Stochastic Restricted Liu Type estimator for SUR model

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Abstract

In this paper, we introduce new Stochastic Restricted Estimator for SUR model, defined by Stochastic Restricted Liu Type SUR estimator (SRLTSE) . The proposed estimator has to deal with multicollinearity in SUR model if there is a degree of uncertainty in the parameters restriction. Moreover, the superiority of (SRLTSE) was derived with respect to mean squared error matrix (MSEM) criterion. Finally, a simulation study was conducted. This simulation used standard mean squares error (MSE) criterion to illustrate the advantage between Stochastic Restricted SUR estimator (SRSE), Stochastic Restricted Ridge SUR estimator (SRRSE), and Stochastic Restricted Liu Type SUR estimator (SRLTSE) at by several factors.

Keywords: SUR model; Stochastic restricted SUR estimator (SRSE); Stochastic restricted ridge SUR estimator (SRRSE); Stochastic restricted Liu type SUR estimator (SRLTSE)

1. Introduction

In the SUR model, there is a wide range of estimators used to deal with multicollinearity. In this context, the ridge estimator, introduced by Alkhamisi (2007) to solve the ill condition for SUR model. (Jibo,2014) proposed the Liu-type estimator in two SUR model, that combined between the Stein estimator and ridge estimator. In many cases, the parameters of the SUR modelmay be surrounded by a degree of uncertainty, so Srivastava and Giles,(1987) suggestedstochastic restricted SUR estimator. Finally, El-Houssainy et al (2010) introduced general stochastic restricted ridge estimator which avoids multicollinearity in stochastic restricted SUR model. In these paper, we propose stochastic restricted Liu-type estimators in SUR model and discuss the priority for SUR liu-type estimators over SUR Ridge estimator in the case of uncertainty restrictions.

2 The Stochastic Restricted Liu-type for SUR model:

Let the SUR model as the form:

$$Y = X\beta + \mu \tag{1}$$

Where $Y = [Y_1, Y_2, \dots Y_q]'$ is nq×1 vector of response variables, X is nq×nq block diagonal matrix of k independent variables, β is nk ×1 vector of unknown coefficients and μ is nq×1 of vector of random error in q equations with $E(\mu) = 0_{nq\times1}$ vector and $E(\mu\mu') = \Sigma \otimes I_n$,$\Sigma$ is q×q variance covariance matrix for errors

$$VarCov\left(\mu_{ij}\right)=\begin{Bmatrix}\sigma_{ii}^2 I_n & i=j, i=1,2,\dots q\\ \sigma_{ij}^2 I_n & i\neq j, i=1,2,\dots q\end{Bmatrix}$$

The OLS estimator is the best linear unbiased estimator (BLUE). However, the high correlation between independent variables causes a rise in the variance. The ridge estimator for linear regression proposed by Hoerl and Kennard(1970) that depended on adding more information to $X'X$matrix to solve the ill condition. In this direction, Srivastava and Giles(1987) introducedridge estimator (RSE)for SUR model as

$$\hat{\beta}_{RSE}(h)=\left(X'\left(\Sigma^{-1}\otimes I_n\right)X+hI_{kq}\right)^{-1}X'\left(\Sigma^{-1}\otimes I_n\right)Y h>0 \qquad (2)$$

In order to reach superior estimator over the other estimators that overcome the problem of ill condition in SUR model, Jibo(2014) propose a Liu-Type estimator for two SUR model.

$$\hat{\beta}_{LTSE}(h,d)=argmin_{\beta}\left((Y-X\beta_i)'(Y-X\beta_i)+\sum_{i}^{q}\sum_{j}^{k}\left(h_j^{\frac{1}{2}}\beta_{ij}-h_j^{-\frac{1}{2}}d\hat{\beta}_{GLS_{ij}}\right)^2\right)$$

$$h>0, -\infty< d<+\infty \qquad (3)$$

Then

$$\hat{\beta}_{LTSE}(h,d)=\left(X'\left(\Sigma^{-1}\otimes I_n\right)X+hI_{kq}\right)^{-1}\left(X'\left(\Sigma^{-1}\otimes I_n\right)Y-d\hat{\beta}_{GLS}\right) \qquad (4)$$

