A Control Chart Based on Two-piece Normal Distribution Using Repetitive Sampling

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Abstract

In this manuscript, a control chart is designed for two-piece normal distribution using repetitive sampling. The necessary measures to determine the average run lengths for in control and out-of-control process are given. The average run lengths are presented for various specified parameters and shift constants. The efficiency of the proposed chart is compared with the existing control chart using single sampling. The application of the proposed chart is given with the help of an example.

Key words

Two-piece normal distribution; control chart; repetitive sampling; average run length

1. Introduction

Statistical analysis plays an important role in the production of high quality product. Among them, a control chart is a useful tool to monitor the manufacturing process. It is used to indicate when the process is going to be out-of-control. Usually, a control chart is based on two natural limits called upper control limit (UCL) and lower control limit (LCL). The process beyond these limits is called out-of-control process. The manufacturing process can be shifted due to some controllable and uncontrollable factors. Due to the shift, the process can be away from the given specifications limit and results in non-conforming products. A quick indication about the out-of-control state helps to minimize the rework and non-conforming product.

Usually, control charts are developed under the assumption that the quality of interest follows the normal distribution. In practice, it is not true that the quality of interest always follows the normal distribution. In this situation, the use of a control chart based on normal distribution assumption increases the false alarms. So, it is necessary to develop control charts designed for non-normal distributions. The details about non-normal control chart can be seen in Santiago and Smith.(2013), Amin et al., (1995) Bai and Choi (1995),Chang and Bai (2013), Al-Oraini et al.,(2002),Riaz et al.,(2014), McCracken, and Chakraborti (2013). The two-piece normal distribution is widely used when data is not symmetric. According to Britton and Fisher (1998) the two-piece normal distribution is used for this type of data. Kimber and Jeynes (1987) used this distribution in measurement of depths of arsenic implants in silicon. Simionescu (2014) used for fan chart to assess uncertainty.

By exploring the literature, we note that there is no work on designing a control chart for two-piece normal distribution. In this paper, we focus on the development of control charts for two-piece normal distribution using single and repetitive sampling. We will present the structure of proposed chart and compared the efficiency of the proposed charts. A simulation study is given for illustration of the proposed chart.

2. Design of Proposed Chart

A random variable, Z, has a two-piece normal (TPN) distribution with parameters (μ, σ_1, σ_2) if it has the probability density function of

$$f(z) = \begin{cases} A\, e^{-\frac{(z-\mu)^2}{2\sigma_1^2}} & ; z \leq \mu \\ A\, e^{-\frac{(z-\mu)^2}{2\sigma_2^2}} & ; z \geq \mu \end{cases} \tag{1}$$

Where $A = \left[\sqrt{2\pi}\left(\frac{\sigma_1+\sigma_2}{2}\right)\right]^{-1}$ John (1982) shows the following relations for average and variance

$$\mu_z = \mu + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} \tag{2}$$

$$\sigma_z^2 = \left(1 - \tfrac{2}{\pi}\right)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2. \tag{3}$$

In the case of equal standard deviations ($\sigma_1 = \sigma_2$) it will be a classical normal distribution, which is symmetric.

In statistical quality control (SQC) most of the commonly used techniques like, control charts and acceptance sampling plans, are frequently carried out under the assumption of normal data, which seldom holds in practice. The several data sets from various areas of application, such as, quality control data from industries, reliability, telecommunications, environment, climatology and finance are observed that this type of data usually display moderate to strong asymmetry as well as light to heavy tails. Hence, even though the simplicity and popularity of the normal distribution, we wind up that in most of the cases, fitting a normal distribution to the data is not the best option. In other words, it is always a very difficult task due to some unmanageable disturbance factors for modeling real data sets, due to some potential asymmetric models for the underlying data distribution. Recently, Figueiredo and Gomes (2013) provided some information about the family of skew-normal distributions in statistical quality control. They presented bootstrap control charts for skew-normal processes and some simulation results about their performance.

