_h(0<\gamma_h<1)$ are pre-determined fractions (vide Rao, 1973). Here we assume that (i) $\tilde{n}$ is so large that $P(\tilde{n}_h \geq 1)=1$ and (ii) r_h are integers. For proportional allocation with fixed sample size $n\left(=\sum_h n_h\right)$, $\gamma_h=\gamma=n/\tilde{n}$ and $n_h=\tilde{n}_h n/\tilde{n}=mr_h$.

Let the ranked set data collected from the h stratum be denoted by

$$d_h=\left\{\left(y_{j\langle j\rangle|t_h}^{(h)}, x_{j(j)|t_h}^{(h)}\right); j=1,..,m;\ t_h=1,..,r_h\right\};\ h=1,...,H \qquad (7)$$

where $x_{j(j)|t_h}^{(h)}$ is the j th ordered statistics for the concomitant variable x of the j th set of t_h th cycle of the h th stratum and $y_{j\langle j\rangle|t_h}^{(h)}$ be the corresponding judgment order statistic for the study variable for y.

2.1. Estimator of the population mean without using auxiliary variable at the estimation stage

We propose the estimator for the population mean μ_y without assuming any auxiliary information at the estimation stage as follows:

$$\hat{\mu}_{yrss}^{st}=\sum_{h=1}^{H}w_h\hat{\mu}_{y\langle rss\rangle}^{h} \tag{8}$$

where $w_h=\frac{\tilde{n}_h}{\tilde{n}}$ and $\hat{\mu}_{y\langle rss\rangle}^{h}=\frac{1}{mr_h}\sum_{t_h=1}^{r_h}\sum_{j=1}^{m}y_{j\langle j\rangle|t_h}^{(h)}$.

Theorem 5.

(i) μ_{yrss}^{st} is an unbiased estimator of μ_y

(ii) $V\left(\hat{\mu}_{yrss}^{st}\right)=\frac{1}{\tilde{n}}\sum_{h=1}^{H}W_h\left[\frac{1}{\gamma_h}\left(\sigma_{hy}^2-\frac{1}{m}\sum_{j=1}^{m}\tau_{h\langle j\rangle|m}^2\right)+\left(\mu_{hy}-\mu_y\right)^2\right]$

where μ_{hy} and σ_{hy}^2 are the population mean and variance of y for the stratum h, $\tau_{h\langle j\rangle|m}=\mu_{h\langle j\rangle|m}-\mu_{hy}$, $\mu_{h\langle j\rangle|m}=E\left(y_{j\langle j\rangle|t_h}^{h}\right)$ and W_h is the proportion of units in the population of the hth stratum.

Proof:

Since $\tilde{n}_h$, the number of units in n that fall into h th stratum is a random variable, we have from Rao (1973)

$E\ w_h\ =W_h$, $V\ w_h\ =\frac{W_h\ 1-W_h}{n}$, $Cov\ w_h,w_{h'}\ =-\frac{W_hW_{h'}}{n}$, $E\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)=\mu_{hy}$ and

$$V\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)=\frac{1}{n_h}\left(\sigma_{hy}^2-\frac{1}{m}\sum_{j=1}^{m}\tau_{h\langle j\rangle|m}^2\right) \tag{9}$$

Using (9), we get the following:

(i) $E\left(\hat{\mu}_{y\langle rss\rangle}^{st}\right)=E\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)\right)=E\left(\sum_{h=1}^{H}w_h\mu_{hy}\right)=\sum_{h=1}^{H}W_h\mu_{hy}=\mu_y$

$$\text{(ii)}\ V\left(\hat{\mu}_{y\langle rss\rangle}^{st}\right)=E\left(\sum_{h=1}^{H}w_h^2V\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)\right)+V\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)\right) \tag{10}$$

Now using Theorem 1, we find

$$E\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)=\mu_{hy}\ \text{and}\ V\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)=\frac{1}{n_h}\left(\sigma_{hy}^2-\frac{1}{m}\sum_{j=1}^{m}\tau_{h\langle j\rangle|m}^2\right) \tag{11}$$

The first component of (10) is obtained from (11) as

$$E\left(\sum_{h=1}^{H}w_h^2V\left(\hat{\mu}_{y\langle rss\rangle}^{h}\mid\tilde{n}_h\right)\right)=E\sum_{h=1}^{H}\left(\frac{\tilde{n}_h}{\tilde{n}}\right)^2\frac{1}{\gamma_h\tilde{n}_h}\left(\sigma_{hy}^2-\frac{1}{m}\sum_{j=1}^{m}\tau_{h\langle j\rangle|m}^2\right)\quad\text{(noting, } n_h=\gamma_h\tilde{n}_h\text{)}$$

$$=\frac{1}{\tilde{n}}\sum_{h=1}^{H}\frac{W_h}{\gamma_h}\left(\sigma_{hy}^2-\frac{1}{m}\sum_{j=1}^{m}\tau_{h\langle j\rangle|m}^2\right) \tag{12}$$

