An Optimal Replenishment Policy for Deteriorating Items with Ramp Type Demand under Permissible Delay in Payments

Sanjay Jain
Department of Mathematical Sciences
Government College
Ajmer - 305001, India

Mukesh Kumar
Department of Mathematics
Government College
Kishangarh - 305802, India

Priya Advani
Department of Mathematics
Government Mahila Engineering College
Ajmer, India
drjainsanjay@gmail.com

Abstract

The aim of this paper is to develop an optimal replenishment policy for inventory models of deteriorating items with ramp type demand under permissible delay in payments. Deterioration of items begins on their arrival in stock. An example is also presented to illustrate the application of developed model.

Keywords: Replenishment policy, Deterioration, Ramp type demand, Permissible delay in payments.

1. Introduction

The economic order quantity model tacitly assumes capital investment in inventories on the receipt of the items by the inventory system. Such an assumption is not quite practical in the real world. Under most market behavior, it is observed that a vendor provides a credit period for buyers to stimulate demand. However, if the items deteriorate while in inventory then purchase of large quantity will be uneconomical.

In the recent past, many researchers have shown a keen interest in the permissible delay in payments associated with other characteristics of inventory model. Goyal (1985) is the pioneer researcher who formed inventory models taking the condition of permissible delay in payments. Mandal and Phaujdar (1989) generalized the inventory model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Shah (1993) developed a lot size inventory model for exponentially decaying inventory when delay in payments is permissible. Aggarwal and Jaggi (1995) extended Goyal's (1985) inventory model by allowing constant rate of deterioration. Jamal et al (1997) further generalized the inventory model by incorporating shortages. Chu et al (1998) investigated the properties of the convexity of the total variable cost function of the inventory model of deteriorating items under permissible delay in payments. Chung (2000) proposed the inventory replenishment policy for deteriorating items under permissible delay in payments. Chang et al (2001) developed an inventory model for deteriorating items with linear trend demand under permissible delay in payments. Oyang et al (2005) gave an optimal policy to minimize cost when the supplier offers not only a permissible delay but also a cash discount. Pal and Ghosh (2007) developed an inventory model for deteriorating items with stock-dependent demand under permissible delay in payments. Kumar et al (2009) developed an inventory model with power demand rate, incremental holding cost and permissible delay in payments.

Time varying demand patterns are commonly used to reflect sales in different phases of the product in the market. But in the real market demand of a product is always in dynamic state due to variability of time, price or even of the instantaneous level of inventory displayed in retail shop. This impressed researchers and marketing practitioners to think about ramp type demand which increases with time up to a certain limit and then ultimately stabilizes and becomes constant. Such a type of demand is observed in items such as newly launched mobile phones, fashion goods, garments and automobiles, cosmetics etc.

Mandal and Pal (1998) developed an order-level inventory model for deteriorating items with ramp type demand. Wu and Ouyang (2000) extended their model by incorporating the concept of shortages followed by inventory. Giri et al (2003) developed an economic order quantity model with Weibull deterioration distribution, shortages and ramp type demand. Jain and Kumar (2007) further generalized the Wu and Ouyang (2000) model by allowing Weibull distribution deteriorating. Panda et al (2008) gave an optimal replenishment policy for perishable seasonal product in a season with ramp type demand rate. Sharma et al (2009) developed an EOQ model for variable rate of deterioration having a ramp type demand rate.

In the present paper, an optimal replenishment policy (ORP) for inventory model of deteriorating items with ramp type demand under permissible delay in payments is studied. The inventory is assumed to deteriorate at a constant rate. The procedure of solving the proposed model is illustrated with numerical examples.

2. Assumptions and Notations

The proposed inventory model having following assumptions and notations:

2.1 Assumptions

1. The shortages are not allowed.
2. The lead-time is zero.
3. The time-horizon of the system is infinite.

4. There is no repair or replacement of the deteriorated inventory during a given cycle.
5. Ramp-type demand rate *f(t)* is given by

$$f(t) = D_0\left[t-(t-\mu)\,H(t-\mu)\right], \qquad D_0 > 0$$

$$\text{where} \quad H(t-\mu) = \begin{cases} 0 & \text{for } t \le \mu \\ 1 & \text{for } t > \mu \end{cases}$$

   is well known Heaviside's unit function.
6. During the fixed credit period *μ*, the unit cost of generated sales revenue is deposited in an interest bearing account. The difference between sales price and unit cost is retained by the system to meet the day-to-day expenses of the system. At the end of the credit period, the account is settled and interest charges are payable on the account in stock.
7. The constant fraction *θ* of on hand inventory gets deteriorated per unit time.