We propose the following Z-chart using repetitive sampling based on the statistic Z by following Aslam et al (2014) and Lee et al., (2014)

Step 1: Select an item randomly at each subgroup and measure its quality characteristic Z.
Step 2: Declare the process as out-of-control if $Z \geq UCL_1 or\ Z \leq LCL_1$. Declare the process as in-control if $LCL_2 \leq Z \leq UCL_2$. Otherwise, go to Step 1 and repeat the process.
The operational procedure of proposed chart is based on four control limits and two control chart coefficients. The limits LCL_1 and UCL_1 are called the inner control limits and the limits LCL_2 and UCL_2 are called the outer control limits. As mentioned by Aslam et al.,(2014)"the proposed control chart does not make a conclusion on the process state if the statistic lies between the inner and outer control limits, in which case repetitive sampling is required.

Let us assume that the outer control limits for the proposed control chart are symmetrically given by

$$\mathrm{LCL}_1 = \mu_z - k_1\sigma_z = \mu_0 + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} - k_1\sqrt{\left(1-\frac{2}{\pi}\right)(\sigma_2-\sigma_1)^2 + \sigma_1\sigma_2} \quad (4)$$

$$\mathrm{UCL}_1 = \mu_z + k_1\sigma_z = \mu_0 + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} + k_1\sqrt{\left(1-\frac{2}{\pi}\right)(\sigma_2-\sigma_1)^2 + \sigma_1\sigma_2}. \quad (5)$$

where μ_0 is the parameter $\mu$when the process is in control. Also, the inner control limits for the proposed control chart are designed by

$$\mathrm{LCL}_2 = \mu_z - k_2\sigma_z = \mu_0 + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} - k_2\sqrt{\left(1-\frac{2}{\pi}\right)(\sigma_2-\sigma_1)^2 + \sigma_1\sigma_2} \quad (6)$$

$$\mathrm{UCL}_2 = \mu_z + k_2\sigma_z = \mu_0 + (\sigma_2 - \sigma_1)\sqrt{\frac{2}{\pi}} + k_2\sqrt{\left(1-\frac{2}{\pi}\right)(\sigma_2-\sigma_1)^2 + \sigma_1\sigma_2}. \quad (7)$$

Note that k_1 and k_2 control limits coefficients and will be determined through simulation later.

For the proposed control chart, the process is declared to be out-of-control if $Z \geq UCL_1 or Z \leq LCL_1$. According to the parameterization by Banerjee and Das (2014), for single sampling, the probability that process is declare out of control when actually in control is denoted by $P_{out,1}^0$ and is given by

$$P_{out,1}^0 = P\{Z \leq LCL_1 | \mu = \mu_0\} + P\{Z \geq UCL_1 | \mu = \mu_0\}$$
$$= 1 - P\{LCL_1 < Z < UCL_1 | \mu = \mu_0\}$$
$$= 1 - \frac{2\sigma_1}{\sigma_1+\sigma_2}\left[\sigma_2\Phi\left(\frac{UCL_1-\mu_0}{\sigma_2}\right) - \sigma_1\Phi\left(\frac{LCL_1-\mu_0}{\sigma_1}\right) + \frac{\sigma_1-\sigma_2}{2}\right] \quad (8)$$

The probability of repetition $\left(P_{rep}^0\right)$ for the proposed control chart is given as follows

$$P_{rep}^0 = P\{UCL_2 \leq Z \leq UCL_1 | \mu = \mu_0\} + P\{LCL_1 \leq Z \leq LCL_2 | \mu = \mu_0\}$$
$$= \frac{2\sigma_1}{\sigma_1+\sigma_2}\left[\varphi\left(\frac{UCL_1-\mu_0}{\sigma_2}\right) - \Phi\left(\frac{UCL_2-\mu_0}{\sigma_2}\right) + \Phi\left(\frac{LCL_2-\mu_0}{\sigma_1}\right) - \Phi\left(\frac{LCL_1-\mu_0}{\sigma_1}\right)\right] \quad (9)$$

