Empirical Characteristic Function Approach to Goodness-of-Fit Tests for the Generalized Exponential Distribution

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Abstract

The goodness-of-fit test of Towhidi and Salmanpour (2007), which is based on the empirical characteristic function is used for testing the fit of the generalized exponential distribution. Monte Carlo procedures are employed to obtain the empirical percentage points for the test statistic. In addition, an extensive power study is conducted to compare the performance of this test against the well known goodness-of-fit tests which are based on empirical distribution function.

Keywords: goodness-of-fit tests, empirical characteristic function, generalized exponential distribution, maximum likelihood estimation, empirical distribution function.

1. Introduction

A statistical problem encountered in many areas of research is the need to assess whether a sample of observations comes from a specified distribution. Typically such situations are known as 'Goodness of Fit' problems. Many goodness-of-fit tests were introduced in the literature. The tests based on the empirical distribution function (EDF) are the most commonly used goodness-of-fit tests. In addition, there are goodness-of-fit tests that are based on the empirical characteristic function (ECF).

The generalized exponential distribution has been introduced and studied quite extensively by Gupta and Kundu (1999, 2001, 2002, 2003). It is observed that this distribution can be considered for situations where a skewed distribution for a non-negative random variable is needed. Also, it is observed that it can be used quite effectively to analyze lifetime data in place of gamma, Weibull and log-normal distributions. The generalized exponential distribution has the following distribution function:

$$F(x;\alpha,\lambda) = (1-e^{-\lambda x})^{\alpha}; \qquad \alpha,\lambda > 0 \tag{1.1}$$

for $x > 0$ and 0 otherwise. The corresponding density function is:

$$f(x;\alpha,\lambda) = \alpha\lambda(1-e^{-\lambda x})^{\alpha-1}e^{-\lambda x}; \qquad \alpha,\lambda > 0 \tag{1.2}$$

for $x > 0$ and 0 otherwise. In addition, the CF of the generalized exponential distribution is given by:

$$\Phi(t) = \frac{\alpha\Gamma\left(1-\frac{it}{\lambda}\right)\Gamma(\alpha)}{\Gamma\left(1-\frac{it}{\lambda}+\alpha\right)} \tag{1.3}$$

Here α is the shape parameter and λ is the scale parameter. If the shape parameter $\alpha = 1$, then the generalized exponential distribution coincides with the exponential distribution with a scale parameter λ. The generalized exponential distribution with the two parameters α and λ will be denoted by GE(α , λ).

Hassan (2005) used different EDF statistics for testing the goodness of fit for the GE(α, λ) with unknown parameters. She obtained the critical values for these test statistics after estimating the unknown parameters by ML method. In addition, Hassan (2005) conducted a power comparison between the Kolmogorov-Smirnov (K-S), Cramer-von Mises (C-M), Anderson-Darling(A-D), Watson (WA), and Liao and Shimokawa (L-S) tests for the GE(α, λ) with the two parameters unknown. She considered different alternatives that are; the standard normal, Weibull, Gamma, exponential, chi-square and uniform distributions. From this power study, Hassan (2005) concluded that the L-S test is generally superior to other tests. Also, she mentioned that the WA test is not appearing to be powerful across this group of alternative distributions.

The present work is interested in applying the test of Towhidi and Salmanpour (2007) for testing the null hypothesis:

H_0: The sample $x_1, x_2, ..., x_n$ comes from GE(α , λ) with unknown parameters α and λ. (1.4)

The unknown parameters of this distribution will be estimated by the maximum likelihood (ML) method. Also, a power comparison between the ECF test and the EDF tests is conducted.

The organization of this article is as follows. Section 2 discusses estimation of the unknown parameters of the GE(α , λ) by the ML method. The goodness-fit-test based on ECF for the GE(α , λ) is provided in section 3. EDF tests for the GE(α , λ) are discussed in Section 4. Then, a power study is conducted at section 5. Finally, conclusions are included in section 6.

2. ML Estimation of the unknown parameters of the Generalized Exponential distribution

For testing H_0 given in (1.4) the unknown parameters of the GE(α , λ) must be estimated first. In this section the ML method for parameter estimation is considered.

Gupta and Kundu (2001b) estimated the two parameters, α and λ, of the GE(α , λ) using the ML method. They concluded the following two equations:

$$\hat{\alpha} = \frac{-n}{\sum_{i=1}^{n} \ln(1 - e^{-\hat{\lambda} x_i})}. \tag{2.1}$$

$$\hat{\lambda}\{[n+\sum_{i=1}^{n}\ln(1-e^{-\hat{\lambda}x_i})][\sum_{i=1}^{n}\{(x_i e^{-\hat{\lambda}x_i})/(1-e^{-\hat{\lambda}x_i})\}]+[\sum_{i=1}^{n}x_i][\sum_{i=1}^{n}\ln(1-e^{-\hat{\lambda}x_i})]\}$$
$$-n\sum_{i=1}^{n}\ln\left(1-e^{-\hat{\lambda}x_i}\right)=0. \qquad (2.2)$$

Where $x_1, x_2, ..., x_n$ is a random sample from the GE(α , λ). Equation (2.2) has to be solved for the unknown $\hat{\lambda}$ by iteration to obtain the ML estimator of λ. Then, the ML estimator of α, $\hat{\alpha}$, can be easily obtained as a function of $\hat{\lambda}$ from equation (2.1).

3. Goodness-of-fit tests based on ECF for the generalized exponential distribution

There are different forms of test statistics that are based on ECF. These tests depend on measuring the distance between the ECF of the sample data and the CF of the hypothesized distribution. Therefore; according to the technique of measuring this distance; these tests can be divided into two major types. First; tests based on the integral of the squared modulus of the difference between the ECF and the CF of the null distribution, such as the test of Wong and Sim (2000). Second, tests based on the difference between the real (or imaginary) part of the ECF and the real (or imaginary) part of the CF of the hypothesized distribution, such as the test of Towhidi and Salmanpour (2007).

