

Impact of Different Repetitive Sampling Schemes on the Performance of X-bar Control Chart

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Abstract

Repetitive sampling has been found very popular in improving the control chart techniques for last couple of years. In repetitive sampling based control charts, there are two additional control limits inside the usual upper control limit (UCL) and lower control limit (LCL). If any subgroup crosses these limits but remain inside outer limits, it is deferred and replaced with another selection. Process is said to be out of control if any subgroup falls outside UCL/LCL. In this article, the technique has been modified by introducing a relation between outer and inner control limits in terms of a ratio and need of this modification has also been justified by highlighting a gap in the existing technique. By using Monte Carlo simulation, several results have been generated relevant to different sample sizes and introduced ratios. The results have been described with the help of average run length (ARL) tables that how the efficiency of control chart is effected by using different ratios. The modification in the technique also provides variety of alternatives within the scope of repetitive based control charts. All the discussed options have summarized to one table to see that how the control limits under this technique behave and impact on detecting shifts in the process average. The schemes have been interpreted in the light of above ratio and their comparison has been described under different sample sizes that facilitate the user to select most appropriate scheme for a desired process control. An example has been included by choosing one of the proposed schemes to show the application and performance of the proposed control chart in a manufacturing process.

Key Words: Average Run Length; Process Control; Repetitive Sampling; Multiple Dependent State Sampling

1. Introduction

The statistical process control (SPC) techniques have been widely used in the industry for the monitoring of product and service. The control chart is one of the most powerful tool of SPC used in various fields, for example, Mason and Antony (2000), Woodall et al. (2006) and Sego (2006) applied control charts respectively, in service industry, health department and medicine & epidemiology. The control chart has been found a best technique to monitor process variation whether quality characteristics is continuous or attribute. The Shewhart control chart has two control limits called upper control limit (UCL) and lower control limit (LCL). The process is said to be out-of-control if the plotting statistic value falls beyond the UCL or LCL. The Shewhart control charts are usually more efficient to detect larger shift in the manufacturing process. These charts are less capable to detect small shifts in the process and may mislead the process monitoring personnel. By exploring the subject of process control, extensive work is found to improve the

efficiency of Shewhart control chart, see for example Koutras (2007). More details about the design and applications of control charts can be seen in Castagliola (2005), Chen et al. (2005), Shu et al. (2007), Yang et al. (2011) and Ali et al. (2016).

In these improving efforts, one of the dimension in control chart is sampling technique. Several sampling techniques so far have been tested and found very useful in improving control charts. In the reviewed literature Haq (2014) applied rank set sampling in his work again Haq (2014) applied rank set sampling related more techniques like ORSS and OIHDRSS in his work and showed improvements due to these techniques. Aslam et al. (2015) developed a control chart based on multiple dependent states sampling technique. Azam et al. (2014) developed a control chart based on repetitive sampling and showed improvements based on used sampling techniques. Aslam et al. (2015) introduced an improved control chart for attribute data using repetitive sampling and Aslam et al. (2015) developed another chart based on multiple dependent states sampling for attribute quality characteristics. Aslam et al. (2014) developed a non-parametric type mixed chart based on repetitive sampling. Aslam et al. (2014) design a t-chart using same sampling technique. Aslam et al. (2015) present a control chart on COM-Poisson distribution with the use of re-sampling technique. Aslam et al. (2015) attempted repetitive sampling on a mixed chart. In two more reviewed articles, Aslam et al. (2016) applied multiple dependent state sampling on variable size control chart and Aslam et al. (2016) applied repetitive sampling on $\bar{X}$ -bar control chart for non-normal correlated data.

From the above review, it is found that repetitive sampling is very popular technique in control chart application. It has been found very effective in sampling plans and control charts. The technique says that in the usual settings of the control charts, two more limits inside the control limits are introduced, may be called as repetitive limits. How it works, as if any point lies between control limit and repetitive limit on any side of chart, such point is deferred unplotted and replaced with another drawing until a well in control or out of control point is obtained.

In fact, process control is symptomatic treatment of the process within which control chart is a problem identifying tool so that, an appropriate dose is administered on the process to keep it on track. As such, in pure manufacturing, control chart is a soft job shop tool like gage repeatability & reproducibility, process capability analysis and Pareto analysis etc. Whenever, an assignable cause is detected in the control chart, immediate action is mandatory to prevent the process from any potential failure or a state where out of specs items are produced. In repetitive sampling it is quite obvious that process owner wants a very precise controls where none of point should lie in repetitive zone. Whenever there is such occurrence, immediately message is passed on to relevant process to trace back the problematic subgroup and remedial measures are taken to maintain the process health.

Exploring some of recent researches particularly on repetitive charts, the extreme limits are mostly at 3(approx.) standard error distant from target when $ARL_0 = 500$ or $ARL_0=370$ in normal distribution case. Since the repetitive limits are inside the extreme limits to declare repetitive zone in the chart, these are lying at less than 3 standard error distance above and below the center line (CL). If the inner limits (repetitive limits) are at 1 times standard error, then as per property of normal distribution the inner limits contain 68.27% subgroups and remaining percentage will require re-sampling. This percentage division of subgroups plays a very important role in the construction and efficiency of control chart. This percentage is dependent on the location of repetitive limits. As the repetitive zone gets wider, it is likely to include more subgroups as deferred or replaced, causing the control criteria stricter and stringent. This division can create an imbalance when most of the subgroups need re-sampling. This situation can be avoided if process owner has complete information that which scheme has better outcome with smaller number of subgroups being re-sampled.