2.2 Notations

C = unit purchase cost in $

C_1 = inventory holding cost per unit per time unit excluding interest charges in $

I = inventory holding charges per unit per year

C_3 = replenishment cost per cycle in $

I_e = interest that can be earned $ per unit time

I_c = interest charges payable per $ per time unit (Ic > Ie)

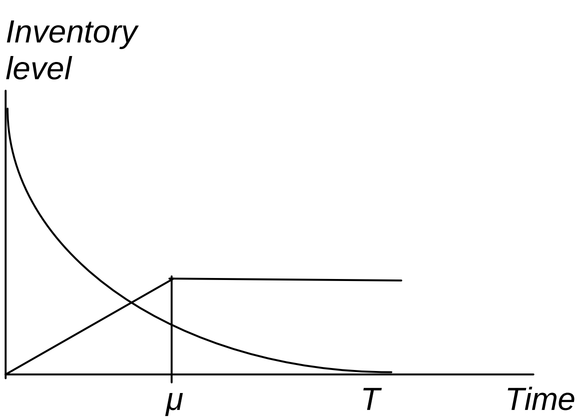
T = cycle time

μ = permissible delay period for settling accounts in time units

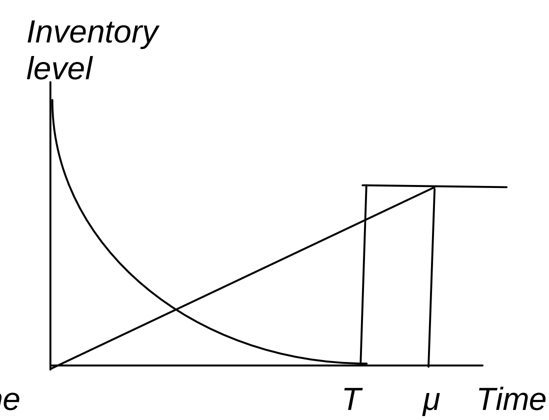
θ = fraction of units that deteriorate per time unit

3. MODEL FORMULATION

For developing mathematical model, we consider two cases as follows:



$\mu < T$



$T \le \mu$

Case I: When $\mu < T$

Let *Q(t)* denote the on hand inventory of the system at any time *t, (0 ≤ t ≤ T)*. Depletion due to demand and deterioration will occur simultaneously. The differential equation that describes the instantaneous state of *Q(t)* is given by

$$\frac{dQ}{dt} + \theta Q(t) = -D_0\, t\ ; \qquad 0 \le t \le \mu \tag{1}$$

$$\frac{dQ}{dt} + \theta Q(t) = -D_0\, \mu\ ; \qquad \mu \le t \le T \tag{2}$$

with the initial and boundary conditions $Q(0) = Q_0$ and $Q(T) = 0$ (3)

where Q_0 is the inventory order quantity. The solutions of equations (1) and (2) subject to the conditions (3) are respectively

$$Q(t) = \begin{cases} Q_0\, e^{-\theta t} - \dfrac{D_0\, t}{\theta} + \dfrac{D_0}{\theta^2}\left(1 - e^{-\theta t}\right) \quad ; \quad 0 \le t \le \mu & (4) \\[2ex] \dfrac{D_0 \mu}{\theta}\left[e^{\theta(T-t)} - 1\right] \quad ; \quad \mu \le t \le T & (5) \end{cases}$$

Substituting $t = \mu$ in equations (4) and (5) and then equating, we get

$$Q_0 = \frac{D_0\, \mu}{\theta} e^{\theta t} - \frac{D_0}{\theta^2}\left(e^{\theta \mu} - 1\right) \tag{6}$$

The total cost will consist of the following components.

