# An Exponential and Log Ratio Estimator of Population Mean Using Auxiliary Information in Double Sampling

Yasir Hassan

Department of Statistics, COMSATS Institute of IT, Lahore, Pakistan yasir.uol@hotmail.com

Muhammad Ismail

Department of Statistics, COMSATS Institute of IT, Lahore, Pakistan drismail39@gmail.com

Aamir Sanaullah

Department of Statistics, COMSATS Institute of IT, Lahore, Pakistan chaamirsanaullah@yahoo.com

#### **Abstract**

In this study an improved version of ratio type exponential estimator is been proposed for estimating average of study variable when the population parameter(s) information of second auxiliary variable is available. The proposed estimator compared with usual unbiased estimator and conventional ratio estimators numerically and hypothetically. The mean square error is also obtained and checked the efficiency of the proposed estimator with usual ratio, Singh and Vishwakarma (2007), Singh et al. (2008), Noor-ul-Amin and Hanif (2012), Yadav et al. (2013) and Sanaullah et al. (2015) estimators.

**Keywords**: Axillary Variable, Exponential, Two Phase and Mean Square Error.

## 1. Introduction

The use of auxiliary variables often increases estimator's efficiency when ratio and regression methods are considered for estimation in survey sampling. The product estimation method may be applied for negative linear relationship and the ratio method of estimation should be applied for the positive linear relationship between response and auxiliary variables. The ratio method for estimating population mean of study variable was introduced by Cochran (1940) using auxiliary variable in single phase sampling design for estimating population mean. Single-phase sampling was used by Robson (1957)-and-Murthy (1964) for estimating the population mean using product estimators. Further contributions under single phased sampling design on the use of auxiliary information include Srivastava (1967), Walsh (1970)-Gupta (1978)-Vos (1980), Kaur (1983), Bahl and Tuteja (1991), Samiuddin and Hanif (2006), Upadhyaya et al. (2011) and Hanif et al. (2009). Sometimes, estimation is done from large sample to observe the auxiliary variable x when the mean value  $\overline{X}$  is unknown and later on a smaller sample is chosen from already drawn large sample for estimating y (study variate). This method is named as two phase or double sampling. Hidiroglou and Särndal (1998) proposed twophase sampling design using two auxiliary variables, which was very cost effective.

Usually the ratio estimators do not work efficiently under situations when the relationship is not linear between the auxiliary and study variables. In such situations the exponential product and ratio estimators proposed by (Bahl & Tuteja, 1991). The exponential ratio estimators were proposed by Singh and Vishwakarma (2007) followed the work of Bahl and Tuteja (1991) in double sampling design. The auxiliary information was used by Samiuddin and Hanif (2007) in two-phase sampling and presented three situations for estimating population parameter i.e. full, partial and no information case. Singh et al. (2008) proposed an unbiased estimator when the information of population parameters was available in two phase sampling. Hanif et al. (2009) extended the work of ratio estimators under two and multi phase sampling schemes. Furthermore, Ahmad et al. (2009) did the comprehensive contribution of ratio estimators. An improved exponential ratio type estimators using two auxiliary variables has been proposed by Sanaullah et al. (2012) for estimating the populations mean in two phase sampling. Sanaullah et al. (2015) also suggested the generalized exponential estimator for estimating population variance under two-phase sampling design.

Let the population  $T = (T_1, T_2, T_3, ..., T_N)$  of N units with x and y, auxiliary and study variables respectively. The population means are  $\overline{Y} = \mathop{\circ}\limits_{i=1}^{N} \frac{y_i}{N}$  and  $\overline{X} = \mathop{\circ}\limits_{i=1}^{N} \frac{x_i}{N}$ .

Consider the usual ratio estimator

$$\overline{Y}_r = \left(\frac{\overline{y}}{\overline{x}}\overline{X}\right); \overline{x} \neq 0 \quad (1.1)$$
Where  $\overline{y} = \mathop{\mathring{\text{a}}}_{i=1}^n \left(\frac{y_i}{n}\right)$  and  $\overline{x} = \mathop{\mathring{\text{a}}}_{i=1}^n \left(\frac{x_i}{n}\right)$ .

