Weighted Analogue of Inverse Gamma Distribution: Statistical Properties, Estimation and Simulation Study

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Abstract

In this article we propose a new weighted version of inverse Gamma distribution known as Weighted Inverse Gamma distribution (WIGD). We examine the Length biased and Area biased versions of Weighted Inverse Gamma distribution. Basic structural properties viz moments, mode, moment generating function (mgf), characteristic function (cf), hazard rate function and measures of uncertainty. The parameters of this model are estimated from both classical (namely, maximum likelihood estimator and method of moments, and compare them by using extensive numerical simulations) and Bayesian point of view. The Bayes estimates are estimated by using non-informative Jeffrey's prior and informative Inverse Chi square prior under different types of loss function (symmetric and asymmetric loss functions). Finally, a simulation study has been conducted for comparing weighted inverse gamma distribution with other competing distributions.

Keywords: Inverse gamma distribution; Moments; Entropy; Hazard rate; Maximum likelihood estimator; Moment estimator.

1. Introduction

The inverse gamma distribution $IG(\alpha, \beta)$ with parameters α and β, is mentioned infrequently in statistical literature, and usually for a specific purpose. One primary use of the *IG* distribution is for Bayesian estimation of the mean of the one parameter exponential distribution (see for example Johnson et al. (1995), as well as estimating variance in a normal regression. A number of brief descriptions of the properties of the distribution are available, mostly in text books on Bayesian methods, often in the econometrics literature, e.g., Koch (2007) and Poirier (1995). Kleiber and Kotz (2003) list some basic structural properties of the *IG* distribution and also model incomes with the distribution. Milevsky and Posner (1998) studied the inverse gamma distribution and point out estimation by method of moments.

The learning of weighted distributions can be used for better comprehension of standard distributions, and provides techniques of spreading distributions for further flexibility to fit the data superior. Rao (1965) proposed the concept of weighted distribution, Patil and Rao (1978) discussed how, for example, truncated distributions and damaged observations can give rise to weighted distributions. Weighted distributions occur frequently in research related to bio-medicine, reliability, ecosystem and branching process can be seen in Patil and Rao (1986), Sharma et al. (2017) studied on Length and Area biased Maxwell distribution, Ahmad et al. (2016) studied length biased Weighted

Lomax distribution with its applications, Das and Roy (2011) discussed the length-biased Weighted Generalized Rayleigh distribution with its properties, also they develop the length-biased Weighted Weibull distribution.

Suppose X is a non-negative random variable with probability density function (pdf) $f(x)$, and then the pdf of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x)\, f(x)}{E(w(x))}\ ,\ x > 0 \qquad (1)$$

where $w(x)$ be a non-negative weight function. On the support of X, where $w(x) > 0$ and $\omega = w(x)\, f(x)\, d\, x$ is a normalizing constant that forces $f^{w}(x)$ to integrate to 1.

Subject upon the choice of weight function $w(x)$, we will get dissimilar weighted distributions. In this paper, we ruminate $w(x) = x^c$ and the model is thus achieved is stated as size biased distribution. Evidently when $c = 1$, the weight function depends on the length of units of interest, then the resultant distribution is called length biased distribution. Correspondingly, for $c = 2$, the resultant distribution is called area biased distribution.

This paper is divided in to two parts: first is to study the structural properties of the weighted inverse gamma distribution (WIG) along with its special cases (for $c = 1$ and 2), and second is to estimate the parameters of the model from both classical and Bayesian view point. Finally, simulation study, summary is provided.

2. Weighted Inverse Gamma distribution

In this section, we build the pdf of weighted Inverse Gamma distribution by taking the weight function as $w(x) = x^c$ and study the behavior of its pdf and hazard function. The probability density function (pdf) and cumulative distribution function (cdf) of the Inverse Gamma distribution is given by

$$f(x;\alpha,\beta) = \frac{\alpha^{\beta}}{\Gamma(\beta)} \frac{1}{x^{\beta+1}} e^{-\frac{\alpha}{x}} \quad , x > 0, \alpha > 0, \beta > 0 \qquad (2)$$

$$F_X(x) = \frac{\gamma\left(\beta, \frac{\alpha}{x}\right)}{\Gamma(\beta)} \qquad (3)$$

where $\gamma\left(\beta, \frac{\alpha}{x}\right)$ denotes the upper incomplete gamma function.

