A Two-Parameter Quasi Lindley Distribution in Acceptance Sampling Plans from Truncated Life Tests

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Abstract

In this paper, acceptance sampling plans are developed when the life test is truncated at a pre-assigned time. For different acceptance numbers, confidence levels and values of the ratio of the fixed experiment time to the specified average life time, the minimum sample sizes required to ensure the specified average life are calculate assuming that the life time variate of the test units follows a two-parameter Quasi Lindley distribution (*QLD*(2)). The operating characteristic function values of the new sampling plans and the corresponding producer's risk are presented.

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1 Introduction

During the Second World War the acceptance sampling plan is used in the US Military for testing the bullets. Lifetime is a substantial quality variable of a product. The acceptance sampling plans are used to locate the acceptability of a product unit, where the consumer can accept or reject the lot based on a random sample selected from the lot. However, the problem is to find the minimum sample size needful to assert a certain average life when the life test is terminated at a pre-assigned time, *t*, and when the observed number of failures overtake a given acceptance number, *c*. Therefore, the decision is to accept a lot if the given life can be established with a pre-determined high probability P^*, which support the consumer. To solve and compute the acceptance sampling parameters it is assumed the life time follow a specific model or distribution. Numerous parametric distributions are used in the analysis of lifetime data (Kantam et al, 2001; Al-Nasser and Al-Omari, 2013; Al-Omari et al, 2018a; Al-Omari et al, 2018b. Al-Nasser et al, 2018a; Al-Nasser et al, 2018b; Aslam et al, 2009 and Aslam et al , 2011).

In this article we suggest of using a two parameters quasi Lindley distribution *QLD*(2) (Shanker and Mishra, 2013) with pdf:

$$f(x;\alpha,\beta)=\frac{\beta(\alpha+\beta x)}{\alpha+1}e^{-\beta x}, \quad \text{if } x>0, \beta>0, \alpha>-1, \tag{1}$$

which is positively skewed distribution and the corresponding cumulative distribution function is given by

$$F(x;\alpha,\beta)=1-\frac{1+\alpha+\beta x}{\alpha+1}e^{-\beta x}, \quad \text{if } x>0, \beta>0, \alpha>-1. \tag{2}$$

The qth moment about origin of the QLD(2) is defined as

$$E\left(X^{q}\right)=\Gamma\left(q+1\right)\frac{\alpha+q+1}{\beta^{q}(\alpha+1)}, \quad q=1,2,3,..., \tag{3}$$

It is of interest to note here that when $\alpha=\beta$, the moments about origin of the *QLD*(2) will be the respective moments of the *QLD*(1) (Lindley, 1958; Ghitany et al., 2008). The mode of the QLD(2) is defined as $Mode=\frac{1-\alpha}{\beta}$, $|\alpha|<1$.The kurtosis of the *QLD*(2) is free of β and is defined as

$$\kappa=\frac{3\left(3\alpha^{4}+24\alpha^{3}+44\alpha^{2}+32\alpha+8\right)}{\left(\alpha^{2}+4\alpha+2\right)^{2}},$$

while the mean residual life and hazard rate functions, respectively are

$$m(x)=\frac{2+\alpha+\beta x}{\beta\left(1+\alpha+\beta x\right)} \text{ and } h(x)=\frac{\beta\left(\alpha+\beta x\right)}{1+\alpha+\beta x}.$$

The method of moment estimate of β is

$$\hat{\beta}=\left(\frac{\alpha+2}{\alpha+1}\right)\frac{1}{\bar{X}}.$$

For more details about the *QLD*(2) see Shanker and Mishra (2013).

