Bayesian Skew Normal Seemingly Unrelated Regression Modelling of Gross Regional Domestic Product

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Abstract

The assumption of the error normality in the regression model was often questioned especially in cases where there was an outlier, which causes the behavior of asymmetric data. To overcome this, without data transformation, we could use skew distribution. This distribution was very important and applicable in various fields of science such as finance, economics, actuarial science, medicine, biology, investment. Skew Normal distributions had been proven to have a convenient for calculating bias in data with asymmetric behavior. This study aims to model SUR with Skew Normal error using Bayesian approach applied to East Java GRDP data. This study would compared two types of models, namely models with Normal distributed errors and models with Skew Normal distributed errors. The result of parameter estimation with Bayesian approach shows that SUR Skew Normal model was more suitable for East Java GRDP modeling rather than using normal error model. This was based on their smaller Root of Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) value.

Keywords: Skew normal distribution; Seemingly unrelated regression; Root of mean square error; Mean absolute error; Mean absolute percentage error.

1. Introduction

In several economic and other areas of knowledge, the seemingly unrelated regression (SUR) model introduced by Zellner (1962) was often used as a tool to explain the occurrence of an economic phenomenon. In most non-Bayesian and Bayesian books, it had provided various explanations of analytical techniques on SUR models, including Judge (1988), Greene (2008), Geweke (2005), and Lancaster (2004). One of the most popular Bayesian approaches for SUR model estimation was the Markov Chain Monte Carlo (MCMC) method, which was a simulation approach technique for calculating the posterior distribution of parameters. The Bayesian approach in the SUR model had been introduced by Zellner (1971) by performing an analysis to obtain posterior distribution of parameters via the Gibbs sampling method on the combined posterior parameters.

As an assumption for the SUR model, normal distributed errors were often incompatible with the condition of an actual data. One alternative solution was transformation to normality, but data transformation could make it difficult in experimental explanations as revealed by Azzalini and Capitanio (1999). One way to overcome this was to use a Skew Normal distribution that had been proven its useful for calculating bias in data with abnormal behavior. The Skew Normal distribution method, that had been disseminated after Azzalini (1985) through a new class of skew-normal distributions, has generated much interest in flexible skewness in multivariate distribution. A popular approach was to build a multivariate Skew Normal distribution by modifying the multiplication of normal probability density function (pdf).

The Skew Normal distribution which was made by Azzalini, although able to provide the relaxation of normality as a pattern that was tilted to the right or tilted to the left, but unable to maintain its stability in its location or stable in the mean (Iriawan, 2012). This means that if the pattern of a residual data was detected rather tilted to the right, then the modeling must bear the shift of its residual center also shifted to the right which was no longer centered in its mode location at zero. It would be work for residual slope when it was tilted to the left. Fernandez and Steel (1998) armed with normal distribution began to think about dividing it into two, the negative side (left side of zero) and the positive side (right side of zero) to be treated with the inverse operator. Based on that thought, Fernandez and Steel made a normal slant that could be stable in its mode of distribution.

This study was using the normal skew distribution of Fernandez and Steel, which has the stability in its mode of distribution and would be applied for the SUR model by using GRDP of East Java province data. Thus, the assumptions of Normal and Skew-Normal errors would be employed to be coupled with the Bayesian approach and the MCMC method for estimating the parameters.

2. Multivariate Skew Normal Distribution

The proposed Skew Normal distribution (Fernandez and Steel, 1998) considers, that δ was a residual model having probable values, $-\infty < \delta < \infty$ was normally, $\delta :\ N(0,\sigma^2)$. Suppose $f(.)$ was the symmetric univariate distribution centered at zero, then the normal skew distribution of residual with the skewness parameters γ was defined as follows :

$$p(\delta|\gamma,f) = \frac{2}{\gamma+\frac{1}{\gamma}}\left\{ f\left(\frac{\delta}{\gamma}\right)\mathrm{I}_{(0,\infty)}(\delta) + f(\gamma\delta)\mathrm{I}_{(-\infty,0)}(\delta)\right\} = \frac{2}{\gamma+\frac{1}{\gamma}} f\left(\delta\gamma^{-sign(\delta)}\right) \qquad (1)$$