Weighted Inverse Gamma distribution (WIGD) is obtained by applying the weights x^c, to the Classical Inverse Gamma distribution. To define Weighted Inverse Gamma distribution if $X \sim WIG(c,\alpha,\beta)$, then pdf of X is given by

$$f_W(x;c,\alpha,\beta) = \frac{\alpha^{\beta-c}}{\Gamma(\beta-c)} \frac{1}{x^{\beta-c+1}} e^{-\frac{\alpha}{x}} \quad , x > 0, \alpha > 0, \beta > c, c > 0 \qquad (4)$$

By substituting $c = 1$ and $c = 2$ in (4), we get the pdfs of length biased Inverse Gamma (LBIG) and area biased Inverse Gamma (ABIG) distributions respectively. Figure (1.1) represents shapes of Weighted Inverse gamma distribution for different values of parameters.

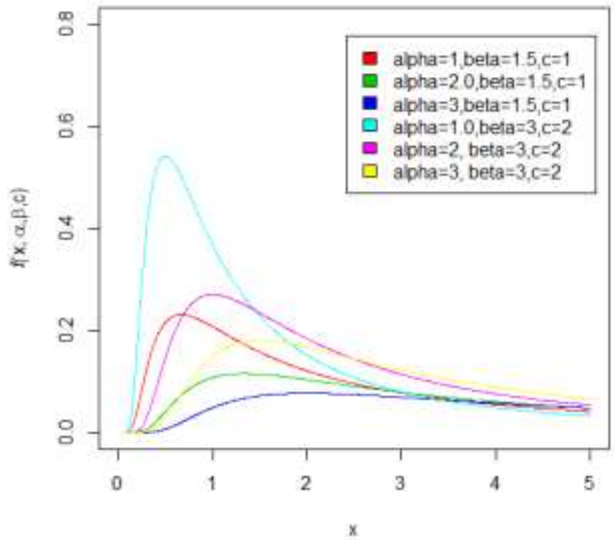

Figure 1.1: Pdf of WIG Distribution under different values to parameters

The cdf, reliability function and hazard function corresponding to the pdf (4) are respectively given by

$$F_{WX}(x) = \frac{\gamma\left(\beta - c, \frac{\alpha}{x}\right)}{\Gamma(\beta - c)}$$

$$R(x) = 1 - \frac{\gamma\left(\beta - c, \frac{\alpha}{x}\right)}{\Gamma(\beta - c)}$$

$$h(x) = \frac{\alpha^{\beta - c}}{\left(\Gamma(\beta - c) - \gamma\left(\beta - c, \frac{\alpha}{x}\right)\right)} \frac{1}{x^{\beta - c + 1}} e^{-\frac{\alpha}{x}}$$

where $\Gamma a = \int_0^\infty e^{-x} x^{a-1} dx$ is gamma function and $\gamma(a,b) = \int_0^b e^{-x} x^{a-1} dx$ is the upper incomplete gamma function.

Relationship with other distributions

Some well-known theoretical distributions can be derived from the proposed WIG distribution. For instance;

1. For $c=0$, equation (4) reduces to the inverse gamma distribution.
2. For $\beta = 1$ and $c=0$, equation (4) reduces to the inverse Exponential distribution.
3. For $\beta = \frac{\beta}{2}$, $\alpha = \frac{1}{2}$ and $c=0$, equation (4) reduces to Inverse Chi-Square distribution.

4. For $\beta = \frac{1}{2}$, $\alpha = \frac{l}{2}$ and *c=0*, equation (4) reduces to Levy distribution.

2.1. Mode of Weighted Inverse Gamma distribution

The pdf of $WIG(c,\alpha,\beta)$ is unimodal for given $c, \alpha \, and \, \beta$ and achieve its maximum at $X_{mode} = \frac{\alpha}{\beta - c + 1}$.