In SUR model, let us suppose that, there are prior information about β as the following

$$r_1=R_1\beta_1+e_1$$

$$r_2=R_2\beta_2+e_2 \qquad (5)$$

$$r_q=R_q\beta_q+e_q$$

We can rewrite the (5) as the stacked form as

$$r=R\beta+e, e\sim MVN(0,\Omega\otimes I_l) \qquad (6)$$

Where $r=\left[r_1', r_2', \dots, r_q'\right]'$ is lq× 1 independent set for known stochastic vector, $l$is number of restriction, $R=Dig\left[R_1, R_2, \dots R_q\right]$ is a lq×kq prior information, $e=\left[e_1', e_2', \dots, e_q'\right]'$ is lq× 1random vector and Ω is known and positive definite.

For (1) and (5), we building mixed model as form

$$\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix} [\beta] + \begin{bmatrix} \mu \\ e \end{bmatrix} \qquad , \begin{bmatrix} \mu \\ e \end{bmatrix} \sim MVN(0, \psi) \tag{7}$$

Where $\psi = \begin{bmatrix} \Sigma \otimes I_n & 0 \\ 0 & \Omega \otimes I_l \end{bmatrix}$

We can rewrite the (6) as the stacked form as

$$Y_r = X_r \beta + \mu_r \tag{8}$$

Srivastava and Giles,(1987) suggested the stochastic restriction estimator for SUR model as

$$\hat{\beta}_{SRSE} = (X'(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R)^{-1}(X'(\Sigma^{-1} \otimes I_n)Y + R'(\Omega^{-1} \otimes I_l)r) \tag{9}$$

Observing

$$(S + R'(\Omega^{-1} \otimes I_l)R)^{-1} = S^{-1} - S^{-1}R'\left((\Omega^{-1} \otimes I_l) + R(\Sigma^{-1} \otimes I_n)R'\right)^{-1} RS^{-1}$$

Where: $S = X'(\Sigma^{-1} \otimes I_n)X$

Then

$$\hat{\beta}_{SRSE} = \hat{\beta}_{GLS} + (X'(\Sigma^{-1} \otimes I_n)X)^{-1} R'\left((\Omega^{-1} \otimes I_l) + R(\Sigma^{-1} \otimes I_n)R'\right)^{-1}\left(r - R\hat{\beta}_{GLS}\right)$$

$$Bise\left[\hat{\beta}_{SRSE}\right] = E\left[\hat{\beta}_{SRSE}\right] - \beta = 0$$

$$MSEM\left[\hat{\beta}_{SRSE}\right] = (X'(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R)^{-1} = C^{-1} \tag{10}$$

Where: $C = (X'(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R)$

For model in (8), we extract The Stochastic Restricted Liu-type estimator for SUR model (SRLTSE) as

$$\hat{\beta}_{SRLTSE}(h, d) = \left(X_r'(\Sigma^{-1} \otimes I_n)X_r + hI_{kq}\right)^{-1}\left(X_r'(\Sigma^{-1} \otimes I_n)Y_r - d\hat{\beta}_{RGLS}\right)$$

$$= \left(X_r'(\Sigma^{-1} \otimes I_n)X_r + hI_{kq}\right)^{-1}\left(X_r'(\Sigma^{-1} \otimes I_n)Y_r - d\hat{\beta}_{RGLS}\right)$$