Where $\mathbf{I}_{S(.)}$ was an indicator function on *S*, and sign $(\cdot)$ was a function mark on $\Re$ with $-\infty < \delta < \infty$ and $\gamma = (0,\infty)$. Parameters λ become divisor transformations, δ/γ, for $\delta \geq 0$,and treat them as multiplier parameters, $\delta\gamma$, for $\delta < 0$. So that the original distribution could still be maintained for, $\gamma = 1$, $f(\delta|\gamma=1) = f(\delta)$, where $\delta :\ N(0,\sigma^2)$. If there was a vector $\boldsymbol{\delta} = (\delta_1, \delta_2, ..., \delta_M)'$, $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_M)'$ and non-singular matrices $\mathbf{A} \in R^{M\times M}$, then the variable $\boldsymbol{\eta} = (\eta_1, \eta_2, ..., \eta_M)' \in R^M$ could be defined as

$$\boldsymbol{\eta} = \mathbf{A}'\boldsymbol{\delta} + \boldsymbol{\xi}, \qquad (2)$$

Then the density function of η, therefore, would follow the general Multivariate Skew Normal. The equation of Skew Normal multivariate of η with the parameters $\boldsymbol{\xi}, \mathbf{A}, \gamma, f$ (Ferreira and Steel, 2007) was defined as:

$$p(\eta \mid \xi, \mathbf{A}, \gamma, f) = \|\mathbf{A}\|^{-1} \prod_{m=1}^{M} p\left[(\eta - \xi)' \mathbf{A}_{.m}^{-1} \mid \gamma_m, f_m \right], \tag{3}$$

where $\mathbf{A}_{.j}^{-1}$ was a member of the jth column of the matrix $\mathbf{A}^{-1}$, $\|\mathbf{A}\|$ was the absolute value of the matrix determinant $\mathbf{A}$, and $p(.\mid \gamma, f)$ was the Skew Normal function as in equation(1).

Considering equations (1) and (2) into equation (3), the general form of Multivariate Skew Normal of $\boldsymbol{\delta}$ could be written in equation (4).

$$p(\boldsymbol{\delta} \mid A, \gamma, f) = \|A\|^{-1} \prod_{m=1}^{M} \left[\frac{2}{\gamma + \frac{1}{\gamma}} f\left(\delta \gamma^{-sign(\delta)} \right) \right] \tag{4}$$

3. Bayes SUR Skew Normal Model

In general, the equation of SUR model with *M* equation could be written as follows (Zellner and Ando, 2010)

$$\mathbf{y}_m = \mathbf{X}_m \boldsymbol{\beta}_m + \boldsymbol{\varepsilon}_m, \quad m = 1, 2, \ldots, M \quad , \tag{5}$$

where $\mathbf{y}_m$ is a $T \times 1$ vector of observations on the dependent variable in the *m*th equation, $\mathbf{X}_m$ is a $T \times k_m$ matrix of independent variables in the *m*th equation with rank k_m, $\boldsymbol{\beta}_m$ is a $k_m \times 1$ vector of unknown parameters in the *m*th equation, $\boldsymbol{\varepsilon}_m$ is an $T \times 1$ vector of unobservable disturbances (Funda and Fikri , 2016) with

$$E(\varepsilon_m) = 0 \text{ and } Cov(\varepsilon_m \varepsilon_n) = \sigma_{mn} I \quad (m, n = 1, 2, \ldots, M)$$

This model had independent variables and different form of error variants. This model also shows the correlation between errors in different equations. The model of equation (5) could be rewritten in matrix form as: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, and $\boldsymbol{\varepsilon} = \mathbf{y} - \mathbf{X}\boldsymbol{\beta}$.