A ratio type exponential estimator suggested by Bahl and Tuteja (1991) in two phase sampling is modified by Singh and Vishwakarma (2007) as:

$$\overline{Y}_{sv} = \overline{y}_2 \exp\left[\frac{\overline{x}_1 - \overline{x}_2}{\overline{x}_1 + \overline{x}_2}\right]. \tag{1.2}$$

The mean square error of Singh and Vishwakarma (2007) is

$$MSE(\overline{Y}_{sv}) = \overline{Y}^{2}C_{y}^{2} \left[ f_{2} + \frac{C_{x}}{4C_{y}} (f_{2} - f_{1}) \left( \frac{C_{x}}{C_{y}} - r_{xy} \right) \right]. \tag{1.3}$$

An exponential estimator ratio-type proposed by Singh et al. (2008) as:

$$\overline{Y}_s = W_{0d}\overline{y}_2 + W_{1d}\overline{y}_{rsd} + W_{2d}\overline{y}_{rsed}$$
(1.4)

where

$$\sum_{i=0}^{2} w_{id} = 1, \ t_{rsd} = \overline{y}_{2} \exp \left[ \frac{a\overline{x}_{(1)} + b}{a\overline{x}_{(2)} + b} \right], \quad t_{rsd} = \overline{y}_{2} \exp \left[ \frac{\left(a\overline{x}_{(1)} + b\right) - \left(a\overline{x}_{(2)} + b\right)}{\left(a\overline{x}_{(1)} + b\right) + \left(a\overline{x}_{(2)} + b\right)} \right]$$

Minimum mean square error of  $\overline{Y}_s$  is

$$MSE(\overline{Y}_s) = \overline{Y}^2 C_v^2 \left( f_2 - \left( f_2 - f_1 \right) \Gamma_{xy}^2 \right) \tag{1.5}$$

The ratio-cum-product exponential estimator in double sampling is proposed by Noor-ul-Amin and Hanif (2012) as:

$$\overline{Y}_{NH_a} = \overline{y}_2 \exp\left[\frac{\overline{Z} - \overline{z}_2}{\overline{Z} + \overline{z}_2} - \frac{\overline{X} - \overline{x}_2}{\overline{X} + \overline{x}_2}\right]. \tag{1.6}$$

The min (MSE) of the  $\overline{Y}_{NH_a}$  is given as:

$$MSE(\overline{Y}_{NH_a}) = \overline{Y}^2 \left[ \frac{f_2}{4} \left( 4C_y^2 + C_z^2 - 4C_y C_z r_{yz} \right) + \frac{f_1}{4} \left( C_x^2 + 4C_y C_x r_{yx} - 2C_x C_z r_{xz} \right) \right]. \tag{1.7}$$

Noor-ul-Amin and Hanif (2012) also proposed chain ratio-type estimator as:

$$\overline{Y}_{NH_b} = \overline{y}_2 \exp \left[ \frac{\overline{X} - \overline{x}_1}{\overline{X} + \overline{x}_1} - \frac{\overline{Z} - \overline{z}_2}{\overline{Z} + \overline{z}_2} \right]. \tag{1.8}$$

The MSE of  $\overline{Y}_{NH_h}$  is:

$$MSE(\overline{Y}_{NH_b}) = \overline{Y}^2 \left[ \frac{f_2}{4} \left( 4C_y^2 + C_z^2 - 4C_yC_z r_{yz} \right) + \frac{f_1}{4} \left( C_x^2 - 4C_yC_x r_{yx} + 2C_xC_z r_{xz} \right) \right]. \tag{1.9}$$

Inspired by the above estimators Sanaullah et al. (2012) proposed modified exponential estimator using two auxiliary variables as:

$$\overline{Y}_{SA} = \overline{y}_2 \exp \left[ \partial \frac{\overline{Z} - \overline{z}_2}{\overline{Z} + \overline{z}_2} - (1 - \partial) \frac{\overline{X} - \overline{x}_2}{\overline{X} + \overline{x}_2} \right]. \tag{1.10}$$