The probability that the process is out of control when actually under the repetitive sampling. it is in control is denoted by P^0_{out} and is defined as follows

$$P^0_{out} = \frac{P^0_{out,1}}{1-P^0_{rep}}. \quad (10)$$

The performance of proposed chart will be measured using the average run length (ARL) criteria. The ARL is used to indicate when on the average process will be out-of-control. The ARL when the process is in control is denoted by ARL_0 and defined as follows

$$ARL_0 = \frac{1}{P^0_{out}}. \quad (11)$$

Suppose now that the process parameter $\mu$is shifted to μ_1. Then, the probability of the process being declared to be out-of-controlbased on the single sample when the process is shifted is

$$P^1_{out,1} = P\{z \le LCL_1 | \mu = \mu_1\} + P\{z \ge UCL_1 | \mu = \mu_1\}$$
$$= 1 - P\{LCL_1 < z < UCL_1 | \mu = \mu_1\}$$
$$= 1 - \frac{2\sigma_1}{\sigma_1+\sigma_2}\left[\sigma_2\varphi\left(\frac{UCL_1-\mu_1}{\sigma_2}\right) - \sigma_1\varphi\left(\frac{LCL_1-\mu_1}{\sigma_1}\right) + \frac{\sigma_1-\sigma_2}{2}\right] \quad (12)$$

Similarly, the probability of resampling for shifted process is given as follows

$$P^1_{rep} = P\{UCL_2 \le z \le UCL_1 | \mu = \mu_1\} + P\{LCL_1 \le z \le LCL_2 | \mu = \mu_1\}$$
$$= \frac{2\sigma_1}{\sigma_1+\sigma_2}\left[\varphi\left(\frac{UCL_1-\mu_1}{\sigma_2}\right) - \varphi\left(\frac{UCL_2-\mu_1}{\sigma_2}\right) + \varphi\left(\frac{LCL_2-\mu_1}{\sigma_1}\right) - \varphi\left(\frac{LCL_1-\mu_1}{\sigma_1}\right)\right] \quad (13)$$

So, the probability of the process being declared to be out of control (P^1_{out}) for the proposed control chart when the process is shifted is given as follows:

$$P^1_{out} = \frac{P^1_{out,1}}{1-P^1_{rep}}. \quad (14)$$

The out of control ARL for the shifted process is given as follows:

$$ARL_1 = \frac{1}{P^1_{out}}. \quad (15)$$

According to Aslam et al.,(2014) "the proposed control chart requires re-sampling when the decision has not been made from the previous sample. The average sample size (ASS) is the expected number of resampling (or sample size) required until the final decision is made, and it can be used as one of the performance measures". The average sample size for the in-control process (ASS_0) and the shifted process (ASS_1) is respectively given by

$$ASS_0 = \frac{1}{1-P^0_{rep}} \quad (16)$$

$$ASS_1 = \frac{1}{1-P^1_{rep}}. \quad (17)$$

Let r_0 be the target in-control ARL. The following algorithm will be used to determine the ARLs values.

1. Select the initial values of k_1 and k_2.
2. Generate a random variable at each subgroup from the two-piece normal distribution having the specified parameters for in-control process, that is, μ_0=0 and different values of $\sigma_1 < \sigma_2$ and $\sigma_1 > \sigma_2$.
3. Adopt the follow the procedure of the proposed control chart and check if the process is declared as out-of-control. If the process is declared as in-control, go to Step 4. If

the process is declared as out-of-control, record the number of subgroups so far as the in-control run length.

4. Repeat Steps 1-3 a sufficient number (10,000 say) of times to calculate the in-control ARL. If the in-control ARL is equal to the specified ARL0(sayr_0), so as minimize ASS_0 such that $ARL_0 \geq r_0$then go to Step 5 with the current values of k_1 and k_2. Otherwise, modify the values of k_1 and k_2 andrepeat Steps 2-4.
5. Then using Eq. (15), we obtain ARL_1 based on the determined values of k_1 and k_2 for various shift values of $\mu_1 = \mu_0 + \delta\sigma_1(\sigma_1 < \sigma_2)$.