In the calculations of control limits, the width of out of control zone is dependent on a constant say k_1 which is multiplier of the standard error. In case ARL_0 is around 370, the value of k_1 is close to 3. Since the repetitive limits are inside the control limits, they are also computed using similar constant say k_2 ($k_2 < k_1$). If we consider a ratio of inner constant (k_2) to outer constant (k_1) i. e. k_2/k_1 , this ratio can guide us that how strict and stringent sampling scheme has been applied in a particular chart. For example, if the ratio (k_2/k_1) is 0.33 given that $k_1=3$, then $k_2=1$, as these constants (k_1, k_2) are z-scores corresponding to limits, it is easy for us to have an idea that in the given case approximately 68% of subgroups will likely to fall in in-control zone whereas others either be re-sampled or go out-of-control and may be used for process diagnostics given that process is in control. This percentage can increase drastically in case of shifted process. Smaller the value of the above mentioned ratio indicates the control criteria to be stricter. For a ratio close to zero, the gap between repetitive limits and control limits on either side of the chart is

too large, due to which number points are expected to fall in the repetitive zone and very few to fall in in-control zone, of the control chart. Because of the narrower in-control zone in the chart, a slight shift in the process, the control criteria deferred the point. As such it can be assumed as a strict criterion. Similarly, different ratios value would have different interpretations about applied control chart.

In the related reviewed literature Azam et al. (2014) used the constants in charts, their ratio (k_2/k_1) remain 0.33, In Aslam et al. (2015) the ratio in different proposals found 0.56 to 0.79, Aslam et al. (2014) applied different charts with ratio 0.33 to 0.40. In other articles related to same subject, quite smaller ratios were observed in repetitive based charts as Aslam et al. (2014) used 0.26, Aslam et al. (2015) further lower the ratio to 0.15 and 0.16. In a mixed control chart by Aslam et al. (2015) different ratios were observed such as 0.27, 0.011, 0.19, 0.08, 0.25, 0.22 and 0.04. In Aslam et al. (2016) the suggested X-bar chart for a correlated data different ratios were used that vary from 0.89 to 0.07.

In the above mentioned review we find smallest ratios as 0.011, 0.04 and 0.07, these indicate very strict control charts where it is obvious that due to very large repetitive zone, number of subgroups might have deferred if these charts are applied to any process. Such charts can be useful when very high precision is required in the product and a slight departure in quality characteristics is crucial for process owner.

In the above mentioned reviewed articles, the mechanism of control chart has not been described that how the outer limits (control limits) and inner limits (repetitive limits) are related mathematically and how many subgroups would be re-sampled. In the proposed schemes a ratio has been defined to show relation between outer and inner limits of the control chart. Moreover, it also provides information that how large percentage of subgroups is expected to be deferred at a particular ratio. By introducing the above mentioned ratio the user has several options to control the efficiency of desired control chart.

2. Design of control chart and Monte Carlo algorithm

Let a sample of size n from a process follow Normal Distribution $N(\mu, \sigma^2)$. When the process is in control at μ_0 the control limits and repetitive limits are calculated as:

$UCL = \mu_0 + k_1\sigma/\sqrt{n}$	out of control	UCL
$URL = \mu_0 + k_2\sigma/\sqrt{n}$	repetitive zone	URL
$CL = \mu_0$	in-control zone	CL
$LRL = \mu_0 - k_2\sigma/\sqrt{n}$	in-control zone	LRL
$LCL = \mu_0 - k_1\sigma/\sqrt{n}$	repetitive zone	LCL
	out of control	

The working above control chart is as follows:

Step1. Selecting a random sample of size n from the process and computing estimator mean as $\bar{x} = \frac{\sum x}{n}$.

Step2. The process is in control if $LRL < \bar{x} < URL$.

Step3. The process is out of control if $\bar{x} > UCL$ or $\bar{x} < LCL$.

If $LCL < \bar{x} \leq LRL$ or $URL \leq \bar{x} < UCL$, go to Step1.

Algorithm: Monte Carlo Simulation Program of Control Chart for in-control and shifted process:

Following are algorithmic steps involved in Monte Carlo Simulation R program;

- (1) Generate a random sample of size n at each subgroup from the Normal Distribution with specified parameters for in-control process. Generate 100,000 such subgroups.
 - (1.1) Obtaining statistic $\bar{X}$ from each sample.
- (2) Setting up control limits.
 - (2.1) Compute UCL and LCL by using a specific Control Limit constant k_1 in the relevant formula.
 - (2.2) Compute URL and LRL by using a specific Control Limit constant k_2 in the relevant formula.
 - (2.3) Keeping in view the working procedures of the control chart and observe whether the process is declared as in-control or out-of-control. When the process is declared as out of control, note down the number of subgroups as run length i.e. the process remained in-control before it is declared to be out-of-control.
 - (2.4) Repeat above Step a sufficient number of times (say 10,000) to calculate the in-control ARL. If the in-control ARL is equal to the desired ARL_0 , then go to Step 3 with the current value of k_1 and k_2 . Otherwise, modify the value of both constants and repeat Steps 2.3 & 2.4.
- (3) Evaluating the out-of-control ARL:
 - (3.1) Generate a random sample of size n at each subgroup from the Normal Distribution with specified parameters for shifted process. Generate 100,000 such subgroups.
 - (3.2) Compute $\bar{x}$ statistic for each sample.
 - (3.3) Applying working criteria of the chart on the obtained 100,000 values of $\bar{x}$
 - (3.4) Repeat Steps 3.1 to 3.3 until the process is declared as out-of-control. Record the number of subgroup as a run length that show out of control signal.
- (4) Repeat all the above mentioned steps 10,000 times to obtain the Average Run Length.