Minimum mean square error of  $\overline{Y}_{SA}$  is

$$MSE(\overline{Y}_{SA}) = f_2\overline{Y}^2 \left[ C_y^2 + \hat{a}^2 C_z^2 - 2C_y C_z \Gamma_{yz} \right] + f_1\overline{Y}^2 \left( 1 - \hat{a} \right) \left[ \left( \frac{1 - \hat{a}}{4} \right) C_x^2 + C_y C_x \Gamma_{yx} - \hat{a} C_x C_z \Gamma_{xz} \right]. \quad (1.11)$$

Yadav et al. (2013) proposed ratio type exponential estimator given as:

$$\overline{Y}_{Yad} = \overline{y} \exp \left[ \frac{\hat{\overline{X}}_{rd} - \overline{x}}{\hat{\overline{X}}_{rd} + \overline{x}} \right]. \tag{1.12}$$

Where  $\hat{\overline{X}}_{rd} = \frac{\overline{x}'}{\left(a\overline{z}' + b\right)} \left(a\overline{Z} + b\right)$ , 'a' and 'b' are chosen positive scalars.

$$MSE\left(\hat{\overline{Y}}_{Yad}\right) = \overline{Y}^{2} \left[ f_{2}C_{y}^{2} + \frac{f_{3}}{4} \left( C_{x}^{2} - 4C_{x}C_{y}\Gamma_{xy} \right) - f_{1}C_{y}^{2}\Gamma_{yz}^{2} \right]. \tag{1.13}$$

Where

$$f_{1} = \left(\frac{1}{n'} - \frac{1}{N}\right), \ f_{2} = \left(\frac{1}{n} - \frac{1}{N}\right), \ \left(f_{2} - f_{1}\right) = f_{3} = \left(\frac{1}{n} - \frac{1}{n'}\right)$$

$$C_{z} = \left(\frac{s_{z}}{Z}\right), \ C_{x} = \left(\frac{s_{x}}{X}\right), \ C_{y} = \left(\frac{s_{y}}{Y}\right) \text{ are coefficient of variations of } Z, X \text{ and } Y$$

$$\text{respectively,} \quad r_{xy} = \left(\frac{s_{xy}}{s_{x}s_{y}}\right), \quad r_{yz} = \left(\frac{s_{yz}}{s_{y}s_{z}}\right) \quad \text{and} \quad r_{xz} = \left(\frac{s_{xz}}{s_{x}s_{z}}\right) \quad \text{the correlation}$$

$$\text{coefficient of } (Y, X), \ (Y, Z) \text{ and } (X, Z), \ K_{01} = r_{xy} \left(\frac{C_{y}}{C_{x}}\right) \quad \text{and} \quad K_{02} = r_{yz} \left(\frac{C_{y}}{C_{z}}\right).$$

The estimators cited above have been extensively used for estimation of population mean in varied situations. The core objective of the study is to proposed an improved ratio type exponential estimator and analyzes its properties. We have used auxiliary information of two variables and proposed a log function estimator in the following section. Efficiency of the proposed estimator has been checked mathematically with some existing estimators i.e.  $\overline{Y}, \overline{Y}_{sv}, \overline{Y}_{s}, \overline{Y}_{NH_a}, \overline{Y}_{NH_b}, \overline{Y}_{SA}$  and  $\overline{Y}_{Yad}$  in section three, while empirical study and conclusion is discussed in section four and five respectively.