Proof:

For the pdf (4),

$$f_w'(x) = \frac{\partial}{\partial x} f_w(x) = \left(\frac{c - \beta - 1}{x} + \frac{\alpha}{x^2} \right) f_w(x)$$

The mode of the weighted inverse gamma can be readily obtained from

$$f_w'(x) = 0 \quad \Rightarrow \left(\frac{c - \beta - 1}{x} + \frac{\alpha}{x^2} \right) f_w(x) = 0$$

Since $f_w(x) \neq 0, \forall x > 0$, we get

$$\left(\frac{c - \beta - 1}{x} + \frac{\alpha}{x^2} \right) = 0$$

Thus the pdf (4) achieve its maximum at $X_{mode} = \frac{\alpha}{\beta - c + 1}$.

2.2. Moments of Weighted Inverse Gamma distribution

Moments helps to determine many properties of the distribution such as Averages, dispersion, skewness and kurtosis. The rth moment about origin of the Weighted Inverse Gamma distribution is given by

$$\mu_r' = \frac{\alpha^{\beta - c}}{\Gamma(\beta - c)} \frac{\Gamma(\beta - c - r)}{\alpha^{\beta - c - r}}, r = 1,2,3,... \qquad (5)$$

The mean and variance of Weighted Inverse Gamma distribution is given by

$$\mu = \frac{\alpha}{\beta - c - 1} \quad \sigma^2 = \frac{\alpha^2}{(\beta - c - 1)^2 \, (\beta - c - 2)}$$

By substituting c =1, 2 in (5) the mean and variance of Length biased Inverse Gamma and Area biased inverse gamma distribution are $\left(\mu = \frac{\alpha}{\beta - 2}, \, \sigma^2 = \frac{\alpha^2}{(\beta - 2)^2 \, (\beta - 3)} \right)$ and $\left(\mu = \frac{\alpha}{\beta - 3}, \, \sigma^2 = \frac{\alpha^2}{(\beta - 3)^2 \, (\beta - 4)} \right)$. The coefficient of skewness and kurtosis of $WIG(c,\alpha,\beta)$ are given by

$$S_k = \beta_1 = \frac{4\sqrt{\beta - c - 2}}{\beta - c - 3} \tag{6}$$

$$K_r = \beta_2 = \frac{30\beta - 30c - 66}{(\beta - c - 3)(\beta - c - 4)} \tag{7}$$

By putting c=1,2 in (6) and (7), the skewness and kurtosis of LBIG and ABIG distribution are $\left(\beta_1 = \frac{4\sqrt{\beta-3}}{\beta-4},\ \beta_2 = \frac{30\beta - 96}{(\beta-4)(\beta-5)}\right)$ and $\left(\beta_1 = \frac{4\sqrt{\beta-4}}{\beta-5},\ \beta_2 = \frac{30\beta - 126}{(\beta-5)(\beta-6)}\right)$ respectively.

Corollary 2.1.
The Moment generating function of $WIG(c,\alpha,\beta)$ does not exists.

Corollary 2.2.
The characteristic function of $WIG(c,\alpha,\beta)$ is given by

$$\phi_X(t) = \frac{2(i\alpha t)^{\frac{\beta-c}{2}}}{\Gamma(\beta - c)} K_\beta\left(\sqrt{-4i\alpha t}\right)$$

Proof:
By definition of moment generating function

$$\phi_X(t) = E(e^{itx}) = \int_0^\infty e^{itx} f_w(x)\,dx$$

Substituting the pdf of $WIG(c,\alpha,\beta)$ in above expression we get the required result.

$$\phi_X(t) = E(e^{itx}) = \frac{\alpha^{\beta-c}}{\Gamma(\beta-c)} \int_0^\infty e^{itx} \frac{1}{x^{\beta-c+1}} e^{-\frac{\alpha}{x}} dx$$

$$\phi_X(t) = \frac{2(i\alpha t)^{\frac{\beta-c}{2}}}{\Gamma(\beta - c)} K_\beta\left(\sqrt{-4i\alpha t}\right)$$