The second component of (10) is

$$V\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{yreg}^{h}\mid\tilde{n}_h\right)\right)=V\left(\sum_{h=1}^{H}w_h\mu_{hy}\right)=\sum_{h=1}^{H}\mu_{hy}^2V\left(w_h\right)+\sum_{h\neq}^{H}\sum_{h'=1}^{H}\mu_{hy}\mu_{h'y}Cov\left(w_h,w_h^{'}\right) \tag{13}$$

Now using (9), we get

$$V\left(\sum_{h=1}^{H} w_h E\left(\hat{\mu}_{yreg}^{h} \mid \tilde{n}_h\right)\right) = \frac{1}{\tilde{n}}\left[\sum_{h=1}^{H} W_h \mu_{hy}^2 - \left(\sum_{h=1}^{H} W_h \mu_{hy}\right)^2\right] = \frac{1}{\tilde{n}}\left[\sum_{h=1}^{H} W_h \left(\mu_{hy} - \mu_y\right)^2\right] \quad (14)$$

Part II of the theorem follows from (10), (12) and (14).

Corollary 1.

For proportional allocation with fixed sample size n at the second phase, $V\left(\hat{\mu}_{yreg}^{st}\right)$ reduces to

$$V\left(\hat{\mu}_{yrss}^{st}\right) = \sum_{h=1}^{H} W_h \left[\frac{1}{n}\left(\sigma_{hy}^2 - \frac{1}{m}\sum_{j=1}^{m} \tau_{h\langle j\rangle|m}^2\right) + \frac{1}{\tilde{n}}\left(\mu_{hy} - \mu_y\right)^2\right]$$

2.2. Using auxiliary information at the estimation stage

Noting $\hat{\mu}_{y\langle rss\rangle}^{h} = \frac{1}{mr_h}\sum_{t_h=1}^{k_h}\sum_{j=1}^{m} y_{j\langle j\rangle|t_h}^{(h)}$ and $\hat{\mu}_{x(rss)}^{h} = \frac{1}{mr_h}\sum_{t_h=1}^{k_h}\sum_{j=1}^{m} x_{j(j)|t_h}^{(h)} = \bar{x}_{h(rss)}$ are unbiased estimators of the population means μ_{hy} and μ_{hx} of y and x of the stratum h respectively, we propose the following regression estimator of the population mean μ_y of y as

$$\hat{\mu}_{yreg}^{st} = \sum_{h=1}^{H} w_h \hat{\mu}_{yreg}^{h}$$

where $w_h = \tilde{n}_h / \tilde{n}$, $\hat{\mu}_{yreg}^{h} = \hat{\mu}_{y\langle rss\rangle}^{h} - \hat{B}^{h}\left(\hat{\mu}_{x(rss)}^{h} - \bar{x}_h^{'}\right)$, $\bar{x}_h^{'} = \frac{1}{\tilde{n}_h}\left(\sum_{i\in\tilde{s}_h} x_i\right)$ and

$$\hat{B}^{h} = \frac{\sum_{t_h=1}^{r_h}\sum_{j=1}^{m}\left(y_{j\langle j\rangle|t_h}^{(h)} - \hat{\mu}_{y\langle rss\rangle}^{h}\right)\left(x_{j(j)|t_h}^{(h)} - \hat{\mu}_{x\langle rss\rangle}^{h}\right)}{\sum_{t_h=1}^{r_h}\sum_{j=1}^{m}\left(x_{j(j)|t_h}^{(h)} - \hat{\mu}_{x\langle rss\rangle}^{h}\right)^2}$$

Assume that $y_i^h\left(x_i^h\right)$, is the value of $y(x)$ for the *i* th unit of the $h(=1,...,H)$ th stratum follows the model.

$$y_i^h = \mu_{hy} + B_h\left(x_i^h - \mu_{hx}\right) + \epsilon_i^h \quad (15)$$

where x_i^h and ϵ_i^h are independent random variables; $E\left(x_i^h\right) = \mu_{hx}$, $V\left(x_i^h\right) = \sigma_{hx}^2$, $E\left(y_i^h\right) = \mu_{hy}$, ; $V\left(y_i^h\right) == \sigma_{hy}^2$; $E(\epsilon_i^h \mid x_i^h) = 0$, $V(\epsilon_i^h \mid x_i^h) = \sigma_{hy}^2(1-\rho_h^2)$, $B_h = \rho_h \frac{\sigma_{hy}}{\sigma_{hx}}$, and ρ_h is the correlation coefficient between x and y of the h th stratum.