$$\hat{\beta}_{SRLTSE}(\lambda,d)=\left([X \quad R]'\left(\begin{bmatrix}\Sigma^{-1}\otimes I_n & 0\\ 0 & \Omega^{-1}\otimes I_l\end{bmatrix}\right)\begin{bmatrix}X\\R\end{bmatrix}+hI_{kq}\right)^{-1}\left([X \quad R]'\begin{bmatrix}\Sigma^{-1}\otimes I_n & 0\\ 0 & \Omega^{-1}\otimes I_l\end{bmatrix}\begin{bmatrix}Y\\r\end{bmatrix} - d\left([X \quad R]'\begin{bmatrix}\Sigma^{-1}\otimes I_n & 0\\ 0 & \Omega^{-1}\otimes I_l\end{bmatrix}\begin{bmatrix}X\\R\end{bmatrix}\right)^{-1}[X \quad R]'\begin{bmatrix}\Sigma^{-1}\otimes I_n & 0\\ 0 & \Omega^{-1}\otimes I_l\end{bmatrix}\begin{bmatrix}Y\\r\end{bmatrix}\right)$$

$$=\left(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R+hI_{kq}\right)^{-1}[X'(\Sigma^{-1}\otimes I_n)Y+R'(\Omega^{-1}\otimes I_l)r - d((X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R)^{-1}X'(\Sigma^{-1}\otimes I_n)Y+R'(\Omega^{-1}\otimes I_l)r)]$$

$$=\left[\left(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R+hI_{kq}\right)^{-1}\left((X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R) - dI_{kq}\right)\right](X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R)^{-1}(X'(\Sigma^{-1}\otimes I_n)Y + R'(\Omega^{-1}\otimes I_l)r)$$

$$\hat{\beta}_{SRLTSE}(h,d)=[\mathrm{A}^{-1}\mathrm{B}]\hat{\beta}_{SRSE}=F_d\hat{\beta}_{SRSE} \qquad (11)$$

Where: $\mathrm{A}=\left(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R+hI_{kq}\right)$, $\mathrm{B}=(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R)-dI_{kq}=\mathrm{C}-dI_{kq}$ *and* $F_d=\left[\left(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R+hI_{kq}\right)^{-1}(X'(\Sigma^{-1}\otimes I_n)X+R'(\Omega^{-1}\otimes I_l)R)-dI_{kq}\right]$

The new estimator in (11) contains a set of special cases

- When d=0, $\hat{\beta}_{SRLTSE}=\hat{\beta}_{SRRSE}(h)$
- When $R=0$, , $\hat{\beta}_{SRLTSE}=\hat{\beta}_{LTSE}(h)$
- When d=0,$R=0$ $\hat{\beta}_{SRLTSE}=\hat{\beta}_{SUR\ Ridge}(h)$

Useful Superiority of the new estimators

In order to illustrate the superiority of the new estimators to the other estimators, we will use mean squared error matrix (MSEM) criteria which defined as

$$\mathrm{MSEM}(\tilde{\beta})=\mathrm{E}(\tilde{\beta}-\beta)(\tilde{\beta}-\beta)'=\mathrm{D}(\tilde{\beta})+\mathrm{Bias}(\tilde{\beta})\mathrm{Bias}(\tilde{\beta})$$

Where $\mathrm{D}(\tilde{\beta})$ is denote the dispersion matrix and $\mathrm{Bias}(\tilde{\beta})=\mathrm{E}(\tilde{\beta})-\beta$ is bias vector. It's clear that, for any two estimators, we say that $\tilde{\beta}_1$is superior to $\tilde{\beta}_2$ in MSEM criteria, iff $\Delta=\mathrm{MSEM}(\tilde{\beta}_2)-\mathrm{MSEM}(\tilde{\beta}_1)\geq 0$

Now for a statement the superiority of $\hat{\beta}_{\mathrm{RSS\,LTRidge}}(h,d)$ over $\hat{\beta}_{\mathrm{RSS\,LT}}(h,d)$, the following lemma can be help.

Lemma (3.1):(Rao et al, 2008) Let N>0, M>0, then N>M, iff $\lambda_{\max}(MN^{-1}) < 1$.

Lemma (3.2):(Rao et al, 2008) Let $\hat{\beta}_j = A_j y, j = 1,2$ be two estimators of β, suppose that $D = \mathrm{Cov}(\hat{\beta}_1) - \mathrm{Cov}(\hat{\beta}_2) > 0$, then $\mathrm{MSEM}(\hat{\beta}_1) - \mathrm{MSEM}(\hat{\beta}_2) > 0$ iff $B_2'(D + B_1 B_1')^{-1} B_2 \leq 1$, where B_j denotes the bias of B_j.