The first type of these tests can be applied only for distributions which have CF in closed form but for other distributions which do not have CF in closed form the computations will be difficult if not impossible. This is because the algorithm will contain more than one integration that does not converge to a solution. On the other hand, the second type of these tests can be applied for any distribution whether its CF is in closed form or not.

Since the CF of the generalized exponential distribution is not in a closed form, then the test of Towhidi and Salmanpour (2007) is the most appropriate in this case. This test is explained in details as follows.

The test of Towhidi and Salmanpour (2007) is given by:

$$T_n^* = \left(Z_n(T_{\underline{m}}) - Z_0(T_{\underline{m}})\right)' B_k \Lambda_k^{-1} B_k' \left(Z_n(T_{\underline{m}}) - Z_0(T_{\underline{m}})\right) \qquad (2.3)$$

Where $Z_n(T_{\underline{m}}) = \begin{bmatrix} \operatorname{Re}\Phi_n(t_1) \\ \operatorname{Re}\Phi_n(t_2) \\ \vdots \\ \operatorname{Re}\Phi_n(t_m) \\ \operatorname{Im}\Phi_n(t_1) \\ \operatorname{Im}\Phi_n(t_2) \\ \vdots \\ \operatorname{Im}\Phi_n(t_m) \end{bmatrix}, \qquad Z_0(T_{\underline{m}}) = \begin{bmatrix} \operatorname{Re}\Phi(t_1) \\ \operatorname{Re}\Phi(t_2) \\ \vdots \\ \operatorname{Re}\Phi(t_m) \\ \operatorname{Im}\Phi(t_1) \\ \operatorname{Im}\Phi(t_2) \\ \vdots \\ \operatorname{Im}\Phi(t_m) \end{bmatrix}.$ (2.4)

i) $T_{\underline{m}} = (t_1, t_2, ..., t_m)$ is an arbitrary point in $\Re^m$, where *m* is a positive integer that is equal to the sample size *n*.

ii) $\Phi(t)$ is the CF of the hypothesized distribution and $\Phi_n(t)$ is the ECF.

iii) $\operatorname{Im}\Phi_n(t) = \frac{1}{n}\sum_{j=1}^{n}\sin(tX_j)$ and $\operatorname{Re}\Phi_n(t) = \frac{1}{n}\sum_{j=1}^{n}\cos(tX_j)$ are the imaginary and real parts of $\Phi_n(t)$, respectively.

iv) $\operatorname{Im}\Phi(t) = E[\sin(tX)]$ and $\operatorname{Re}\Phi(t) = E[\cos(tX)]$ are the imaginary and real parts of $\Phi(t)$, respectively.

v) Λ_k is a $k \times k$ diagonal matrix whose diagonal elements, $\lambda_1, \lambda_2, ..., \lambda_k$, are the k largest eigen values of Ω_0 . Ω_0 be the $2m \times 2m$ variance-covariance matrix

of $Y_j(T_{\underline{m}})$ under H_0. Where
$$Y_j(T_{\underline{m}}) = \begin{bmatrix} \cos(t_1 X_j) - E\left[\cos(t_1 X_j)\right] \\ \vdots \\ \cos(t_m X_j) - E\left[\cos(t_m X_j)\right] \\ \sin(t_1 X_j) - E\left[\sin(t_1 X_j)\right] \\ \vdots \\ \sin(t_m X_j) - E\left[\sin(t_m X_j)\right] \end{bmatrix} \quad (2.5)$$

And the integer value k is chosen such that the ratio $\sum_{j=1}^{k}\lambda_j \Big/ \sum_{j=1}^{2m}\lambda_j$ is near to 1.

vi) B_k is a $2m \times k$ matrix whose columns are the eigen vectors, $\underline{B}_1, \underline{B}_2, ..., \underline{B}_k$, corresponding to $\lambda_1, \lambda_2, ..., \lambda_k$.

It is known that the standard procedure for testing H_0 , given in (1.4), is to compare the value of the test statistic for the observed sample with its theoretical critical values (percentage points) under H_0.

The exact distribution of the test statistic, T_n^*, under H_0 is difficult; if not impossible; to obtain. In this section, Monte Carlo procedures are employed to obtain the empirical critical values of T_n^* under the GE(α, λ) null distribution. Two significance levels will be considered that are $\gamma = 0.05$ and 0.01. Different sample sizes are considered $n = 10$, 20 and 30 with N= 10,000 replications. In addition, different values for the shape parameter α were considered, $\alpha = 0.5, 1, 1.5, 2, 2.5,$ and 3 with a scale parameter $\lambda = 1$ in all the cases.

To find the empirical critical values for the different significance levels, the following steps are performed:

Step (1): Generate *N* random samples of size *n* from the GE(α,λ). This can be done by firstly generating a random sample from the uniform(0,1) distribution, $u_1,u_2,...,u_n$. Then the uniform random numbers can be transformed to generalized exponential random numbers using the following transformation:

$$x_i = (-\frac{1}{\lambda})\ln[1-(u_i)^{\frac{1}{\alpha}}], \qquad i = 1,2,\dots,n \tag{2.6}$$

Step (2): Estimate the parameters of the GE(α,λ) using the ML method.

Step (3): Calculate the value of T_n^* for each random sample. This is done as follows:

i) Select the values $t_1,...,t_m$ by the relation: $t_i = F^{-1}\left(\frac{i}{m+1}\right)$, $i=1,...,m$, where F^{-1} is the inverse distribution function of the GE(α,λ) and $m=n$ (the sample size).

ii) Define the vector $T'_m = (t_1,...,t_m)$ and evaluate the vector $Z_n(T_m)-Z_0(T_m)$. Where $Z_n(T_m)$ and $Z_0(T_m)$ are defined in (2.4).

iii) Determine the matrix Ω_0 which is the $2m\times 2m$ variance-covariance matrix of $Y_j(T_m)$, which is defined in (2.5), under H_0. Then derive its eigen values and eigen vectors.

iv) Arrange the eigen values, β_i, such that $\beta_1 \ge \beta_2 \ge ... \ge \beta_{2m}$. Then, choose an integer k so that the ratio $\frac{\sum_{i=1}^{k}\beta_i}{\sum_{i=1}^{2m}\beta_i}$ is large enough (near to one).