Based on equation (4), the general form of Multivariate Skew Normal of $\boldsymbol{\varepsilon}$ could be written as:

$$p(\varepsilon \mid \mathbf{A}, \gamma, f) = \|\mathbf{A}\|^{-1} \prod_{m=1}^{M} \left[\frac{2}{\gamma + \frac{1}{\gamma}} \left(\frac{1}{\sqrt{2\pi}} \right) \exp\left[\frac{1}{2} \left(\varepsilon \mathbf{A}^{-1} \gamma^{-sign(\varepsilon \mathbf{A}^{-1})} \right)^2 \right] \right]. \tag{6}$$

Given that the Jacobian from ε to y was 1, the density of **y** could be written as follow:

$$p(\mathbf{y} \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma, \mathbf{X}) = \|\boldsymbol{\Sigma}\|^{-1/2} \prod_{m=1}^{M} \left[\frac{2}{\gamma + \frac{1}{\gamma}} \left(\frac{1}{\sqrt{2\pi}} \right) \exp\left[\frac{1}{2} \left((\mathbf{y} - \mathbf{X}\boldsymbol{\beta})' (\boldsymbol{\Sigma})^{-1} (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \gamma^{-sign((\mathbf{y}-\mathbf{X}\boldsymbol{\beta})\boldsymbol{\Sigma}^{-1/2})} \right) \right] \right]. \tag{7}$$

Equation (7) was a function of the likelihood SUR Skew Normal model, which would be used to obtain the estimated parameters of SUR Skew Normal model through the mean posterior model. Skew Normal distribution was non standard distribution, and therefore, there were not facilitated by WinBUGS package program. To overcome this problem, the "Zeros trick" scenario would be used. Zeros Trick in WinBUGS uses the log of its likelihood function. The syntax code could be written as follow (Ntzoufras, 2009)

```
C<-10000
for(i in 1:n){
zeros[i]<-0
zeros[i]~dpois(zeros.mean[i])
zeros.mean[i]<- -l[i]+ C
l[i]<- #Log Likelihood function
...}
```

Log Likelihood function of SUR Skew Normal model in the syntax above could be substituted with the following equation:

$$\log p\left(\mathbf{y}\,|\,\boldsymbol{\beta},\boldsymbol{\Sigma},\gamma,\mathbf{X}\right)= \log\left\|\Sigma\right\|^{-1/2}+\sum_{j=1}^{m}\log\left[\frac{2}{\gamma+\frac{1}{\gamma}}\left(\frac{1}{\sqrt{2\pi}}\right)\exp\left[\frac{1}{2}\left(\left(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\right)'\left(\boldsymbol{\Sigma}\right)^{-1}\left(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\right)\gamma^{-sign\left(\left(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\right)\boldsymbol{\Sigma}^{-1/2}\right)}\right)\right]\right] \quad (8)$$

The prior of γ in SUR Skew Normal model follows the Gamma distribution with parameters p and q which could be written as $\gamma \sim Gamma\left(p,q\right)$ and was defined as follows:

$$p(\gamma)= f(\gamma\,|\,\alpha,\beta)=\frac{\gamma^{p-1}e^{-\gamma/q}}{\Gamma\left(p\right)q^{p}}, \qquad \gamma \geq 0, p>0, q>0. \quad (9)$$

The use of prior distribution parameters model uses an independent prior distribution to avoid problems in modeling (Box and Tiao, 1973). The prior distribution for parameters $\boldsymbol{\beta}$ was as follows:

$$\begin{aligned} p(\boldsymbol{\beta}) &= p(\boldsymbol{\beta}_1)\,p(\boldsymbol{\beta}_2)....p(\boldsymbol{\beta}_M) \\ p(\boldsymbol{\beta}) &= \prod_{m=1}^{M} p(\boldsymbol{\beta}_m) \end{aligned} \quad (10)$$