Lemma (3.3): (Farebrother, 1976) Let M be a p.d matrix, α be a non-zero vector, then $M - \alpha\alpha' \geq 0$ iff $\alpha' M^{-1} \alpha \leq 1$.

The properties of the $\hat{\beta}_{SRLTSE}(h,d)$estimator is given as follows:

The expected value, the bias and $MSEM$ for $\hat{\beta}_{SRLTSE}(h,d)$ estimator are given by:

$$E\left[\hat{\beta}_{\mathrm{SRLTSE}}(h,d)\right] = F_d E\left[\hat{\beta}_{\mathrm{SRSE}}\right]$$

$$Bias\left[\hat{\beta}_{\mathrm{SRLTSE}}(h,d)\right] = F_d \beta - \beta = \left(F_d - I_{kq}\right)\beta = \left(A^{-1}B - I_{kq}\right)\beta$$

$$=\left[\left(X'\,(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R + hI_{kq}\right)^{-1}\left(X'\,(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R - dI_{kq}\right) - I_{kq}\right]\beta$$

$$=\left[\left(X'\,(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R + hI_{kq}\right)^{-1}\left[X'\,(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R - dI_{kq} - X'\,(\Sigma^{-1} \otimes I_n)X - R'(\Omega^{-1} \otimes I_l)R - hI_{kq}\right]\right]\beta$$

$$E\left[\hat{\beta}_{\mathrm{SRLTSE}}(\lambda,d)\right] = -\left[dI_{kq} + hI_{kq}\right]\left(X'\,(\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R + hI_{kq}\right)^{-1}\beta \quad (12)$$

And the dispersion matrix or variance–covariance matrix is

$$D\left[\hat{\beta}_{\mathrm{SRLTSE}}(h,d)\right] = \mathrm{Cov}\left[\hat{\beta}_{\mathrm{RSS\,LT}}(h,d)\right] = F_d C^{-1} F_d' = A^{-1} B C^{-1} B' A^{-1'}$$

$$\mathrm{MSEM}\left[\hat{\beta}_{\mathrm{SRLTSE}}(h,d)\right] = A^{-1} B C^{-1} B' A^{-1'} + (A^{-1}B - I)\beta\beta'(A^{-1}B - I)'$$

$$= F_d C^{-1} F_d' + (A^{-1}B - I)\beta\beta'(A^{-1}B - I)$$

$$= F_d C^{-1} F_d' + \left[dI_{kq} + hI_{kq}\right] A^{-1}\beta\beta' A^{-1'}\left[dI_{kq} + hI_{kq}\right] \quad (13)$$

For (10) and (13) we illustrate the difference as a following

$$\Delta = \mathrm{MSEM}\left(\hat{\beta}_{\mathrm{SRSE}}\,(h,d)\right) - \mathrm{MSEM}\left(\hat{\beta}_{\mathrm{SRLTSE}}(h,d)\right)$$

$$= C^{-1} - F_d C^{-1} F_d' - \left[dI_{kq} + hI_{kq}\right] A^{-1}\beta\beta' A^{-1'}\left[dI_{kq} + hI_{kq}\right]' = D - LL \quad (14)$$

Where:

$D = C^{-1} - F_d C^{-1} F_d'$, $L = [dI_{kq} + hI_{kq}]A^{-1}\beta$

Since $C^{-1} > 0$, $F_d C^{-1} F_d' > 0$, then by lemma (3-1)

When $\lambda_{max}[(F_d C^{-1} F_d')C^{-1}] < 1$, $C^{-1} > F_d C^{-1} F_d'$

By lemma (3.3) , when $LDL' \leq 1, \Delta \geq 0$.