Construct the matrices Λ_k and B_k.

v) Calculate the value of T_n^* as in (2.3).

Step (4): Arrange the *N* values of T_n^* in an ascending order

Step (5): Calculate the 100(1-γ)*th* percentile from the ordered T_n^* values.

These steps were performed using the Mathcad (version 13) software. The results are shown in table (1).

4. Goodness-of-fit tests based on EDF for the generalized exponential distribution

The most commonly used goodness-of-tests are the tests based on EDF. These test statistics generally measure in a different way the distance between a CDF; $F_0(x)$; and the EDF ; denoted by $F_n(x)$; which is calculated from the sample. There are many tests based on EDF statistics such as the K-S, C-M and A-D tests. In the last two decades, many authors (Lawless, 1982; Liao and Shimokawa, 1999b; Littell *et al*., 1979; Park and Seoh, 1994; Stephens, 1974) have reported that the A-D and C-M tests are more powerful than the K-S test. At present, the A-D test is mostly applied in the statistical analysis of lifetime data (Liao and Shimokawa, 1999a).

When $F_0(x)$ is completely specified (i.e. does not contain unknown parameters) and the data are uncensored, the EDF tests are all distribution free and percentage points for these test statistics are generally known. However, this is no longer the case when data are censored or $F_0(x)$ involves unknown parameters. Therefore; if the hypothesized distribution contains unknown parameters that must be estimated from the sample data, then the standard tables used for the EDF statistics are no longer valid and the resulting test would be inaccurate. In this case, these statistics can be modified by inserting estimates of unknown parameters in $F_0(x)$. However, it is not possible to obtain an analytic expression for the distribution of these modified statistics. Hence, the critical values for these modified statistics are usually obtained by Monte Carlo simulation methods (Abd-Elfattah *et al*., 2010).

In this section; the empirical percentage points of several modified EDF statistics, under the null hypothesis that the data are from the GE(α , λ), will be obtained by Monte Carlo simulations. The considered EDF statistics are:

1. The K-S test statistic:

$$D = \max\left(D^{+}, D^{-}\right), \tag{4.1}$$

where: $D^{+} = \max_i \left\{\left(i/n\right) - F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right)\right\}$

and

$$D^{-} = \max_i \left\{F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right) - \left[(i-1)/n\right]\right\}$$

$F_0\left(x_i, \hat{\alpha}, \hat{\lambda}\right)$ is the CDF of the GE(α , λ) given in (1.1); and $\hat{\alpha}$ and $\hat{\lambda}$ are the ML estimates of the parameters of the GE(α , λ).

2. The C-M statistic:

$$W^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left[F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right) - \frac{2i-1}{2n} \right]^2 . \tag{4.2}$$

3. The WA statistic:

$$U^2 = W^2 - \left[\frac{\sum_{i=1}^{n} F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right)}{n} - \frac{1}{2} \right]^2 . \tag{4.3}$$

4. The A-D statistic:

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left\{ \ln F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right) + \ln\left[1 - F_0\left(x_{(n-i+1)}, \hat{\alpha}, \hat{\lambda}\right)\right] \right\} . \tag{4.4}$$

5. The L-S statistic:

$$L_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \frac{\max\left[\frac{i}{n} - F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right), F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right) - \frac{i-1}{n} \right]}{\sqrt{F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right)\left[1 - F_0\left(x_{(i)}, \hat{\alpha}, \hat{\lambda}\right)\right]}} . \tag{4.5}$$

The steps for obtaining critical values for these five EDF statistics are the same steps followed for obtaining critical values for the T_n^* statistic in the previous section. Except for step (5) where the D, W^2, U^2, A^2 and L_n statistics must be calculated from each sample. These steps were performed using the Mathcad (version 13) software. Different sample sizes were considered: $n = 10$, 20 and 30. Two significance levels considered: $\gamma = 0.05$ and 0.01. For the GE(α, λ); different values for the shape parameter α were considered that are: $\alpha = 0.5(0.5)\,3$, while λ was always considered to be 1.

The critical values for the different modified EDF statistics are listed in table (1) in the appendix with the critical values of the T_n^* statistic.

Table (1): Critical values of the EDF and ECF Statistics

n	Statistic	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	*D*	0.05	0.748	0.847	0.899	0.938	0.962	0.976
		0.01	0.942	0.991	0.999	1	1	1
	W^2	0.05	2.079	2.679	2.97	3.126	3.213	3.262
		0.01	3.071	3.303	3.33	3.333	3.333	3.333
	A^2	0.05	13.303	22.269	31.263	40.109	49.083	58.086
		0.01	30.878	59.165	90.529	125.136	164.164	207.316
	U^2	0.05	1.917	2.472	2.743	2.889	2.97	3.016
		0.01	2.839	3.054	3.08	3.083	3.083	3.083
	L_n	0.05	10.64	18.60	31.64	57.34	104.83	202.07
		0.01	103.44	253.50	1344.43	13997.04	226276.89	5730672.82
	T_n^*	0.05	9.044	8.957	8.38	8.38	8.267	8.114
		0.01	50.361	22.702	20.467	20.467	21.183	19.106
20	*D*	0.05	0.489	0.569	0.621	0.660	0.690	0.714
		0.01	0.660	0.759	0.818	0.857	0.884	0.906
	W^2	0.05	1.827	2.627	3.187	3.616	3.957	4.238
		0.01	3.432	4.636	5.280	5.675	5.947	6.131
	A^2	0.05	10.012	15.377	20.063	24.373	28.544	32.466
		0.01	19.671	32.797	44.691	55.855	67.393	78.512
	U^2	0.05	1.753	2.521	3.057	3.469	3.798	4.068
		0.01	3.297	4.455	5.075	5.455	5.718	5.897
	L_n	0.05	4.964	6.372	7.917	9.440	11.199	13.068
		0.01	16.043	17.001	22.258	30.864	45.057	66.663
	T_n^*	0.05	2.942	3.598	4.582	4.831	4.968	4.61
		0.01	9.754	11.161	10.644	10.296	9.437	8.658
30	*D*	0.05	0.388	0.453	0.497	0.529	0.554	0.570
		0.01	0.512	0.599	0.652	0.688	0.718	0.740
	W^2	0.05	1.668	2.436	3.022	3.498	3.892	4.155
		0.01	3.113	4.509	5.464	6.131	6.611	6.846
	A^2	0.05	9.178	13.797	17.530	21.200	24.235	26.484
		0.01	16.647	26.291	34.883	43.131	50.966	55.744
	U^2	0.05	1.625	2.372	2.941	3.407	3.788	4.040
		0.01	3.031	4.388	5.319	5.969	6.437	6.665
	L_n	0.05	3.971	4.943	5.786	6.650	7.420	7.844
		0.01	8.085	9.499	11.323	13.282	15.796	17.586
	T_n^*	0.05	1.687	1.833	2.298	2.552	2.669	2.742
		0.01	5.469	5.057	5.367	6.459	6.049	5.822