The problem of acceptance sampling based on truncated life tests is considered by many authors, and there are Numerous articles are available on the single sampling plan based on the truncated life test for various statistical distributions; for example, Sobel and Tischendrof (1959) for exponential distribution, Al-Omari (2014) considered for three parameter kappa distribution Al-Nasser and Al-Omari (2013) investigated the problem of acceptance sampling plan in truncated life tests for exponentiated Fréchet distribution. Baklizi et al. (2005) considered the acceptance sampling plans based on truncated life tests Rayleigh model. Kantam et al. (2001) considered truncated life tests for log-logistic distribution. Al-Omari (2018a,b) proposed acceptance sampling plans for the Garima and transmuted generalized inverse Weibull distributions, respectively. Al-Omari and Al-Hadhrami (2018) suggested acceptance sampling plans based on truncated life tests for Extended Exponential distribution.

The rest of the paper is organized as follows. In Section 2, the suggested sampling plans, along with the operating characteristic function are presented. The results are explained and discussed by an example in Section 3. Our conclusions are summarized in Section 4.

2 Acceptance sampling plan

We assume that the lifetime t of the product follows a $QLD(2)$. The single sampling plan is consists of the following: (1) the number of units n, on test; (2) an acceptance number c, where if c or less failures happen during the test time, the lot is accepted; (3)the maximum test duration time, t; (4) a ratio t/μ_0, where μ_0 is the specified average life.

The producer's risk is the probability of rejecting the lot when $\mu \geq \mu_0$ (is fixed not to exceed $1-P^*$, i.e., the one for which the true average life is below the specified life μ_0) and the producer's risk is known as the probability of rejecting a good lot.

Consider that the lot size is sufficiently large to obtainthe probability of accepting a lot using the binomial distribution. Here, the problem is to determine the smallest sample size n required to satisfy the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \leq 1-P^*, \tag{9}$$

up to c for given values of $P^*\left(0<P^*<1\right)$, where $p=F(t;\mu_0)$ is the probability of a failure observed during the time t which depends only on the ratio t/μ_o; where the population of the QLD(2) distribution is given by:

$$\mu_1^{'}=\frac{1}{\beta}\left(\frac{\alpha+2}{\alpha+1}\right).$$

In Table 1, for $\alpha=1$, we provide the minimum values of the sample size, satisfying Inequality (9) with $P^*=0.75$, 0.9, 0.95, 0.99for $t/\mu_o=0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712. These choices are consistent with that of Gupta and Groll (1961), Kantam and Rosaiah (2001),and Baklizi et al., (2005); Al-Nasser and Gogah, (2017); Gogah and Al-Nasser, (2018); Al-Omari et al, (2016).

If the number of observed failures before the time t is less than or equal to the acceptance number c, then based on (12) we may have

$$F\left(t;\mu\right) \leq F\left(t;\mu_0\right) \quad \text{iff } \mu \geq \mu_0. \tag{10}$$

The probability $L(p)$ of acceptance the lot can be determined by the operating characteristic (OC) function of the sampling plan $\left(n,c,t/\mu_0\right)$ as

$$L(p)=P(\text{Accepting a lot})=\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}, \tag{11}$$

where $p=F(t;\mu)$ is considered as a function of μ (the lot quality parameter). The operating characteristic function values as a function of $\mu \geq \mu_0$ are given in Table 2 for $\alpha=1$.

The producer's risk is defined as the probability of rejecting the lot when $\mu > \mu_0$. Based on the considered sampling plan and a given value of the producer's risk, say 0.05, one may be interested in knowing what value of μ / μ_0 will ensure the producer's risk less than or equal to 0.05 if the sampling plan under study is adopted. The value of μ / μ_0 is the smallest positive number for which $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ satisfies the inequality

$$\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \geq 0.95. \tag{12}$$

For a given sampling plan $(n, c, t/\mu_0)$ at specified confidence level P^*, the minimum values of μ / μ_0 satisfying inequality (10) are summarized in Table 3 when $\alpha = 1$.