Choice for d and h

To choose stochastic shrinking parameter (d),In the (1), we defined $X_r(\Sigma^{-1} \otimes I_n) = X_r{}^*, U' X_r{}^{*\prime} X_r{}^* U = \Lambda = \mathrm{diag}(\delta_{11}, \delta_{12}, \dots, \delta_{qk}), U' U = UU' = I \text{ and } \alpha_r = U'\beta$, where U, Λ are the diagonal matrix of eigenvalue and the eigenvector of $X_r{}^{*\prime} X_r{}^*$. The optimal estimators for stochastic shrinking parameter is

$$d_{ij} = \frac{\alpha_{ij}^2}{\frac{1}{\delta_{ij}} + \alpha_{ij}^2} \qquad , i = 1,2, \dots . q, j = 1.2 \dots . k$$

In this paper, we develop the ideas for Kristofer et al (2012) and use the single value of $\hat{d}$ as:

$$\hat{d} = max\left(0, median\left(\frac{\hat{\alpha}_{r_{ij}}^{\ 2}}{\frac{1}{\delta_{ij}} + \hat{\alpha}_{r_i}^{\ 2}}\right)\right), i = 1,2, \dots . q, j = 1.2 \dots . k$$

For estimate the *h*parameter, we develop the ideas for Jibo and Yasin (2017) as:

$$\hat{h} = \frac{1}{kq} \sum_{i=1}^{q} \sum_{j=1}^{k} \frac{\delta_{ij} - \hat{d}\left(1 + b_{ij} \hat{\sigma}_{ij}{}_i^2\right)}{\hat{\alpha}_{r_i}^{\ 2}}, i = 1,2, \dots . q, j = 1.2 \dots . k$$

Where b_{ij} is eigenvalues of C *and*

$$(X' (\Sigma^{-1} \otimes I_n)X + R'(\Omega^{-1} \otimes I_l)R) = U\mathrm{diag}(b_{11}, b_{12}, \dots, b_{qk})U$$

3 The simulation study

In this section, we use a simulation study to illustrate the Stochastic Restricted Liu-type estimators for SUR model. This simulation is slightly based on Alkamisi (2010) and Al-Houssainy etal. (2012). We generate data for the following equation $y_{ij} = x_{ilj}{}' \beta_i + \mu_{ij}$

and $r_{ij} = R_{ilj}{'}\beta_i + e_{ij}$ $i = 1,2,\dots,q$, $i = 1,2,\dots,n, l = 1,2,\dots,k$ and we set the initial value for $\beta = [\beta_1,\beta_2,\beta_3] = [(3,4,5),(1,2,3),(2,3,4)]$. The independent variables are generated from multivariate normal distribution $MVN_P(0,\Sigma_X)$ where $\text{diag}(\Sigma_X) = 1$ and off − diag $(\Sigma_X) = \rho_{x_{ij}}$, $i,j = 1,2,3, i \neq j$ Where $\rho_{x_{ij}}$ denoted the correlation between the explanatory variables and we assume that $\rho_{x_{ij}} = 0.85$ for high correlation and $\rho_{x_{ij}} = 0.25$ for low correlation. We generate the random error terms from multivariate normal distribution $MVN_q(0,\Sigma_\mu)$ where $\text{diag}(\Sigma_\mu) = 1$ and off − diag $(\Sigma_\mu) = \rho_{\mu_{ij}}$, $i,j = 1,2,3, i \neq j$ and $\rho_{\mu_{ij}}$ denoted the correlation between error for i,j equations. We assume that $\rho_{\mu_{ij}} = 0.80$ for high correlation and and $\rho_{\mu_{ij}} = 0.20$ for low correlation. The restriction matrix for each equations are is given in Table (1,2) and we generate the random restriction error terms from multivariate normal distribution $MVN_q(0,\Omega_e)$ where $\text{diag}(\Omega_e) = 1$ and off − diag $(\Omega_e) = \rho_{e_{ij}}$, $i,j = 1,2,3, i \neq j$ Where where $\rho_{e_{ij}}$ denoted the correlation between restriction error for i,j equations and we assume that $\rho_{e_{ij}} = 0.80$.