3. Performance of control chart under different schemes

In the following table, ARLs have been computed by using Monte Carlo simulation procedure to observe the performance of the chart at different settings. The tables contain sufficient information to choose a particular scheme for a process. Table1 containing schemes with sample sizes 5, 10, 20 and 30 where ratios k_2/k_1 i.e. a ratio of inner constant (k_2) to outer constant (k_1) are smaller 0.05 to 0.30. By the term smaller we understand that when ratio is 0.05 the repetitive zone in the chart is wide enough to include sequence of subgroups for re-sampling. This scheme is good enough with $ARL_0=100$ only. If the desired ARL_0 is greater, it suggests switching the scheme to some other ratio scheme where repetitive zone is shorter. Tables1a and 1b contain the schemes under different sample sizes where the above discussed ratio varies 0.33 to 0.90. and $ARL_0=370$. Table 2a and 2b contain results at $ARL_0=500$ using sample sizes 5, 10, 20, 30 and 40. The ratio describing different schemes is from 0.33 to 0.90.

Table 3 is summary of all the given schemes showing capabilities of charts by applying different settings of two constants and their potential impact on the process control. By going through the table one can decide about which scheme can be suitable for specific process requirements.

Table 1: ARLs for ratio (k_2/k_1) 0.05 to 0.30

n	5	10	20	30	5	10	20	30
k_1	3.40	3.40	3.40	3.40	3.40	3.40	3.40	3.40
shift	$k_2 = 0.05 k_1$				$k_2 = 0.10 k_1$			
0.00	98.45	98.99	99.06	103.82	202.95	198.92	200.50	202.95
0.10	95.48	89.83	70.23	50.26	194.85	174.79	141.12	104.20
0.15	86.29	63.39	35.45	19.69	168.58	126.90	65.95	40.24
0.20	71.43	39.87	16.39	8.18	142.48	76.21	29.08	15.16
0.25	50.52	22.90	7.51	3.73	101.14	44.36	14.20	6.11
0.30	35.52	13.08	3.79	1.95	67.58	25.70	6.70	2.80
0.35	22.59	7.69	2.18	1.33	45.02	14.98	3.43	1.67
0.40	15.47	4.62	1.53	1.10	30.53	8.54	2.07	1.21
0.45	10.68	3.07	1.22	1.04	20.11	5.38	1.46	1.09
0.50	7.55	2.18	1.09	1.02	13.63	3.32	1.22	1.03
0.70	2.24	1.11	1.00	1.00	3.45	1.20	1.01	1.00
0.90	1.20	1.01	1.00	1.00	1.44	1.02	1.00	1.00
1.00	1.10	1.00	1.00	1.00	1.19	1.00	1.00	1.00
n	5	10	20	30	5	10	20	30
k_1	3.40	3.40	3.40	3.40	3.21	3.21	3.21	3.21

shift	$k_2 = 0.15 k_1$					$k_2 = 0.20 k_1$			
0.00	298.24	299.97	301.14	298.83	301.06	298.28	299.24	298.18	
0.10	288.39	251.78	199.50	147.44	255.85	205.79	138.20	106.98	
0.15	258.34	187.11	98.18	59.66	203.85	139.45	67.91	42.21	
0.20	202.77	116.69	44.77	23.65	142.87	82.01	33.84	17.62	
0.25	145.23	65.43	20.54	8.45	103.30	47.09	16.07	7.46	
0.30	101.47	37.06	9.47	3.88	71.23	28.31	8.08	3.52	
0.35	67.57	21.00	4.71	2.01	50.49	16.33	4.34	1.92	
0.40	45.39	12.23	2.62	1.39	32.72	10.17	2.57	1.37	
0.45	30.38	7.55	1.71	1.12	22.57	6.18	1.76	1.14	
0.50	20.88	4.29	1.30	1.04	16.67	4.16	1.31	1.06	
0.70	4.80	1.35	1.01	1.00	4.26	1.36	1.01	1.00	
0.90	1.74	1.03	1.00	1.00	1.71	1.03	1.00	1.00	
1.00	1.31	1.01	1.00	1.00	1.34	1.01	1.00	1.00	
n	5	10	20	30	5	10	20	30	
k_1	3.21	3.21	3.21	3.21	3.15	3.15	3.15	3.15	
shift	$k_2 = 0.25 k_1$				$k_2 = 0.30 k_1$				
0.00	371.31	370.98	368.73	367.12	368.12	369.92	368.23	367.43	
0.10	312.77	260.53	170.49	135.16	303.62	245.59	171.52	125.05	
0.15	247.19	160.44	89.11	53.31	230.68	156.20	84.99	50.90	
0.20	173.53	99.26	41.59	21.71	170.02	95.05	42.44	22.29	
0.25	128.42	57.82	19.84	8.95	118.28	59.90	20.09	9.89	
0.30	83.26	34.54	9.91	4.29	83.40	35.39	10.17	4.50	
0.35	59.41	20.94	5.16	2.32	58.91	21.06	5.68	2.47	
0.40	41.91	12.60	2.95	1.53	43.37	13.04	3.34	1.65	
0.45	29.23	8.04	1.98	1.20	29.36	8.45	2.12	1.25	
0.50	20.25	5.11	1.44	1.09	20.64	5.33	1.58	1.10	
0.70	5.29	1.51	1.03	1.00	5.67	1.59	1.03	1.00	
0.90	2.00	1.06	1.00	1.00	2.11	1.07	1.00	1.00	
1.00	1.42	1.02	1.00	1.00	1.52	1.02	1.00	1.00	