Theorem 6.

Under the model (15)

(i) μ_{yreg}^{st} is an unbiased estimator of μ_y

(ii) $V\left(\hat{\mu}_{yreg}^{st}\right) = \frac{1}{\tilde{n}}\sum_{h=1}^{H} W_h\left[\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{\gamma_h}\left\{1 + E\left(\frac{\left(\bar{Z}_{hrss} - \bar{Z}_h\right)^2}{S_{hz}^2}\right)\right\} + \frac{\rho_h^2\sigma_{hy}^2}{m\gamma_h}\right] + \frac{1}{\tilde{n}}\left[\sum_{h=1}^{H} W_h\left(\mu_{hy} - \mu_y\right)^2\right]$

$$=\frac{1}{\tilde{n}}\sum_{h=1}^{H}W_h\left[\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{\gamma_h}\left\{1+E\left(\frac{\left(\bar{x}_{h(rss)}-\bar{x}_h^{'}\right)^2}{s_{hx(rss)}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m\gamma_h}\right]+\frac{1}{\tilde{n}}\left[\sum_{h=1}^{H}W_h\left(\mu_{hy}-\mu_y\right)^2\right]$$

where $\bar{Z}_{h(rss)}=\frac{1}{mr_h}\sum_{k=1}^{r_h}\sum_{j=1}^{m}z_{j(j)|k}^{h}$, $S_{hz}^2=\frac{1}{mr_h}\sum_{k=1}^{r_h}\sum_{j=1}^{m}\left(z_{j(j)|k}^{h}-\bar{Z}_{h(rss)}\right)^2$,

$s_{hx(rss)}^2=\frac{1}{mr_h}\sum_{k=1}^{r_h}\sum_{j=1}^{m}\left(x_{j(j)|k}^{h}-\bar{x}_{h(rss)}\right)^2$, $z_{j(j)|k}^{h}=\frac{x_{j(j)|k}^{h}-\mu_{hx}}{\sigma_{hx}}$, $\bar{Z}_h=\frac{\bar{x}_h^{'}-\mu_{hx}}{\sigma_{hx}}$ and $\bar{x}_h^{'}=\frac{1}{\tilde{n}_h}\sum_{i\in\tilde{s}_h}x_{hi}$.

Proof:

(i) $E\left(\hat{\mu}_{yreg}^{st}\right)=E\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{yreg}^{h}\mid\tilde{n}_h\right)\right)=E\left(\sum_{h=1}^{H}w_h\mu_{hy}\right)=\sum_{h=1}^{H}W_h\mu_{hy}=\mu_y$

(ii) $$Var\left(\hat{\mu}_{yreg}^{st}\right)=E\left(\sum_{h=1}^{H}w_h^2V\left(\hat{\mu}_{yreg}^{h}\mid\tilde{s}_h\right)\right)+V\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{yreg}^{h}\mid\tilde{s}_h\right)\right)\qquad(16)$$

Now using (6), we find

$$V\left(\hat{\mu}_{yreg}^{h}\mid\tilde{s}_h\right)=\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{mr_h}\left[1+E\left(\frac{\left(\bar{Z}_{hrss}-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right]+\frac{\rho_h^2\sigma_{hy}^2}{r_hm^2}$$

$$=\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{n_h}\left[1+E\left(\frac{\left(\bar{Z}_{hrss}^2-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right]+\frac{\rho_h^2\sigma_{hy}^2}{mn_h}\qquad(17)$$

Equation (17) yields the first component of (16) as

$$E\left(\sum_{h=1}^{H}w_h^2V\left(\hat{\mu}_{yreg}^{h}\mid\tilde{n}_h\right)\right)=E\sum_{h=1}^{H}\left(\frac{\tilde{n}_h}{\tilde{n}}\right)^2\left[\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{\gamma_h\tilde{n}_h}\left\{1+E\left(\frac{\left(\bar{Z}_{hrss}-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m\gamma_h\tilde{n}_h}\right]$$

$$=\frac{1}{\tilde{n}}\sum_{h=1}^{H}W_h\left[\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{\gamma_h}\left\{1+E\left(\frac{\left(\bar{Z}_{hrss}^2-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m\gamma_h}\right]\qquad(18)$$

$$=\frac{1}{\tilde{n}}\sum_{h=1}^{H}W_h\left[\frac{\sigma_{hy}^2\left(1-\rho_h^2\right)}{\gamma_h}\left\{1+E\left(\frac{\left(\bar{Z}_{hrss}^2-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m\gamma_h}\right]$$