5. Power Study

The power of a goodness-of-fit test is defined as the probability that a statistic will lead to the rejection of the null hypothesis, H_0, when it is false. The power of a goodness-of-fit test at a significance level γ is denoted by $1-\beta$, where β is the probability of committing a type-II error, that is failing to reject a false null hypothesis.

In this section a power comparison is conducted between the different EDF tests and the ECF test of Towhidi and Salmanpour (2007), for the GE(α,λ) with unknown parameters. For the alternative hypothesis, H_a, that the random sample comes from another distribution; several alternative distributions were considered. These distributions are:

i) Chi-Square distribution with shape parameter α (=1.5 and 3).
ii) Exponential distribution with scale parameter λ (=1).
iii) Gamma distribution with shape parameter α (=1.5 and 2.5).
iv) Lognormal distribution with scale parameter λ (=1).
v) Weibull distribution with shape parameter α (=1.5).

The steps for obtaining the power of any test are as follows:

Step (1): Generate *N* (=10,000) random samples of size *n* from each alternative distribution.

Step (2): Calculate the value of the test statistic from each sample.

Step (3): Arrange the *N* values of the test statistic in an ascending order.

Step (4): From table (1), and for each significance level (γ); get the critical value, call it *c*, of each test statistic under the null distribution GE(α , λ).

Step (5): Calculate *L* = the number of the test statistic values that are greater than **c**.

Step (6): Calculate the power of the test:

$$1-\beta = L/N$$

These steps were performed using the Mathcad (version 13) software. Different sample sizes were considered: $n=10$, 20 and 30 with two significance levels: $\gamma=0.01$ and 0.05. The results of the power comparison are shown, for each alternative distribution, in tables (2) to (9).

From tables (2) to (9) it is noticed that, for the modified EDF tests, the power increase as the sample size increase in most of the cases. Some exceptions happen for some alternatives where the powers are approximately equal for $n=10$, 20 and 30. This happens, for example, when the alternative is Chi-square ($\alpha=1.5$) [only when $\gamma=0.01$], exponential ($\lambda=1$) or gamma ($\alpha=1.5$) for the K-S, C-M, A-D and WA tests. On the other hand, for the ECF test, the power decrease as the sample size increase except when the alternative distribution is gamma ($\alpha=2.5$), Chi-square ($\alpha=$1.5,3) or Weibull ($\alpha=1.5$) where the power increase as the sample size increase.

Table (2): Power values when the alternative distribution is Chi-square ($\alpha = 1.5$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	*D*	0.05	0.277	0.007	0.004	0.003	0.002	0.002
		0.01	0.003	0.001	0.001	0	0	0
	W^2	0.05	0.022	0.006	0.004	0.003	0.002	0.001
		0.01	0.003	0.001	0.001	0	0	0
	A^2	0.05	0.205	0.090	0.048	0.032	0.026	0.024
		0.01	0.049	0.024	0.020	0.005	0.003	0.003
	U^2	0.05	0.022	0.006	0.004	0.003	0.002	0.001
		0.01	0.003	0.001	0.001	0	0	0
	L_n	0.05	0.366	0.302	0.256	0.217	0.185	0.158
		0.01	0.185	0.149	0.101	0.062	0.037	0.022
	T_n^*	0.05	0.247	0.247	0.256	0.256	0.257	0.259
		0.01	0.105	0.161	0.169	0.169	0.166	0.175
20	*D*	0.05	0.264	0.107	0.045	0.019	0.008	0.004
		0.01	0.019	0.001	0	0	0	0
	W^2	0.05	0.248	0.075	0.022	0.007	0.003	0.001
		0.01	0.011	0.0002	0	0	0	0
	A^2	0.05	0.484	0.328	0.233	0.167	0.125	0.094
		0.01	0.238	0.092	0.044	0.027	0.017	0.011
	U^2	0.05	0.248	0.075	0.022	0.007	0.003	0.001
		0.01	0.011	0	0	0	0	0
	L_n	0.05	0.600	0.543	0.496	0.460	0.428	0.400
		0.01	0.368	0.359	0.325	0.292	0.260	0.229
	T_n^*	0.05	0.579	0.543	0.494	0.483	0.479	0.493
		0.01	0.340	0.311	0.322	0.328	0.346	0.366
30	*D*	0.05	0.503	0.299	0.182	0.115	0.076	0.056
		0.01	0.146	0.029	0.005	0.002	0.001	0.0002
	W^2	0.05	0.527	0.297	0.165	0.089	0.046	0.029
		0.01	0.148	0.014	0.001	0.0001	0	0
	A^2	0.05	0.678	0.537	0.434	0.348	0.288	0.251
		0.01	0.457	0.255	0.144	0.085	0.050	0.037
	U^2	0.05	0.527	0.297	0.166	0.089	0.047	0.030
		0.01	0.149	0.014	0.001	0.0001	0	0
	L_n	0.05	0.751	0.694	0.645	0.606	0.579	0.562
		0.01	0.554	0.518	0.476	0.441	0.410	0.391
	T_n^*	0.05	0.790	0.777	0.745	0.727	0.719	0.714
		0.01	0.589	0.605	0.591	0.555	0.566	0.575