Prior distribution for each $\boldsymbol{\beta}_m$, m=1,2,..,M defined as $p\left(\boldsymbol{\beta}_m\right)$ was a normal distribution which was a pseudo informative prior defined as follows:

$\boldsymbol{\beta}_m : N(\mu_{\boldsymbol{\beta}_m}, \sigma^2_{\boldsymbol{\beta}_m})$

$\sigma^2_{\boldsymbol{\beta}_m} = 1/\tau_{\boldsymbol{\beta}_m}$, or it could be written as follows :

$$p(\boldsymbol{\beta}_m) = \sqrt{\frac{\tau_{\boldsymbol{\beta}_m}}{2\pi}} exp[-\frac{\tau_{\boldsymbol{\beta}_m}}{2}(\boldsymbol{\beta}_m - \mu_{\boldsymbol{\beta}_m})^2], \quad (11)$$

$$\propto \sqrt{\tau_{\boldsymbol{\beta}_m}} exp[-\frac{\tau_{\boldsymbol{\beta} m}}{2}(\boldsymbol{\beta}_m - \mu_{\boldsymbol{\beta}_m})^2].$$

In the Bayesian approach, Inverse Wishart as a prior distribution was often used as a prior for sigma $(\boldsymbol{\Sigma})$, in equation (8) when the prior distribution was a conjugate of its likelihood function. Since this Inverse Wishart distribution was not conjugated the likelihood function of the SUR Skew Normal model, the prior distribution for the sigma $(\boldsymbol{\Sigma})$ was designed as a combination of prior of tau (τ) and rho (ρ). Prior of tau (τ) uses Gamma distribution (*m*, *n*) which could be written $\tau \sim$ Gamma(m,n) and the density as follows

$$p(\tau \mid m,n) = \frac{\tau^{m-1} e^{-\tau/n}}{\Gamma(m) n^m}, \quad (12)$$

and prior of rho (ρ) was defined to follow the Uniform distribution (*a,b*) or $\rho \sim$ *Uniform*(a,b) and had the density as follows :

$$p(\rho \mid a,b) = \frac{1}{b-a}. \quad (13)$$

The joint posterior distribution for the SUR Skew Normal model, therefore, was obtained by combining the above information from Skew Normal likelihood and prior distribution, which could be written as follows

$$g(\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma | \mathbf{y}, \mathbf{X}) \propto p(y \mid \boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma, \mathbf{y}, \mathbf{X}).p(\boldsymbol{\beta}_1)p(\boldsymbol{\beta}_2)....p(\boldsymbol{\beta}_M).p(\tau).p(\rho).p(\gamma). \quad (14)$$

Finally, the marginal posterior distribution of each coefficient of Bayes SUR model was as follows

$$p(\boldsymbol{\beta}_1|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_2) p(\boldsymbol{\beta}_3)...p(\boldsymbol{\beta}_M) p(\tau) p(\rho) p(\gamma) d\boldsymbol{\beta}_2 d\boldsymbol{\beta}_3...d\boldsymbol{\beta}_M d\tau d\rho d\gamma$$

$$p(\boldsymbol{\beta}_2|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_1) p(\boldsymbol{\beta}_3)...p(\boldsymbol{\beta}_M) p(\tau) p(\rho) p(\gamma) d\boldsymbol{\beta}_1 d\boldsymbol{\beta}_3...d\boldsymbol{\beta}_M d\tau d\rho d\gamma$$

M

$$p(\boldsymbol{\beta}_M|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_1) p(\boldsymbol{\beta}_2)...p(\boldsymbol{\beta}_{M-1}) p(\tau) p(\rho) p(\gamma) d\boldsymbol{\beta}_1 d\boldsymbol{\beta}_2...d\boldsymbol{\beta}_{M-1} d\tau d\rho d\gamma \quad (15)$$

$$p(\tau|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_1) p(\boldsymbol{\beta}_2)...p(\boldsymbol{\beta}_M) p(\rho) p(\gamma) d\boldsymbol{\beta}_1 d\boldsymbol{\beta}_2...d\boldsymbol{\beta}_M d\rho d\gamma$$