i) Total interest earned during cycle time

$$= I_e \int_0^{\mu} C\, D_0\, t\, dt = \frac{C\, D_0\, I_e\, \mu^2}{2} \tag{7}$$

ii) Total interest paid during cycle time

$$= I_c \int_{\mu}^{T} C\, Q(t)\, dt = \frac{C\, D_0\, I_c\, \mu}{\theta}\left\{\left(\mu - T\right) + \frac{1}{\theta}\left[e^{\theta(T-\mu)} - 1\right]\right\} \tag{8}$$

iii) Total inventory holding cost

$$= C_1 \int_0^{T} Q(t)\, dt$$

$$= C_1 \int_0^{\mu} Q(t)\, dt + C_1 \int_{\mu}^{T} Q(t)\, dt$$

$$= \frac{C_1\, D_0\, \mu}{\theta}\left\{\left(\frac{\mu}{2} - T\right) + \frac{e^{\theta t}}{\theta} + \frac{\left(1 - e^{\theta \mu}\right)}{\mu\, \theta^2}\right\} \tag{9}$$

iv) Number of units *D(T)* that deteriorate during the cycle is given by

$$D(t)=Q_0-\left[\int_0^{\mu} D_0\, t\, dt+\int_{\mu}^{T} D_0\, \mu\, dt\right]$$

$$=\frac{D_0\,\mu}{\theta}e^{\theta T}+\frac{D_0}{\theta^2}\left(1-e^{\theta\mu}\right)+D_0\,\mu\left(\frac{\mu}{2}-T\right)$$

The cost due to deterioration during the cycle = *C D(T)*

$$=\frac{C\,D_0\,\mu}{\theta}e^{\theta T}+\frac{C\,D_0}{\theta^2}\left(1-e^{\theta\mu}\right)+C\,D_0\,\mu\left(\frac{\mu}{2}-T\right) \tag{10}$$

v) Replenishment cost per cycle = C_3 (11)

The total cost $\xi_1(T)$ per unit time is given as

$\xi_1(T)$ = [{Inventory holding cost + cost of deterioration + interest paid + replenishment cost – interest earned}/T] (12)

Using (7) to (11) in (12), we get

$$\xi_1(t)=\frac{D_0}{T\,\theta}\left(C_1+C\,\theta\right)\left[\frac{\mu\,e^{\theta T}}{\theta}+\frac{\left(1-e^{\theta\mu}\right)}{\theta^2}+\mu\left(\frac{\mu}{2}-T\right)\right]$$

$$+\frac{CI_c D_0\mu}{\theta^2 T}\left[e^{\theta(T-\mu)}-\theta(T-\mu)-1\right]+\frac{C_3}{T}-\frac{C\,D_0 I_e\mu^2}{2T} \tag{13}$$

For minimization of cost, we set $\frac{\partial\xi_1(T)}{\partial T}=0$

$$\Rightarrow D_0\left(C_1+C\theta\right)\left[\mu\,\theta\left\{2\,e^{\theta T}(\theta T-1)-\mu\,\theta\right\}-2\,(1-e^{\mu\theta})\right]$$

$$+2CI_c D_0\mu\,\theta\left\{e^{\theta(T-\mu)}(\theta T-1)-\mu\,\theta+1\right\}+\left[CD_0 I_e\mu^2-2C_3\right]\theta^3=0 \tag{14}$$

Case II: When $T\leq\mu$

The differential equation that describes the instantaneous state of *Q(t)* is given by

$$\frac{dQ(t)}{dt}+\theta\,Q(t)=-D_0\,t\,;\qquad 0\leq t\leq T \tag{15}$$

with the initial and boundary conditions $Q(0) = Q_0$ and $Q(T) = 0$ (16)

where Q_0 is the inventory order quantity. The solutions of equations (15) subject to conditions $Q(0) = Q_0$ is

$$Q(t)=Q_0\,e^{-\theta t}-\frac{D_0\,t}{\theta}+\frac{D_0}{\theta^2}\left(1-e^{-\theta t}\right) \tag{17}$$

using *Q(T) = 0* in (17), we get

$$Q_0 = \frac{D_0}{\theta}\left[T\,e^{\theta T} - \frac{1}{\theta}\left(e^{\theta T}-1\right)\right] \tag{18}$$

In this case, the customer earns interest on sales revenue up to the permissible period and pays no interest for the items kept in stock.