Table (3): Power values when the alternative distribution is Chi-square ($\alpha = 3$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	D	0.05	0.332	0.104	0.023	0.010	0.005	0.003
		0.01	0.009	0.001	0	0	0	0
	W^2	0.05	0.337	0.078	0.028	0.012	0.006	0.003
		0.01	0.017	0.001	0	0	0	0
	A^2	0.05	0.657	0.476	0.331	0.229	0.162	0.123
		0.01	0.336	0.119	0.082	0.058	0.024	0.020
	U^2	0.05	0.337	0.078	0.028	0.012	0.006	0.003
		0.01	0.017	0.001	0	0	0	0
	L_n	0.05	0.716	0.643	0.579	0.517	0.461	0.410
		0.01	0.462	0.394	0.296	0.198	0.125	0.080
	T_n^*	0.05	0.586	0.588	0.599	0.599	0.600	0.604
		0.01	0.241	0.397	0.418	0.418	0.412	0.433
20	D	0.05	0.906	0.800	0.689	0.570	0.473	0.385
		0.01	0.570	0.225	0.075	0.025	0.007	0.002
	W^2	0.05	0.918	0.811	0.687	0.551	0.430	0.324
		0.01	0.610	0.185	0.041	0.008	0.002	0.001
	A^2	0.05	0.952	0.909	0.860	0.811	0.755	0.704
		0.01	0.864	0.699	0.526	0.394	0.282	0.206
	U^2	0.05	0.918	0.811	0.688	0.552	0.431	0.325
		0.01	0.609	0.185	0.041	0.008	0.002	0.001
	L_n	0.05	0.954	0.935	0.915	0.899	0.878	0.857
		0.01	0.830	0.823	0.786	0.744	0.690	0.639
	T_n^*	0.05	0.928	0.913	0.892	0.886	0.884	0.891
		0.01	0.795	0.767	0.777	0.783	0.799	0.814
30	D	0.05	0.984	0.964	0.936	0.908	0.873	0.847
		0.01	0.924	0.783	0.617	0.471	0.339	0.246
	W^2	0.05	0.988	0.972	0.947	0.918	0.882	0.851
		0.01	0.943	0.797	0.595	0.395	0.250	0.182
	A^2	0.05	0.991	0.982	0.973	0.960	0.947	0.935
		0.01	0.975	0.936	0.884	0.818	0.743	0.691
	U^2	0.05	0.988	0.972	0.947	0.918	0.881	0.851
		0.01	0.943	0.798	0.595	0.395	0.251	0.183
	L_n	0.05	0.992	0.987	0.982	0.977	0.973	0.969
		0.01	0.968	0.956	0.943	0.928	0.911	0.899
	T_n^*	0.05	0.948	0.945	0.931	0.925	0.920	0.919
		0.01	0.865	0.873	0.867	0.846	0.854	0.858

Table (4): Power values when the alternative distribution is Exponential ($\lambda = 1$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	D	0.05	0.087	0.050	0.035	0.026	0.019	0.015
		0.01	0.025	0.010	0.005	0.000	0.000	0.000
	W^2	0.05	0.092	0.050	0.034	0.024	0.018	0.013
		0.01	0.027	0.010	0.005	0.003	0.003	0.003
	A^2	0.05	0.101	0.050	0.030	0.020	0.014	0.010
		0.01	0.031	0.010	0.005	0.002	0.001	0.001
	U^2	0.05	0.092	0.050	0.034	0.024	0.018	0.013
		0.01	0.027	0.010	0.005	0.003	0.003	0.003
	L_n	0.05	0.082	0.050	0.034	0.023	0.016	0.012
		0.01	0.016	0.011	0.005	0.002	0.001	0.001
	T_n^*	0.05	0.463	0.465	0.476	0.476	0.479	0.483
		0.01	0.186	0.305	0.323	0.323	0.317	0.332
20	D	0.05	0.090	0.052	0.034	0.025	0.019	0.015
		0.01	0.025	0.010	0.005	0.003	0.002	0.001
	W^2	0.05	0.091	0.050	0.032	0.023	0.017	0.013
		0.01	0.027	0.009	0.005	0.003	0.002	0.001
	A^2	0.05	0.101	0.051	0.029	0.018	0.012	0.009
		0.01	0.031	0.009	0.004	0.002	0.001	0.001
	U^2	0.05	0.091	0.050	0.032	0.023	0.017	0.013
		0.01	0.027	0.009	0.005	0.003	0.002	0.001
	L_n	0.05	0.084	0.050	0.030	0.022	0.017	0.012
		0.01	0.010	0.009	0.006	0.004	0.003	0.002
	T_n^*	0.05	0.099	0.079	0.060	0.056	0.055	0.059
		0.01	0.021	0.017	0.018	0.019	0.022	0.025
30	D	0.05	0.093	0.048	0.032	0.024	0.018	0.016
		0.01	0.027	0.011	0.007	0.004	0.003	0.002
	W^2	0.05	0.095	0.050	0.034	0.024	0.019	0.015
		0.01	0.032	0.011	0.006	0.003	0.002	0.002
	A^2	0.05	0.100	0.049	0.030	0.020	0.015	0.011
		0.01	0.035	0.011	0.005	0.002	0.001	0.001
	U^2	0.05	0.094	0.050	0.034	0.024	0.019	0.015
		0.01	0.032	0.011	0.006	0.003	0.002	0.002
	L_n	0.05	0.093	0.052	0.034	0.023	0.016	0.014
		0.01	0.013	0.009	0.006	0.004	0.003	0.002
	T_n^*	0.05	0.183	0.174	0.144	0.133	0.128	0.125
		0.01	0.062	0.068	0.064	0.051	0.056	0.058