$$p(\rho|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_1) p(\boldsymbol{\beta}_2)...p(\boldsymbol{\beta}_M) p(\tau) p(\gamma) d\boldsymbol{\beta}_1 d\boldsymbol{\beta}_2...d\boldsymbol{\beta}_M d\tau d\gamma$$

$$p(\gamma|\mathbf{y}) \propto \int ... \int f_L(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \gamma) p(\boldsymbol{\beta}_1) p(\boldsymbol{\beta}_2)...p(\boldsymbol{\beta}_M) p(\tau) p(\rho) d\boldsymbol{\beta}_1 d\boldsymbol{\beta}_2...d\boldsymbol{\beta}_M d\tau d\rho$$

Due to the complexity of the integration process in equation (15), the estimation would be done by using MCMC and Gibbs Sampling which the employing the repeated sampling scenario through the form of full conditional posterior distribution. The full conditional posterior distribution of each model parameter was as follow

i. full conditional posterior distribution of $\boldsymbol{\beta}$:

$$p\left(\boldsymbol{\beta}_m \middle| \boldsymbol{\beta}_{\setminus m}, \boldsymbol{\Sigma}, \gamma \middle| \mathbf{y}\right) \propto \mathbf{A} \times \mathbf{B}, \tag{16}$$

ii. full conditional posterior distribution of $\boldsymbol{\Sigma}$:

$$p\left(\boldsymbol{\Sigma}_m \middle| \boldsymbol{\beta}, \boldsymbol{\Sigma}_{\setminus m}, \gamma \middle| \mathbf{y}\right) \propto \mathbf{A} \times \mathbf{B} \times \frac{\tau^{m-1} e^{-\tau/n}}{\Gamma(m) n^m}, \tag{17}$$

iii. full conditional posterior distribution of γ:

$$p\left(\gamma \middle| \boldsymbol{\beta}, \boldsymbol{\Sigma}, \mathbf{y}\right) \propto \mathbf{A} \times \left(\frac{\gamma^{p-1} e^{-\gamma/q}}{\Gamma(p) q^p} \right), \tag{18}$$

where

$$\mathbf{A} = \exp\left[\frac{1}{2} \sum_{m=1}^{M} \left(\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right)' \left(\boldsymbol{\Sigma}\right)^{-1} \left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right) \gamma^{-sign\left(\left(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\right)\boldsymbol{\Sigma}^{-1/2}\right)} \right) \right], \tag{19}$$

$$\mathbf{B} = exp\left[-\frac{1}{2} \sum_{m=1}^{M} \tau_{\boldsymbol{\beta}m} (\boldsymbol{\beta}_m - \mu_{\boldsymbol{\beta}_m})^2 \right], \tag{20}$$

$m = 1, 2, ..., M$.

The Gibbs Sampling iteration procedure was performed with the following stages:

Step 1: Determine the initial value for each parameter to be estimated.

Step 2: Generate the sample by executing iterations of M for each parameter using the full conditional posterior, ie:

i) Generating $\boldsymbol{\beta}$ using equation (16),

ii) Generating $\boldsymbol{\Sigma}$ using equations (17),

iii) Generating γ using equation (18).

iv) Processes i) up to iii) this was done iterative until convergent.

Repeating step 2.i) up to 2.iii) until each parameter convergence.

4. Application And Result

This research uses data of GRDP (response variable) and data of labor force, labor and investment (domestic investment and foreign investment) as the predictor variables, for the three main sectors in East Java, which refer to AB Santosa et all (2017). In detail, the types of research variables that were employed in this study were shown in Table 1.

In this Bayes SUR model, parameter estimation was done by estimating probability distribution parameters with Skew Normal error by using WinBUGS. Bayesian estimates were considered to be significant at 5% of the probability, if the credibility of the interval from the SUR model coefficients for the posterior did not have a zero value. The marginal posterior distribution for all parameters was obtained based on the 100,000 iteration MCMC processes.