Table 1a: ARLs for ratio (k_2/k_1) 0.33 to 0.67 with $ARL_0=370$

n	5	10	20	30	40	5	10	20	30	40
k_1	3.126	3.126	3.126	3.126	3.126	3.09	3.09	3.09	3.09	3.09
shift	$k_2=0.33k_1$					$k_2=0.40k_1$				
0.00	367.82	372.74	373.94	374.64	367.37	375.38	371.96	373.65	378.16	367.92
0.10	300.89	242.64	174.23	126.23	98.53	305.31	242.93	171.85	131.96	97.61
0.15	225.69	152.58	82.08	54.72	35.39	242.16	160.08	86.61	57.94	38.74
0.20	170.94	96.87	41.69	22.24	13.35	174.58	98.38	46.15	24.61	15.34
0.25	120.11	56.46	20.85	9.78	5.79	118.39	61.59	23.33	11.26	6.45
0.30	83.03	35.84	10.99	5.04	2.80	87.04	37.56	12.69	5.58	3.13
0.35	60.14	22.14	5.95	2.69	1.65	62.85	24.82	6.80	3.04	1.89
0.40	40.63	13.64	3.44	1.77	1.23	44.29	15.29	3.99	1.88	1.34
0.45	30.66	8.95	2.21	1.28	1.09	32.58	9.73	2.50	1.42	1.12
0.50	20.64	5.63	1.61	1.13	1.03	23.84	6.47	1.82	1.18	1.06
0.60	10.93	2.72	1.16	1.02	1.00	12.55	3.17	1.21	1.03	1.01
0.70	5.97	1.64	1.04	1.00	1.00	6.69	1.85	1.06	1.00	1.00
0.80	3.37	1.24	1.01	1.00	1.00	4.12	1.31	1.01	1.00	1.00
0.90	2.18	1.09	1.00	1.00	1.00	2.53	1.12	1.00	1.00	1.00
1.00	1.62	1.02	1.00	1.00	1.00	1.81	1.06	1.00	1.00	1.00
n	5	10	20	30	40	5	10	20	30	40
k_1	3.055	3.055	3.055	3.055	3.055	3.02	3.02	3.02	3.02	3.02
shift	$k_2=0.50k_1$					$k_2=0.67k_1$				
0.00	373.52	372.38	377.45	376.69	364.31	375.18	373.61	371.82	370.62	368.27
0.10	308.64	254.02	171.58	128.87	103.98	298.61	246.31	178.82	132.15	110.47
0.15	239.01	168.07	92.20	59.62	42.12	232.63	163.33	92.78	63.67	46.09
0.20	178.04	102.81	48.47	26.65	17.42	171.86	105.60	53.37	33.60	20.54

0.25	129.01	64.47	26.29	13.31	7.94	134.31	67.91	30.60	16.14	10.29
0.30	91.11	41.68	14.61	6.56	3.95	98.37	46.12	17.31	8.81	5.06
0.35	67.37	25.28	8.26	3.71	2.23	70.11	30.09	10.43	5.04	2.97
0.40	49.50	17.80	4.91	2.31	1.53	51.31	20.74	6.46	3.19	2.01
0.45	35.42	11.46	3.03	1.61	1.22	38.28	14.43	4.10	2.11	1.46
0.50	24.89	8.04	2.13	1.30	1.10	29.24	10.29	2.96	1.60	1.22
0.60	14.36	3.94	1.34	1.06	1.01	17.65	5.30	1.65	1.17	1.03
0.70	7.84	2.25	1.10	1.01	1.00	10.61	3.04	1.26	1.04	1.01
0.80	4.77	1.52	1.03	1.00	1.00	6.47	1.99	1.08	1.01	1.00
0.90	3.06	1.22	1.01	1.00	1.00	4.22	1.51	1.03	1.00	1.00
1.00	2.17	1.08	1.00	1.00	1.00	2.92	1.24	1.01	1.00	1.00