For example, for $\alpha = 1$ assume that the experimenter wants to establish the true unknown average life to be at least 1000 hours with confidence $P^* = 0.95$ and it is desired to stop the experiment at $t = 1571$ hours when the acceptance number $c = 2$. Then the required sample size n from Table 1 is $n = 6$. Now, the 6 units have to be put on test. If within 1000 hours no more than 2 failures out of 6 units are noticed, then the experimenter can assert with confidence of 0.95 that the average life is at least 1000 hours.

From Table 2, the operating characteristic values for the sampling plan $(n = 6, c = 2, t/\mu_0 = 1.571)$ are given by

μ/μ_0	2	4	6	8	10	12
OC	0.323651	0.777318	0.910457	0.956358	0.975719	0.985173

This means that if the true mean life is twice the specified mean life $(\mu/\mu_0 = 2)$, the producer’s risk is about 0.676349, and the producer’s risk is about 0.222682, 0.089543, 0.043642, 0.024281, 0.014827 for $\mu/\mu_0 = 4, 6, 8, 10, 12$, respectively.

From Table 3, we can get the value of μ/μ_0 for various choices of c, t/μ_0, such that the producer’s risk may not exceed 0.05. Thus in the above example, the value of μ/μ_0 is 7.587 for $c = 2$, $t/\mu_0 = 1.571$, and $P^* = 0.95$. This means that the product can have an average life of 7.587 times the specified average lifetime of 1000 hours in order that the product be accepted with probability at least 0.95. The actual average lifetime required to accept 95% of the lots is presented in Table 3.

Table 1: Minimum sample size *n* to be tested for time *t* in order to ensure with probability P^* and acceptance number *c* that μ / μ_0 when $\alpha = 1$ in *QLD*(2).

		t / μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
	1	6	4	3	3	2	2	2	2
	2	9	6	5	4	3	3	3	3
	3	11	8	6	6	5	4	4	4
	4	14	10	8	7	6	5	5	5
	5	17	12	10	8	7	6	6	6
	6	19	14	11	10	8	7	7	7
	7	22	16	13	11	9	8	8	8
	8	24	17	14	12	10	9	9	9
	9	27	19	16	14	11	11	10	10
	10	29	21	17	15	13	12	11	11
0.90	0	5	3	2	2	1	1	1	1
	1	8	5	4	4	3	2	2	2
	2	11	8	6	5	4	3	3	3
	3	14	10	8	7	5	5	4	4
	4	17	12	9	8	6	6	5	5
	5	20	14	11	9	8	7	6	6
	6	23	16	13	11	9	8	7	7
	7	25	18	14	12	10	9	8	8
	8	28	20	16	14	11	10	9	9
	9	31	22	17	15	12	11	11	10
	10	34	24	19	16	13	12	12	11
0.95	0	6	4	3	2	2	1	1	1
	1	10	6	5	4	3	3	2	2
	2	13	9	7	6	4	4	3	3
	3	16	11	9	7	6	5	4	4
	4	19	13	10	9	7	6	6	5
	5	22	15	12	10	8	7	7	6
	6	25	17	14	12	9	8	8	7
	7	28	19	15	13	10	9	9	8
	8	31	21	17	15	12	10	10	9
	9	34	23	19	16	13	11	11	10
	10	36	25	20	17	14	12	12	11
0.99	0	9	6	4	3	2	2	2	1
	1	13	9	7	5	4	3	3	2
	2	17	11	9	7	5	4	4	4
	3	20	14	10	9	6	5	5	5
	4	24	16	12	10	8	7	6	6
	5	27	18	14	12	9	8	7	7
	6	30	20	16	13	10	9	8	8
	7	33	23	18	15	11	10	9	9
	8	36	25	19	16	13	11	10	10
	9	39	27	21	18	14	12	11	11
	10	42	29	23	19	15	13	12	12

Table 2: Operating characteristic values for the sampling plan $(n, c, t/\sigma_0)$ for a given probability P^* with acceptance number *c* = 2 when $\alpha = 1$ in *QLD*(2).