The contribution of all of three main sectors was about 72 percent of the total East Java's GRDP. Therefore, these three main sectors were used as an indicator of economic development in this province. Based on the equation (7), SUR models in this study were:

$$\begin{aligned}
y_{1t} &= \beta_{10} + \beta_{11} X_{1t,1} + \beta_{12} X_{1t,2} + \beta_{13} X_{1t,3} + \beta_{14} X_{1t,4} + u_{1t} \\
y_{2t} &= \beta_{20} + \beta_{21} X_{2t,1} + \beta_{22} X_{2t,2} + \beta_{23} X_{2t,3} + \beta_{24} X_{2t,4} + u_{2t} \\
y_{3t} &= \beta_{30} + \beta_{31} X_{3t,1} + \beta_{32} X_{3t,2} + \beta_{33} X_{3t,3} + \beta_{34} X_{3t,4} + u_{3t}.
\end{aligned} \tag{21}$$

Table 1: Response variables (Y) and predictor variables (X) used in the study

Variables		Unit
Y_{1t}	GRDP of Agriculture Sector	billion Rupiahs
Y_{2t}	GRDP of Manufacturing Industrial Sector	billion Rupiahs
Y_{3t}	GRDP of Trade, Hotel and Restaurant Sector	billion Rupiahs
$X_{1t,1}$	labors amount of Agriculture Sector	thousand heads
$X_{2t,1}$	labors amount of Manufacturing Industrial Sector	thousand heads
$X_{3t,1}$	labors amount of Trade, Hotel and Restaurant Sector	thousand heads
$X_{1t,2}$	Labor Wages of Agriculture Sector	thousand Rupiahs
$X_{2t,2}$	Labor Wages of Manufacturing Industrial Sector	thousand Rupiahs
$X_{3t,2}$	Labor Wages of Trade, Hotel and Restaurant Sector	thousand Rupiahs
$X_{1t,3}$	Domestic Investment of Agriculture Sector	billion Rupiahs
$X_{2t,3}$	Domestic Investment of Manufacturing Industrial Sector	billion Rupiahs
$X_{3t,3}$	Domestic Investment of Trade, Hotel and Restaurant Sector	billion Rupiahs
$X_{1t,4}$	Foreign Investment of Agriculture Sector	billion Rupiahs
$X_{2t,4}$	Foreign Investment of Manufacturing Industrial Sector	billion Rupiahs

$X_{3t,4}$	Foreign Investment of Trade, Hotel and Restaurant Sector	billion Rupiahs

Estimation parameters of SUR models with Bayesian MCMC Normal and Skew Normal methods were shown in Table 2. All estimated parameters were positive, both for the GRDP model of agriculture, manufacturing industry and trade, hotels and restaurants. These results were consistent with economic theory, which states that the higher of increasing amount of labor, labor wages, domestic investment and foreign investment, the greater of increasing amount of GRDP. While the partial test shows that there was only one variable, namely investment domestic (X_{23}), which was not significant in affecting the GRDP of the manufacturing sector.

Table 2: A Result of Bayes SUR Model Parameters with Norma and Skew Normal Model Assumption