The second component of (16) is obtained from (14) as

$$V\left(\sum_{h=1}^{H}w_hE\left(\hat{\mu}_{yreg}^{h}\mid\tilde{n}_h\right)\right)=V\left(\sum_{h=1}^{H}w_h\mu_{hy}\right)=\frac{1}{\tilde{n}}\left[\sum_{h=1}^{H}W_h\left(\mu_{hy}-\mu_y\right)^2\right]\qquad(19)$$

Part II of the theorem follows from (16), (18) and (19).

Corollary 2.

For proportional allocation with fixed sample size n, $V\left(\hat{\mu}_{yreg}^{st}\right)$ reduces to

$$V\left(\hat{\mu}_{yreg}^{st}\right)=\frac{1}{n}\sum_{h=1}^{H}W_h\left[\sigma_{hy}^2\left(1-\rho_h^2\right)\left\{1+E\left(\frac{\left(\bar{Z}_{hrss}-\bar{Z}_h\right)^2}{S_{hz}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m}\right]+\frac{1}{\tilde{n}}\left[\sum_{h=1}^{H}W_h\left(\mu_{hy}-\mu_y\right)^2\right]$$

$$=\frac{1}{n}\sum_{h=1}^{H}W_h\left[\sigma_{hy}^2\left(1-\rho_h^2\right)\left\{1+E\left(\frac{\left(\bar{x}_{h(rss)}-\bar{x}_h^{'}\right)^2}{s_{hx(rss)}^2}\right)\right\}+\frac{\rho_h^2\sigma_{hy}^2}{m}\right]+\frac{1}{\tilde{n}}\left[\sum_{h=1}^{H}W_h\left(\mu_{hy}-\mu_y\right)^2\right]$$

where $\bar{Z}_{h(rss)}=\frac{1}{mr_h}\sum_{k=1}^{r_h}\sum_{j=1}^{m}z_{j(j)|k}^{h}=\frac{1}{mr_h}\sum_{k=1}^{r_h}\sum_{j=1}^{m}\frac{x_{j(j)|k}^{h}-\mu_{hx}}{\sigma_{hx}}=\frac{\bar{x}_{h(rss)}-\mu_{hx}}{\sigma_{hx}}$ and

$\bar{Z}_{hrss}-\bar{Z}_h=\frac{\bar{x}_{h(rss)}-\mu_{hx}}{\sigma_{hx}}-\frac{\bar{x}_h^{'}-\mu_{hx}}{\sigma_{hx}}=\frac{\bar{x}_{h(rss)}-\bar{x}_h^{'}}{\sigma_{hx}}$.

3. Comparison of stratified and un-stratified ranked set sampling strategies

It is very difficult to compare the performances of the proposed estimators $t_2=\hat{\mu}_{yreg}^{st}$ and $t_3=\hat{\mu}_{yrss}^{st}$ theoretically with the existing estimators $t_0=\hat{\mu}_{y\langle rss\rangle}$ (Stokes, 1977) and $t_1=\hat{\mu}_{YM}$ (Yu and Lam, 1997), since the expressions of the variances $Var(t_j);\, j=0,1,2$ involve several unknown parameters. Hence, we will compare the performances using simulation studies. For the simulation studies, we have considered both real and simulated data.

The real tree data related to the diameter in centimetres at breast height (x) and entire height (y) in feet of 396 trees which was originally collected by Platt et al. (1988) and used later by Chen et al. (2003) and Zamanzade and Mahdizadeh (2018). The mean diameter and heights of 396 (= N) trees are μ_x = 20.970 cm and μ_y = 52.967 feet respectively. The tree were portioned into two strata with diameter less than equal 13.6 cm (stratum 1) and more than 13.6 cm (stratum 2) respectively.

The number of trees belonging to the stratum 1 and stratum 2 are equal to 198. In the first-phase, a sample $\tilde{s}$ of size $\tilde{n}$ is selected from the entire population by the SRSWOR method and information only on the auxiliary variable x is collected. Let $\tilde{s}_i$ be the sample of size $\tilde{n}_i$ that falls in the stratum $i(=1,2)$. From the sample $\tilde{s}_i$, a sub-sample of size s_i of size $n_i=[n\tilde{n}_i/\tilde{n}]$ is collected using proportional allocation by RSS sampling with $m(=2,3,4)$ as set-size and n as a predetermined number. For our simulation studies we take the following combinations of $(\tilde{n},n,m)$: (250,160,2), (250,160,3), (250,160,4); (250,100,2), (250,100,3), (250,100,4); (200,125,2), (200,125,3), (200,125,4); (200,80,2), (150,80,3), (200,80,4); (150,80,2), (150,80,3), (150,80,4); (150,60,2), (150,60,3), (150,60,4).