# 2. The Proposed Estimator

Following Srivastava (1971), we proposed an log in exponential type estimator in two double sampling using x and z auxiliary variables. Let  $n\mathfrak{l}$  and n be the first phase and second phase sample sizes respectively where  $n < n\mathfrak{l}$  using the transformation  $\overline{x}^* = \frac{n\mathfrak{l}\left(\overline{x}_{(1)}\right) - n\left(\overline{x}_{(2)}\right)}{\left(n\mathfrak{l} - n\right)}$ , which is an unbiased estimator of  $\overline{X}$ , the proposed estimator expressed as:

$$\overline{Y}_{g}^{(1)} = \overline{y} \left[ \partial \exp \left\{ \frac{1}{\partial q_{1}} \log \left( \frac{\overline{Z}}{\overline{z}_{(1)}} \right) \right\} + (1 - \partial) \exp \left\{ \frac{1}{1 - \partial q_{2}} \log \left( \frac{\overline{x}^{*}}{\overline{x}_{(1)}} \right) \right\} \right]. \tag{2.1}$$

To acquire the properties of the proposed estimator, such as the mean square error we consider the following as,

$$\overline{y} = \overline{Y} + \overline{e}_v, \ \overline{x} = \overline{X} + \overline{e}_x, \ \overline{z}_1 = \overline{Z} + \overline{e}_{z_1}, \ \overline{z}_2 = \overline{Z} + \overline{e}_{z_2}$$

where

$$E(\overline{e}_{y}) = E(\overline{e}_{z}) = E(\overline{e}_{x_{1}}) = E(\overline{e}_{x_{2}}) = 0$$

$$E(\overline{e}_{y}^{2}) = f_{2}\overline{Y}^{2}C_{y}^{2}, \quad E(\overline{e}_{z}^{2}) = f_{1}\overline{Z}^{2}C_{z}^{2}, \quad E(\overline{e}_{x_{1}} - \overline{e}_{x_{2}})^{2} = (f_{2} - f_{1})\overline{X}^{2}C_{x}^{2},$$

$$E(\overline{e}_{y}\overline{e}_{z}) = f_{1}\overline{Y}\overline{Z}C_{y}C_{z}\rho_{yz}, \quad E(\overline{e}_{y}(\overline{e}_{x_{1}} - \overline{e}_{x_{2}})) = (f_{1} - f_{2})\overline{X}\overline{Y}C_{y}C_{x}\rho_{xy}$$

An Exponential and Log Ratio Estimator of Population Mean Using Auxiliary Information in Double Sampling Stating (2.1) in terms of e's as,

$$\overline{Y}_{g}^{(1)} = \left(\overline{Y} + \overline{e}_{y_{2}}\right) \left\{ \frac{1}{a} q_{1} \log \left(\frac{1}{1 + \frac{\overline{e}_{z_{1}}}{\overline{Z}}}\right) \right\} + \left(1 - a\right) \exp \left\{ \frac{1}{1 - a} q_{2} \log \left(\frac{n'\left(\overline{X} + \overline{e}_{x_{1}}\right) - n\left(\overline{X} + \overline{e}_{x_{2}}\right)}{\frac{n' - n}{\overline{X} + \overline{e}_{x_{1}}}}\right) \right\} \tag{2.2}$$

On further simplification of (2.2) up to the order  $O(n^{-1})$ , we may get as,

$$\left(\overline{Y}_{g}^{(1)} - \overline{Y}\right) @ \left[\overline{e}_{y_{2}} - \overline{Y}q_{1}\left(\frac{\overline{e}_{z_{1}}}{\overline{Z}}\right) + \frac{n\overline{Y}}{n' - n}q_{2}\left(\frac{\overline{e}_{x_{1}} - \overline{e}_{x_{2}}}{\overline{X}}\right)\right]$$

$$(2.3)$$

From (2.3), the expression of MSE of  $\overline{Y}_g^{(1)}$  may be given as up to  $O(n^{-1})$ ,

$$MSE\left(\overline{Y}_{g}^{(1)}\right) @ \overline{Y}^{2} \begin{bmatrix} f_{2}C_{y}^{2} + q_{1}^{2}f_{1}C_{z}^{2} + \left(\frac{n}{(n'-n)}q_{2}\right)^{2} \left(f_{2} - f_{1}\right)C_{x}^{2} - 2q_{1}f_{2}C_{y}C_{z}\Gamma_{yz} + \left(\frac{n}{(n'-n)}q_{2}\left(f_{1} - f_{2}\right)C_{y}C_{x}\Gamma_{xy} + \left(\frac{n}{(n'-n)}q_{2}\right)C_{y}C_{x}\Gamma_{xy} + \left(\frac{n}{(n'-n)}q_{2}\right)C_{y}C_{x}\Gamma_{xy$$