where

$$K_\beta\left(\sqrt{z}\right) = \frac{1}{2}\left(\frac{1}{2}z\right)^{-\beta} \sum_{k=0}^{\beta-1} \frac{(\beta-k-1)!}{k!}\left(-\frac{1}{4}z^2\right)^k + (-1)^{\beta+1}\ln\left(\frac{1}{2z}\right) I_\beta(z) +$$

$$(-1)^\beta \left(\frac{1}{2}\right)\left(\frac{1}{2}z\right)^\beta \sum_{k=0}^{\infty} [\psi(k+1) + \psi(\beta+k+1]\frac{\left(\frac{1}{2}z^2\right)^k}{k!(\beta+k)!}$$

is modified Bessel function of second kind and $\psi(.)$ is digamma function.

2.3 Measures of Uncertainty

The entropy of a random variable X with probability density $WIG(c,\alpha,\beta)$ is a measure of variation of the uncertainty.

2.3.1 Shannon's Entropy

The Shannon's entropy is given by

$$H(X) = -E\left(\log(f_w(x))\right)$$

Using pdf (4) in above equation we get

$$\begin{aligned} H(X) &= \log\Gamma(\beta-c) - (\beta-c)\log\alpha - (\beta-c+1)E\left(\log(1/x)\right) + \alpha E(1/x) \\ &= (\beta-c) + \log(\alpha\,\Gamma(\beta-c)) - (\beta-c+1)\psi(\beta-c) \qquad (8) \end{aligned}$$

where

$$\psi(x) = \frac{\Gamma' x}{\Gamma x} \text{ and } \int_0^\infty \log(x)\; x^{p-1}\, e^{-x}\, dx = \Gamma' p$$

2.3.2. Rényi entropy

The Rényi entropy (1960), denoted by $I_R(\rho)$ is defined as

$$I_R(\rho) = \frac{1}{1-\rho}\log\left\{\int_{-\infty}^{\infty} f_w{}^{\rho}(x)\,dx\right\}$$

where $\rho > 0$ and $\rho \neq 1$.

By substituting pdf (4) in above expression, we obtain

$$\begin{aligned} I_R(\rho) &= \frac{1}{1-\rho}\log\left\{\int_0^\infty \left(\frac{\alpha^{\beta-c}}{\Gamma(\beta-c)}\right)^{\rho} \frac{1}{x^{\rho(\beta-c+1)}}\, e^{-\frac{\rho\alpha}{x}}\, dx\right\} \\ &= \frac{1}{1-\rho}\log\left\{\frac{\alpha^{\rho(\beta-c)}}{\Gamma^{\rho}(\beta-c)} I\left(\rho(\beta-c+1)-1, \rho\alpha\right)\right\} \qquad (9) \end{aligned}$$

Where

$$I(a,b) = \int_0^\infty \frac{1}{x^{a+1}}\, e^{-\frac{b}{x}}\, dx = \frac{\Gamma a}{b^a}$$

3. Methods of Estimation

In this section, parameters of weighted inverse gamma distribution are estimated by various methods of estimation viz, method of moments, maximum likelihood estimation and Bayesian method of estimation.

3.1. Method of Moments

In order to estimate the unknown parameters of $WIG(c,\alpha,\beta)$ model by the method of moments, we equate the sample moments with the corresponding population moments.

$$\mu_r{}' = \frac{1}{n}\sum_{i=0}^{n} x^r{}_i$$

Replacing sample moments with population moments, we get

$$\frac{1}{n}\sum_{i=1}^{n} x_i = \mu_1'$$

$$\Rightarrow \bar{x} = \frac{\alpha}{\beta - c - 1} \tag{10}$$

and $$\frac{1}{n}\sum_{i=1}^{n} x_i^2 = \mu_2'$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2 = \mu_2$$

$$\Rightarrow s^2 = \frac{\alpha^2}{(\beta - c - 1)^2 (\beta - c - 2)} \tag{11}$$

On solution the equations (10) and (11) we obtain the estimates for α and β say $\hat{\alpha}$ and $\hat{\beta}$ respectively.