Table 1shows the ARLs according to the shift value$\delta = 0.0\ to\ 2.0$ with the interval of 0.1 whenr_0 is 200 or 250, while Table 2reports on the ARLs when r_0 is 300 or 370.Whereas, Tables 3 and 4 obtained the ARL_1 values based on the determined values of k_1 and k_2 for various shift values of $\mu_1 = \mu_0 - \delta\sigma_1(\sigma_1 > \sigma_2)$ according to the shift value $\delta = 0.0$ to 2.0 with the interval of 0.1. From Tables 1, 2, 3 and 4, we note the following behavior of ARL_1.

1. The case of $\mu_1 = \mu_0 = 0$, corresponds to the in-control ARL, which is obtained very close to the target r_0 values.
2. As the shift δ increases (the process mean increases), the out-of-control ARLs decrease rapidly when $\sigma_1 < \sigma_2$. Similar trend can be observed from Tables 3 and 4 when $\sigma_1 > \sigma_2$ whereas decreasing speed seems to get faster in the later case.
3. The decreasing speed in ARLs seems to get faster as r_0 increases. The average sample size for the out of control process (ASS_1) increases as shift δ increases in both case ($\sigma_1 < \sigma_2$ or $\sigma_1 > \sigma_2$) .
4. The control constant k_1 increases as r_0 increases, while $k_2$not follows this tendency according to different r_0's when $\sigma_1 < \sigma_2$whereas control constant k_2 increases as r_0 increases in case of $\sigma_1 > \sigma_2$, while k_1 not follows this tendency according to different r_0's.

Table 1: Average run lengths for proposed control chart when r_0 is 200 or 250 (when$\sigma_1 < \sigma_2$).

δ	r_0=200 k1=3.0456 k2=0.7307		r_0=250 k1=3.0859 k2=0.8724	
	ARL1	ASS1	ARL1	ASS1
0.0	200.00	1.639	250.00	1.467
0.1	154.01	1.625	191.04	1.458
0.2	116.91	1.620	144.06	1.456
0.3	88.24	1.624	108.11	1.462
0.4	66.54	1.636	81.09	1.475
0.5	50.27	1.656	60.95	1.496
0.6	38.10	1.683	45.97	1.523
0.7	29.00	1.718	34.82	1.558
0.8	22.19	1.760	26.51	1.598
0.9	17.08	1.807	20.30	1.645
1.0	13.24	1.859	15.65	1.697

1.1	10.34	1.915	12.15	1.752
1.2	8.15	1.971	9.51	1.811
1.3	6.48	2.027	7.52	1.870
1.4	5.22	2.079	6.00	1.927
1.5	4.25	2.124	4.85	1.980
1.6	3.51	2.159	3.97	2.026
1.7	2.94	2.182	3.29	2.061
1.8	2.50	2.189	2.77	2.084
1.9	2.16	2.181	2.37	2.092
2.0	1.89	2.156	2.05	2.084

Table 2. Average run lengths for proposed control chart when r_0 is 300 or 370 (when $\sigma_1 < \sigma_2$).

δ	r_0=300 k1=3.1740 k2=0.7856		r_0=370 k1=3.2587 k2=0.7474	
	ARL1	ASS1	ARL1	ASS1
0.0	300.00	1.567	370.00	1.618
0.1	226.73	1.555	276.65	1.606
0.2	169.46	1.552	204.96	1.602
0.3	126.18	1.558	151.44	1.607
0.4	93.97	1.571	111.98	1.620
0.5	70.14	1.592	83.02	1.642
0.6	52.54	1.621	61.76	1.672
0.7	39.53	1.658	46.15	1.710
0.8	29.89	1.701	34.66	1.756
0.9	22.73	1.752	26.17	1.809
1.0	17.39	1.808	19.89	1.868
1.1	13.41	1.868	15.22	1.933
1.2	10.42	1.932	11.75	2.001
1.3	8.18	1.996	9.15	2.070
1.4	6.49	2.059	7.20	2.138
1.5	5.20	2.116	5.73	2.201
1.6	4.23	2.166	4.63	2.256
1.7	3.49	2.204	3.78	2.300
1.8	2.92	2.228	3.14	2.328
1.9	2.48	2.235	2.65	2.339
2.0	2.14	2.225	2.27	2.331