Table (5): Power values when the alternative distribution is Exponential ($\lambda = 2$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	D	0.05	0.380	0.248	0.184	0.135	0.103	0.082
		0.01	0.130	0.054	0.024	0.000	0.000	0.000
	W^2	0.05	0.388	0.240	0.164	0.120	0.090	0.072
		0.01	0.136	0.051	0.024	0.014	0.014	0.014
	A^2	0.05	0.380	0.218	0.136	0.090	0.063	0.047
		0.01	0.138	0.045	0.019	0.009	0.005	0.004
	U^2	0.05	0.388	0.240	0.165	0.120	0.091	0.072
		0.01	0.136	0.052	0.025	0.014	0.014	0.014
	L_n	0.05	0.213	0.128	0.083	0.056	0.039	0.028
		0.01	0.040	0.025	0.013	0.006	0.004	0.002
	T_n^*	0.05	0.798	0.799	0.807	0.807	0.808	0.810
		0.01	0.480	0.668	0.687	0.687	0.679	0.697
20	D	0.05	0.669	0.526	0.434	0.360	0.309	0.267
		0.01	0.360	0.203	0.128	0.085	0.063	0.047
	W^2	0.05	0.680	0.523	0.419	0.342	0.287	0.246
		0.01	0.374	0.189	0.112	0.074	0.050	0.036
	A^2	0.05	0.659	0.485	0.365	0.281	0.218	0.171
		0.01	0.374	0.168	0.088	0.048	0.028	0.017
	U^2	0.05	0.680	0.523	0.420	0.342	0.288	0.246
		0.01	0.373	0.189	0.112	0.074	0.051	0.036
	L_n	0.05	0.478	0.329	0.230	0.165	0.123	0.094
		0.01	0.064	0.058	0.036	0.022	0.012	0.007
	T_n^*	0.05	0.240	0.197	0.156	0.147	0.144	0.155
		0.01	0.064	0.054	0.058	0.060	0.068	0.076
30	D	0.05	0.839	0.723	0.630	0.561	0.505	0.469
		0.01	0.597	0.405	0.293	0.225	0.177	0.145
	W^2	0.05	0.853	0.739	0.638	0.559	0.497	0.456
		0.01	0.622	0.403	0.269	0.193	0.148	0.128
	A^2	0.05	0.831	0.693	0.582	0.485	0.415	0.366
		0.01	0.606	0.370	0.228	0.141	0.088	0.066
	U^2	0.05	0.852	0.738	0.638	0.558	0.497	0.456
		0.01	0.621	0.403	0.269	0.193	0.148	0.129
	L_n	0.05	0.723	0.563	0.444	0.343	0.273	0.242
		0.01	0.225	0.152	0.095	0.061	0.036	0.026
	T_n^*	0.05	0.635	0.612	0.546	0.519	0.504	0.495
		0.01	0.358	0.372	0.362	0.336	0.345	0.350

Table (6): Power values when the alternative distribution is Gamma ($\alpha = 1.5$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	D	0.05	0.075	0.045	0.032	0.024	0.019	0.016
		0.01	0.022	0.011	0.006	0.000	0.000	0.000
	W^2	0.05	0.077	0.045	0.031	0.023	0.018	0.015
		0.01	0.026	0.012	0.006	0.004	0.004	0.004
	A^2	0.05	0.111	0.055	0.033	0.023	0.017	0.013
		0.01	0.034	0.013	0.007	0.004	0.003	0.002
	U^2	0.05	0.077	0.045	0.031	0.023	0.018	0.015
		0.01	0.025	0.012	0.006	0.004	0.004	0.004
	L_n	0.05	0.121	0.079	0.054	0.038	0.028	0.021
		0.01	0.028	0.019	0.010	0.005	0.003	0.002
	T_n^*	0.05	0.301	0.303	0.316	0.316	0.319	0.323
		0.01	0.050	0.149	0.166	0.166	0.161	0.174
20	D	0.05	0.084	0.037	0.022	0.016	0.011	0.009
		0.01	0.016	0.007	0.004	0.003	0.002	0.002
	W^2	0.05	0.082	0.034	0.021	0.014	0.011	0.009
		0.01	0.017	0.007	0.004	0.003	0.002	0.002
	A^2	0.05	0.131	0.063	0.033	0.020	0.014	0.011
		0.01	0.034	0.010	0.004	0.003	0.002	0.002
	U^2	0.05	0.082	0.034	0.021	0.014	0.011	0.009
		0.01	0.017	0.007	0.004	0.003	0.002	0.002
	L_n	0.05	0.146	0.099	0.068	0.052	0.041	0.034
		0.01	0.027	0.025	0.020	0.014	0.011	0.007
	T_n^*	0.05	0.279	0.244	0.204	0.196	0.193	0.204
		0.01	0.102	0.086	0.091	0.095	0.105	0.115
30	D	0.05	0.115	0.048	0.025	0.016	0.011	0.008
		0.01	0.020	0.006	0.003	0.002	0.001	0.001
	W^2	0.05	0.121	0.045	0.024	0.014	0.010	0.008
		0.01	0.022	0.006	0.003	0.002	0.001	0.001
	A^2	0.05	0.169	0.081	0.047	0.028	0.019	0.015
		0.01	0.054	0.016	0.006	0.002	0.002	0.001
	U^2	0.05	0.121	0.045	0.024	0.014	0.010	0.008
		0.01	0.022	0.006	0.003	0.002	0.001	0.001
	L_n	0.05	0.183	0.120	0.085	0.064	0.052	0.047
		0.01	0.044	0.033	0.024	0.018	0.014	0.012
	T_n^*	0.05	0.145	0.132	0.101	0.088	0.085	0.081
		0.01	0.031	0.036	0.033	0.023	0.027	0.029