Table1b: ARLs for ratio (k_2/k_1) 0.75 to 0.90 with $ARL_0=370$

n	5	10	20	30	40	5	10	20	30	40
k_1	3.01	3.01	3.01	3.01	3.01	3.00	3.00	3.00	3.00	3.00
shift	$k_2=0.75 k_1$					$k_2=0.80 k_1$				
0.00	367.23	368.72	373.20	367.57	367.22	365.30	366.70	369.39	368.47	368.29
0.10	305.56	243.56	187.05	134.40	107.78	297.13	230.15	175.68	138.93	105.83
0.15	239.80	166.65	99.92	67.38	47.07	234.72	163.85	96.97	64.76	47.34
0.20	179.38	105.74	55.23	32.16	21.81	174.25	108.33	52.94	32.85	21.98
0.25	127.30	71.44	30.89	17.52	10.66	130.77	69.24	29.78	17.00	11.15
0.30	97.51	48.25	18.68	9.62	6.01	96.74	46.42	18.57	9.67	6.18
0.35	73.81	31.26	11.50	5.61	3.29	72.10	32.16	11.49	5.87	3.62
0.40	52.68	22.14	7.06	3.55	2.27	54.70	22.53	7.50	3.80	2.43
0.45	41.22	15.34	4.84	2.40	1.64	41.58	15.32	4.92	2.61	1.76
0.50	29.98	11.14	3.26	1.79	1.32	31.54	11.15	3.64	1.93	1.40
0.60	18.32	5.93	1.91	1.25	1.10	17.96	6.05	2.02	1.30	1.10
0.70	10.91	3.54	1.35	1.06	1.02	11.53	3.63	1.41	1.09	1.02
0.80	7.01	2.28	1.13	1.02	1.00	7.67	2.41	1.17	1.02	1.00
0.90	4.76	1.65	1.04	1.00	1.00	5.07	1.75	1.06	1.00	1.00
1.00	3.35	1.33	1.01	1.00	1.00	3.44	1.40	1.02	1.00	1.00
n	5	10	20	30	40	5	10	20	30	40
k_1	3.00	3.00	3.00	3.00	3.00	2.995	2.995	2.995	2.995	2.995
shift	$k_2=0.83 k_1$					$k_2=0.90 k_1$				
0.00	368.75	367.50	365.69	367.08	368.06	372.15	367.19	368.62	367.81	367.60
0.10	281.30	246.08	174.63	133.41	106.97	287.36	237.77	169.93	137.03	105.78
0.15	234.92	162.52	93.67	65.32	45.18	228.46	164.69	94.67	66.64	48.07
0.20	170.13	103.42	52.24	32.40	22.21	167.59	110.65	55.35	33.48	24.77
0.25	129.59	69.53	31.44	18.44	11.77	123.51	69.66	31.56	18.43	11.83
0.30	94.84	47.73	18.36	10.24	6.24	94.48	47.39	20.29	10.64	6.85
0.35	70.10	31.49	12.40	6.29	3.81	72.69	32.69	12.36	6.53	4.08
0.40	53.86	21.52	7.73	3.95	2.53	54.79	23.30	8.16	4.38	2.81
0.45	41.10	15.85	5.23	2.74	1.81	41.21	15.90	5.59	2.91	1.96
0.50	31.68	11.22	3.73	1.99	1.44	32.09	12.10	4.17	2.24	1.59
0.60	19.41	6.39	2.14	1.33	1.12	19.28	6.99	2.33	1.46	1.17
0.70	11.39	3.82	1.46	1.12	1.02	12.53	4.08	1.61	1.15	1.05
0.80	7.76	2.56	1.20	1.03	1.00	8.35	2.76	1.27	1.05	1.01
0.90	5.27	1.82	1.07	1.01	1.00	5.53	2.01	1.10	1.02	1.00
1.00	3.70	1.43	1.02	1.00	1.00	4.10	1.57	1.03	1.00	1.00

Table2a: ARLs for ratio (k_2/k_1) 0.33 to 0.67 with $ARL_0=500$

n	5	10	20	30	40	5	10	20	30	40
k_1	3.27	3.27	3.27	3.27	3.27	3.21	3.21	3.21	3.21	3.21