We may get the optimum values of  $q_1$  and  $q_2$  respectively.

$$q_{1} = \left(\frac{C_{y}}{C_{z}} r_{yz}\right) = q_{1}^{*}(say), \ q_{2} = \left(\frac{n' - n}{n}\right) \left(\frac{C_{y}}{C_{x}} r_{xy}\right) = q_{2}^{*}(say), \ (2.5)$$

and the expression for the proposed estimator  $\overline{Y}_g^{(1)}$  using optimum values of  $q_1^*$  and  $q_2^*$  as

$$\min MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right) @ \overline{Y}^{2}C_{y}^{2} \left\lceil f_{2}\left(1-\Gamma_{xy}^{2}\right)+f_{1}\left(\Gamma_{xy}^{2}-\Gamma_{yz}^{2}\right)\right\rceil$$

$$(2.6)$$

Let,  $n_1, n_2$  samples be chosen at the first and second phases to get information on q auxiliary variables  $z_{(1)_1}, z_{(1)_2}, \dots, z_{(1)_q}$  and  $x_{(1)_1}, x_{(1)_2}, \dots, x_{(1)_s}$  respectively as well as on  $\overline{Y}_g^{(1)^*}$ . The generalized form of ratio estimator for estimating population average of Y is given as under when partial information on q+s auxiliary information is known:

$$\overline{Y}_{g}^{(1)^{*}} = \overline{y}_{(2)} \left[ \partial \exp \left\{ \frac{1}{\partial z} \sum_{i=1}^{q} q_{i} \log \left( \frac{\overline{Z}_{i}}{\overline{Z}_{(1)i}} \right) \right\} + (1 - \partial) \exp \left\{ \frac{1}{1 - \partial z} \sum_{k=1}^{s} q_{k} \log \left( \frac{\overline{x}_{(1)k}^{*}}{\overline{x}_{(1)k}} \right) \right\} \right]. \tag{2.7}$$

Applying expectation under both phases when the information from both phases is used, then MSE of (2.7) given as:

$$MSE\left(\overline{Y}_{g}^{(1)^{*}}\right) = E_{1}E_{2/1}\left[\overline{e}_{y_{(2)}} - \overline{Y}\sum_{i=1}^{q}q_{i}\left(\frac{\overline{e}_{z_{(1)i}}}{\overline{Z}_{i}}\right) - \frac{n\overline{Y}}{N-n}\sum_{k=1}^{s}q_{k}\left(\frac{\overline{e}_{x_{(1)k}} - \overline{e}_{x_{(2)k}}}{\overline{X}_{k}}\right)\right]^{2}$$
(2.8)

Where optimum values of  $q_i$  and  $q_k$  are

$$q_{i} = \left(-1\right)^{i+1} \frac{C_{y}}{C_{z_{i}}} \frac{\left|R_{yz_{i}}\right|_{yz_{q}}}{\left|R\right|_{z_{q}}} \tag{2.9}$$

and

$$q_{k} = (-1)^{k+1} \frac{C_{y}}{C_{x_{k}}} \frac{(n\mathfrak{l} - n)}{n} \frac{|R_{yx_{k}}|_{yx_{s}}}{|R|_{x_{s}}}.$$
(2.10)

# 3. Mathematical Comparison of $\overline{Y}_g^{(1)}$ with some Existing Estimators

The proposed estimator has been given in (2.1) with the mean square error in (2.4). Now we are comparing proposed estimator with the estimators given by Singh and Vishwakarma (2007), Singh et al. (2008), Noor-ul-Amin and Hanif (2012), Sanaullah et al. (2012) and Yadav et al. (2013). The comparison is done on the basis of MSE and checked the efficiency of proposed estimator with others.