The simulated data comprises with 5 bivariate normal populations of sizes 600 $(=N)$ each of which have the same $\mu_x=10,\ \mu_y=25,\ \sigma_x=2.5,\ \sigma_y=4.5$ but different values of $\rho\ (=0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9)$. We divide each population into two strata as the tree population. From each of the 5 populations, ranked set samples of parameters $(\tilde{n},n,m)$: (400,125,2), (400,125,3), (400,125,4); (400,80,2), (400,80,3), (400,80,4); (250,80,2), (250,80,3), (250,80,4) are selected.

From the selected sample s, we compute estimates t_0, t_1 , t_2 and t_3. We will call the process of selection of two-phase sample s and obtaining estimates from the selected sample as an iteration. The iteration is repeated $R=100,000$ times. Let the values of the t_0

, t_1 , t_2 and t_3 based on the qth iteration be denoted by $t_0(q)$, $t_1(q)$, $t_2(q)$ and $t_3(q)$ respectively. The relative biases (RB) and the mean square errors (MSE) of the estimators are computed using the following formula:

$$RB\left(t_j\right)=\frac{1}{\mu_y}\left(\frac{1}{R}\sum_{q=1}^{R}t_j(q)-\mu_y\right) \quad \text{and} \quad MSE\left(t_j\right)=\frac{1}{R}\sum_{q=1}^{R}\left(t_j(q)-\mu_y\right)^2 \quad ; \quad j=0,1,2,3 \tag{20}$$

The percentage relative efficiency of the estimator t_j compared with the conventional estimator $t_0=\hat{\mu}_{y\langle rss\rangle}$ given by

$$PRE(t_j)=100\times MSE\left(t_0\right)/MSE\left(t_j\right)\% \tag{21}$$

The values of $RB\left(t_j\right)$ and $PRE(t_j)$ are computed for the five populations with $m=2,3,4$ for the live and simulated populations and are given in the Table 1 and Table 2.

Table 1: Tree data: Relative bias (RB) and Relative Efficiency (RE)

		RB	PRE	RB	PRE	RB	PRE
		$(\tilde{n},n)$ = (250,160)		**$(\tilde{n},n)$ = (200, 125)**		**$(\tilde{n},n)$ = (150, 80)**	
2	$\hat{\mu}_{y\langle rss\rangle}$	-0.0376	100.0000	-0.0348	100.0000	-0.0012	100.0000
	$\hat{\mu}_{YM}$	-0.0493	183.6407	-0.0874	155.9507	-0.1540	150.5943
	$\hat{\mu}_{yreg}^{st}$	-0.0545	189.0201	-0.0329	160.8565	-0.0529	155.3266
	$\hat{\mu}_{yrss}^{st}$	-0.0209	113.7285	-0.0045	102.6377	-0.0014	100.5208
3	$\hat{\mu}_{y\langle rss\rangle}$	-0.0009	100.0000	-0.0311	100.0000	0.0084	100.0000
	$\hat{\mu}_{YM}$	-0.0495	155.1414	-0.0731	133.6771	-0.1249	128.3830
	$\hat{\mu}_{yreg}^{st}$	-0.0252	160.9543	-0.0303	137.5143	-0.0492	131.4918
	$\hat{\mu}_{yrss}^{st}$	-0.0233	105.9229	0.0078	96.1193	0.0126	92.7720
4	$\hat{\mu}_{y\langle rss\rangle}$	0.0253	100.0000	0.0213	100.0000	0.0234	100.0000
	$\hat{\mu}_{YM}$	-0.0715	137.5715	-0.0920	115.6215	-0.0962	111.0767
	$\hat{\mu}_{yreg}^{st}$	-0.0166	140.6034	-0.0078	119.7614	-0.0885	113.1531
	$\hat{\mu}_{yrss}^{st}$	-0.0058	98.9452	0.0124	87.2825	-0.0619	82.9030
		$(\tilde{n},n)$ = (250,100)		**$(\tilde{n},n)$= (200,80)**		**$(\tilde{n},n)$ = (150, 60)**	
2	$\hat{\mu}_{y\langle rss\rangle}$	-0.0581	100.0000	0.0100	100.0000	-0.0621	100.0000
	$\hat{\mu}_{YM}$	-0.1029	233.4535	-0.1195	202.9745	-0.2260	179.3489
	$\hat{\mu}_{yreg}^{st}$	-0.0236	239.2576	-0.0953	209.1225	-0.1061	185.3866
	$\hat{\mu}_{yrss}^{st}$	0.0012	129.7157	-0.0493	120.9196	-0.0592	113.1328
3	$\hat{\mu}_{y\langle rss\rangle}$	0.0011	100.0000	0.0201	100.0000	0.0153	100.0000
	$\hat{\mu}_{YM}$	-0.0899	198.1569	-0.0761	169.4921	-0.1678	149.3681
	$\hat{\mu}_{yreg}^{st}$	0.0150	207.1916	-0.0245	179.0657	-0.0896	153.6236
	$\hat{\mu}_{yrss}^{st}$	0.0518	123.5520	0.0001	113.6298	-0.0592	103.0698
4	$\hat{\mu}_{y\langle rss\rangle}$	-0.0186	100.0000	0.0074	100.0000	-0.0402	100.0000
	$\hat{\mu}_{YM}$	-0.0783	173.7545	-0.0743	149.7989	-0.1302	133.4380
	$\hat{\mu}_{yreg}^{st}$	-0.0212	178.7264	0.0064	155.2155	-0.0762	138.0304
	$\hat{\mu}_{yrss}^{st}$	-0.0282	113.9226	0.0439	103.7253	-0.0282	96.2106