shift	$k_2=0.33k_1$					$k_2=0.40k_1$				
0.00	503.17	500.41	505.49	502.54	502.03	496.26	498.99	499.68	496.64	498.39
0.10	454.54	364.40	270.72	190.57	150.43	409.57	344.44	241.91	179.39	144.54
0.15	361.62	247.01	129.13	81.52	52.58	331.19	223.32	125.36	75.09	50.60
0.20	269.25	152.11	61.29	33.86	19.84	244.76	135.87	61.40	33.22	20.44
0.25	191.47	89.73	30.57	13.68	7.50	169.10	85.73	31.18	14.23	8.11
0.30	129.98	53.94	15.35	6.10	3.33	126.02	51.59	16.17	6.79	3.80
0.35	95.25	32.22	7.80	3.32	1.88	88.64	32.45	8.44	3.59	2.09
0.40	64.74	19.68	4.35	1.90	1.31	63.27	19.63	4.76	2.15	1.41
0.45	44.33	12.00	2.63	1.38	1.11	41.93	12.57	2.97	1.54	1.17
0.50	32.27	7.77	1.82	1.18	1.04	30.98	8.07	2.09	1.22	1.07
0.60	14.80	3.44	1.18	1.02	1.00	16.26	3.82	1.27	1.03	1.01
0.70	7.91	1.89	1.04	1.00	1.00	8.81	2.10	1.07	1.01	1.00
0.80	4.41	1.31	1.01	1.00	1.00	4.91	1.40	1.02	1.00	1.00
0.90	2.60	1.11	1.00	1.00	1.00	2.93	1.16	1.01	1.00	1.00
1.00	1.80	1.05	1.00	1.00	1.00	2.01	1.06	1.00	1.00	1.00
n	5	10	20	30	40	5	10	20	30	40
k_1	3.155	3.155	3.155	3.155	3.155	3.112	3.112	3.112	3.112	3.112
shift	$k_2=0.50k_1$					$k_2=0.67k_1$				
0.00	523.18	527.36	513.50	494.42	512.07	503.70	500.05	502.26	518.33	518.74
0.10	417.94	337.75	237.99	174.66	140.99	420.16	338.07	236.94	187.60	148.84
0.15	308.41	221.16	130.52	75.98	53.92	315.20	225.02	134.73	83.38	59.29
0.20	241.83	139.86	63.84	35.67	22.26	243.93	147.33	68.78	40.19	25.42
0.25	174.84	85.14	33.83	16.36	9.68	176.50	90.28	38.92	20.67	11.87
0.30	123.12	55.67	18.51	8.47	4.41	125.41	59.28	21.80	10.69	6.24
0.35	86.08	33.71	9.89	4.36	2.52	93.69	39.80	12.88	5.95	3.33
0.40	62.36	22.06	5.80	2.67	1.65	68.53	25.75	8.08	3.53	2.23
0.45	46.40	14.63	3.75	1.77	1.26	51.46	17.24	4.80	2.37	1.58
0.50	33.01	9.55	2.46	1.39	1.13	37.78	11.95	3.41	1.75	1.27
0.60	18.82	4.63	1.44	1.08	1.01	22.36	6.20	1.84	1.21	1.06
0.70	9.82	2.53	1.11	1.02	1.00	12.97	3.43	1.31	1.06	1.01
0.80	5.80	1.60	1.04	1.00	1.00	7.74	2.14	1.10	1.01	1.00
0.90	3.62	1.25	1.01	1.00	1.00	4.90	1.56	1.04	1.00	1.00
1.00	2.47	1.12	1.00	1.00	1.00	3.35	1.27	1.01	1.00	1.00

Table2b: ARLs for ratio (k_2/k_1) 0.75 to 0.90 with $ARL_0=500$

n	5	10	20	30	40	5	10	20	30	40
k_1	3.112	3.112	3.112	3.112	3.112	3.11	3.11	3.11	3.11	3.11
shift	$k_2=0.75 k_1$					$k_2=0.80 k_1$				
0.00	498.95	501.35	501.19	498.58	502.04	503.34	502.17	499.98	503.60	499.65
0.10	410.31	331.68	229.94	181.09	147.17	402.05	340.23	247.59	185.55	147.63
0.15	318.06	225.35	126.54	86.07	58.15	315.14	227.47	133.14	84.36	62.60
0.20	229.78	142.03	68.76	42.01	27.80	244.38	146.64	69.00	43.80	28.64
0.25	169.89	91.66	37.97	22.12	13.03	179.62	93.22	40.70	22.67	13.39
0.30	133.08	59.61	22.76	12.00	6.98	130.69	61.45	23.63	11.97	7.39
0.35	95.20	39.49	13.71	6.71	3.90	96.91	41.37	14.23	7.08	4.18
0.40	72.03	27.81	8.71	3.96	2.52	75.06	28.59	8.61	4.39	2.79
0.45	51.39	18.70	5.80	2.72	1.76	54.60	19.41	6.20	2.92	1.88
0.50	38.53	13.73	3.84	1.93	1.40	40.82	13.37	4.17	2.18	1.46
0.60	23.28	6.98	2.10	1.32	1.10	23.67	7.47	2.29	1.37	1.12
0.70	13.33	4.04	1.46	1.10	1.02	14.83	4.24	1.49	1.15	1.02
0.80	8.76	2.52	1.14	1.02	1.00	9.05	2.71	1.19	1.03	1.01
0.90	5.66	1.80	1.06	1.00	1.00	6.19	1.92	1.08	1.01	1.00
1.00	3.90	1.39	1.01	1.00	1.00	4.15	1.47	1.03	1.00	1.00
n	5	10	20	30	40	5	10	20	30	40