Table (1):The restrictions for each equations in simulation study when *l*=3

R_1			R_2			R_3		
2	1	1	2	2	4	2	1	1
1	5	1	1	1	3	5	2	0
0	2	3	2	1	0	2	2	1

Table (2):The restrictions for each equations in simulation study when *l*=10

R_1			R_2			R_3			R_4			R_5		
2	1	1	2	2	4	2	1	1	5	2	0	2	1	3
1	5	1	1	1	3	5	2	0	4	2	3	4	1	3
0	2	3	2	1	0	2	2	1	2	1	1	2	3	7
R_6			R_7			R_8			R_9			R_{10}		
3	1	3	2	1	6	2	1	1	2	1	3	5	4	0
2	5	5	5	2	1	5	5	4	4	2	2	2	1	5
2	2	0	2	1	2	3	1	5	3	3	1	4	6	2

The factors that are used in the simulation are summarized in the table (3).

Table (3):The factors in simulation study

Factors	The alternative value
Number of observations	30, 100
Number of equations	3,10
Correlation between variables	0.85 , 0.15
Correlation between errors	0.80,0.20

The simulation is repeated 1000 times using Matlap software and we to illustrate the superiority between estimator, we use the standard mean squares error (MSE) criterion which is defined as follows:

$$MSE(\hat{\beta}) = \frac{\sum_{i=1}^{1000}(\hat{\beta}_i - \beta)'(\hat{\beta}_i - \beta)}{1000}$$

4 The simulation results

Table (4):The value for MSE for different estimators$(\rho_x = 0.85)$

Factors	$\hat{\beta}_{SRSE}$	$\hat{\beta}_{SRRSE}$	$\hat{\beta}_{SRLTSE}$
T=30, q=2, $\rho_\mu = 0.20$	25.954	18.716	15.824
T=100, q=2,$\rho_\mu = 0.20$	23.841	16.874	11.254
T=30, q=10 , $\rho_\mu = 0.20$	31.085	23.314	17.726
T=100, q=10 , $\rho_\mu = 0.20$	27.231	20.517	18.658
T=30, q=2, $\rho_\mu = 0.80$	24.185	17.622	14.212
T=100, q=2, $\rho_\mu = 0.80$	21.325	15.852	10.571
T=30, q=10 , $\rho_\mu = 0.80$	29.321	21.325	16.325
T=100, q=10 , $\rho_\mu = 0.80$	25.365	19.902	17.057

Table (5):The value for MSE for different estimators$(\rho_x = 0.15)$

Factors	$\hat{\beta}_{SRSE}$	$\hat{\beta}_{SRRSE}$	$\hat{\beta}_{SRLTSE}$
T=30, q=2, $\rho_\mu = 0.20$	21.521	14.182	11.245
T=100, q=2,$\rho_\mu = 0.20$	20.518	12.941	8.215
T=30, q=10 , $\rho_\mu = 0.20$	27.321	16.324	14.879
T=100, q=10 , $\rho_\mu = 0.20$	23.854	18.052	15.365
T=30, q=2, $\rho_\mu = 0.80$	21.325	16.321	13.987
T=100, q=2, $\rho_\mu = 0.80$	18.056	13.214	7.481
T=30, q=10 , $\rho_\mu = 0.80$	26.325	20.005	15.210
T=100, q=10 , $\rho_\mu = 0.80$	23.104	17.258	15.021

For Table (4), (5), we observe that the $\hat{\beta}_{SRLTSE}$ has the best performance according to MSE for all the situations and it works better at the higher degrees for correlation between variables and at the lower degrees for correlation between errors. The increase in the number of observations and number of equations have a good effect on all estimators.

5 Conclusions

In this paper, we presented a liu-type estimator for the restricted SUR model in case of multicollinearity. This estimator was defined the Stochastic Restricted Liu Type SUR estimator . The simulation study is use to evaluate the Stochastic Restricted SUR estimator, Stochastic Restricted Ridge SUR estimator and Stochastic Restricted Liu Type SUR estimator. The MSE criterion used as a criterion to show the superiority of estimators. The MSE criterion shows that, the Stochastic Restricted Liu Type SUR estimator (SRLTSE) has a better performance than the others. estimators.

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