Table 3. Average run lengths for proposed control chart when r_0 is 200 or 250 (when $\sigma_1 > \sigma_2$).

δ	r_0=200 k1=3.5179 k2=0.7071		r_0=250 k1=3.5131 k2=0.7732	
	ARL1	ASS1	ARL1	ASS1
0.0	200.00	3.911	250.00	3.090
0.1	151.81	3.695	187.45	2.956
0.2	113.94	3.562	139.41	2.877
0.3	84.68	3.499	102.96	2.845
0.4	62.38	3.499	75.54	2.858
0.5	45.57	3.562	55.08	2.916
0.6	33.00	3.691	39.91	3.021
0.7	23.69	3.894	28.72	3.180
0.8	16.85	4.184	20.53	3.401
0.9	11.85	4.586	14.55	3.700
1.0	8.24	5.133	10.23	4.097
1.1	5.65	5.877	7.12	4.622
1.2	3.81	6.901	4.90	5.318
1.3	2.52	8.331	3.34	6.246
1.4	1.63	10.369	2.24	7.495
1.5	1.03	13.333	1.49	9.178
1.6	0.64	17.689	0.98	11.423
1.7	0.39	23.941	0.65	14.299
1.8	0.24	31.900	0.44	17.609
1.9	0.17	38.782	0.32	20.556
2.0	0.14	39.254	0.25	21.779

Table 4. Average run lengths for proposed control chart when r_0 is 300 or 370 (when $\sigma_1 > \sigma_2$).

δ	r_0=300 k1=3.5491 k2=0.8033		r_0=370 k1=3.4795 k2=0.9874	
	ARL1	ASS1	ARL1	ASS1
0.0	300.00	2.829	370.00	1.914
0.1	223.32	2.718	271.81	1.873
0.2	165.08	2.655	199.10	1.855
0.3	121.28	2.632	145.49	1.859
0.4	88.59	2.649	106.08	1.884

0.5	64.35	2.706	77.18	1.930
0.6	46.48	2.805	56.03	2.001
0.7	33.37	2.953	40.58	2.098
0.8	23.80	3.156	29.31	2.225
0.9	16.86	3.429	21.12	2.388
1.0	11.84	3.790	15.17	2.594
1.1	8.25	4.264	10.86	2.852
1.2	5.68	4.891	7.75	3.174
1.3	3.87	5.724	5.51	3.573
1.4	2.60	6.842	3.92	4.066
1.5	1.73	8.348	2.78	4.666
1.6	1.14	10.373	1.98	5.382
1.7	0.75	13.019	1.43	6.202
1.8	0.50	16.216	1.04	7.078
1.9	0.35	19.405	0.79	7.901
2.0	0.27	21.353	0.62	8.510