Table (7): Power values when the alternative distribution is Gamma ($\alpha = 2.5$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	*D*	0.05	0.090	0.033	0.020	0.016	0.013	0.012
		0.01	0.016	0.009	0.006	0.000	0.000	0.000
	W^2	0.05	0.086	0.030	0.021	0.016	0.014	0.012
		0.01	0.018	0.009	0.006	0.004	0.004	0.004
	A^2	0.05	0.236	0.106	0.051	0.030	0.020	0.015
		0.01	0.053	0.014	0.008	0.006	0.004	0.003
	U^2	0.05	0.086	0.030	0.021	0.016	0.014	0.012
		0.01	0.018	0.009	0.006	0.004	0.004	0.004
	L_n	0.05	0.262	0.185	0.135	0.098	0.072	0.053
		0.01	0.072	0.048	0.025	0.012	0.006	0.004
	T_n^*	0.05	0.220	0.221	0.230	0.230	0.232	0.235
		0.01	0.069	0.121	0.129	0.129	0.125	0.134
20	*D*	0.05	0.390	0.218	0.125	0.073	0.042	0.027
		0.01	0.073	0.009	0.003	0.001	0.001	0.001
	W^2	0.05	0.407	0.215	0.110	0.057	0.029	0.014
		0.01	0.077	0.005	0.002	0.001	0.001	0.001
	A^2	0.05	0.494	0.348	0.248	0.179	0.127	0.090
		0.01	0.255	0.087	0.031	0.013	0.005	0.002
	U^2	0.05	0.407	0.215	0.111	0.057	0.029	0.014
		0.01	0.076	0.005	0.002	0.001	0.001	0.001
	L_n	0.05	0.492	0.404	0.328	0.277	0.237	0.205
		0.01	0.168	0.158	0.120	0.089	0.064	0.046
	T_n^*	0.05	0.253	0.222	0.184	0.177	0.173	0.183
		0.01	0.093	0.080	0.085	0.088	0.096	0.104
30	*D*	0.05	0.601	0.442	0.322	0.246	0.192	0.162
		0.01	0.285	0.107	0.041	0.017	0.006	0.003
	W^2	0.05	0.630	0.462	0.340	0.249	0.185	0.148
		0.01	0.321	0.106	0.028	0.006	0.001	0.001
	A^2	0.05	0.671	0.543	0.448	0.364	0.303	0.264
		0.01	0.470	0.268	0.158	0.090	0.047	0.032
	U^2	0.05	0.630	0.462	0.340	0.248	0.185	0.148
		0.01	0.320	0.107	0.028	0.006	0.001	0.001
	L_n	0.05	0.668	0.580	0.506	0.444	0.395	0.372
		0.01	0.360	0.301	0.242	0.200	0.164	0.147
	T_n^*	0.05	0.567	0.554	0.517	0.506	0.501	0.499
		0.01	0.427	0.432	0.427	0.415	0.420	0.423

Table (8): Power values when the alternative distribution is Lognormal ($\lambda = 1$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	*D*	0.05	0.114	0.059	0.038	0.028	0.022	0.017
		0.01	0.027	0.012	0.006	0.000	0.000	0.000
	W^2	0.05	0.127	0.062	0.040	0.028	0.021	0.017
		0.01	0.032	0.012	0.006	0.004	0.004	0.004
	A^2	0.05	0.230	0.138	0.100	0.084	0.075	0.070
		0.01	0.101	0.070	0.057	0.018	0.013	0.012
	U^2	0.05	0.126	0.062	0.039	0.028	0.021	0.017
		0.01	0.032	0.012	0.006	0.004	0.004	0.004
	L_n	0.05	0.326	0.274	0.240	0.211	0.188	0.167
		0.01	0.189	0.161	0.128	0.098	0.074	0.058
	T_n^*	0.05	0.602	0.602	0.609	0.609	0.610	0.611
		0.01	0.500	0.542	0.548	0.548	0.546	0.552
20	*D*	0.05	0.222	0.126	0.082	0.057	0.041	0.033
		0.01	0.057	0.020	0.009	0.006	0.004	0.003
	W^2	0.05	0.246	0.140	0.088	0.059	0.043	0.033
		0.01	0.071	0.021	0.011	0.007	0.004	0.003
	A^2	0.05	0.341	0.236	0.176	0.140	0.119	0.103
		0.01	0.180	0.101	0.078	0.068	0.049	0.033
	U^2	0.05	0.245	0.140	0.088	0.059	0.043	0.033
		0.01	0.071	0.021	0.010	0.007	0.004	0.003
	L_n	0.05	0.450	0.405	0.371	0.348	0.329	0.315
		0.01	0.296	0.292	0.275	0.252	0.234	0.219
	T_n^*	0.05	0.403	0.357	0.305	0.292	0.287	0.303
		0.01	0.152	0.128	0.136	0.141	0.158	0.175
30	*D*	0.05	0.301	0.194	0.134	0.100	0.080	0.068
		0.01	0.117	0.048	0.025	0.013	0.008	0.006
	W^2	0.05	0.346	0.227	0.162	0.121	0.091	0.077
		0.01	0.152	0.063	0.029	0.014	0.009	0.007
	A^2	0.05	0.427	0.319	0.255	0.210	0.178	0.160
		0.01	0.268	0.162	0.113	0.088	0.074	0.065
	U^2	0.05	0.345	0.226	0.162	0.120	0.091	0.077
		0.01	0.152	0.063	0.029	0.013	0.009	0.007
	L_n	0.05	0.537	0.489	0.458	0.432	0.415	0.407
		0.01	0.402	0.379	0.360	0.346	0.332	0.323
	T_n^*	0.05	0.434	0.416	0.370	0.348	0.339	0.335
		0.01	0.213	0.224	0.215	0.186	0.196	0.202