k_1	3.11	3.11	3.11	3.11	3.11	3.11	3.11	3.11	3.11	3.11
shift	$k_2=0.83k_1$					$k_2=0.90k_1$				
0.00	499.83	501.06	501.90	507.80	503.37	501.76	502.02	503.41	504.06	502.92
0.10	403.22	333.64	239.91	187.51	153.14	404.34	345.92	252.16	182.59	148.07
0.15	326.51	224.25	127.86	85.47	61.23	328.35	231.96	134.38	90.34	63.41
0.20	234.52	146.40	69.41	42.66	27.92	246.24	147.08	73.33	44.49	29.77
0.25	179.82	96.06	42.60	22.49	14.38	184.39	95.77	41.07	23.48	14.62
0.30	133.50	62.81	23.62	12.37	7.76	138.04	66.85	23.96	13.29	8.05
0.35	99.02	41.80	14.71	7.57	4.59	101.02	43.14	15.62	8.06	4.93
0.40	72.37	28.48	9.43	4.59	2.80	70.38	30.24	9.89	5.11	3.21
0.45	54.15	19.82	6.14	3.08	2.03	55.41	21.56	7.03	3.38	2.14
0.50	40.30	14.64	4.41	2.19	1.55	41.05	14.83	4.77	2.51	1.68
0.60	23.61	7.86	2.39	1.42	1.14	25.11	8.13	2.59	1.55	1.21
0.70	15.26	4.40	1.54	1.13	1.04	15.04	4.78	1.72	1.19	1.05
0.80	9.32	2.85	1.23	1.04	1.01	9.76	3.14	1.31	1.06	1.01
0.90	6.19	2.01	1.09	1.01	1.00	6.62	2.19	1.13	1.02	1.00
1.00	4.30	1.53	1.03	1.00	1.00	4.64	1.73	1.05	1.00	1.00

Table 3: Summarization of all schemes

Scheme	k_1	Ratio $=k_2/k_1$	ARL_0	Expected % of resampled subgroups	Shift sizes detectable before 10 th run length on average				
					n=5	n=10	n=20	n=30	n=40
1	3.40	0.05	100	86.43	0.50 & above	0.35 & above	0.25 & above	0.20 & above	0.20 & above
2	3.40	0.10	200	73.32	0.70 & above	0.40 & above	0.30 & above	0.25 & above	0.20 & above
3	3.40	0.15	300	60.94	0.70 & above	0.45 & above	0.30 & above	0.25 & above	0.25 & above
4	3.21	0.20	300	51.95	0.70 & above	0.40 & above	0.30 & above	0.25 & above	0.25 & above
5	3.21	0.25	370	42.09	0.70 & above	0.45 & above	0.30 & above	0.25 & above	0.25 & above
6	3.15	0.30	370	34.30	0.70 & above	0.45 & above	0.30 & above	0.25 & above	0.25 & above
7	3.126	0.33	370	30.05	0.70 & above	0.45 & above	0.35 & above	0.25 & above	0.25 & above
8	3.09	0.40	370	21.45	0.70 & above	0.45 & above	0.35 & above	0.30 & above	0.25 & above
9	3.055	0.50	370	12.44	0.70 & above	0.50 & above	0.35 & above	0.30 & above	0.25 & above
10	3.02	0.67	370	4.05	0.80 & above	0.50 & above	0.35 & above	0.30 & above	0.25 & above
11	3.01	0.75	370	2.14	0.80 & above	0.60 & above	0.40 & above	0.30 & above	0.30 & above
12	3.00	0.80	370	1.37	0.80 & above	0.60 & above	0.40 & above	0.30 & above	0.30 & above
13	3.00	0.83	370	1.01	0.80 & above	0.60 & above	0.40 & above	0.30 & above	0.30 & above
14	2.995	0.90	370	0.43	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above
15	3.27	0.33	500	27.95	0.70 & above	0.50 & above	0.35 & above	0.30 & above	0.25 & above
16	3.21	0.40	500	19.78	0.70 & above	0.50 & above	0.35 & above	0.30 & above	0.25 & above
17	3.155	0.50	500	11.31	0.70 & above	0.50 & above	0.35 & above	0.30 & above	0.25 & above

18	3.122	0.67	500	3.47	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above
19	3.112	0.75	500	1.77	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above
20	3.11	0.80	500	1.10	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above
21	3.11	0.83	500	0.80	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above
22	3.11	0.90	500	0.33	0.80 & above	0.60 & above	0.40 & above	0.35 & above	0.30 & above

4. Discussion on different schemes

In all the above listed schemes, the shifts have been included in the above mentioned table that can be detectable less than $ARL_1=10$. For the other shifts, the detailed tables of ARLs need to be reviewed. In case resample percentage gets higher, it means there might need to defer a sequence of subgroups that can may fall in repetitive zone of the control chart. Now discussing these schemes individually, in the scheme number 1 in above mentioned table the ratio of inner limits constant to outer limits constant is $k_2/k_1=0.05$ i.e. inner limit constant (k_2) is almost 0.05 times of the extreme limits constant (k_1). In the literature reviewed, in some of the cases the ratio has been even below 0.05. If this scheme is used in normal processes, it is expected that around 86% of the subgroups may need to be deferred and re-sampled since they fall in the repetitive zone of the control chart. Though this scheme can detect 0.5sigma shift at a sample size of 5 only and with a sample size of 30 it can detect 0.20sigma shift. It should be discouraged to apply due to very higher expectancy of resample percentage of drawn subgroups for charting purpose. Similarly, the same arguments prevail for the scheme 2.

From scheme 2 to 9, the control chart is capable to detect 0.70sigma shift at $n=5$ and 0.25sigma shift at $n=40$. Best choice here can be the scheme 9 because percentage of resample subgroups is 12.44 only and having $ARL_0=370$. Considering scheme 14 at $ARL_0=370$, this is also very good choice as it can detect 0.80sigma shift at $n=5$ and 0.30sigma shift at $n=40$ whereas it has expected percentage of 0.43 only for subgroups being re-sampled. Moreover, considering the cases from 18 to 22 all the schemes are performing equally yet the choice can be optimized to option 22 if the precision requirement in related product is lenient, here only 0.33% of the subgroups are expected to lie in repetitive zone at $ARL_0=500$.