			σ/σ_0					
P^*	n	t/σ_0	2	4	6	8	10	12
0.75	9	0.628	0.660516	0.922079	0.971936	0.986976	0.992945	0.995762
	6	0.942	0.682325	0.930897	0.975760	0.988922	0.994059	0.996456
	5	1.257	0.635269	0.917613	0.970929	0.986704	0.992872	0.995751
	4	1.571	0.671066	0.929433	0.975690	0.989037	0.994177	0.996552
	3	2.356	0.688039	0.934024	0.977644	0.990050	0.994768	0.996926
	3	3.141	0.500400	0.866791	0.951391	0.977651	0.988046	0.992905
	3	3.927	0.341571	0.782024	0.913979	0.959000	0.977639	0.986573
	3	4.712	0.223248	0.688039	0.866758	0.934024	0.963248	0.977644
0.90	11	0.628	0.52322	0.871503	0.950981	0.976577	0.987087	0.992152
	8	0.942	0.470641	0.852008	0.943248	0.972872	0.985053	0.990924
	6	1.257	0.488847	0.862699	0.948493	0.975709	0.986736	0.991997
	5	1.571	0.479681	0.860197	0.947888	0.975557	0.986710	0.992007
	4	2.356	0.386885	0.816073	0.929471	0.966619	0.981795	0.989043
	3	3.141	0.500400	0.866791	0.951391	0.977651	0.988046	0.992905
	3	3.927	0.341571	0.782024	0.913979	0.959000	0.977639	0.986573
	3	4.712	0.223248	0.688039	0.866758	0.934024	0.963248	0.977644
0.95	13	0.628	0.400847	0.812491	0.924360	0.962807	0.979136	0.987171
	9	0.942	0.379600	0.805377	0.922079	0.961925	0.978742	0.986976
	7	1.257	0.362957	0.799170	0.920053	0.961143	0.978396	0.986807
	6	1.571	0.323651	0.777318	0.910457	0.956358	0.975719	0.985173
	4	2.356	0.386885	0.816073	0.929471	0.966619	0.981795	0.989043
	4	3.141	0.190881	0.671258	0.858784	0.929490	0.960384	0.975711
	3	3.927	0.341571	0.782024	0.913979	0.959000	0.977639	0.986573
	3	4.712	0.223248	0.688039	0.866758	0.934024	0.963248	0.977644
0.99	17	0.628	0.218112	0.681267	0.856972	0.925646	0.956847	0.972852
	11	0.942	0.236228	0.704638	0.871503	0.934452	0.962433	0.976577
	9	1.257	0.184732	0.660038	0.848489	0.921930	0.955028	0.971878
	7	1.571	0.209166	0.688027	0.865103	0.931749	0.961159	0.975921
	5	2.356	0.193071	0.675493	0.860265	0.929837	0.960355	0.975571
	4	3.141	0.190881	0.671258	0.858784	0.92949	0.960384	0.975711
	4	3.927	0.084719	0.521646	0.769834	0.878413	0.929456	0.955866
	4	4.712	0.035049	0.386885	0.671194	0.816073	0.889649	0.929471

Table 3: Minimum ratio of μ / μ_0 for the acceptability of a lot with producer's risk of 0.05 when $\alpha = 1$ in *QLD*(2).