3.2. Maximum Likelihood Estimation

Let $x_1, x_2, x_3.......x_n$ be a random sample from the Weighted Inverse Gamma distribution, then the corresponding likelihood function is given as

$$l(X;\alpha,\beta) = \left(\frac{\alpha^{\beta-c}}{\Gamma(\beta-c)}\right)^n \prod_{i=1}^{n}\left(\frac{1}{x_i^{\beta-c+1}}\right)\exp\left(-\frac{\alpha}{\sum_{i=1}^{n} x_i}\right)$$

The log-likelihood function is given as:

$$\log l(x;\alpha,\beta) = n(\beta-c)\log\alpha - n\log\Gamma(\beta-c) + \sum_{i=1}^{n}\log\left(\frac{1}{x_i^{\beta-c+1}}\right) - \frac{\alpha}{\sum_{i=1}^{n} x_i} \tag{12}$$

Now, differentiate above equation with respect to parameters α and β and equate to zero, we obtain the normal equations

$$\frac{\partial \log l}{\partial \alpha} = 0 \Rightarrow \frac{n(\beta-c)}{\alpha} - \frac{1}{\sum_{i=1}^{n} x_i} = 0 \tag{13}$$

$$\frac{\partial \log l}{\partial \beta} = 0 \Rightarrow n\log\alpha - n\left(\log(\beta-c) - \frac{1}{2(\beta-c)}\right) + \sum_{i=1}^{n}\log x_i = 0 \tag{14}$$

The MLE $\hat{\eta} = (\hat{\alpha}, \hat{\beta})$ of $\eta = (\alpha, \beta)$ is obtained by solving the above nonlinear system of equations. It is usually more convenient to use nonlinear optimization algorithms such as quasi-Newton algorithm to numerically maximize the log likelihood function given in (12). Applying the usual large sample approximation, the MLE $\hat{\eta}$ can be treated as being approximately bivariate normal with variance-covariance matrix equal to the inverse of the expected information matrix, i.e.

$$\sqrt{n}\,(\hat{\eta}-\eta)\rightarrow N\left(0,n\,I^{-1}(\eta)\right)$$

where $I^{-1}(\eta)$ is the limiting variance-covariance matrix of $\hat{\eta}$.

3.3. Bayesian Method of Estimation

In this section, we construct Bayes estimator of the scale parameter α of WIG distribution using non informative Jeffrey's prior and informative Inverse Chi square prior under different loss functions.

3.3.1 Posterior distribution using Jeffrey's prior

Assuming that α has Jeffery prior i.e $g(\alpha)\propto\frac{1}{\alpha}$ the posterior distribution is given by

$$P_J(\alpha\,|\,\underset{-}{x})=\frac{t_1^{\,n(\beta-c)}}{\Gamma(n(\beta-c))}\alpha^{n(\beta-c)-1}e^{-\alpha t_1} \qquad (15)$$

which is density function of gamma distribution with shape parameter (t_1) and scale parameter $n(\beta-c)$, where $t_1=\frac{1}{\sum_{i=1}^{n}x_i}$.

The Bayes estimate of α using Jeffrey's prior under SELF, Entropy and LINEX are given by

$$\hat{\alpha}_S=\frac{n(\beta-c)}{t_1}$$

$$\hat{\alpha}_E=\frac{(n(\beta-c)-1)}{t_1}$$

and

$$\hat{\alpha}_L=\frac{1}{a}\log\left(\frac{a+t_1}{t_1}\right)^{n(\beta-c)}$$

3.3.2 Posterior Risk Functions:

The risk functions of the estimators $\hat{\alpha}_S$, $\hat{\alpha}_E$ and $\hat{\alpha}_L$ relative to SELF, entropy loss function and LINEX loss function are denoted by $R(\hat{\alpha}_S)$, $R(\hat{\alpha}_E)$ and $R(\hat{\alpha}_L)$ are given by

$$R(\hat{\alpha}_S)=\hat{\alpha}^2+\frac{(n(\beta-c)+1)(n(\beta-c))}{t_1^{\,2}}-\frac{2n(\beta-c)\hat{\alpha}}{t_1}$$

$$R(\hat{\alpha}_E)=\frac{\hat{\alpha}t_1}{(n(\beta-c))}-\log\hat{\alpha}+\psi(n(\beta-c))-1$$

$$R(\hat{\alpha}_E) = e^{a\hat{\alpha}}\left(\frac{t_1}{t_1+a}\right)^{n(\beta-c)} - a\hat{\alpha} + \frac{a(n(\beta-c))}{t_1} - 1$$

where $\psi(n(\beta-c))$ is a digamma function.