	MCMC NORMAL				MCMC SKEW NORMAL			
	Mean	2.50%	97.50%	Elasticity	Mean	2.50%	97.50%	Elasticity
β_{10}	24,124.57	14,700.00	33,470.00		7,761.20	-10,759.30	26,148.11	
β_{11}	1.55	0.32	2.79	0.2848	3.64	1.59	5.71	0.6688
β_{12}	18.62	15.48	21.71	0.1199	16.92	13.38	20.48	0.1090
β_{13}	9.25	0.05	19.72	0.0108	16.90	0.86	32.94	0.0197
β_{14}	40.75	15.34	67.81	0.0158	50.34	10.30	90.48	0.0195
β_{20}	17,693.25	-4,958.00	40,880.00		18,507.85	-17,288.00	54,029.07	
β_{21}	13.03	0.67	24.67	0.4563	12.14	0.87	23.53	0.4250
β_{22}	25.33	14.29	37.01	0.1789	24.71	13.01	36.35	0.1745
β_{23}	0.18	-0.14	0.52	0.0220	0.31	0.00	0.62	0.0375
β_{24}	4.39	2.26	6.65	0.0697	4.86	2.73	7.02	0.0771
β_{30}	-8,259.73	-39,160.00	19,970.00		-39,221.31	-91,352.17	12,944.92	
β_{31}	15.44	5.39	26.71	0.7910	26.59	14.61	38.58	1.3621
β_{32}	39.26	21.02	56.88	0.2652	24.03	3.86	44.13	0.1623
β_{33}	6.36	1.11	12.18	0.0243	10.97	1.61	20.33	0.0419
β_{34}	55.40	30.80	81.34	0.0507	61.28	15.25	107.20	0.0561

Source: Data analysis using Program R danWinBUGS

SUR models using Bayesian MCMC Normal method for the model of the agricultural sector, total employment had the highest level of elasticity, that was 0, 2848. This means that the growth of total employment by one percent, the GRDP of the agricultural sector in East Java would grow 0.2848 percent. Similarly, MCMC Skew Normal approach reported that the number of workers has the highest level of elasticity on the agricultural sector value, that was 0.6688. Likewise, in the agricultural sector, MCMC Normal and Skew Normal methods in modeling of industrial sector and trade, hotel and restaurant sectors, were syncronously shown that the amount of labor in this sector also had the highest level of elasticity, but in different level. MCMC reports 0.4563 for elasticity of the number workers in industry sectors and 0.7910 for trading sectors, while MCMC Skew Normal reports for 0.4250 and 1.3621 for each sector respectively.

Some selection criteria, i.e. Root Mean Square Error (RMSE), Mean Absolute Deviation (MAD) and Mean Absolute Percentage Error (MAPE) the MCMC Skew Normal approach shows a better value than the MCMC Normal approach. Table 3 and Figure 1 shows these comparisons.