$$MSE\left(\hat{\overline{Y}}_{sv}\right) \ge MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right)$$

$$\hat{\overline{Y}}_{g}^{(1)} \text{ is more efficent than } \hat{\overline{Y}}_{sv}$$

$$. iff$$

$$f_{1}\left(\Gamma_{xy}^{2} + \frac{K^{2}}{4} - K\Gamma_{xy}\right) - f_{2}\left(\Gamma_{xy}^{2} + \Gamma_{yz}^{2} + \frac{K^{2}}{4} - K\Gamma_{xy}\right) \ge 0$$

$$where \ K = \frac{C_{x}}{4C_{xy}}$$
(3.1)

ii. 
$$MSE\left(\hat{\overline{Y}}_{s}\right) \stackrel{3}{\sim} MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right) = f_{1}\left(\Gamma_{yz}\right)^{2} \stackrel{3}{\sim} 0$$
 (3.2)

iii. 
$$MSE\left(\hat{\bar{Y}}_{NH_{a}}\right) \ge MSE\left(\hat{\bar{Y}}_{g}^{(1)}\right)$$

$$iff$$

$$f_{1}\left(\frac{C_{x}^{2}}{4} + C_{x}C_{y}\rho_{xy} - \frac{1}{2}C_{x}C_{z}\rho_{xz} - C_{y}^{2}\left(\rho_{xy}^{2} - \rho_{yz}^{2}\right)\right) + f_{2}\left(\frac{C_{z}^{2}}{4} - C_{y}C_{z}\rho_{yz} + C_{y}^{2}\rho_{xy}^{2}\right) \ge 0$$
(3.3)

iv. 
$$MSE\left(\hat{\overline{Y}}_{NH_{b}}\right) \ge MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right)$$

$$f_{1}\left(\frac{C_{x}^{2}}{4} - C_{y}C_{x}\Gamma_{xy} - C_{y}^{2}\left(\Gamma_{xy}^{2} - \Gamma_{yz}^{2}\right) + \frac{1}{2}C_{x}C_{z}\Gamma_{xz}\right) + f_{2}\left(\frac{C_{z}^{2}}{4} - C_{y}C_{z}\Gamma_{yz} + C_{y}^{2}\Gamma_{xy}^{2}\right) \ge 0$$

$$(3.4)$$

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$$MSE\left(\hat{\overline{Y}}_{SA}\right) \ge MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right)$$
v.  $f_{1}\left(1-\hat{a}\right)\left(\frac{\left(1-\hat{a}\right)}{4}C_{x}^{2} + C_{y}C_{x}\Gamma_{xy} - \hat{a}C_{x}C_{z}\Gamma_{xz} - C_{y}^{2}\left(r_{xy}^{2} - r_{yz}^{2}\right)\right)$  (3.5)
$$+f_{2}\left(\hat{a}C_{z}^{2} - 2C_{y}C_{z}\Gamma_{yz} + C_{y}^{2}\Gamma_{xy}^{2}\right) \ge 0$$
vi.  $MSE\left(\hat{\overline{Y}}_{yad}\right)^{3}MSE\left(\hat{\overline{Y}}_{g}^{(1)}\right) => 2f_{1}\left(2C_{y}\Gamma_{yz}\right)^{2} + f_{3}\left(C_{x} - 2C_{y}\Gamma_{xy}\right)^{2} \ge 0$  (3.6)

If the conditions above in (3.1) to (3.6) are met then proposed estimator  $\overline{Y}_g^{(1)}$  are more efficient than the estimators  $\overline{Y}, \overline{Y}_{sv}, \overline{Y}_{s}, \overline{Y}_{NH_a}, \overline{Y}_{NH_b}, \overline{Y}_{SA}$  and  $\overline{Y}_{Yad}$ .