The total cost will consists of the following components.

i) Total interest earned during the cycle =

interest earned up to T + interest earned during (μ-T)

$$= CI_e\int_0^T D_0 t\,dt + C\,I_e D_0 T\,(\mu - T) = C\,I_e D_0 T\,(\mu - \frac{T}{2}) \tag{19}$$

ii) Total inventory holding cost = $C_1\int_0^T Q\,dt = \frac{C_1 D_0 T}{\theta}\left\{\frac{e^{\theta T}}{\theta} + \frac{\left(1-e^{\theta T}\right)}{\theta^2 T} - \frac{T}{2}\right\}$ (20)

iii) Number of units that deteriorate

$$D(t) = Q_0 - \int_0^T D_0\,t\,dt$$

$$= \frac{D_0\,T}{\theta}e^{\theta T} + \frac{D_0}{\theta^2}\left(1-e^{\theta T}\right) - D_0\frac{T^2}{2}$$

Cost of deterioration = $CD(t) = \frac{CD_0\,T}{\theta}e^{\theta T} + \frac{CD_0}{\theta^2}\left(1-e^{\theta T}\right) - CD_0\frac{T^2}{2}$ (21)

iv) Replenishment cost = C_3 (22)

The total cost $\xi_2(T)$ per unit time is given as

$\xi_2(T)$ = [{Inventory holding cost + cost of deterioration + replenishment cost – interest earned}/T] (23)

using (19) to (22) in (23), we get

$$\xi_2(T) = \frac{\left(C_1 + C\theta\right)D_0}{\theta}\left\{\frac{e^{\theta T}}{\theta} + \frac{\left(1-e^{\theta T}\right)}{\theta^2 T} - \frac{T}{2}\right\} + \frac{C_3}{T} - CI_e D_0\left(\mu - \frac{T}{2}\right) \tag{24}$$

For minimization of cost, we set

$$\frac{\partial \xi_2(T)}{\partial T} = 0$$

$$\Rightarrow D_0\left(C_1 + C\theta\right)\left[T\,\theta\left\{2\,e^{\theta T}(T\theta - 1) - T\,\theta\right\} - 2\left(1-e^{T\theta}\right)\right] + \left[2C_3 - CD_0 I_e T^2\right]\theta^3 = 0 \tag{25}$$

4. Algorithm

In order to obtain optimum total cost and optimum cycle time interval between two consecutive orders, we proceed as follows:

Step 1: Compute T_1 from equation (14). If $T_1 \geq \mu$, then T_1 is the optimum value of T and obtain total optimum cost from equation (13).

Step 2: If $T_1 < \mu$, then compute T_2 from equation (25). Again if $T_2 < \mu$ then T_2 is the optimum value of T and optimum total cost can be computed from equation (24).

5. Numerical Example

To illustrate the result of the derived model we consider the following example.

C = \$ 150 per unit per annum, I = 0.1 year, R = 1000 units,
C_3 = \$ 500 per unit per annum, Ie = 12% per annum, Ic = 20% per annum,
$C_1 = C^*I$, μ = 1/4, 1/8 year, θ = 0.01 (.01) 0.10

M	θ	T_1	ξ_1
1/8	0.01	0.3626	1327.64
	0.02	0.3566	1352.041
	0.03	0.3506	1376.847
	0.04	0.3449	1401.465
	0.05	0.3395	1425.902
	0.06	0.3343	1450.19
	0.07	0.3293	1474.356
	0.08	0.3245	1498.417
	0.09	0.3199	1522.398
	0.10	0.3154	1546.319

Table – 1

μ	θ	T_1	T_2	ξ_2
1/4	0.01	0.1485	0.2353	47.59
	0.02	0.1441	0.2349	74.49
	0.03	0.1396	0.2345	101.10
	0.04	0.1352	0.2339	127.38
	0.05	0.1308	0.2334	153.29
	0.06	0.1266	0.2328	178.83
	0.07	0.1225	0.2321	203.97
	0.08	0.1185	0.2314	228.68
	0.09	0.1145	0.2306	252.96
	0.10	0.1106	0.2298	276.79

Table - 2

6. Conclusion

Here an optimal replenishment policy is developed for inventory model of deteriorating items with ramp type demand under permissible delay in payments.

Table-1 reveals that as rate of deterioration increases, the cycle time decreases while total cost per time unit of inventory increases.

When permissible delay period is greater than optimum cycle time it follows from table-2 that total cost per time unit increases as rate of deterioration increases.

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