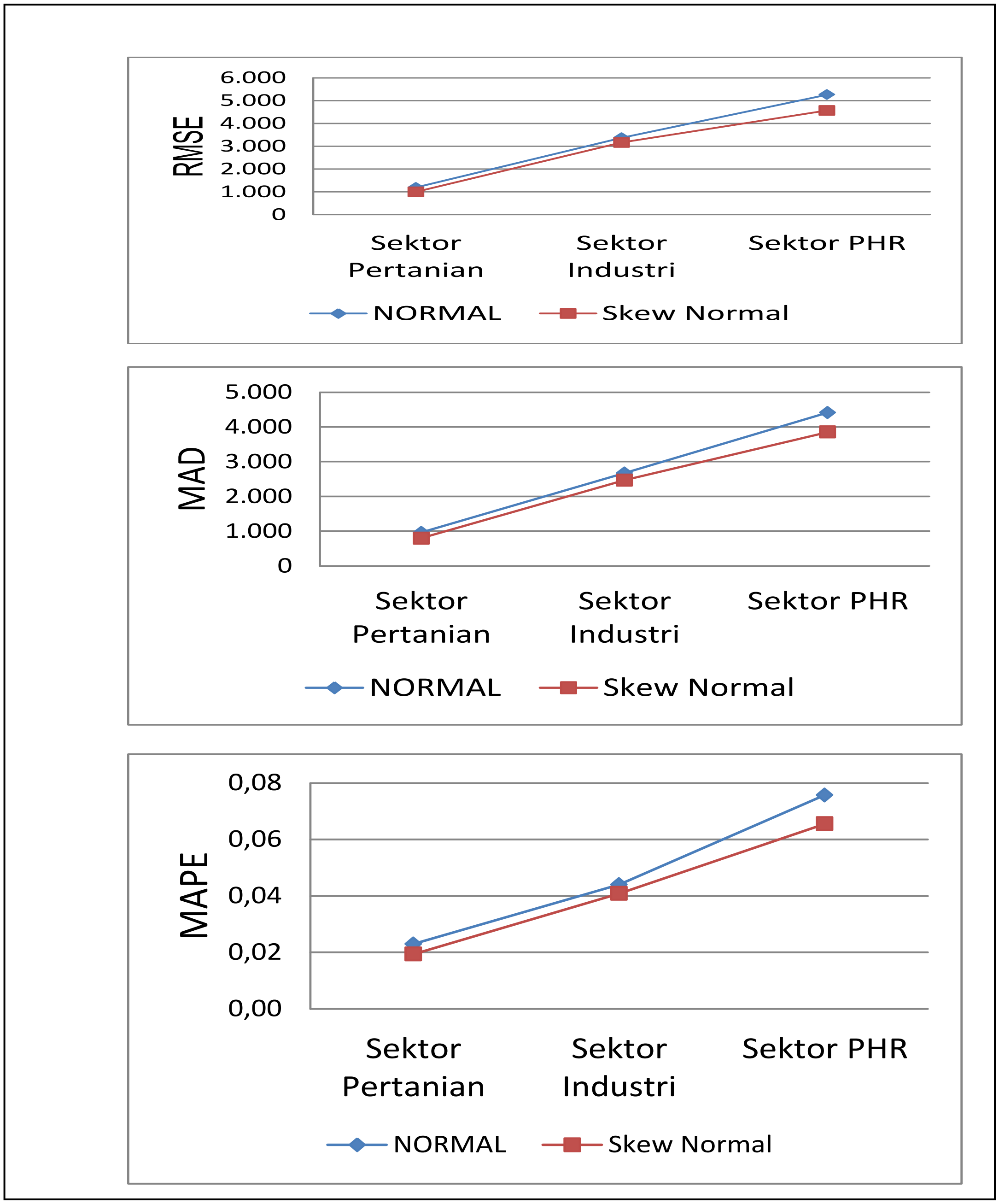

Figure 1. Some of the selection criteria models for MCMC Approach

Table 3: The RMSE, MAD and MAPE of The SUR model with MCMC NORMAL and SKEW NORMAL

Model of Bayesian SUR	Bayesian SUR of MCMC Normal			Bayesian SUR of MCMC Skew Normal		
	RMSE	MAD	MAPE	RMSE	MAD	MAPE
GRDP of the agricultural sector	1.192,7752	962,2523	0,0229	998,8476	795,7055	0,0194
GRDP of the industrial sector	3.369,5928	2.670,9537	0,0440	3.161,2334	2.466,2381	0,0409
GRDP of trade, hotel and restaurant sectors	5.258,3207	4.413,4124	0,0758	4.565,5748	3.853,0332	0,0655

Source: Data analysis using R Package danWinBUGS

5. Conclusion

All estimated parameters for MCMC Normal and Skew Normal method were positive. It was conforming to an economic theory, that says the higher increasing the amount of labor, wage labor, domestic investment and foreign investment, the greater growing the GRDP. All of variables were significant to influence the GRDP, except the domestic investment of manufacturing industry sector (X_{23}).

The results of parameter estimation using MCMC Normal and Skew Normal approach show almost the same level of elasticity; the amount of labor has the highest elasticity in figuring the East Java's GRDP model of agriculture, manufacturing industry and trade, hotels and restaurant's sectors. However, comparing the goodness of a model based on several criteria for selecting the model (RMSE, MAD and MAPE) shows that the MCMC Skew Normal approach was better than the MCMC Normal approach. SUR model with Skew Normal error was an alternative method for GRDP model in East Java. This was because SUR models with Skew Normal errors were more reliable than models with Normal errors.

Future research

The MCMC Skew Normal error model approach could be applied to more complex variants on the SUR model which were obtained in GRDP cases.

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