		t / μ_0							
*P**	*c*	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	27.779	27.891	37.218	23.528	35.284	47.041	58.812 7	0.568
	1	7.476	7.196	6.890	8.611	7.680	10.239	12.801 1	5.360
	2	4.791	4.550	4.879	4.577	4.453	5.936	7.422	8.905
	3	3.450	3.585	3.334	4.167	4.835	4.411	5.515	6.617
	4	3.042	3.088	3.111	3.239	3.841	3.634	4.543	5.451
	5	2.786	2.784	2.951	2.697	3.253	3.161	3.952	4.741
	6	2.462	2.578	2.520	2.753	2.864	2.840	3.551	4.260
	7	2.356	2.428	2.48	2.440	2.586	2.607	3.259	3.911
	8	2.170	2.155	2.222	2.208	2.377	2.429	3.037	3.644
	9	2.117	2.086	2.218	2.283	2.214	2.952	2.861	3.433
	10	1.994	2.029	2.041	2.114	2.470	2.777	2.717	3.260
0.90	0	46.146	41.668	37.218	46.514	35.284	47.041	58.812 7	0.568
	1	10.140	9.209	9.602	12.001	12.913	10.239	12.801 1	5.360
	2	5.953	6.311	6.071	6.098	6.864	5.936	7.422	8.905
	3	4.497	4.647	4.783	5.080	4.835	6.446	5.515	6.617
	4	3.771	3.829	3.619	3.888	3.841	5.121	4.543	5.451
	5	3.337	3.346	3.335	3.200	4.045	4.337	3.952	4.741
	6	3.047	3.027	3.137	3.149	3.514	3.818	3.551	4.260
	7	2.719	2.800	2.736	2.773	3.139	3.448	3.259	3.911
	8	2.582	2.631	2.660	2.777	2.86	3.169	3.037	3.644
	9	2.474	2.499	2.409	2.530	2.643	2.952	3.690	3.433
	10	2.386	2.393	2.378	2.335	2.47	2.777	3.471	3.260
0.95	0	55.329	55.443	55.602	46.514	69.757	47.041	58.812 7	0.568
	1	12.798	11.214	12.289	12.001	12.913	17.215	12.801 1	5.360
	2	7.112	7.186	7.250	7.587	6.864	9.151	7.422	8.905
	3	5.193	5.174	5.495	5.080	6.249	6.446	5.515	6.617
	4	4.255	4.197	4.120	4.523	4.856	5.121	6.403	5.451
	5	3.702	3.625	3.715	3.688	4.045	4.337	5.422	4.741
	6	3.338	3.250	3.440	3.538	3.514	3.818	4.773	4.260
	7	3.081	2.985	2.990	3.099	3.139	3.448	4.310	3.911
	8	2.889	2.788	2.875	3.053	3.310	3.169	3.962	3.644
	9	2.740	2.635	2.783	2.772	3.042	2.952	3.690	3.433
	10	2.542	2.514	2.544	2.551	2.828	2.777	3.471	3.260
0.99	0	82.877	82.993	73.983	69.491	69.757	92.999	116.27	70.568
	1	16.781	17.204	17.631	15.358	17.997	17.215	21.523	15.36
	2	9.425	8.929	9.589	9.061	9.145	9.151	11.440	13.727
	3	6.580	6.746	6.201	6.867	6.249	6.446	8.059	9.670
	4	5.460	5.293	5.110	5.150	5.831	6.474	6.403	7.682
	5	4.612	4.455	4.465	4.643	4.798	5.392	5.422	6.506
	6	4.064	3.912	4.039	3.920	4.128	4.684	4.773	5.727
	7	3.681	3.716	3.737	3.736	3.658	4.185	4.310	5.172
	8	3.398	3.410	3.300	3.325	3.743	3.812	3.962	4.754
	9	3.181	3.175	3.152	3.246	3.424	3.524	3.690	4.428
	10	3.009	2.990	3.033	2.972	3.170	3.293	3.471	4.165

5. Conclusion

In this paper, acceptance sampling plans based on truncated lifetime tests are developed when the lifetime tests follow a two parameter Quasi Lindley distribution. The minimum sample size needed to assert a certain mean life of the test units, the OC values of the sampling plan, along with the minimum ratio to the specified mean life for accepting a lot with assured producer's risk are calculated for the Quasi Lindley distribution QLD(2) when $\alpha = 1$.The results obtained in this paper encourage the practitioners to use the suggested sampling plans. However, the suggested sampling plan discussed in this paper can be extended by considering different types of the sampling plans such as the group acceptance sampling plan (Rao, 2009). Other extension can be made by considering repetitive sampling plan (Sherman, 1965).

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