Lemma:
For given posterior distribution (15), we have

$$E(\alpha \mid \underset{-}{x}) = \sum_{i=1}^{n} x_i \,(n(\beta-c))$$

$$E(\alpha^2 \mid \underset{-}{x}) = \left(\sum_{i=1}^{n} x_i\right)^2 (n(\beta-c))(n(\beta-c)+1)$$

$$E(\alpha^{-1} \mid \underset{-}{x}) = \frac{1}{\sum_{i=1}^{n} x_i \,(n(\beta-c)-1)}$$

$$E(\alpha^{-2} \mid \underset{-}{x}) = \frac{1}{\left(\sum_{i=1}^{n} x_i\right)^2 (n(\beta-c)-1)(n(\beta-c)-2)}$$

3.3.3 Posterior distribution using inverse Chi-square prior

Assuming that α has inverse chi-square defined by $g(\alpha) = \frac{(\nu/2)^{\frac{\nu}{2}}}{\Gamma(\nu/2)} \frac{e^{-\frac{\nu}{2x}}}{x^{1+\nu/2}}$ the posterior distribution is given by

$$P_{IC}(\alpha \mid \underset{-}{x}) = \frac{t_2^{\,n(\beta-c)+2}}{\Gamma(n(\beta-c)+2)} \alpha^{n(\beta-c)+1} e^{-t_2\alpha} \qquad (16)$$

which is density function of gamma distribution with shape parameter (t_2) and scale parameter $(n(\beta-c)+2)$, where $t_2 = \left(\theta + \frac{1}{\sum_{i=1}^{n} x_i}\right)$.

The Bayes estimate of α using Inverse Chi-square prior under SELF, Entropy and LINEX are given by

$$\hat{\alpha}_S = \frac{n(\beta-c)+2}{t_2}$$

$$\hat{\alpha}_E = \frac{(n(\beta-c)+1}{t_2}$$

And

$$\hat{\alpha}_L = \frac{1}{a}\log\left(\frac{a+t_2}{t_2}\right)^{n(\beta-c)+2}$$

3.3.4 Posterior Risk Functions:

The risk functions of the estimators $\hat{\alpha}_S$, $\hat{\alpha}_E$ and $\hat{\alpha}_L$ relative to SELF, entropy loss function and LINEX loss function are denoted by $R(\hat{\alpha}_S)$, $R(\hat{\alpha}_E)$ and $R(\hat{\alpha}_L)$ are given by

$$R(\hat{\alpha}_S) = \hat{\alpha}^2 + \frac{(n(\beta-c)+3)(n(\beta-c)+2)}{t_2^{\,2}} - \frac{2(n(\beta-c)+2)\hat{\alpha}}{t_2}$$

$$R(\hat{\alpha}_E) = \frac{\hat{\alpha}\, t_2}{(n(\beta-c)+1)} - \log\hat{\alpha} + \psi(n(\beta-c)+2) - 1$$

$$R(\hat{\alpha}_E) = e^{a\hat{\alpha}}\left(\frac{t_2}{a+t_2}\right)^{n(\beta-c)+2} - a\hat{\alpha} + \frac{a(n(\beta-c)+2)}{t_2} - 1$$

where $\psi(n(\beta-c)+2)$ is a digamma function.

4. Simulation Study

The most common and simplest method for generating random sample is based on the inverse cumulative distribution function (cdf). For arbitrary cdf, define $G^{-1}(u)$ = min {x; $G(x) \geq u$}. The inverse cdf method can't be directly applied for WIG distribution because of the closed form expression for its quantile function is not available. Here, we intend to use Newton's method for the calculation of the quantile function numerically. The algorithm used for this determination is as follows:

Algorithm:

Step 1. Set n, c, α, β and initial value x^0.

Step 2. Generate U ~ Uniform(0, 1).