# 4. Empirical Study

Real data sets have been selected from the literature to check the performance of proposed estimator. The comparison of proposed estimator w.r.t usual unbiased estimator y, Singh and Vishwakarma (2007), Singh et al. (2008), Sanaullah et al. (2012), Noor-ul-Amin and Hanif (2012) and Yadav et al. (2013) has been made. The description of populations is as under:

**Table 4.1: Variables depiction for each population** 

Pop.	Y	X	Z	Source of Populations
1	Head length of second son	Head length of first son(SEP)	Head breadth of first son	Anderson (1958), pp.97
2	Hospital beds total numbers	Active physicians total numbers	Total Population	Nachtshemim, Neter and Kutner (2004), pp.1350
3	Residence sale value	Finished area of residence (square feet)	Total number of bedrooms	Nachtshemim, Neter and Kutner (2004), pp.1353

The formula is used to calculate percent relative efficiencies of  $\overline{Y}_{sv}, \overline{Y}_{s}, \overline{Y}_{NH_{a}}, \overline{Y}_{NH_{b}}, \overline{Y}_{SA}$  and  $\overline{Y}_{Yad}$  and proposed estimator  $\overline{Y}_{g}^{(1)}$  w.r.t estimator  $\overline{Y}$  is given as,

$$PRE(.,\overline{y}) = \frac{MSE(\overline{y})}{MSE(.)} \cdot 100 \quad where(.) = \hat{\overline{Y}}_{sv}, \hat{\overline{Y}}_{s}, \hat{\overline{Y}}_{NH_a}, \hat{\overline{Y}}_{NH_b}, \hat{\overline{Y}}_{SA}, \hat{\overline{Y}}_{yad} \text{ and } \hat{\overline{Y}}_{g}^{(1)}$$

**Table 4.2:** 

S. No.	Estimators	Population 1	Population 2	Population 3
1	$\hat{\overline{Y}}$	100	100	100
2	$\hat{ar{Y}}_{SV}$	104.9957	101.2964	102.5081
3	$\hat{\overline{Y}}_{S}$	137.9253	398.8051	226.4482
4	$\hat{ar{Y}}_{{\scriptscriptstyle NH}_a}$	102.1974	203.9909	106.0967
5	$\hat{ar{Y}}_{_{N\!H_{_{b}}}}$	138.2796	254.1943	119.9574
6	$\hat{ar{Y}}_{S\!A}$	310.4039	486.9430	101.8165
7	$\hat{\overline{Y}}_{Yad}$	273.1810	452.9864	141.9751
8	$\hat{ar{Y}}_g^{(1)}$	312.1419	950.8994	242.2356

The proposed estimator preformed better than the prior estimators  $\overline{Y}_{sv}, \overline{Y}_{s}, \overline{Y}_{NH_a}, \overline{Y}_{NH_b}, \overline{Y}_{SA}$  and  $\overline{Y}_{Yad}$  which have been taken into account in the previous literature based on percent relative efficiencies as given in table 4.2. For practical purposes the decision of estimator depending upon the accessibility of the population parameter(s).

### 5. Conclusion

An exponential type ratio estimator in double sampling has been proposed in this study. Its mean square error is obtained up-to the first degree of approximation. In some situations the suggested estimator  $\overline{Y}_g^{(1)}$  is found to have better efficiency than the ratio-type exponential estimator  $\hat{Y}_{sv}$  Singh and Vishwakarma (2007),  $\hat{Y}_s$  Singh et al. (2008), Noor-ul-Amin and Hanif (2012)  $(\hat{Y}_{NH_a}, \hat{Y}_{NH_b})$  in chain ratio type estimator,  $\hat{Y}_{sA}$  Sanaullah et al. (2012) and  $\hat{Y}_{yad}$  Yadav et al. (2013) ratio type exponential estimator. An empirical study is conducted to support proposed estimator and it is found that the precision of  $\hat{Y}_g^{(1)}$  (suggested estimator) is more than  $\hat{Y}$  (unbiased estimator), Singh and Vishwakarma (2007), Singh et al. (2008), Sanaullah et al. (2012) Noor-ul-Amin and Hanif (2012) and Yadav et al. (2013). Thus for its use in practice this study is recommended.

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