Table (9): Power values when the alternative distribution is Weibull ($\alpha = 1.5$)

n	Test	γ	Value of α the shape parameter of the null distribution					
			0.5	1	1.5	2	2.5	3
10	*D*	0.05	0.442	0.3198	0.253	0.203	0.168	0.142
		0.01	0.198	0.103	0.058	0.000	0.000	0.000
	W^2	0.05	0.457	0.3204	0.244	0.194	0.159	0.134
		0.01	0.212	0.104	0.060	0.0402	0.0402	0.0402
	A^2	0.05	0.466	0.318	0.229	0.173	0.135	0.108
		0.01	0.233	0.105	0.056	0.035	0.023	0.016
	U^2	0.05	0.457	0.3204	0.244	0.194	0.159	0.135
		0.01	0.212	0.104	0.061	0.0405	0.0405	0.0405
	L_n	0.05	0.335	0.240	0.180	0.137	0.107	0.083
		0.01	0.108	0.077	0.049	0.030	0.019	0.013
	T_n^*	0.05	0.061	0.063	0.083	0.083	0.086	0.092
		0.01	0.001	0.001	0.001	0.001	0.001	0.001
20	*D*	0.05	0.696	0.568	0.481	0.418	0.370	0.333
		0.01	0.418	0.267	0.185	0.137	0.108	0.085
	W^2	0.05	0.714	0.573	0.482	0.413	0.361	0.317
		0.01	0.442	0.264	0.179	0.131	0.100	0.079
	A^2	0.05	0.707	0.556	0.453	0.374	0.311	0.265
		0.01	0.461	0.262	0.165	0.111	0.076	0.053
	U^2	0.05	0.713	0.573	0.482	0.413	0.361	0.318
		0.01	0.442	0.264	0.180	0.131	0.100	0.079
	L_n	0.05	0.577	0.454	0.352	0.285	0.234	0.193
		0.01	0.152	0.141	0.105	0.074	0.048	0.033
	T_n^*	0.05	0.466	0.409	0.343	0.330	0.324	0.342
		0.01	0.150	0.118	0.129	0.136	0.158	0.180
30	*D*	0.05	0.828	0.733	0.655	0.591	0.542	0.509
		0.01	0.624	0.448	0.343	0.276	0.227	0.193
	W^2	0.05	0.846	0.749	0.667	0.604	0.549	0.509
		0.01	0.654	0.458	0.336	0.257	0.210	0.188
	A^2	0.05	0.834	0.722	0.636	0.556	0.491	0.447
		0.01	0.654	0.451	0.316	0.223	0.164	0.135
	U^2	0.05	0.845	0.748	0.667	0.604	0.549	0.509
		0.01	0.653	0.458	0.336	0.257	0.210	0.188
	L_n	0.05	0.769	0.649	0.553	0.460	0.392	0.361
		0.01	0.345	0.263	0.196	0.145	0.106	0.086
	T_n^*	0.05	0.606	0.581	0.506	0.466	0.450	0.442
		0.01	0.253	0.269	0.257	0.211	0.227	0.236

When comparing the power values of the EDF tests with the ECF test, from tables (2) to (9), it can be noticed that:

1. When $n=10$, The ECF test is the most powerful test in most of the cases. Some exceptions happen as in the case for the Weibull ($\alpha=1.5$) alternative where it can be seen that the EDF tests are powerful than the ECF test.

2. When $n=20$,

   i. The ECF test is the most powerful test when the alternative distribution is gamma ($\alpha=1.5$). Also, it is the most powerful test when the alternative is the exponential ($\lambda=1$) except when the shape parameter of the null distribution $\alpha=0.5$ where the A-D test has the highest power value.

   ii. The L-S test is the most powerful test when the alternative distribution is lognormal. Also, it is the most powerful test when the alternative is the Chi-square ($\alpha=3$) distribution except when γ=0.01 and the shape parameter of the null distribution $\alpha=0.5$ where the A-D test has the highest power value.

   iii. For the Chi-square ($\alpha=1.5$) alternative, the ECF test is the most powerful when the shape parameter of the null distribution $\alpha=2$, 2.5 and 3. On the other hand, the L-S test is the most powerful test when the shape parameter of the null distribution $\alpha=0.5$, 1 and 1.5.

   iv. For the Chi-square ($\alpha=3$) alternative, exponential ($\lambda=2$) and Weibull ($\alpha=1.5$) alternatives, the EDF tests have higher power than the ECF test. Except for large values of the shape parameter value of the null distribution $\alpha=$2.5 and 3 where the ECF test has higher power.

3. When $n=30$,

   i. The ECF test is the most powerful test when the alternatives are the Chi-square ($\alpha=1.5$) and exponential $(\lambda=1)$.

   ii. The L-S test is the most powerful test when the alternative is the lognormal distribution.

   iii. For the Chi-square ($\alpha=3$) alternative; the L-S test is the most powerful except when $\gamma=0.01$ and the shape parameter of the null distribution is $\alpha=0.5$ where the A-D test is the most powerful.

   iv. For the exponential ($\lambda=2$) alternative, the following is noticed:

      a. At $\gamma=0.05$ and when the shape parameter of the null distribution is $\alpha=0.5$,1 and 1.5; the C-M test is the most powerful. In addition; when the shape parameter of the null distribution is $\alpha=$2 and 2.5 the K-S test is the most powerful.

b. At $\gamma = 0.01$ and when the shape parameter of the null distribution is $\alpha = 0.5$; the C-M test is the most powerful. In addition; when the shape parameter of the null distribution is $\alpha = 1$ the K-S test is the most powerful.

c. For the rest of the cases, the ECF test has the greatest power.

v. For the gamma $(\alpha = 1.5)$ alternative; the ECF test is the most powerful test in most of the cases except when the value of the shape parameter of the null distribution is $\alpha = 0.5$ where the L-S test is the most powerful at $\gamma = 0.05$, but, when $\gamma = 0.01$ the A-D test is the most powerful.

vi. For the gamma $(\alpha = 2.5)$ alternative; the ECF test is the most powerful test in most of the cases except when the value of the shape parameter of the null distribution is $\alpha = 0.5$ where the A-D test is the most powerful. Also, when $\gamma = 0.05$ and $\alpha = 1$ the L-S test is the most powerful.

vii. For the Weibull ($\alpha = 1.5$) alternative, it can be seen that the EDF tests are powerful than the ECF test. The only exception happens is when the value of the shape parameter of the null distribution is $\alpha = 3$, where the ECF test is the most powerful.

6. Conclusions

Generally it can be concluded that there is no one test that is optimal for all of the cases. It was shown that the most powerful test depends on the sample size, the alternative distribution, the significance level and the value of the shape parameter of the null distribution. Also, it is concluded that the ECF test is the best when the sample size is small ($n = 10$) except when the alternative distribution is Weibull ($\alpha = 1.5$) where the EDF tests are better.

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