3. Illustrative Examples

3.1 Real data

We use a real data set about fracture toughness from the material Sialon ($Si_{6-x}Al_xO_xN_{8-x}$). The data gives Fracture toughness for Sialon ($Si_{6-x}Al_xO_xN_{8-x}$) material (in the units of MPa $m^{1/2}$): 3.05, 2.9, 2.75, 2.7, 2.65, 3.15, 3.75, 3.8, 3.72, 3.52, 3.44, 3.26, 2.99, 2.79, 3, 3.18, 3.66, 3.2, 3.29, 3.5, 3.1, 3.65, 3.42, 3.38, 3.29. The data was obtained from Nadarajah and Kotz (2007). According to Nadarajah and Kotz (2007), the data is well fitted to Neville and Kennedy's distribution, Burr type XII distribution and the Burr type III distribution. We have estimated the parameters using method of maximum likelihood suggested by John (1982). The estimates for the given data are $\hat{\mu} = 3.290, \hat{\sigma}_1 = 0.3605385\ and \hat{\sigma}_2 = 0.3052052$.We used the Kolmogorov-Smirnov (K-S) test for each data sets to the fitted model. It is observed that for data set, K-S distance is 0.0908 with the corresponding p value is 0.9861. It indicates that the two-piece normal distribution provides reasonable fit for given data set and it show QQ plot in Figure 1. For constructing the proposed control chart using this data, consider the target ARL0 (r_0) be 370. Thus from Table 2, we have k_1 =3.2587 and k_2 = 0.7474. The outer and inner control limits are given by

LCL1 = 2.159423
UCL1 = 4.332277
LCL2 = 2.996672
UCL2 = 3.495028

The control chart as well as the outer and inner control limits for fracture toughness of Sialon material data constructed in figure 2. This figure shows that the process is in control.

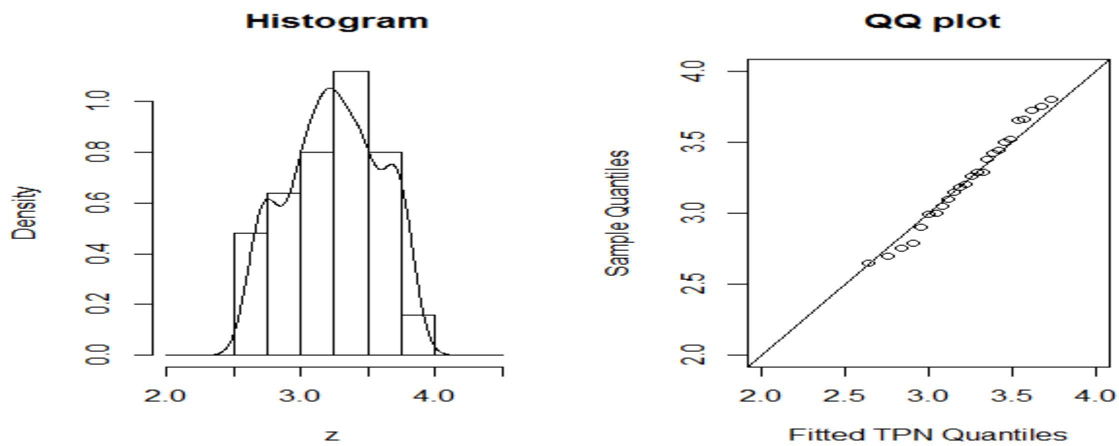

Figure 1.Fitted density (left picture) and QQ plot (right picture) for Fracture Toughness data