Step 3. Update x^0 by using the Newton's formula, $x^* = x^0 - R(x^0, c, \alpha, \beta)$, where $R(x^0, c, \alpha, \beta) = \frac{G(x^0, c, \alpha, \beta) - U}{g(x^0, c, \alpha, \beta)}$, where G(.) and g(.) are cdf and pdf WIG distribution respectively.

Step 4. If $|x^0 - x^*| < \in$, (very small, $\in > 0$ tolerance limit), store $x=x^*$ as a sample from $WIG(c, \alpha, \beta)$.

Step 5. If $|x^0 - x^*| > \in$, then set $x^0=x^*$ and go to step 3.

Step 6. Repeat steps 2-5, n times for $x_1, x_2, x_3, ..., x_n$ respectively.

On the basis of generated sample from the above algorithm, we use R code and check the goodness of fit of the LWIG and AWIG distributions and compare the appropriateness with some other class of weighted distributions.

We apply KS test and AIC criterion for goodness of fit. The summary of the fitting is presented in Table 4.1. From the table, we observe that the family of weighted Inverse Gamma distribution is suitable for modelling than the other competing distributions.

Table 4.1: Estimation of Parameters and Comparison Criteria

Distribution	Parameter estimate	Standard Error	*-2logl*	AIC	BIC	AICC	Shannon Entropy
IGD	$\hat{\alpha}$ = 1.1643 $\hat{\beta}$ =0.8367	0.3823 0.2050	136.479	140.479	139.273	141.024	2.729
LBIGD	$\hat{\alpha}$ = 1.8826 $\hat{\beta}$ =2.3149	0.5814 0.3351	109.488	113.488	112.282	114.033	2.1806
ABIGD	$\hat{\alpha}$ = 1.9084 $\hat{\beta}$ =3.4394	0.5841 0.3694	100.871	104.871	103.665	105.416	2.017
Inverse Chi Square	$\hat{\alpha}$ =0.9387	0.1705	219.610	221.610	220.989	221.783	4.392
Inverse Exponential	$\hat{\alpha}$ =1.3106	0.2621	123.086	125.086	124.483	125.259	2.461
Levy	$\hat{\alpha}$ =0.0357	0.0101	354.681	356.681	356.078	356.854	7.093

From Table 4.1, it has been observed that the family of Weighted Inverse Gamma distribution have the lesser AIC, AICC and BIC values as compared to other competing distributions. Hence we can concluded that the Weighted Inverse Gamma distribution leads to a better fit and is suitable for modeling than the other competing distributions.

5. Application

The data set represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal (1960). The data are as follows:
0.1, 0.33, 0.44, 0.56, 0.59, 0.59, 0.72, 0.74, 0.92, 0.93, 0.96, 1, 1, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15, 1.16, 1.2, 1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.6, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55, 2.54, 0.77.

Distribution	Parameter Estimate	Standard Error	*-2logl*	AIC	BIC	AICC
IGD	$\hat{\alpha}$ = 0.2730 $\hat{\beta}$ =1.3008	0.1868 1.71105	112.7648	118.764	124.5007	119.2865
LBIGD	$\hat{\alpha}$ = 0.0332 $\hat{\beta}$ =0.0421	0.0223 0.0166	77.04523	83.0452	88.7812	83.5669
ABIGD	$\hat{\alpha}$ =0.0151 $\hat{\beta}$ = 0.0343	0.0028 0.0051	41.6281	47.6281	53.3641	48.1498
Inverse Chi Square	$\hat{\alpha}$ =11.4351	6.5347	114.6634	116.6634	118.5760	116.7467
Inverse Exponential	$\hat{\alpha}$ =13.1806	18.4316	94.6973	98.6973	102.5213	98.9526
Levy	$\hat{\alpha}$ =0.1772	0.0289	246.1711	248.1711	250.0837	248.2544

From above Table, it has been clearly observed that the family of Weighted Inverse Gamma distribution have the lesser AIC, AICC and BIC values as compared to other competing distributions. Hence we can concluded that the Weighted Inverse Gamma distribution leads to a better fit and is suitable for modeling than the other competing distributions.

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