Table 2: Bivariate Normal data: Relative bias (RB) and Relative Efficiency (RE)

m	Estimators	ρ = 0.5		ρ = 0.6		ρ = 0.7		ρ = 0.8		ρ = 0.9	
		RB	**PRE**	**RB**	**PRE**	**RB**	**PRE**	**RB**	**PRE**	**RB**	**PRE**
						$(\tilde{n}, n)$ = (400, 125)					
2	$\hat{\mu}_{y\langle rss\rangle}$	-0.007	100.000	0.000	100.000	-0.001	100.000	0.000	100.000	-0.009	100.000
	$\hat{\mu}_{YM}$	0.001	105.711	0.012	123.375	0.002	133.247	0.001	166.876	-0.002	260.631
	$\hat{\mu}_{yreg}^{st}$	0.002	106.214	-0.001	123.803	-0.016	133.173	-0.004	166.398	-0.002	260.938
	$\hat{\mu}_{yrss}^{st}$	-0.001	97.338	0.002	109.578	-0.002	109.906	0.001	122.497	-0.002	153.668
3	$\hat{\mu}_{y\langle rss\rangle}$	-0.001	100.000	0.004	100.000	-0.001	100.00	0.002	100.000	-0.006	100.000
	$\hat{\mu}_{YM}$	0.002	101.634	0.011	113.628	0.001	122.631	0.000	146.398	-0.002	211.965
	$\hat{\mu}_{yreg}^{st}$	0.003	101.947	-0.005	113.161	-0.006	122.289	-0.008	147.418	-0.004	213.146
	$\hat{\mu}_{yrss}^{st}$	0.001	95.157	-0.006	102.884	0.006	103.991	-0.004	113.718	-0.003	138.658
4	$\hat{\mu}_{y\langle rss\rangle}$	0.000	100.000	-0.006	100.000	-0.007	100.000	0.001	100.000	0.006	100.000
	$\hat{\mu}_{YM}$	0.002	100.109	0.003	109.169	0.002	115.823	0.007	135.626	-0.003	186.343
	$\hat{\mu}_{yreg}^{st}$	0.007	100.293	0.001	108.531	0.000	113.804	-0.002	135.412	-0.001	185.834
	$\hat{\mu}_{yrss}^{st}$	0.007	93.206	0.002	100.129	0.010	100.961	0.004	109.483	0.002	127.097
						$(\tilde{n}, n)$ = (400, 80)					
2	$\hat{\mu}_{y\langle rss\rangle}$	-0.004	100.000	-0.005	100.000	0.004	100.000	0.002	100.000	0.006	100.000
	$\hat{\mu}_{YM}$	-0.007	120.563	-0.011	122.744	-0.009	138.491	-0.006	179.296	0.004	280.103
	$\hat{\mu}_{yreg}^{st}$	0.008	119.793	0.017	120.831	-0.014	137.887	-0.007	177.535	0.005	278.194
	$\hat{\mu}_{yrss}^{st}$	-0.003	107.712	-0.002	107.774	-0.006	115.066	0.005	131.598	-0.006	158.69
3	$\hat{\mu}_{y\langle rss\rangle}$	-0.014	100.000	-0.001	100.000	-0.002	100.000	0.011	100.000	0.005	100.000
	$\hat{\mu}_{YM}$	0.004	113.308	-0.008	115.974	-0.005	127.063	-0.004	158.161	-0.001	233.942
	$\hat{\mu}_{yreg}^{st}$	0.006	113.476	0.010	115.598	-0.018	126.77	-0.005	157.756	0.003	233.319
	$\hat{\mu}_{yrss}^{st}$	0.000	105.135	-0.007	105.988	-0.011	109.547	0.005	122.871	-0.006	145.776
4	$\hat{\mu}_{y\langle rss\rangle}$	-0.003	100.000	0.008	100.000	-0.003	100	0.002	100.000	-0.004	100.000
	$\hat{\mu}_{YM}$	-0.005	108.406	-0.004	111.904	-0.001	121.196	-0.004	146.037	0.003	205.491
	$\hat{\mu}_{yreg}^{st}$	0.007	108.357	0.019	110.235	0.003	120.798	-0.010	144.377	0.004	203.829
	$\hat{\mu}_{yrss}^{st}$	-0.001	100.626	0.006	101.836	0.010	106.276	-0.005	116.793	0.000	133.482
						$(\tilde{n}, n)$ = (250, 80)					
2	$\hat{\mu}_{y\langle rss\rangle}$	0.000	100.000	-0.001	100.000	-0.007	100.000	-0.001	100.000	-0.007	100.000
	$\hat{\mu}_{YM}$	-0.005	98.858	0.001	105.934	-0.004	123.857	-0.004	141.428	0.005	188.716
	$\hat{\mu}_{yreg}^{st}$	-0.002	97.786	0.000	105.888	-0.013	123.717	-0.012	139.53	0.003	188.326
	$\hat{\mu}_{yrss}^{st}$	-0.005	90.903	0.001	98.253	-0.002	106.137	-0.007	108.078	-0.001	123.835
3	$\hat{\mu}_{y\langle rss\rangle}$	0.001	100.000	-0.003	100.000	0.007	100.000	0.002	100.000	0.000	100.000
	$\hat{\mu}_{YM}$	0.003	93.089	-0.010	99.610	0.002	111.199	-0.004	125.223	0.011	159.688
	$\hat{\mu}_{yreg}^{st}$	0.006	93.481	0.002	101.023	-0.013	110.429	0.005	123.969	0.006	159.736
	$\hat{\mu}_{yrss}^{st}$	0.004	88.131	0.005	95.075	0.001	97.778	0.003	101.009	0.012	113.072
	$\hat{\mu}_{y\langle rss\rangle}$	-0.003	100.000	0.003	100.000	0.006	100.000	0.000	100.000	0.004	100.000