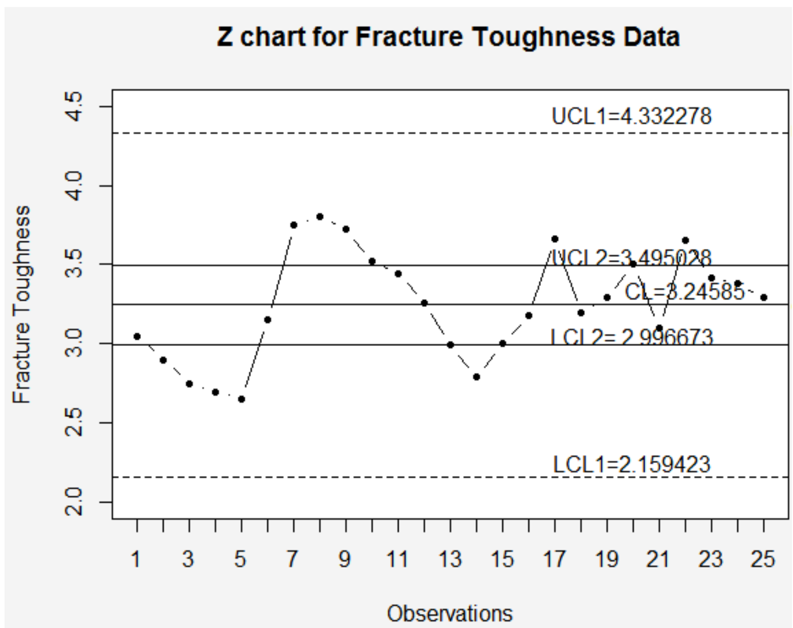

Figure 2. Proposed control chart for Fracture Toughness data

3.2. Simulated data

In this section, we compare proposed control chart with existing chart using simulated data. For this purpose, the first 28 observations are generated from two-piece normal distribution (TPN) with parameters μ=0, σ_1=1 and σ_2=1.5 (i.e. the in-control situation) and the second set of the 28 observations from two-piece normal distribution with parameters μ=1.0, σ_1=1 and σ_2=1.5 (i.e. out-of-control situation having a shift of δ =1.0.To fix the ARL_0 at 370, we have used k_1=3.2587and k_2=0.7474 for the proposed chart from Table 2 and k=3.0891 for existing chart from Table 5. The graphical display of the proposed Z-control chart is presented in the Figure3. Then, again generate 28 values from two-piece normal distribution from in control and 28 from out of control process (with same shift as in proposed chart, δ =1.0). Figure 4 shows the existing chart using single sampling.

Table 5: Comparison of ARLs when r_0 is 300 or 370.

δ	r_0= 300				r_0= 370			
	Proposed using Single sampling k=3.0137		Proposed with k1=3.1740 k2=0.7856		Proposed using Single sampling k=3.0891		Proposed with k1=3.2587 k2=0.7474	
	ARL1	ASS1	ARL1	ASS1	ARL1	ASS1	ARL1	ASS1
0.0	300.03	1.00	300.00	1.57	370.06	1.00	370.00	1.62
0.1	230.19	1.00	226.73	1.56	281.88	1.00	276.65	1.61
0.2	174.86	1.01	169.46	1.55	211.52	1.00	204.96	1.60
0.3	132.70	1.01	126.18	1.56	160.32	1.01	151.44	1.61
0.4	101.08	1.01	93.97	1.57	121.06	1.01	111.98	1.62
0.5	77.49	1.01	70.14	1.59	92.84	1.01	83.02	1.64
0.6	59.86	1.02	52.54	1.62	71.52	1.01	61.76	1.67
0.7	46.62	1.02	39.53	1.66	55.60	1.02	46.15	1.71
0.8	36.62	1.03	29.89	1.70	43.65	1.02	34.66	1.76
0.9	29.02	1.04	22.73	1.75	34.60	1.03	26.17	1.81
1.0	23.19	1.05	17.39	1.81	27.71	1.04	19.89	1.87
1.1	18.70	1.06	13.41	1.87	22.42	1.05	15.22	1.93
1.2	15.21	1.07	10.42	1.93	18.32	1.06	11.75	2.00
1.3	12.47	1.09	8.18	2.00	14.93	1.08	9.15	2.07
1.4	10.32	1.11	6.49	2.06	12.26	1.09	7.20	2.14
1.5	8.61	1.13	5.20	2.12	10.16	1.12	5.73	2.20
1.6	7.24	1.16	4.23	2.17	9.07	1.14	4.63	2.26
1.7	6.14	1.19	3.49	2.20	7.81	1.17	3.78	2.30
1.8	5.25	1.24	2.92	2.23	6.80	1.21	3.14	2.33
1.9	4.53	1.28	2.48	2.24	5.20	1.25	2.65	2.34
2.0	3.93	1.34	2.14	2.23	4.97	1.30	2.27	2.33

From Figure 3, we can see that the proposed chart gives out-of-control signals at samples 48, thus giving a one out-of-control signal. Figure 4depicts that the existing single sample classical control chart gives no out-of-control signal. It exemplifies the ability of the proposed chart to quickly detect small shifts in the process. We see that the proposed chart detects the shift more quickly than the existing chart and the number of signals given by the proposed chart is also greater than the existing chart.