5. Example from a process of blade manufacturing

Following chart is based on data that have been obtained from quality assurance department of a blade manufacturing process where blades are manufactured from an imported mettle strip. Due to variations in the process, the machine damages some of portion of strip which is called scrape. Scrape data have been obtained for process control as it is one of the quality concerned variable. The data when process is in control follow normal distribution. The parameters estimates from in-control process are $\hat{\mu} = 580 \text{ grams}$ and $\hat{\sigma} = 256.40 \text{ grams}$. The sampling has been done with two scenarios i.e. repetitive sampling and random sampling. The subgroup size remains 5 for all the selected samples. The control chart with repetitive sampling based on ratio $k_2/k_1 = 0.75$ indicates that technique is better in detecting assignable causes compare to simple technique.

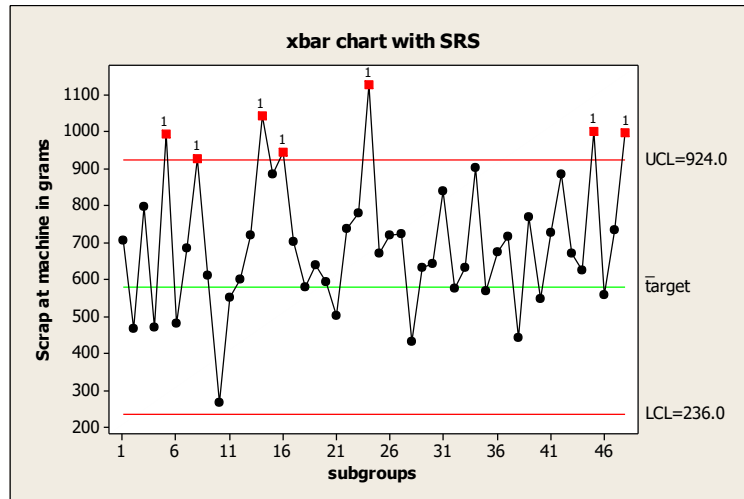


Figure1: X-bar chart with simple random sampling (SRS)

In the above mentioned chart, it is indicated that first out of control alarm is detected at 5th subgroup and then few more out of control alarms have been detected. At subgroup 24th a point is detected then none of point indicated as out of control till 45th. In the entire chart seven points in total were detected as out of control alarms. The out of control limits in the above chart are at 3.00 standard error.

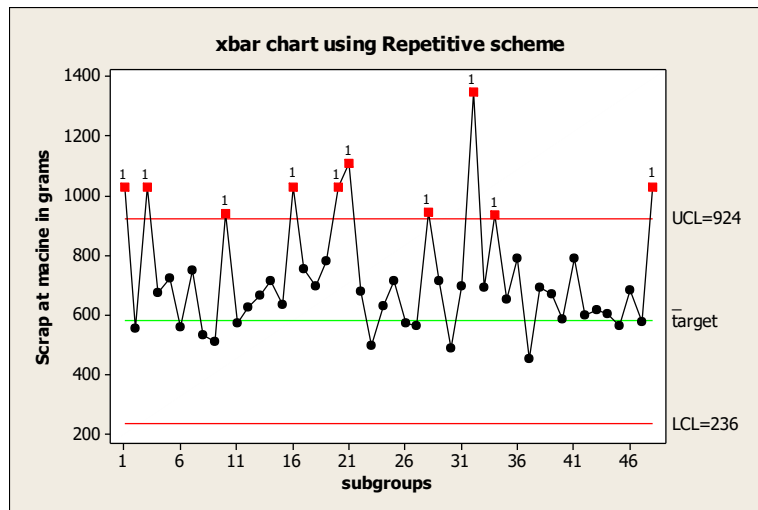


Figure 2: X-bar chart based on repetitive sampling

The figure 2 has been constructed on the basis of repetitive sampling; here k_2/k_1 ratio is 0.75 with $k_1=3.00$ and the chart proved perform better as it is showing out of control alarm at first subgroup and from subsequent pattern of points it is quite clear that repetitive schemes are better in out of control detection capability. Attempting different ratios of k_2/k_1 we can experience several capabilities of chart to detect the shifts in the process average.

6. Concluding remarks

This article is giving another direction to repetitive sampling based control charts. It has been already proved in the referenced articles that repetitive sampling based control charts are more efficient than usual simple random sampling based control charts. The modification done in the repetitive technique in this article enables the user to regulate the efficiency of control chart by selecting different repetitive control limits. It also guides that how a repetitive based control chart characterizes different behaviors at various settings of repetitive zone. It provides an insight to mechanism of repetitive sampling which facilitate the user to determine how large numbers of subgroups may require

re-sampling if the proposed scheme is used for a certain process. Another merit of this study is that it can be applied to many variable and attribute control charts to improve their performance. As the proposed provides a variety of alternatives within the scope of repetitive control charts, it certainly improves the efficiency of repetitive technique and adds value to process control tools. This exercise has been performed on normally distributed process, for the non-normal cases, opportunity is there for researchers to apply it on some skewed distributions.

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