4	$\hat{\mu}_{YM}$	-0.004	91.438	0.002	95.469	0.012	103.284	0.009	115.114	0.000	139.226
	$\hat{\mu}^{st}_{yreg}$	0.007	90.762	-0.001	95.006	-0.002	102.485	-0.003	113.655	0.001	140.013
	$\hat{\mu}^{st}_{yrss}$	0.003	86.252	0.002	89.996	0.005	92.301	0.001	95.570	0.002	103.763

3.1. Simulation Results

Relative biases of all the estimators are very low in general. For the tree data it ranges from -0.2260 to 0.0518. The relative biases for the simulated bivariate normal data is much lower than that of the tree data and it varies from -0.014 to 0.023. The estimators $\hat{\mu}_{YM}$ and $\hat{\mu}^{st}_{yreg}$ using auxiliary information possess higher relative efficiency than the naïve estimator $\hat{\mu}_{y\langle rss\rangle}$ in almost all situations. The proposed two-phase stratified regression estimator $\hat{\mu}^{st}_{yreg}$ performs the best, the next place is occupied by two-phase un-stratified regression estimator $\hat{\mu}_{YM}$. The estimator $\hat{\mu}^{st}_{yreg}$ performs the best in all situations for the tree data with a maximum PRE 239.2576. The stratified estimator $\hat{\mu}^{st}_{yrss}$ performs better than naïve estimator in general but in some isolated situations, it possesses lower efficiency (with the minimum PRE = 86.252) than the naïve estimator. For a given combination of $(\tilde{n}, n)$, PREs of all the estimators for both the tree data and simulated data decrease with m. For a given $\tilde{n}$ and m, PRE of the estimators decreases with n. The relative efficiencies of all the estimators for the simulated data increase with the correlation coefficient ρ. The Yule-estimator $\hat{\mu}_{YM}$ performs slightly better than the proposed two-phase estimator $\hat{\mu}^{st}_{yreg}$ in scanty occasions.