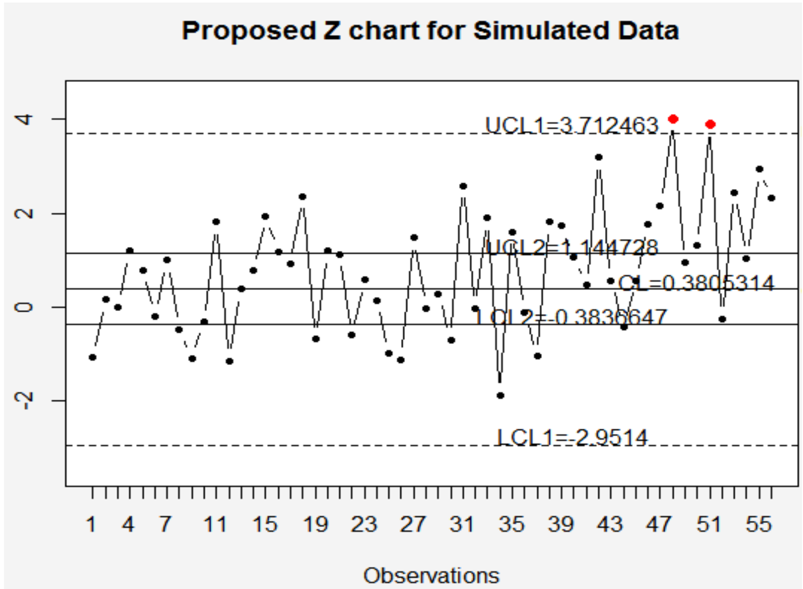

Figure 3. Proposed control chart for simulated data

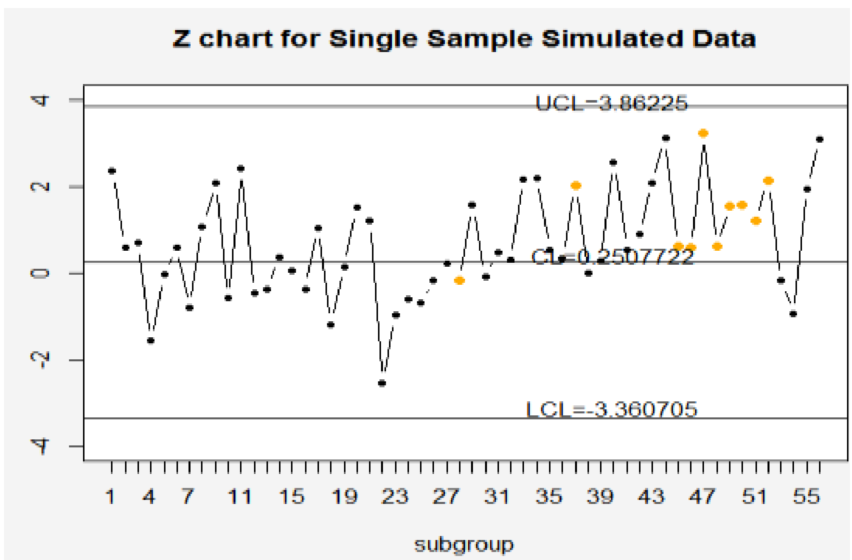

Figure 4. Single sample control chart for simulated data

4. Concluding Remarks

In this manuscript, a control chart two-piece normal distribution using repetitive sampling. The complete structure of proposed chart is developed for two-piece normal distribution. The tables are presented for practical use. The application of proposed chart is given with the help of real data. The simulation study shows that the proposed chart is more efficient than chart using single sampling chart in terms of ARL. The proposed chart can be used in the industry for the monitoring of the process. The proposed chart using cost model can be extended as future research.

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