4. Conclusion

Stokes (1977) recommended regression estimator for the ranked set sampling when the population mean of the auxiliary variable is known. Yu and Lam (1997) proposed the regression estimator in two-phase sampling when the population mean of the auxiliary variable is unknown. We also propose an alternative two-phase stratified ranked set sampling. On the basis of real and simulated data, it is found that the proposed regression estimator outperforms the other estimators in most situations, especially for the real tree data. We suggest therefore that instead of using two-phase sampling, one should use two-phase stratified sampling for small strata for improving efficiency of the Yu-Lam estimator. Determination of the asymptotic distribution of the proposed estimators, coverage probabilities of the asymptotic confidence intervals and Bootsrap confidence intervals using Akgul et al. (2018) are subjects of our future research.

Acknowledgement

The authors would like to thank Editor and the referees for their constructive comments and suggestions which improved and enriched the presentation of the paper.

This article was finalized during the R & D leave granted to Prof J. O. Olaomi by the University of South Africa. The views expressed in this paper are those of the authors alone, and they do not necessarily reflect the views of the University of South Africa.

References

1. Akgul, F.G., Acitas,S., and Senoglu, B. (2018). Inferences on stress-strength reliability based on rank set sampling data in case of Lindley distribution. Journal of Statistical Computation and Simulation, 88 (15), 3018-3032.
2. Chen, Z., Bai, Z. Sinha, B.K. (2003). Ranked Set Sampling theory and Applications. Springer, New York.
3. Dell, T.R. and Clutter, J.L. (1972). Ranked-set sampling theory with order statistic background. Biometrics, 28, 545-555.
4. Frey, J., Ozturk, O., and Deshpande, J. (2007). Nonparametric tests for perfect judgment rankings. Journal of the American Statistical Association, 102: 708-717.
5. Kadilar, C., Unyazici, Y. and Cingi, H. (2006). Ratio estimator for the population mean using ranked set sampling. Statistical Papers, 50, 301-309.
6. Mahdizadeh, M. and Zamanzade, E. 2018. Stratified pair ranked set sampling. *Communications in Statistics-Theory and Methods*, 47(24), 5904-5915.
7. McIntyre, G.A. (1952). A method of unbiased selective sampling using ranked sets. Australian Journal of Agricultural Research, 3, 385-390.
8. Platt, W.J., Evans, G.M., and Rathbun, S.L. (1988). The population dynamics of long-lived confier (Pinus plustris). American Naturalist, 131, 491-525.
9. Raghunath Arnab and J. O. Olaomi (2015). Improved estimation from ranked set sampling. Hacettepe Journal of Mathematics and Statistics, 44 (6), 1513 – 1525
10. Stokes, S.L. (1977). Ranked set sampling with concomitant variables, Communications in Statistics- Theory and Methods, 12, 1207-1211.
11. Stokes, S.L. (1980 a). Inference on correlation coefficient in bivariate normal populations from ranked- set sampling. Journal of the American Statistical Association,75, 989-995.
12. Stokes, S.L. (1980). Estimation of variance using judgment ordered ranked-set samples. Biometrics, 36, 35-42.
13. Takahasi, K. (1970). Practical note on estimation of population means based on samples stratified by means of ordering. Annals of the Institute of Statistical Mathematics, 22, 421-428.
14. Takahasi, K. and Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. Annals of the Institute of Statistical Mathematics, 20, 1-31.
15. Yu, P.L.H. and Lam, K. (1997). Regression estimator in ranked set sampling, Biometrics, 53, 1070- 1080.
16. Zamanzade, E., and Al-Omari, A.I. (2016): New ranked set sampling for estimating the population mean and variance. *Hacettepe Journal of Mathematics and Statistics*, 45(6), 1881-1905.
17. Zamanzade, E., Arghami, N.R., and Vock, M. 2012. Permutation-based tests of perfect ranking, *Statistics and Probability Letters*, 82(12), 2213-2220.
18. Zamanzade, E., and Mahdizadeh, M. (2018): Distribution function estimation using concomitant-based ranked set sampling. *Hacettepe Journal of Mathematics and Statistics*, 47(3), 755-761.
19. Zamanzade, E**.**, and Vock, M. 2018. Some nonparametric tests of perfect Judgment ranking for judgment post stratification. *Statistical Papers*, 59(3), 1085-1100.